CSC420: Assignment 2

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Problem 1

See *question_1.py* for implementation.

1. The result of applying linear interpolation to up-sample the image by 4 times are shown in Figure 1.

   Figure 1a shows the first round of convolution along the first axis.

   Figure 2b shows the second round of convolution along the second axis on top of the result of the first round.

   Comparing to the original image, the up-sampled image maintains most of the information because the images look alike.

(a) First round (b) Second Round (c) Original Image

Figure 1

To achieve the same operation (up-sampling by a factor of 4), we can multiply the 1-dimensional filter by itself to get a 2-dimensional filter. This filter will be separable, and will produce the same result as convolving with the 1-dimensional filter twice on different directions.

In this specific case of up-sampling by a factor of 4, we calculate that

$$h = \begin{bmatrix} 0.25 & 0.5 & 0.75 & 1 & 0.75 & 0.5 & 0.25 \end{bmatrix}$$

$$h \times h^T = \begin{bmatrix} 0.0625 & 0.125 & 0.1875 & 0.25 & 0.1875 & 0.125 & 0.0625 \\ 0.125 & 0.25 & 0.375 & 0.5 & 0.375 & 0.25 & 0.125 \\ 0.1875 & 0.375 & 0.5625 & 0.75 & 0.5625 & 0.375 & 0.1875 \\ 0.25 & 0.5 & 0.75 & 1 & 0.75 & 0.5 & 0.25 \\ 0.1875 & 0.375 & 0.5625 & 0.75 & 0.5625 & 0.375 & 0.1875 \\ 0.125 & 0.25 & 0.375 & 0.5 & 0.375 & 0.25 & 0.125 \\ 0.0625 & 0.125 & 0.1875 & 0.25 & 0.1875 & 0.125 & 0.0625 \end{bmatrix}$$

2. The generalized two dimensional linear interpolation reconstruction filter is

$$h = \begin{bmatrix} \frac{1}{d} & \dots & \frac{d-1}{d} & 1 & \frac{d-1}{d} & \dots & \frac{1}{d} \end{bmatrix}$$

$$h \times h^T = \begin{bmatrix} \frac{1}{d}^2 & \dots & \frac{d-1}{d} \times \frac{1}{d} & 1 \times \frac{1}{d} & \frac{d-1}{d} \times \frac{1}{d} & \dots & \frac{1}{d}^2 \\ \frac{d-1}{d} \times \frac{1}{d} & \ddots & \dots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & 1 & \dots & \dots & \vdots \\ \frac{d-1}{d} \times \frac{1}{d} & \dots & \dots & \dots & \dots & \dots & \frac{d-1}{d}^2 \end{bmatrix}$$

Since this is a separable filter (composed of $h \times h^T$), it is equivalent of applying the 1D linear interpolation filter twice in both direction.

The results of the 2D linear interpolation filter convolution is shown in Figure 2a. Comparing to the result of applying 1D linear interpolation filter twice in two directions (Figure 2c), the results do not have any difference.

(a) 2D Filter (b) 1D Filter (c) Original Image

Figure 2

Problem 2

1. The computational cost for $h * I$ when h is not separable is $O(n^2m^2)$.

   This is because each pixel in I gets computed for m^2 times, and there are n^2 pixels in total.

2. The computational cost for $h * I$ when h is separable is $O(m^2 2n)$.

   This is because each pixel in I gets computed for $2m$ times (only vertical and horizontal edge detecting vectors), and there are n^2 pixels in total