A1 Soln

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Question 1

Read Data

```r
# Read the crime show ratings data
crime_show_file = "crime_show_ratings.RDS"
crime_show_data = readRDS(crime_show_file)
```

Question 1.a

Let y_i denote season rating for sample i.

Let $x_{i,2000}$ be indicator variable that is set to 1 if the decade for the sample i is 2000, 0 otherwise.

Let $x_{i,2010}$ be indicator variable that is set to 1 if the decade for the sample i is 2010, 0 otherwise.

Equation for linear model:

$$y_i = \beta_0 + \beta_1 x_{i,2000} + \beta_2 x_{i,2010} + \epsilon_i$$

Anova Assumptions:

1. Errors (ϵ_i) are independent
2. Errors are Normally distributed with $E[\epsilon_i] = 0$
3. Errors have constant variance $var[\epsilon] = \sigma^2$

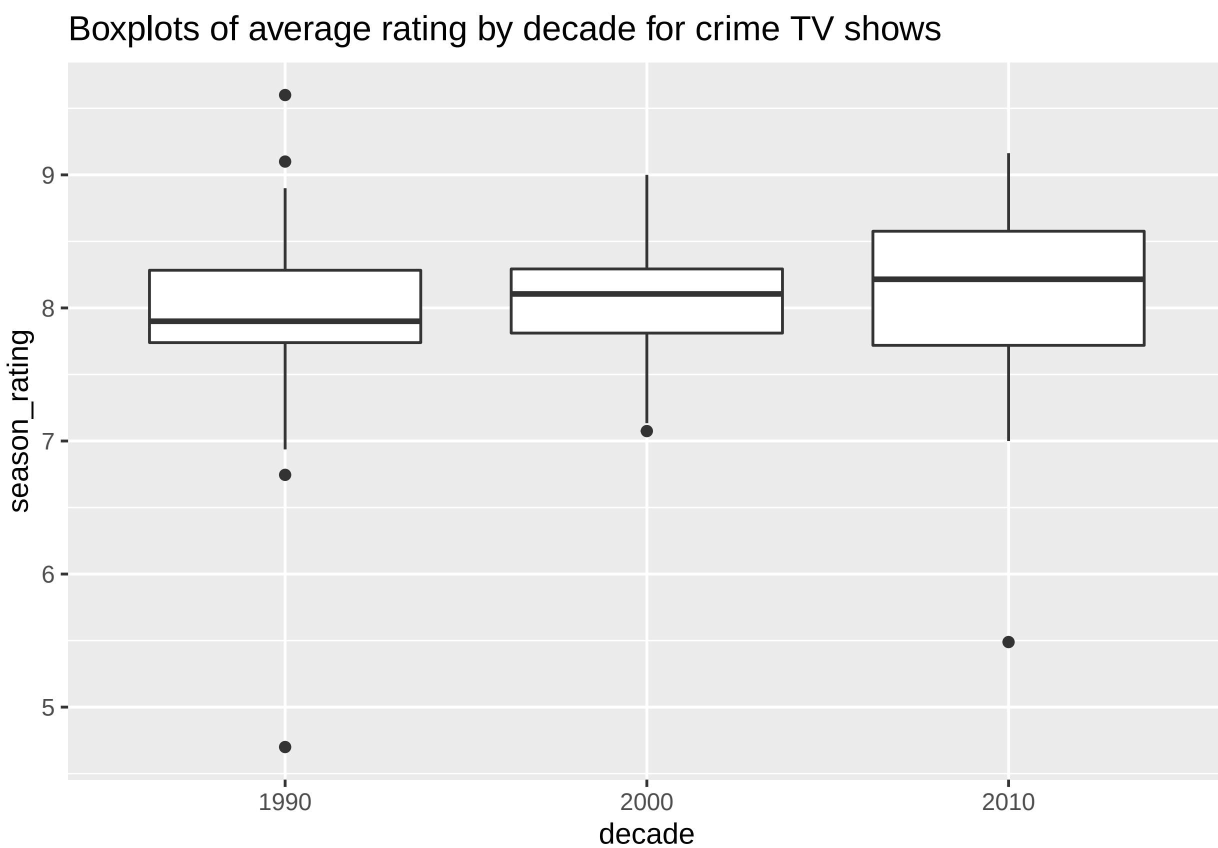
Question 1.b

The hypotheses for ANOVA are listed and can be described as follows:

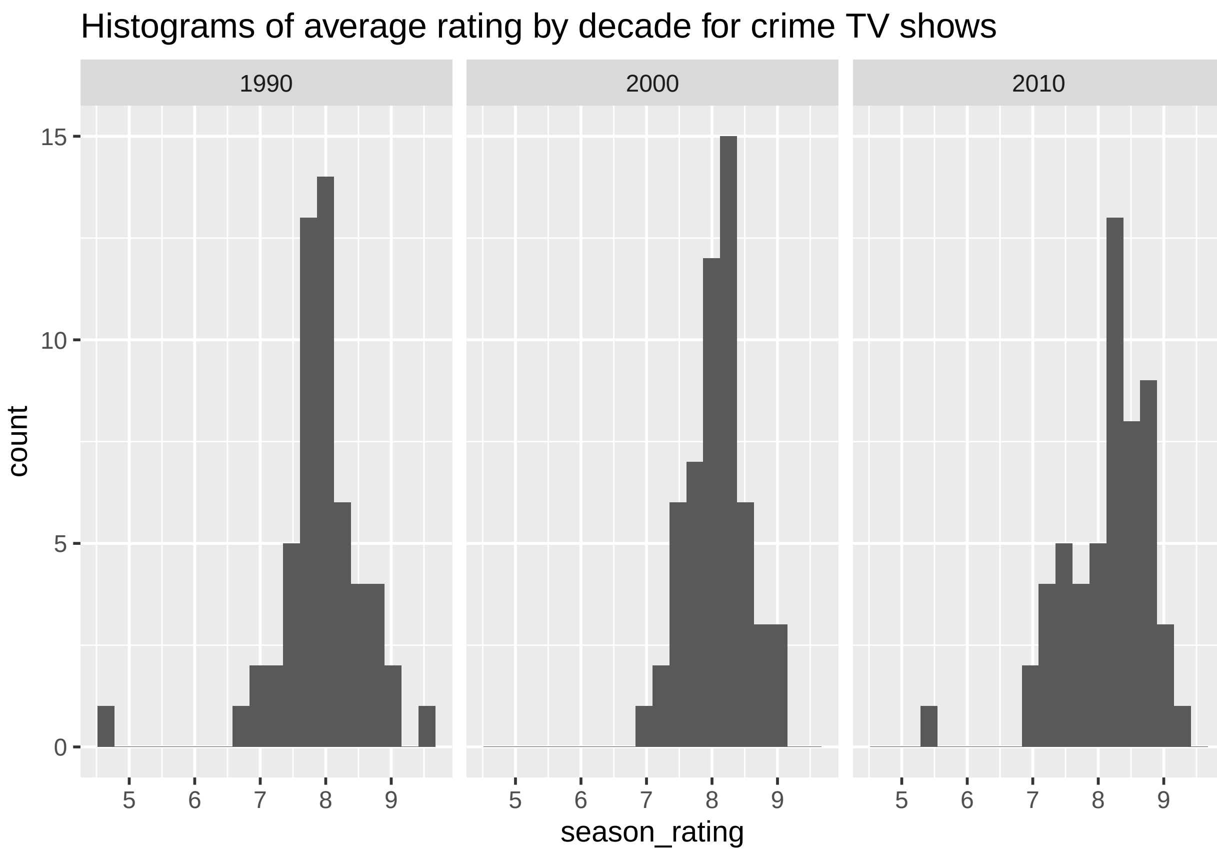
- H_0: The mean season rating for crime shows are the same across different decades. I.e. Different decades do not have effect on average season ratings
- H_1: Across different decades, least one mean is different from the others: The mean season rating for crime shows are different across different decades. I.e. Different decades have at least some effect on average season ratings

Question 1.c

```r
# Side by side box plots
crime_show_data %>%
  ggplot(aes(x = decade, y = season_rating)) +
  geom_boxplot() +
  ggtitle("Boxplots of average rating by decade for crime TV shows")
```



```r
# Facetted histograms
crime_show_data %>%
  ggplot(aes(x = season_rating)) +
  geom_histogram(bins = 20) +
  facet_wrap(~ decade) +
  ggtitle("Histograms of average rating by decade for crime TV shows")
```



The box plot provides a better visualization of the data because it shows comparison across three decades' basic statistics (maximum, minimum, quartiles, median) side by side.

On the other side, with the histograms, it is harder to tell which decade has a higher average rating because it only provides visualization over frequencies within each decade, and provides a relatively poor visualization for comparing between different decades.

One improvement for the box plot could be to sanitizing the data before plotting. In the plot, we observe that there are some outliers, especially with decades 1990 and 2010. Removing those outliers may provide a better visualization.

According to the box plot, we can see that the boxes are roughly on the same level. There is no sign of extremely skewed data except for some outliers, so their means are similar to the median (all around 8). Thus it does not suggest a significant difference between the means.

Question 1.d

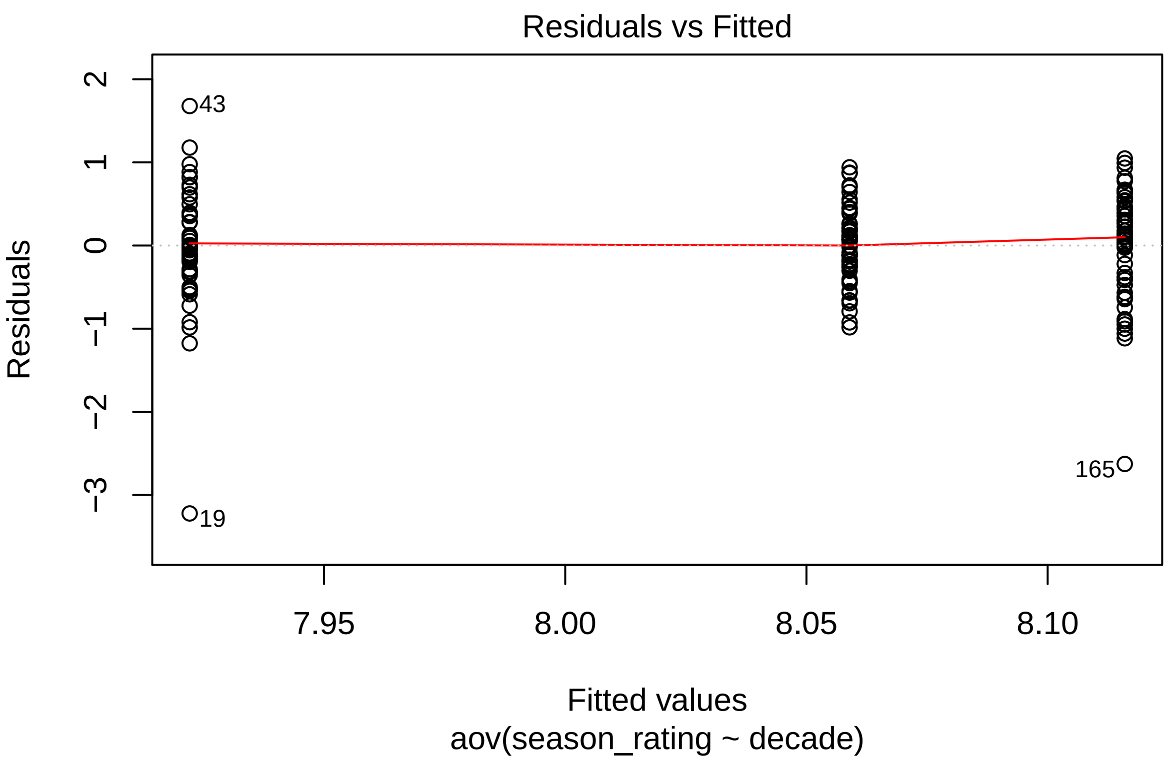
```r
# one way anova
one_way_anova <- aov(season_rating ~ decade, data = crime_show_data)

summary(one_way_anova)
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## decade        2   1.09  0.5458   1.447  0.238
## Residuals   162  61.08  0.3771
```

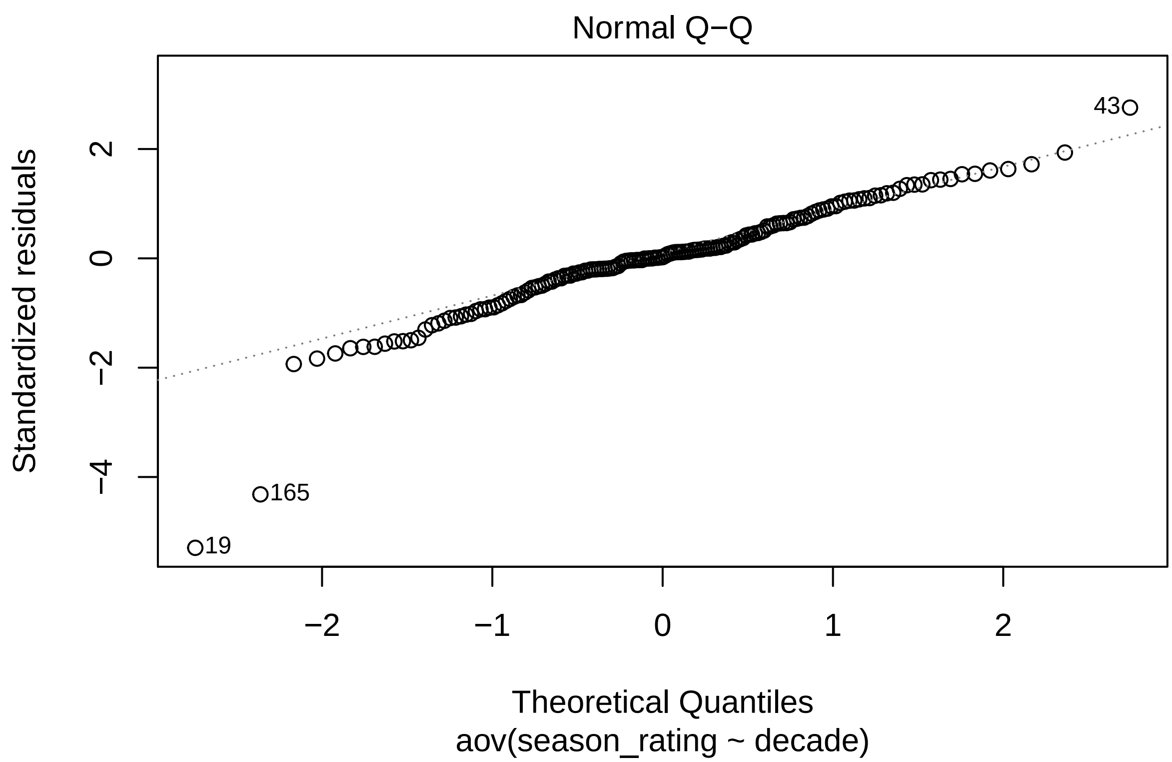
From the one way anova we can see that the p-value for the F-test is 0.238. We can interpret this as: The probability of observing the current sample given the assumption that the three decades having the same mean season ratings is 0.238. In this case, the p-value is not significant enough for us to reject H_0. Thus we cannot reject the statement that different decades do not have any effect on mean season ratings.

Question 1.e

```r
# first plot of one way anova
plot(one_way_anova, 1)
```



```
# second plot for one way anova
plot(one_way_anova, 2)
```




```
# variance for different decades
crime_show_data %>%
  group_by(decade) %>%
  summarise(var_rating = sd(season_rating) ^ 2)
```

```
## # A tibble: 3 x 2
##   decade var_rating
##   <chr>       <dbl>
## 1 1990        0.480
## 2 2000        0.203
## 3 2010        0.447
```

The first plot is the Residual vs Fitted plot. The plot shows that except for some ouliers, the residuals are roughly randomly scattered around the 0-line, and they do not indicate any pattern. This shows that the data follows a linear relationship, have equal error variances, and have a few outliers.

The second plot is the normal q-q plot. From the plot, we can see that except for points 19, 165, and 43, the points form a relatively straight line, indicating that the data follows a normal distribution with a few outliers.

From the standard deviations, we calculate that the ratio of the largest within-group and biggest within-group variance estimate is $\frac{\sigma_{1990}}{\sigma_{2000}} = \frac{0.480}{0.203} = 2.365 < 3$. According to the rule of thumb from Dean and Voss, the assumption for equality of variances is satisfied.

Question 1.f

```r
# linear model
lm_rating_decade = lm(season_rating ~ decade, data = crime_show_data)

summary(lm_rating_decade)
```

```
##
## Call:
## lm(formula = season_rating ~ decade, data = crime_show_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2222 -0.2589  0.0135  0.3862  1.6778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.9222     0.0828  95.679   <2e-16 ***
## decade2000    0.1368     0.1171   1.168   0.2444
## decade2010    0.1938     0.1171   1.655   0.0998 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6141 on 162 degrees of freedom
## Multiple R-squared:  0.01756,    Adjusted R-squared:  0.005426
## F-statistic: 1.447 on 2 and 162 DF,  p-value: 0.2382
```

The linear model can be expressed as

$$y = \beta_0 + \beta_1 x_{2000} + \beta_2 x_{2010}$$

where y is the season rating, β_is are the coefficients, x_{2000} is the indicator variable for decade 2000, and x_{2010} is the indicator variable for decade 2010.

β_0 is the intercept of the regression line, which is equal to the sample mean for decade 1990.

β_1 is the amount of score increase when the indicator variable x_{2000} is set to 1.

β_2 is the amount of score increase when the indicator variable x_{2010} is set to 1.

Then the sample mean for decade 1990 $\hat{\mu}_{1990} = \beta_0 = 7.9222$.

The sample mean for decade 2000 $\hat{\mu}_{2000} = \beta_0 + \beta_1 = 7.9222 + 0.1368 = 8.059$.

The sample mean for decade 2000 $\hat{\mu}_{2010} = \beta_0 + \beta_2 = 7.9222 + 0.1938 = 8.116$.

Question 2

Read Data

```r
# Read the crime show ratings data
smokeFile = 'smokeDownload.RData'
if (!file.exists(smokeFile)) {
  download.file('http://pbrown.ca/teaching/303/data/smoke.RData',
                smokeFile)
}
(load(smokeFile))
```

```
## [1] "smoke"        "smokeFormats"
```

```r
smokeFormats[smokeFormats[, 'colName'] == 'chewing_tobacco_snuff_or',
             c('colName', 'label')]
```

```
##                      colName
## 151 chewing_tobacco_snuff_or
##                                                                           label
## 151 RECODE: Used chewing tobacco, snuff, or dip on 1 or more days in the past 30 days
```

```r
# Data sanitization
smokeSub = smoke[which(smoke$Age > 10 & !is.na(smoke$Race)), ]
smokeSub$ageC = smokeSub$Age - 16

# Poisson GLM
smokeModel = glm(
  chewing_tobacco_snuff_or ~ ageC + RuralUrban + Race + Sex,
  data = smokeSub,
  family = binomial(link = 'logit')
)
knitr::kable(summary(smokeModel)$coef, digits = 3)
```

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-2.700	0.082	-32.843	0.000
ageC	0.341	0.021	16.357	0.000
RuralUrbanRural	0.959	0.088	10.934	0.000
Raceblack	-1.557	0.172	-9.068	0.000
Racehispanic	-0.728	0.104	-6.981	0.000
Raceasian	-1.545	0.342	-4.515	0.000
Racenative	0.112	0.278	0.404	0.687
Racepacific	1.016	0.361	2.814	0.005
SexF	-1.797	0.109	-16.485	0.000

```r
# odds table
logOddsMat = cbind(est = smokeModel$coef, confint(smokeModel, level = 0.99))
oddsMat = exp(logOddsMat)
oddsMat[1, ] = oddsMat[1, ] / (1 + oddsMat[1, ])
rownames(oddsMat)[1] = 'Baseline prob'
knitr::kable(oddsMat, digits = 3)
```

	est	0.5 %	99.5 %
Baseline prob	0.063	0.051	0.076

	est	0.5 %	99.5 %
ageC	1.407	1.334	1.485
RuralUrbanRural	2.610	2.088	3.283
Raceblack	0.211	0.132	0.320
Racehispanic	0.483	0.367	0.628
Raceasian	0.213	0.077	0.466
Racenative	1.119	0.509	2.163
Racepacific	2.761	0.985	6.525
SexF	0.166	0.124	0.218

Question 2.a

The statistical model that corresponds to `smokeModel` is

$$Y_i \sim Binomial(N_i, \mu_i)$$
$$h(\mu_i) = log(\frac{\mu_i}{1-\mu_i}) = X_i^T \beta$$

where

- μ_i is the sample mean $E(Y_i)$
- N_i is the population size
- $h(\mu_i)$ is the logit link function

Within X_i, there is an intercept of all 1's, a numeric variable of age centered around 16, and indicator variables for Region (Rural / Urban), Race (Black, Hispanic, Asian, Native, Pacific), and Sex (Male, Female).

Question 2.b

For the `Baseline prob` row,

- The value under `est` refers to the probability of observing the subset of individuals who are 16-year-old urban white males that have used chewing tobacco, snuff, or dip on 1 or more days in the past 30 days.
- The value under 0.5% and 99.5% represents the 99% confidence interval for the estimated probability

Question 2.c

```
# new data to predict
newData = data.frame(
  Sex = rep(c('M', 'F'), c(3, 2)),
  Race = c('white', 'white', 'hispanic', 'black', 'asian'),
  ageC = 0,
  RuralUrban = rep(c('Rural', 'Urban'), c(1, 4))
)

# predicted data
smokePred = as.data.frame(predict(smokeModel, newData,
                                  se.fit = TRUE, type = 'link'))[, 1:2]

smokePred$lower = smokePred$fit - 3 * smokePred$se.fit
smokePred$upper = smokePred$fit + 3 * smokePred$se.fit
smokePred
```

```
##         fit     se.fit     lower     upper
## 1 -1.740164 0.05471340 -1.904304 -1.576024
## 2 -2.699657 0.08219855 -2.946253 -2.453062
## 3 -3.427371 0.10692198 -3.748137 -3.106605
## 4 -6.053341 0.19800963 -6.647370 -5.459312
## 5 -6.041103 0.35209311 -7.097383 -4.984824
```

```r
# predicted odds
expSmokePred = exp(smokePred[, c('fit', 'lower', 'upper')])
knitr::kable(cbind(newData[, -3], 1000 * expSmokePred / (1 + expSmokePred)),
             digits = 1)
```

Sex	Race	RuralUrban	fit	lower	upper
M	white	Rural	149.3	129.6	171.4
M	white	Urban	63.0	49.9	79.2
M	hispanic	Urban	31.5	23.0	42.8
F	black	Urban	2.3	1.3	4.2
F	asian	Urban	2.4	0.8	6.8
<!– T	ODO: are th	ese logodds a	nd odds?	–>	

The claim that rural white males are the group most likely to use chewing tobacco is likely to be true. This is because the fitted value for rural white male is the highest among all groups. Also, the confidence interval of rural white males do not overlap with that of any other group.

The claim that less than half of one percent of ethnic-minority urban women and girls chew tobacco is likely to be faulse given the data. Considering black and asian urban female, the fitted probability of them chewing tobacco is in 99% confidence interval of $(0.13\%, 0.42\%)$ and $(0.08\%, 0.68\%)$. We can see that the 99% confidence interval for black urban women is less than half of one percent. However, the 99% confidence interval for asian urban women includes 0.5% of its population. Thus claim is false.

Question 3

Read Data

```r
# Read the fiji data
fijiFile = 'fijiDownload.RData'
if (!file.exists(fijiFile)) {
  download.file('http://pbrown.ca/teaching/303/data/fiji.RData',
                fijiFile)
}
(load(fijiFile))
```

```
## [1] "fiji"     "fijiFull"
```

```r
# data sanitization
fijiSub = fiji[fiji$monthsSinceM > 0 & !is.na(fiji$literacy), ]
fijiSub$logYears = log(fijiSub$monthsSinceM / 12)
fijiSub$ageMarried = relevel(fijiSub$ageMarried, '15to18')
fijiSub$urban = relevel(fijiSub$residence, 'rural')
```

```r
# first poisson glm
fijiRes = glm(
  children ~ offset(logYears) + ageMarried + ethnicity + literacy + urban,
  family = poisson(link = log),
```

```
  data = fijiSub
)
logRateMat = cbind(est = fijiRes$coef, confint(fijiRes, level = 0.99))
knitr::kable(cbind(summary(fijiRes)$coef,
                   exp(logRateMat)),
             digits = 3)
```

	Estimate	Std. Error	z value	Pr(>|z|)	est	0.5 %	99.5 %
(Intercept)	-1.181	0.017	-69.196	0.000	0.307	0.294	0.321
ageMarried0to15	-0.119	0.021	-5.740	0.000	0.888	0.841	0.936
ageMarried18to20	0.036	0.021	1.754	0.079	1.037	0.983	1.093
ageMarried20to22	0.018	0.024	0.747	0.455	1.018	0.956	1.084
ageMarried22to25	0.006	0.030	0.193	0.847	1.006	0.930	1.086
ageMarried25to30	0.056	0.048	1.159	0.246	1.057	0.932	1.195
ageMarried30toInf	0.138	0.098	1.405	0.160	1.147	0.882	1.462
ethnicityindian	0.012	0.019	0.624	0.533	1.012	0.964	1.061
ethnicityeuropean	-0.193	0.170	-1.133	0.257	0.824	0.514	1.242
ethnicitypartEuropean	-0.014	0.069	-0.206	0.837	0.986	0.822	1.171
ethnicitypacificIslander	0.104	0.055	1.884	0.060	1.110	0.959	1.276
ethnicityroutman	-0.033	0.132	-0.248	0.804	0.968	0.675	1.336
ethnicitychinese	-0.380	0.121	-3.138	0.002	0.684	0.492	0.920
ethnicityother	0.668	0.268	2.494	0.013	1.950	0.895	3.622
literacyno	-0.017	0.019	-0.857	0.391	0.984	0.936	1.034
urbansuva	-0.159	0.022	-7.234	0.000	0.853	0.806	0.902
urbanotherUrban	-0.068	0.019	-3.513	0.000	0.934	0.888	0.982

```
# second poisson glm
fijiSub$marriedEarly = fijiSub$ageMarried == '0to15'
fijiRes2 = glm(
  children ~ offset(logYears) + marriedEarly + ethnicity + urban,
  family = poisson(link = log),
  data = fijiSub
)
logRateMat2 = cbind(est = fijiRes2$coef, confint(fijiRes2, level = 0.99))
knitr::kable(cbind(summary(fijiRes2)$coef,
                   exp(logRateMat2)),
             digits = 3)
```

	Estimate	Std. Error	z value	Pr(>|z|)	est	0.5 %	99.5 %
(Intercept)	-1.163	0.012	-93.674	0.000	0.313	0.303	0.323
marriedEarlyTRUE	-0.136	0.019	-7.189	0.000	0.873	0.832	0.916
ethnicityindian	-0.002	0.016	-0.154	0.877	0.998	0.958	1.039
ethnicityeuropean	-0.175	0.170	-1.034	0.301	0.839	0.524	1.262
ethnicitypartEuropean	-0.014	0.068	-0.202	0.840	0.986	0.823	1.171
ethnicitypacificIslander	0.102	0.055	1.842	0.065	1.107	0.957	1.273
ethnicityroutman	-0.038	0.132	-0.285	0.775	0.963	0.672	1.330
ethnicitychinese	-0.379	0.121	-3.130	0.002	0.684	0.493	0.921
ethnicityother	0.681	0.268	2.545	0.011	1.976	0.907	3.667
urbansuva	-0.157	0.022	-7.162	0.000	0.855	0.808	0.904
urbanotherUrban	-0.066	0.019	-3.414	0.001	0.936	0.891	0.984

```r
# lrtest
lmtest::lrtest(fijiRes2, fijiRes)
```

```
## Likelihood ratio test
##
## Model 1: children ~ offset(logYears) + marriedEarly + ethnicity + urban
## Model 2: children ~ offset(logYears) + ageMarried + ethnicity + literacy +
##     urban
##   #Df  LogLik Df  Chisq Pr(>Chisq)
## 1  11 -9604.3
## 2  17 -9601.1  6 6.3669     0.3834
```

Question 3.a

The statistical model that corresponds to `fijiRes` is

$$Y_i \sim Poisson(\lambda)$$
$$h(\mu_i) = h(\lambda_i) = log(\frac{\lambda_i}{O_i}) = X_i^T \beta$$

where

- μ_i is the sample mean $E(Y_i)$.
- O_i is the offset term, which is the number of months since married.
- $h(\mu_i)$ is the log link function.

Within X_i, there is an intercept of all 1's, and indicator variables for Age range of marriage (15-18, 0-15, 18-20, 0-22, 22-25, 25-30, 30+), ethnicity (Fijian, Indian, European, Part Eruopean, Pacific Islander, Routman, Chinese, Others) + literacy (Yes, No) + urban (Rural, Suva, Other Urban).

The intercept represents predicted rates of children per month for the subset of females who are between 15 to 18 years old, Fijian, rural and literate.

Question 3.b

The model `fijiRes2` is nested within `fijiRes`.

`fijiRes2` can be viewed as `fijiRes` stripping ethnicity, and only retaining one age group indicator variable. The marriedEarly variable can simply be seen as the indicator variable for Age range of marriage 0-15. Thus the coefficients β for `fijiRes2` will have 6 less rows and is a subset of the coefficients for `fijiRes`.

Question 3.c

From the lmtest result above, we can see that the model being compared is `fijiRes2` and `fijiRes`.

`fijiRes` takes into account for women of different ages and their literacy.

`fijiRes2` is nested within `fijiRes`, and does not account for women of different ages above 15 years old, and their literacy.

The p-value for the test is 0.3834. This is not significant enough to reject the null hypothesis that adding literacy and age range improves how well the model explains the data.

Thus the second claim is likely to be true.