Rotary Position Embeddings

arxiv.org/abs/2104.09864

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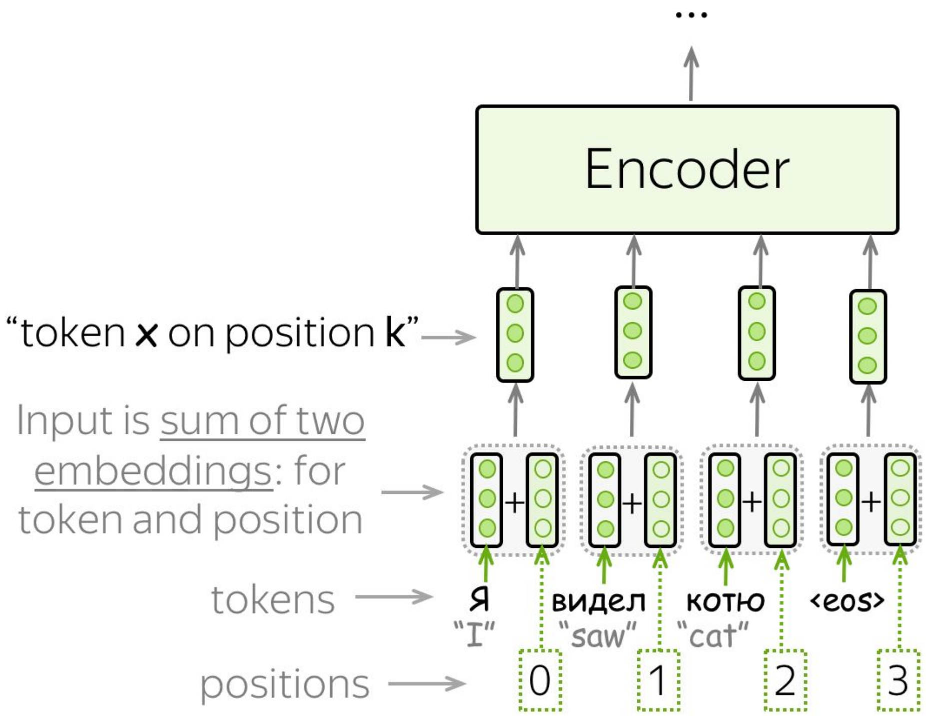
8 декабря 2021

Positional Encoding

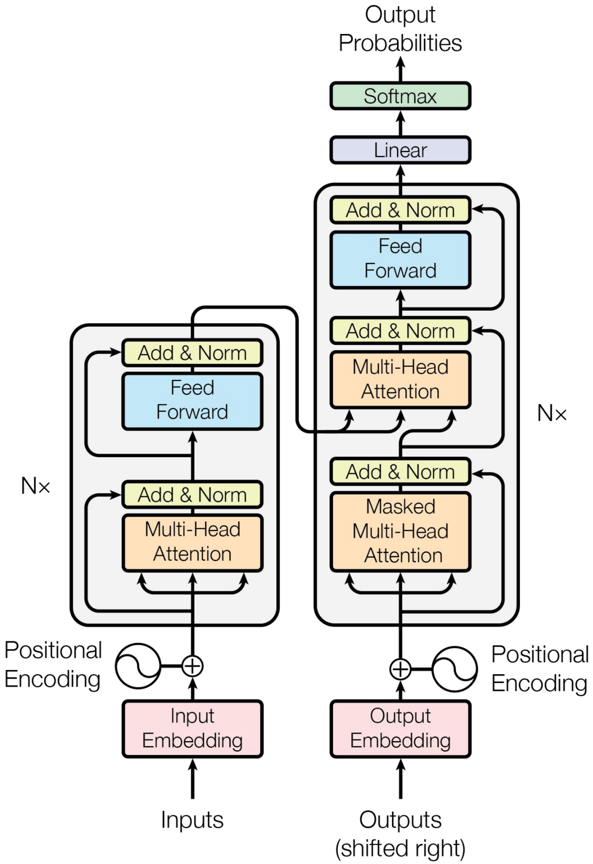


Positional Encoding

$$\overrightarrow{p_t}^{(i)} = f(t)^{(i)} := \begin{cases} \sin(\omega_k . t), & \text{if } i = 2k \\ \cos(\omega_k . t), & \text{if } i = 2k+1 \end{cases}$$

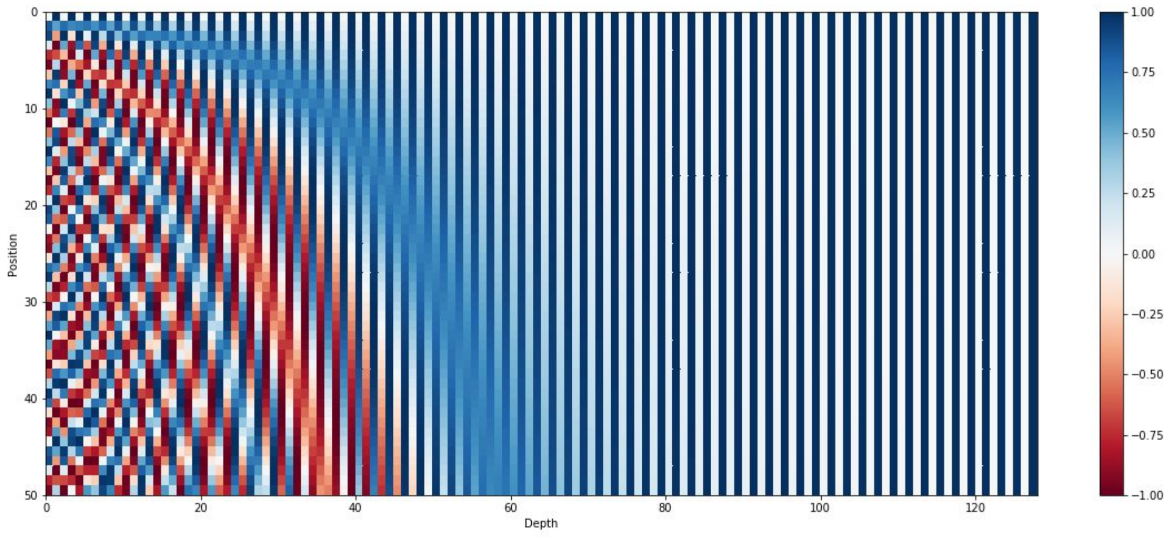
$$\omega_k = \frac{1}{10000^{2k/d}}$$

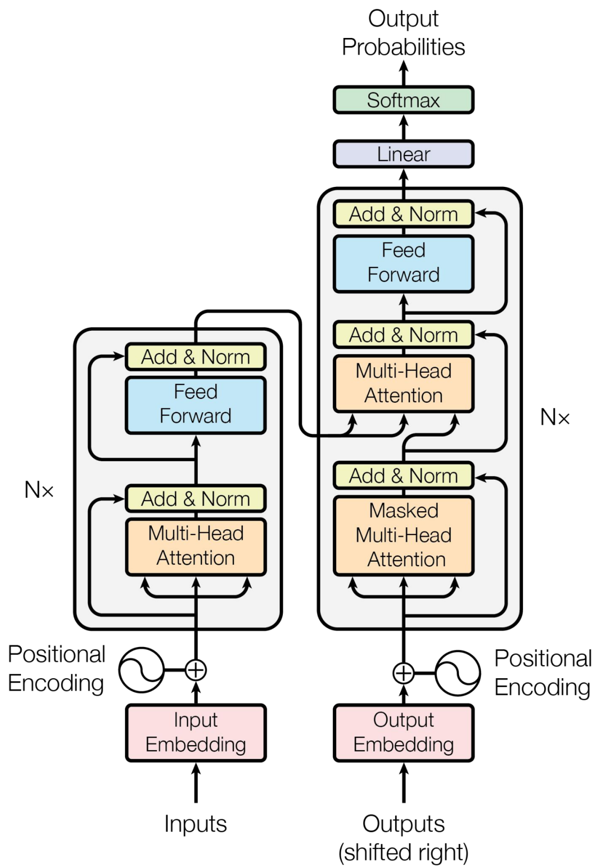
$$\psi'(w_t) = \psi(w_t) + \overrightarrow{p_t}$$



arxiv.org/abs/1706.03762

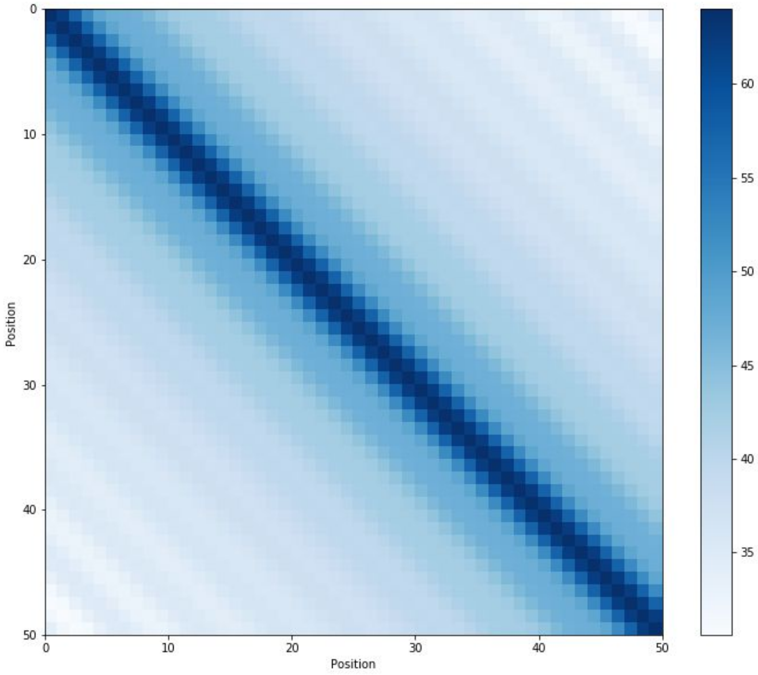
Positional Encoding

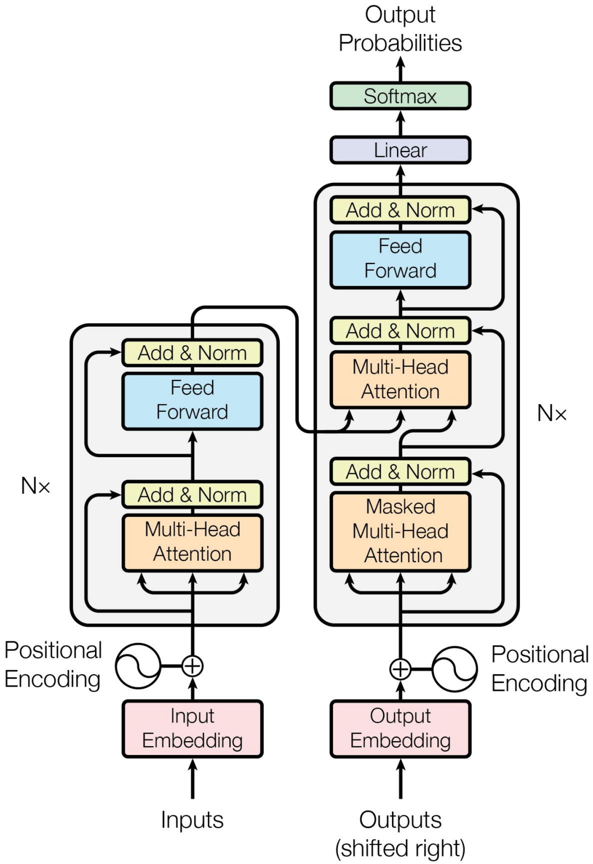




kazemnejad.com/blog/transformer_architecture_positional_encoding

Positional Encoding





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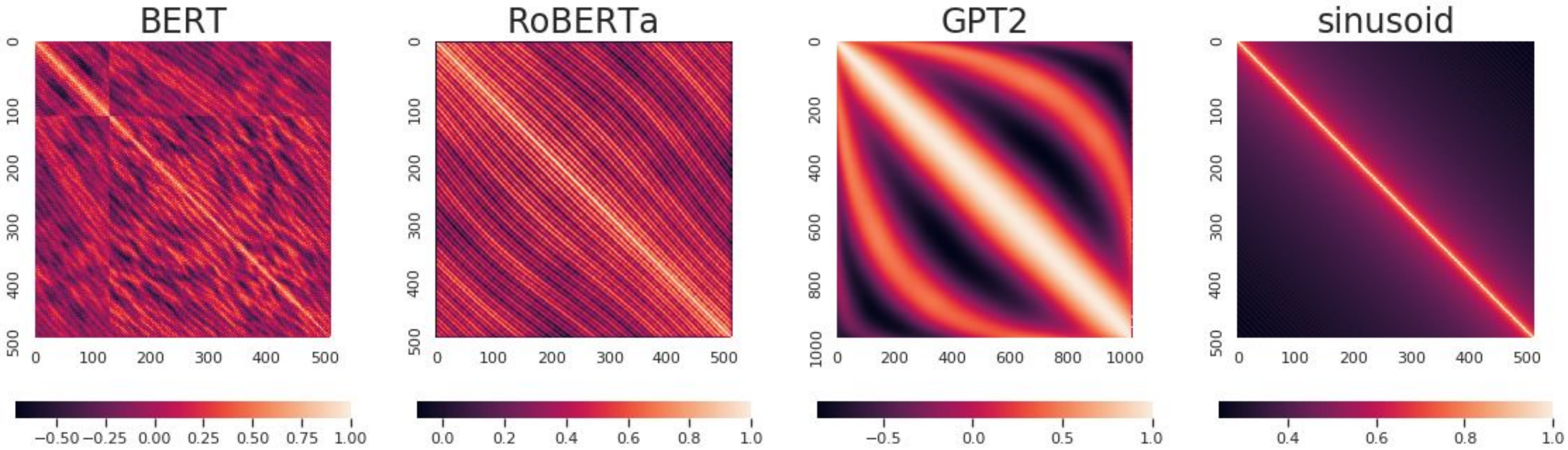
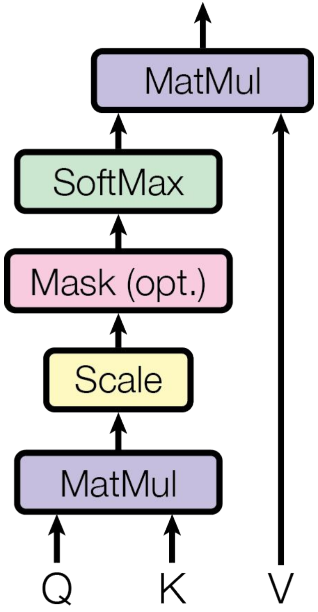
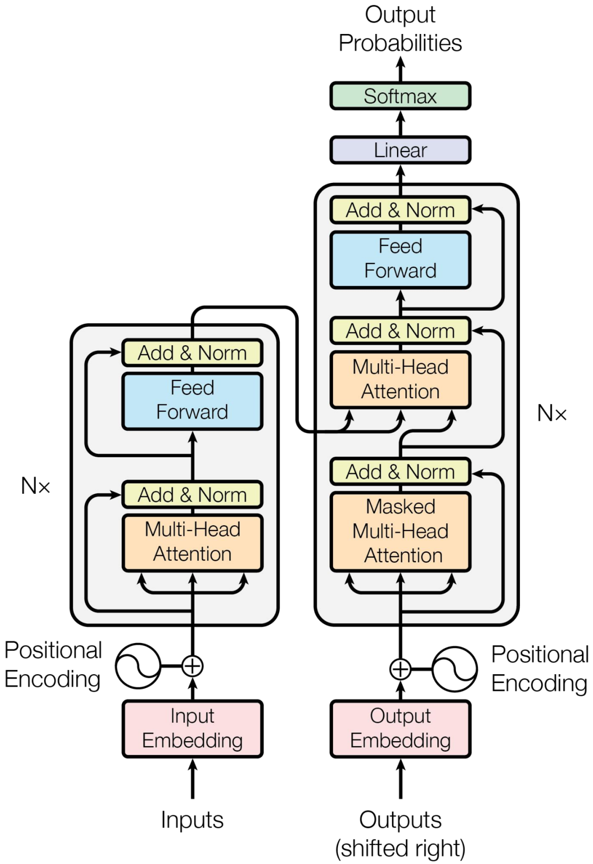
Trainable Position Embedding



arxiv.org/abs/2010.04903

Attention



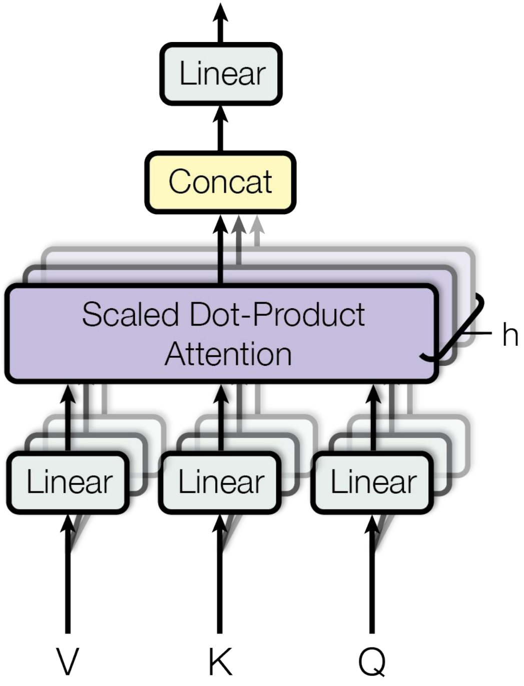


$$\mathrm{Attention}(Q, K, V) = \mathrm{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$

arxiv.org/abs/1706.03762

Multihead Attention

$$\mathrm{MultiHead}(Q, K, V) = \mathrm{Concat}(\mathrm{head_1}, ..., \mathrm{head_h})W^O$$
$$\text{where } \mathrm{head_i} = \mathrm{Attention}(QW_i^Q, KW_i^K, VW_i^V)$$



arxiv.org/abs/1706.03762

Attention

$$\boldsymbol{q}_m = f_q(\boldsymbol{x}_m, m)$$
$$\boldsymbol{k}_n = f_k(\boldsymbol{x}_n, n)$$
$$\boldsymbol{v}_n = f_v(\boldsymbol{x}_n, n)$$

$$\mathbf{o}_m = \sum_{n=1}^{N} a_{m,n} \boldsymbol{v}_n$$

$$a_{m,n} = \frac{\exp\left(\frac{\boldsymbol{q}_m^\intercal \boldsymbol{k}_n}{\sqrt{d}}\right)}{\sum_{j=1}^{N} \exp\left(\frac{\boldsymbol{q}_m^\intercal \boldsymbol{k}_j}{\sqrt{d}}\right)}$$

Relative Position Encoding

$$f_q(\boldsymbol{x}_m) := \boldsymbol{W}_q \boldsymbol{x}_m$$
$$f_k(\boldsymbol{x}_n, n) := \boldsymbol{W}_k(\boldsymbol{x}_n + \tilde{\boldsymbol{p}}_r^k)$$
$$f_v(\boldsymbol{x}_n, n) := \boldsymbol{W}_v(\boldsymbol{x}_n + \tilde{\boldsymbol{p}}_r^v)$$

$$r = \text{clip}(m - n, r_{\text{min}}, r_{\text{max}})$$

Model	Position Information	EN-DE BLEU	EN-FR BLEU
Transformer (base)	Absolute Position Representations	26.5	38.2
Transformer (base)	Relative Position Representations	**26.8**	**38.7**
Transformer (big)	Absolute Position Representations	27.9	41.2
Transformer (big)	Relative Position Representations	**29.2**	**41.5**

arxiv.org/abs/1803.02155

Formulation

$$\langle f_q(\boldsymbol{x}_m, m), f_k(\boldsymbol{x}_n, n)\rangle = g(\boldsymbol{x}_m, \boldsymbol{x}_n, m-n)$$

Solution (2D)

$$f_q(\boldsymbol{x}_m, m) = (\boldsymbol{W}_q \boldsymbol{x}_m) e^{im\theta}$$
$$f_k(\boldsymbol{x}_n, n) = (\boldsymbol{W}_k \boldsymbol{x}_n) e^{in\theta}$$

$$f_{\{q,k\}}(\boldsymbol{x}_m, m) = \begin{pmatrix} \cos m\theta & -\sin m\theta \\ \sin m\theta & \cos m\theta \end{pmatrix} \begin{pmatrix} W^{(11)}_{\{q,k\}} & W^{(12)}_{\{q,k\}} \\ W^{(21)}_{\{q,k\}} & W^{(22)}_{\{q,k\}} \end{pmatrix} \begin{pmatrix} x^{(1)}_m \\ x^{(2)}_m \end{pmatrix}$$

General Solution

$$\boxed{f_{\{q,k\}}(\boldsymbol{x}_m, m) = \boldsymbol{R}^d_{\Theta,m} \boldsymbol{W}_{\{q,k\}} \boldsymbol{x}_m}$$

$$\boldsymbol{R}^d_{\Theta,m} = \begin{pmatrix} \cos m\theta_1 & -\sin m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\ \sin m\theta_1 & \cos m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cos m\theta_2 & -\sin m\theta_2 & \cdots & 0 & 0 \\ 0 & 0 & \sin m\theta_2 & \cos m\theta_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cos m\theta_{d/2} & -\sin m\theta_{d/2} \\ 0 & 0 & 0 & 0 & \cdots & \sin m\theta_{d/2} & \cos m\theta_{d/2} \end{pmatrix}$$

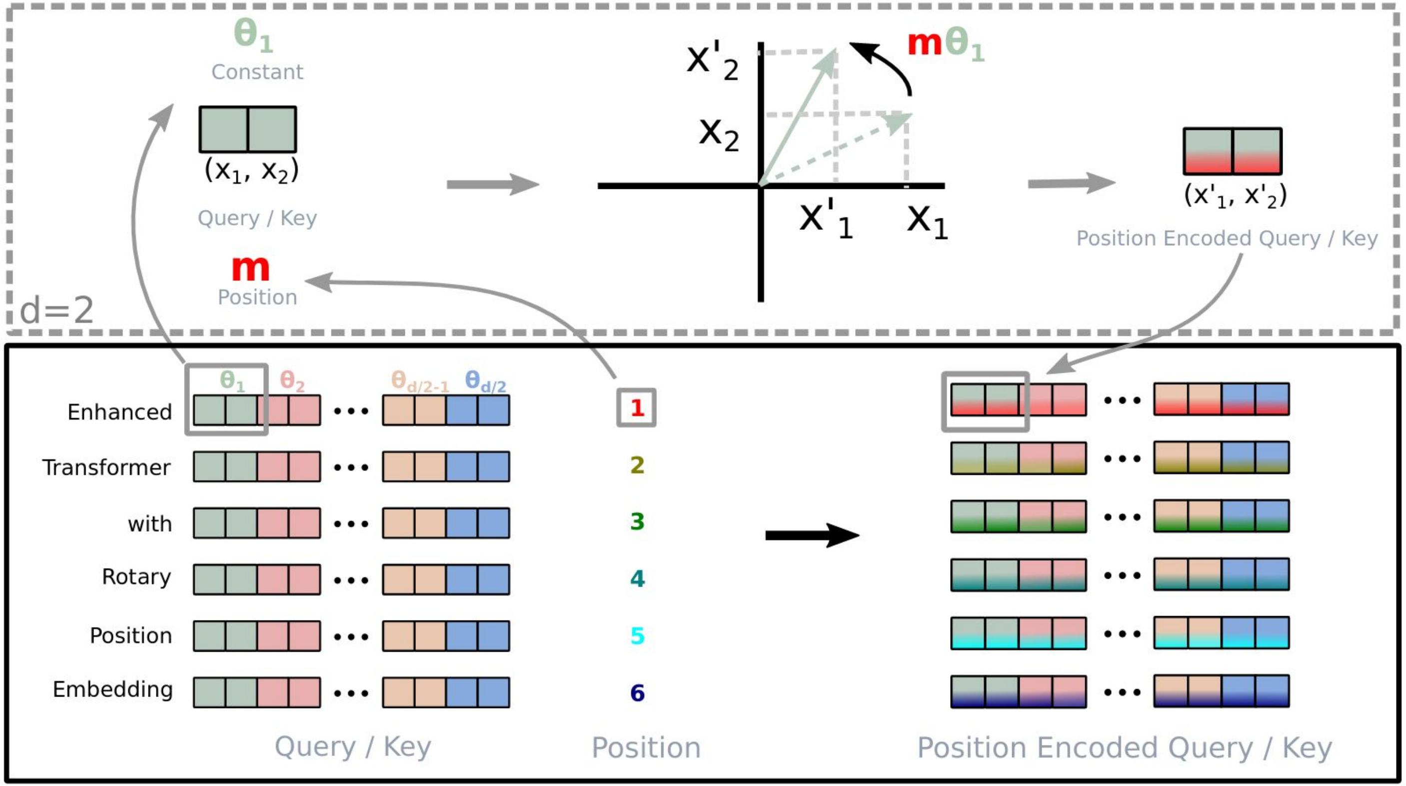
$$\Theta = \{\theta_i = 10000^{-2(i-1)/d}, i \in [1, 2, ..., d/2]\}$$

General Solution

$$\boxed{f_{\{q,k\}}(\boldsymbol{x}_m, m) = \boldsymbol{R}^d_{\Theta,m}\boldsymbol{W}_{\{q,k\}}\boldsymbol{x}_m} \qquad \boldsymbol{R}^d_{\Theta,n-m} = (\boldsymbol{R}^d_{\Theta,m})^\intercal \boldsymbol{R}^d_{\Theta,n}$$

$$\boldsymbol{q}_m^\intercal \boldsymbol{k}_n = (\boldsymbol{R}^d_{\Theta,m}\boldsymbol{W}_q\boldsymbol{x}_m)^\intercal(\boldsymbol{R}^d_{\Theta,n}\boldsymbol{W}_k\boldsymbol{x}_n) = \boldsymbol{x}^\intercal \boldsymbol{W}_q R^d_{\Theta,n-m}\boldsymbol{W}_k\boldsymbol{x}_n$$

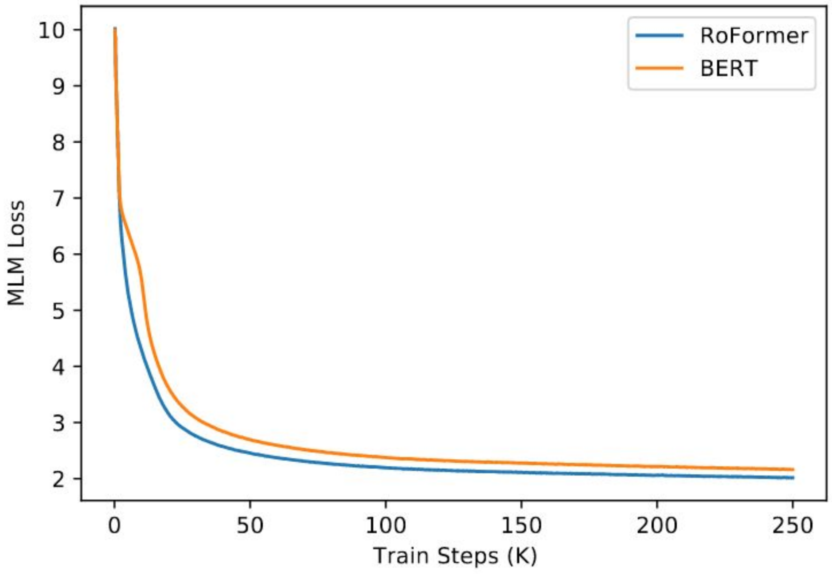
Implementation

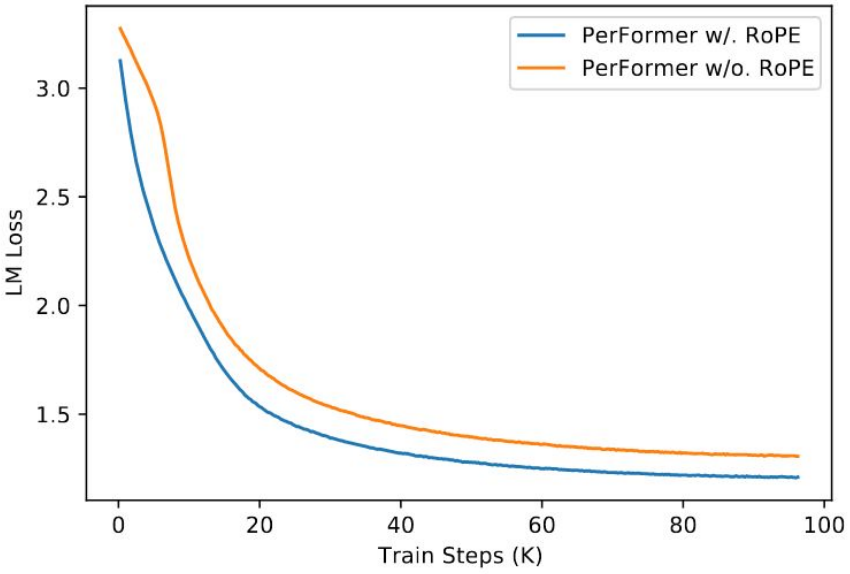


Implementation

$$\boldsymbol{R}^d_{\Theta,m}\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{d-1} \\ x_d \end{pmatrix} \otimes \begin{pmatrix} \cos m\theta_1 \\ \cos m\theta_1 \\ \cos m\theta_2 \\ \cos m\theta_2 \\ \vdots \\ \cos m\theta_{d/2} \\ \cos m\theta_{d/2} \end{pmatrix} + \begin{pmatrix} -x_2 \\ x_1 \\ -x_4 \\ x_3 \\ \vdots \\ -x_{d-1} \\ x_d \end{pmatrix} \otimes \begin{pmatrix} \sin m\theta_1 \\ \sin m\theta_1 \\ \sin m\theta_2 \\ \sin m\theta_2 \\ \vdots \\ \sin m\theta_{d/2} \\ \sin m\theta_{d/2} \end{pmatrix}$$

Experiments: Pretraining





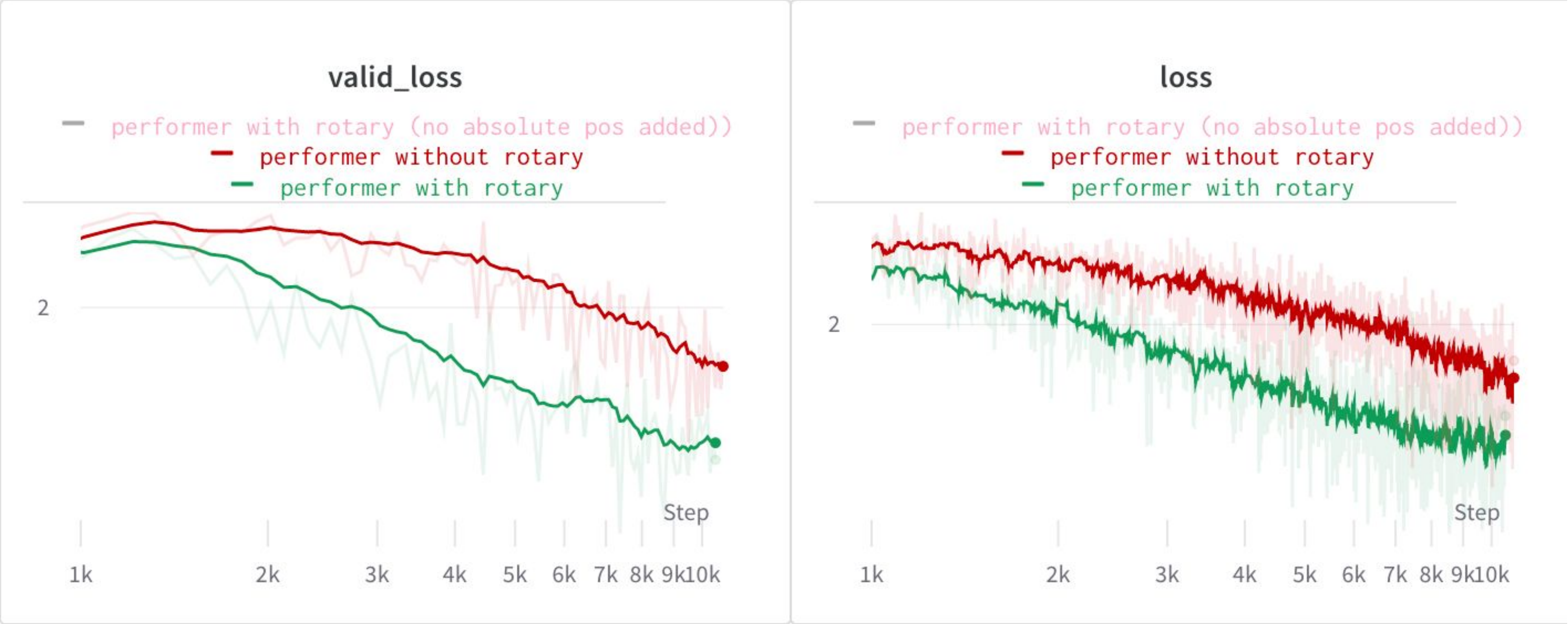
Experiments: Downstream Tasks

Model	MRPC	SST-2	QNLI	STS-B	QQP	MNLI(m/mm)
BERT[8]	88.9	93.5	90.5	85.8	71.2	84.6/83.4
RoFormer	**89.5**	90.7	88.0	**87.0**	**86.4**	80.2/79.8

Model	validation	test
BERT-512	64.13%	67.77%
WoBERT-512	64.07%	68.10%
RoFormer-512	64.13%	68.29%
RoFormer-1024	**66.07%**	**69.79%**

Model	BLEU
Transformer-base[37]	27.3
RoFormer	**27.5**

Experiments: Performer



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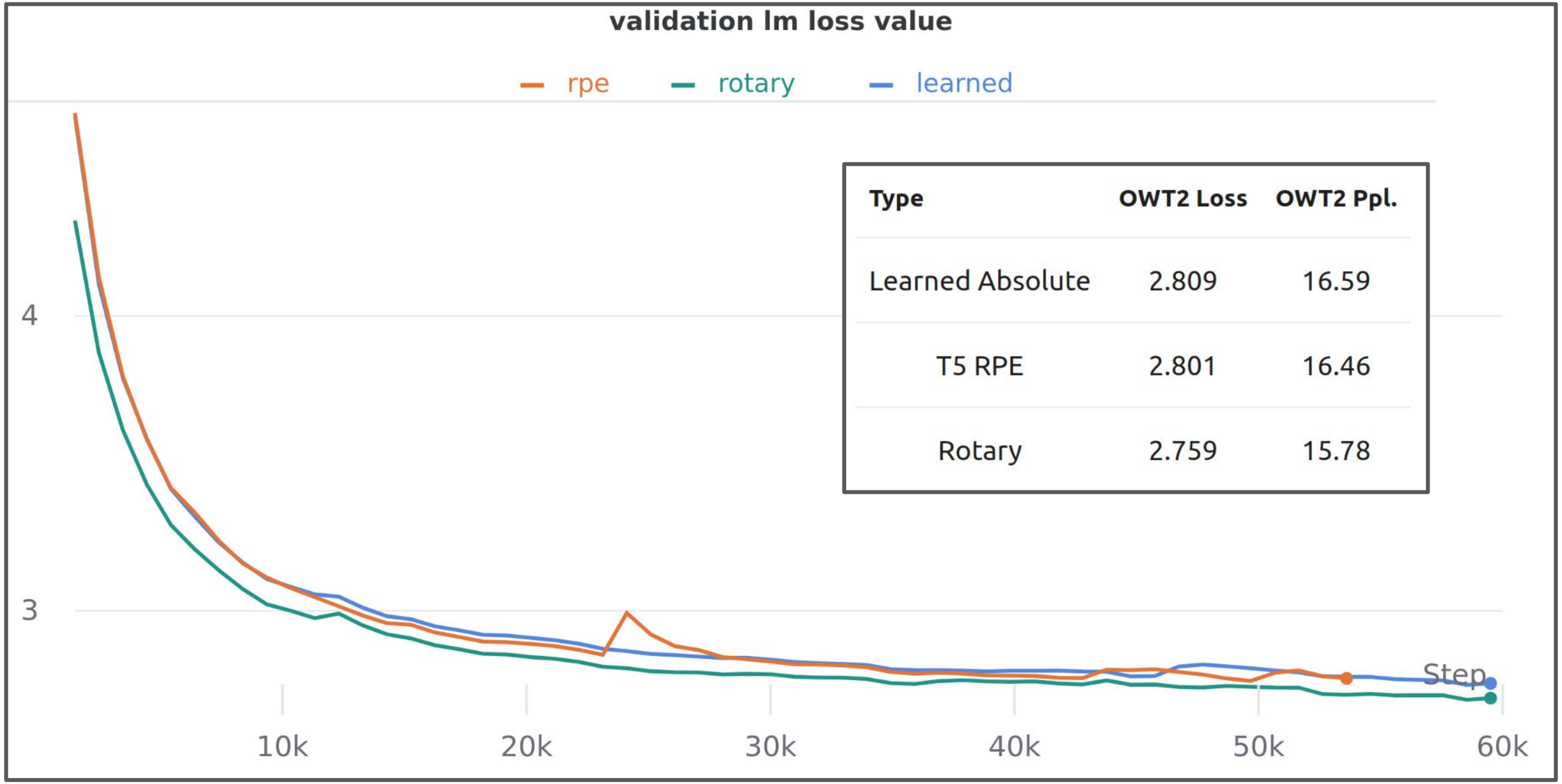
Experiments: LM



Type	OWT2 Loss	OWT2 Ppl.
Learned Absolute	2.809	16.59
T5 RPE	2.801	16.46
Rotary	2.759	15.78

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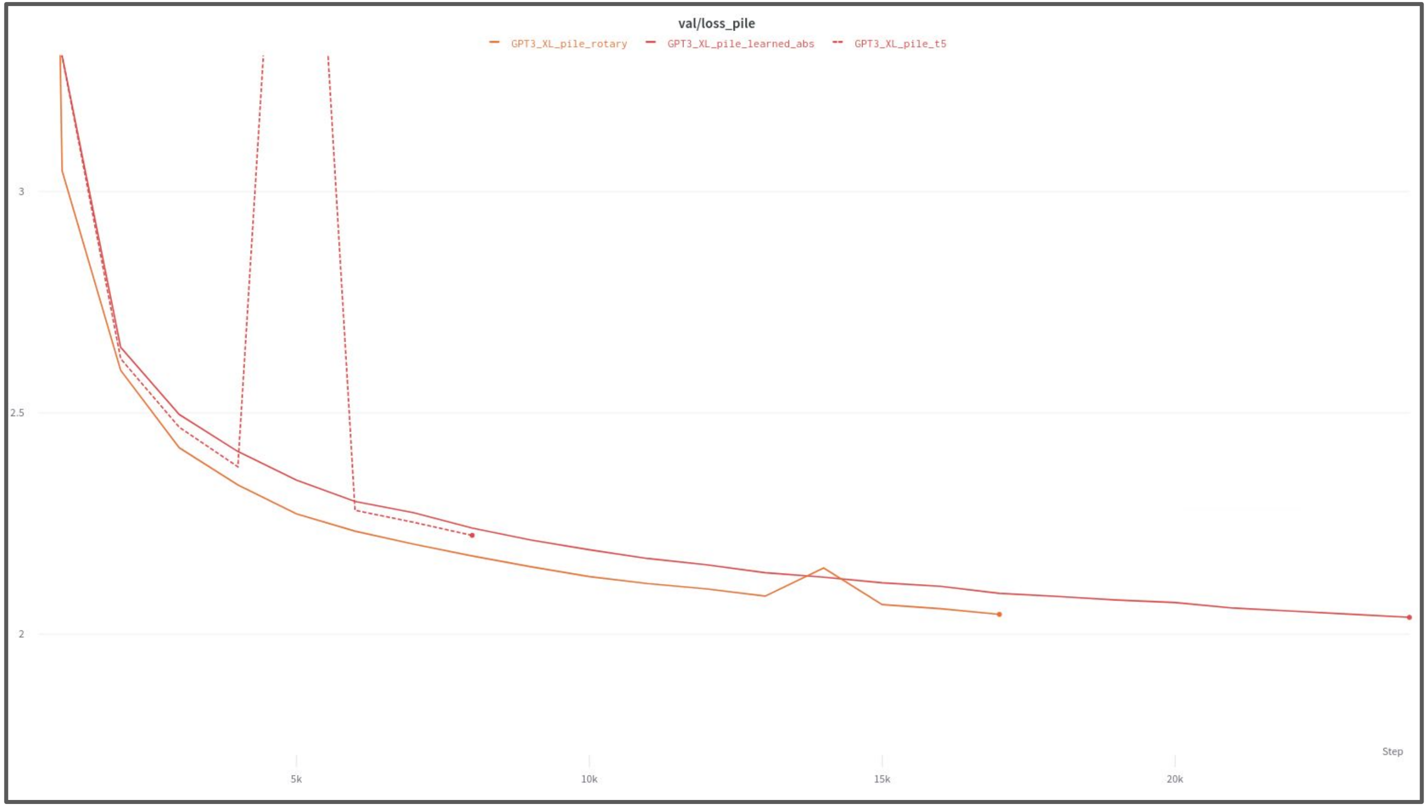
Experiments: LM



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Conclusion: Rotary Embeddings

- Relative position in self-attention encoded through rotation matrix
- No training
- Faster convergence
- Greater stability