CSC420: Assignment 1

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Problem 1

1. The computational cost for $h * I$ when h is not separable is $O(n^2m^2)$.

   This because each pixel in I gets computed for m^2 times, and there are n^2 pixels in total.

2. The computational cost for $h * I$ when h is not separable is $O(m^2 2n)$.

   This is because each pixel in I gets computed for $2m$ times (only vertical and horizontal edge detecting vectors), and there are n^2 pixels in total

Problem 2

Canny Edge Detection Steps:

1. Filter the image with derivative of Gaussian in both horizontal and vertical directions.
   - The purpose of this is to smooth the image and remove the noise.
   - To do this, we apply Gaussian filter to convolve with the image.

2. Find the magnitude and direction for the gradients
   - The purpose of this is to find the possible edges
   - To do this, we apply edge detection filters (for example, Sobel) with different directions and convolve with the image.

3. Non-maximum suppression
   - Get rid of the spurious response from edge detection produced by noise.
   - To do this, we only take local maximum or minimum of the edges.

4. Linking and Thresholding
   - The purpose of this is to connect the unlinked edges.
   - To do this, define 2 thresholds low and high. We use the high threshold's results to start the edge curves (get rid of the weak edges) and use low threshold's results to connect the unlinked edges.

Problem 3

Laplacian of Gaussian is the second derivative of Gaussian. This means that all the zero-crossing pixels are either local maxima or local minima. In an image, if a pixel is a local maximum or minimum, then this pixel is on an edge.

Laplacian of Gaussian filters are also symmetrical. It has the shape of Figure 1 both vertically and horizontally, so it can detect edges both vertically and horizontally.

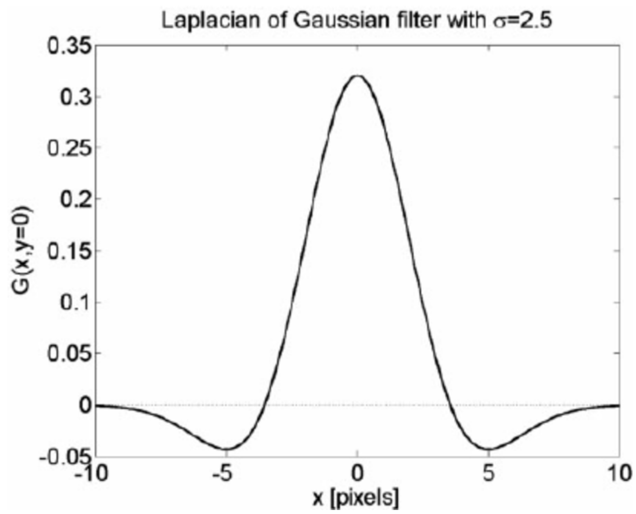


Figure 1: Laplacian of Gaussian with $\sigma = 2.5$

Problem 4

See code implementations in question_4.py.

1. See results of my_correlation in Figure 2a. The result is produced by correlating the original image (Figure 2b) with the filter $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   Note that the image is sharper comparing to the original image. (One specific location to look for would be on the dome.)

2. See results of my_convolution in Figure 2c. The result is produced by convolving the original image (Figure 2b) with the filter $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   Note that the image is sharper comparing to the original image. (One specific location to look for would be on the dome.)

(a) my_correlation (b) Original image (c) my_correlation

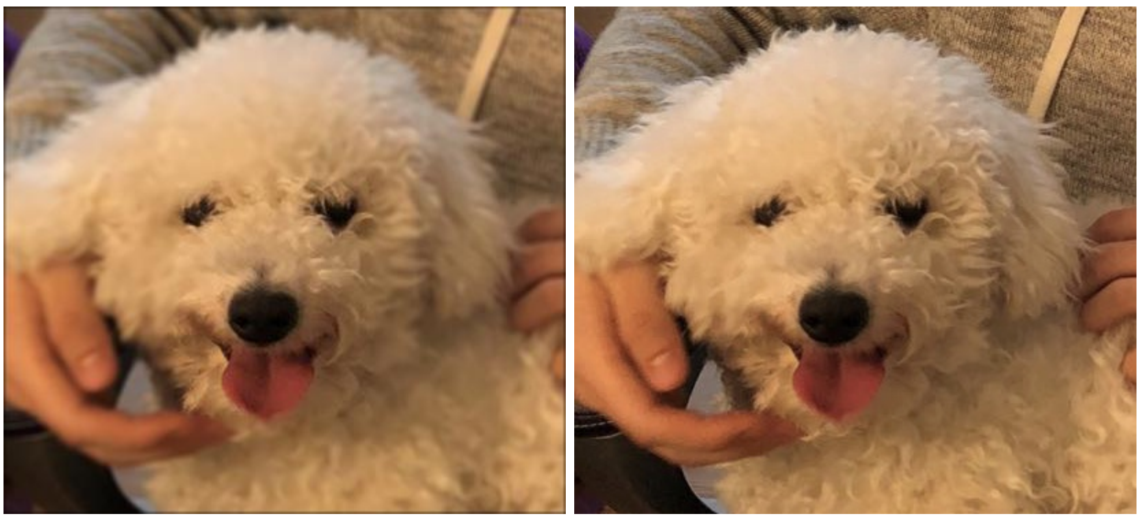
Note: Since the sharpening filter is symmetrical, the results for correlation and convolution are the same. The difference between convolution and correlation is that the filter is flipped.

3. To achieve portrait mode, we want to blur the background. To do this a blur filter is chosen.

   The filter chosen was $\frac{1}{25}\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}$.

   Note that the normal 3×3 blur matrix was not used. This is because the output for $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ is not obvious enough.

   See portrait mode (Figure 3a) and original photo (Figure 3b) bellow. The face for the dog is the focus rectangle. Notice that the hand and the ears are fuzzed out.



(a) Portrait mode (b) Original image

Problem 5

See code implementations in question_5.py.

1. Separable Filters allow correlation and convolution operations to be performed at $2K$ operation cost instead of K^2. This is because separable filters are able to be split into $1D$ horizontal and $1D$ vertical filters. The result of convolving the image with the 2 $1D$ filters is the same as convolving the image with the original separable filter.

   This is achieved when when the Singular Value Decomposition has only one non-zero single value.

$$F = U\Sigma V^T = \Sigma_{i=0}^{k} \sigma_i u_i v_i^T$$

   where $\sqrt{\sigma_1}\boldsymbol{u}_1$ and $\sqrt{\sigma_1}\boldsymbol{v}_1$ are the vertical and horizontal filters.

2. • The Separable filter is $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

   The output for the separable filter is

```
[[-0.25]
[-0.5  ]
[-0.25]]

[[-0.25  -0.5    -0.25]]
True
```

   • The Inseparable filter is $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$.

   The output for the separable filter is

```
False
```

Problem 6

See code implementations in question_6.py.

1. The image with noise is shown in Figure 4a.

(a) add_rand_correlation (b) Original image

2. The recovered image is shown in Figure 5b. The chosen filter was the mean filter $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$. This was chosen because averaging the pixels will reduce the effect of the noise and make the noisy pixel resemble the average of its neighbors.

(a) add_rand_correlation (b) The recovered image (c) Original image

Figure 5

3. The image with salt and pepper noise is shown in Figure 6a. Notice that there are random black and white pixels across the image.

(a) add_salt_and_pepper (b) Original image

Figure 6

4. Trying to recover salt and pepper noisy image with the mean filter does not yield a very good result: the output image is still noisy. The result is shown in 7a.

   The chosen filter is the median filter. This is achieved with OpenCV's *medianBlur* method. This works because salt and pepper noise is very extreme. The pixel has signal of either 1 or 0 (white or black). This affects the mean vastly, where as it does not really affect the median of the nearby pixels. By taking the mean of the near by pixels, we will likely get a better resemblance of the neighbor pixels. The result is shown in Figure 7b.

(a) mean filter (b) median filter (c) Original image

Figure 7

5. The salt and pepper noisy image applied to the color image is shown in 8a. This is achieved by adding salt and pepper noise to each channel and then combining them.

   When applying the median filter as in part d, most of the noise are gone.

   As in part d, this is achieved with OpenCV's *medianBlur* method. By taking the mean of the near by pixels, we will likely get a better resemblance of the neighbor pixels. The result is shown in Figure 8b.

   We can see that the median filter image (Figure 8b) is slightly blurred when comparing to the original image (Figure 8c). This is because by taking the median, the pixel will be affected by its neighbor pixels.

(a) salt_and_pepper noisy image (b) median filter (c) Original image

Figure 8