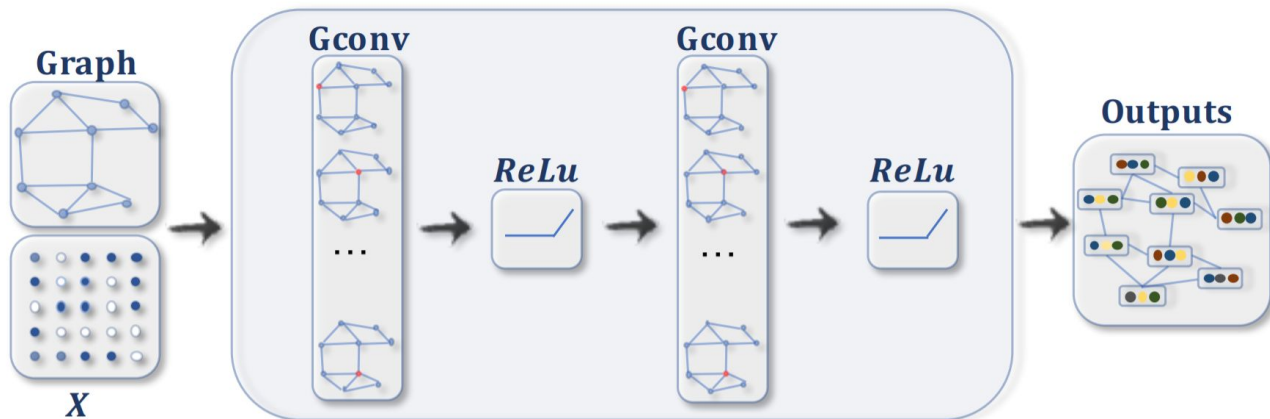


# MultiSage: Empowering GCN with Contextualized Multi-Embeddings on Web-Scale Multipartite Networks

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# Graph Neural Networks?



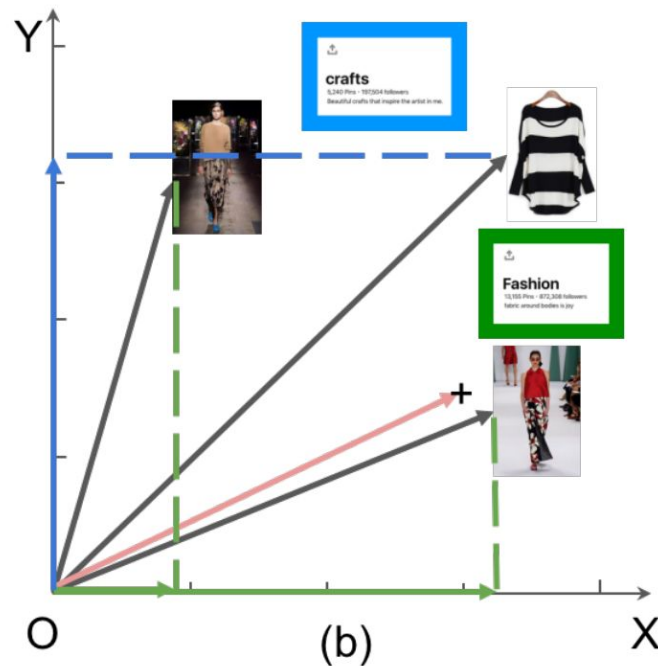
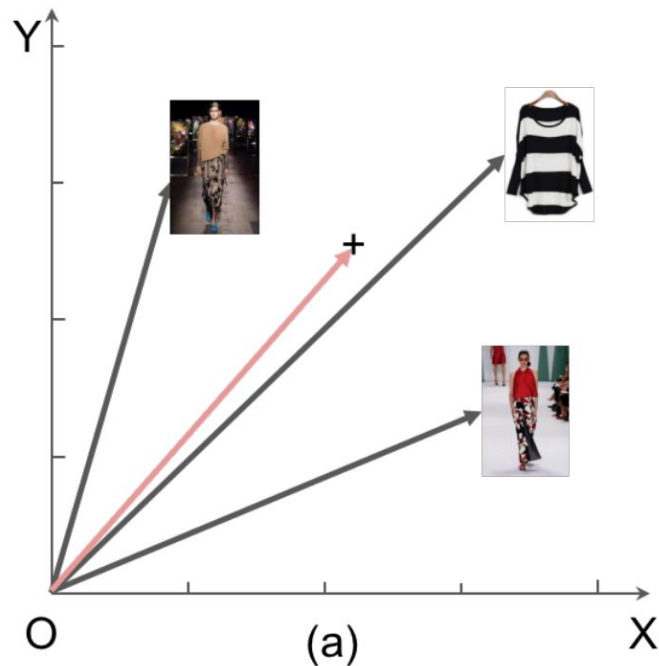
$$\mathbf{h}_{\mathcal{N}(v)}^k \leftarrow \text{AGGREGATE}_k(\{\mathbf{h}_u^{k-1}, \forall u \in \mathcal{N}(v)\})$$
$$\mathbf{h}_v^k \leftarrow \sigma \left( \mathbf{W}^k \cdot \text{CONCAT}(\mathbf{h}_v^{k-1}, \mathbf{h}_{\mathcal{N}(v)}^k) \right)$$

# Target and Context Nodes



Dataset	Target	Context	Other context
IMDB [40]	movie	genre	director, actor
TCGA [43]	gene	pathway	disease, species
OAG [27]	paper	venue	author, keyword
Pinterest [6]	pin	board	<b>user</b> , session

# Contextual Graph Embeddings



# Architecture

Raw feature transformation:

$$\mathbf{z}_t = \text{ReLU}(\mathbf{W}_t^{(K)} \dots \text{ReLU}(\mathbf{W}_t^{(1)} \mathbf{x}_t + \mathbf{b}_t^{(1)}) \dots + \mathbf{b}_t^{(K)})$$

$$\mathbf{z}_c = \text{ReLU}(\mathbf{W}_c^{(K)} \dots \text{ReLU}(\mathbf{W}_c^{(1)} \mathbf{x}_c + \mathbf{b}_c^{(1)}) \dots + \mathbf{b}_c^{(K)})$$

Contextual masking:

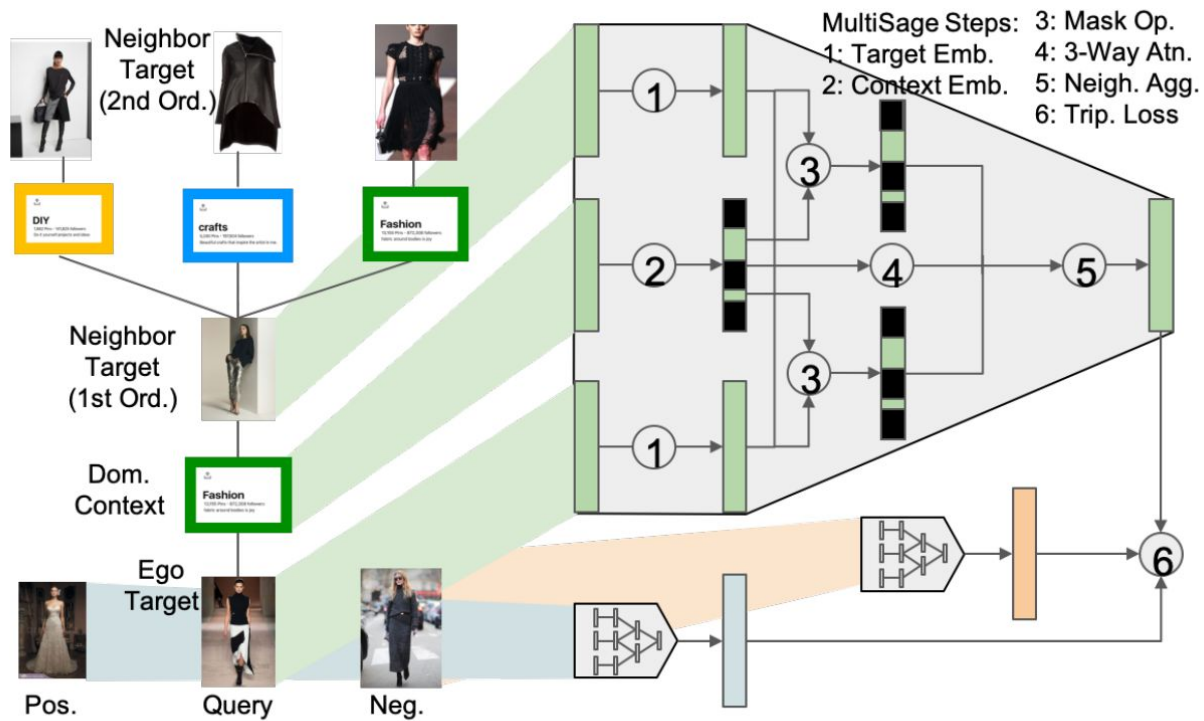
$$\mathbf{z}_{t|c} = \mathbf{z}_t \otimes \mathbf{z}_c$$

Contextual attention:

$$\alpha(v, o, u) = \frac{\exp\left(\tau(\mathbf{a}^T [\mathbf{W}_{at}\mathbf{z}_t(v) \odot \mathbf{W}_{ac}\mathbf{z}_c(o) \odot \mathbf{W}_{at}\mathbf{z}_t(u)])\right)}{\sum_{u' \in \mathcal{N}_v, o' \sim (v, u')} \exp\left(\tau(\mathbf{a}^T [\mathbf{W}_{at}\mathbf{z}_t(v) \odot \mathbf{W}_{ac}\mathbf{z}_t(o') \odot \mathbf{W}_{at}\mathbf{z}_t(u')])\right)}$$

$$\mathbf{z}_{\mathcal{N}_v}(x) = \sigma\left(\frac{1}{D} \sum_{d=1}^D \sum_{u \in \mathcal{N}_v, o \sim (v, u)} \alpha^{(d)}(v, o, u) \mathbf{z}_{t|c}(x, o)\right)$$

# Architecture



# Learning pipeline

Loss:  $\mathcal{J}(v_q, v_p, v_n) = \max\{0, \mathbf{h}_{v_q}^T \mathbf{h}_{v_n} - \mathbf{h}_{v_q}^T \mathbf{h}_{v_p}^L + \delta\}$

Negative sampling with PersPageRank



**Query**



**Positive Example**



**Random Negative**



**Hard Negative**

# Personalized PageRank

$$\boldsymbol{\pi}_{\text{pr}} = \mathbf{A}_{\text{rw}} \boldsymbol{\pi}_{\text{pr}}.$$

$$\mathbf{A}_{\text{rw}} = \mathbf{A} \mathbf{D}^{-1}$$

$$\boldsymbol{\pi}_{\text{ppr}}(\mathbf{i}_x) = (1 - \alpha) \hat{\tilde{\mathbf{A}}} \boldsymbol{\pi}_{\text{ppr}}(\mathbf{i}_x) + \alpha \mathbf{i}_x.$$

$$\boldsymbol{\pi}_{\text{ppr}}(\mathbf{i}_x) = \alpha \left( \mathbf{I}_n - (1 - \alpha) \hat{\tilde{\mathbf{A}}} \right)^{-1} \mathbf{i}_x$$

$$\boldsymbol{\Pi}_{\text{ppr}} = \alpha \left( \mathbf{I}_n - (1 - \alpha) \hat{\tilde{\mathbf{A}}} \right)^{-1}$$

$$\mathbf{A} \in \mathbb{R}^{n \times n}. \tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}_n$$

$$\hat{\tilde{\mathbf{A}}} = \tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}}$$

$$\mathbf{Z}^{(0)} = \mathbf{H} = f_{\theta}(\mathbf{X}),$$

$$\mathbf{Z}^{(k+1)} = (1 - \alpha) \hat{\tilde{\mathbf{A}}} \mathbf{Z}^{(k)} + \alpha \mathbf{H},$$

$$\mathbf{Z}^{(K)} = \text{softmax} \left( (1 - \alpha) \hat{\tilde{\mathbf{A}}} \mathbf{Z}^{(K-1)} + \alpha \mathbf{H} \right)$$



# Neighborhood Sampling

- $\text{vis}[v]$  - visiting count
- $\text{ctx}[v]$  - dominant context
- $\text{dom}[v]$  - 'dominance' of the context
- $\text{path}[1]$  - the first context node
- $\text{path}[-1]$  - the last target node

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## Algorithm 2 Parallel Contextualized Random Walk

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**Input:** graph  $\mathcal{G} = \{\mathcal{T}, \mathcal{C}, \mathcal{E}\}$ , walk length  $\zeta$ , number of walks  $\kappa$ , number of threads  $\xi$ , number of neighbors to keep  $s$   
**Output:** sampled contextualized neighbor lists  $\mathcal{S} = \{\{v : [(c, u), \forall u \in \mathcal{N}_v, c \sim (v, u)]\}, \forall v \in \mathcal{T}\}$

```
1:  $\forall v \in \mathcal{T}, \mathcal{S}[v] \leftarrow \emptyset, \text{vis}[v], \text{ctx}[v], \text{dom}[v] \leftarrow \text{defaultdict(int)}$   
2: Start a pool  $\Omega$  of  $\xi$  threads in parallel  
3: for  $v$  in  $|\mathcal{T}|$  do  
4:   with  $\omega = \text{rand}(\Omega)$   
5:   for  $i$  in  $\kappa$  do  
6:      $\text{path} \leftarrow \text{rand\_walk}(v, \zeta)$   
7:      $\text{vis}[v][\text{path}[-1]] ++$   
8:     if  $\text{ctx}[v][\text{path}[-1]] == \text{path}[1]$  then  
9:        $\text{dom}[v][\text{path}[-1]] ++$   
10:    else  
11:      if  $\text{dom}[v][\text{path}[-1]] > 0$  then  
12:         $\text{dom}[\text{path}[-1]] --$   
13:      else  
14:         $\text{dom}[v][\text{path}[-1]] ++$   
15:         $\text{ctx}[v][\text{path}[-1]] = \text{path}[1]$   
16:      end if  
17:    end if  
18:  end for  
19:   $\mathcal{S}[v] \leftarrow \text{top } s \text{ target-context pairs w.r.t. vis}[v] \ \& \ \text{ctx}[v]$   
20: end for
```

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# Results

Pinterest	MRR	REC@1	REC@10	DC+	DC*	DE+	DE*	INT	UNI	JAC
Visual	0.4406	0.1710	0.3606	0.4194	0.6337	0.9101	1.1255	23.32	174.67	0.1506
Textual	0.5741	0.1888	0.4965	0.3414	0.7614	0.7549	1.2050	31.78	166.21	0.1917
Combined	0.4438	0.1731	0.3635	0.4190	0.6340	0.9096	1.1258	23.44	174.53	0.1512
Pixie	0.3093	0.0418	0.2169	N/A	N/A	N/A	N/A	21.32	176.65	0.1351
PinSage	0.8759	0.4928	0.8234	0.2655	0.9279	0.7161	1.3593	47.30	150.69	0.3302
GAT	0.8880	0.5357	0.8665	0.2532	0.9343	0.7060	1.3618	48.70	149.24	0.3572
HAN	0.9013	0.5653	0.8838	0.2501	0.9415	0.6907	1.3558	50.29	148.83	0.3672
MULTISAGE-2	<b>0.9569</b>	<b>0.6215</b>	<b>0.9326</b>	<b>0.2316</b>	<b>0.9655</b>	<b>0.6660</b>	<b>1.3871</b>	<b>53.95</b>	<b>144.04</b>	<b>0.3906</b>
OAG	MRR	REC@1	REC@10	DC+	DC*	DE+	DE*	INT	UNI	JAC
Textual	0.1418	0.0273	0.0399	0.1081	0.4788	0.2814	1.0557	33.10	164.87	0.2193
Pixie	0.3126	0.1054	0.2642	N/A	N/A	N/A	N/A	36.58	160.76	0.2517
PinSage	0.5682	0.1845	0.5193	0.1238	0.6381	0.3179	1.1577	41.13	156.80	0.2935
GAT	0.6059	0.2355	0.5498	0.1104	0.6416	0.2908	1.2022	43.02	155.70	0.3144
HAN	0.6214	0.2641	0.5749	0.1005	0.6543	0.2869	1.2383	44.96	154.49	0.3200
MULTISAGE-2	0.6874	0.3270	0.6455	0.0836	0.6989	<b>0.2542</b>	1.2769	48.63	148.97	0.3602
MULTISAGE-3	<b>0.7026</b>	<b>0.3614</b>	<b>0.6875</b>	<b>0.0814</b>	<b>0.7127</b>	0.2583	<b>1.3058</b>	<b>51.63</b>	<b>145.40</b>	<b>0.3891</b>

Table 2: Performance of state-of-the-art large-scale embedding methods for general recommendation.

Method	Home Decoration			Women's Fashion		
	MRR	REC@1	REC@10	MRR	REC@1	REC@10
PinSage	0.8021/0.8067	0.4195/0.4257	0.7439/0.7460	0.7537/0.7545	0.3754/0.3759	0.6838/0.6976
MULTISAGE	<b>0.8407/0.8488</b>	<b>0.4899/0.5160</b>	<b>0.7954/0.8146</b>	<b>0.8058/0.8294</b>	<b>0.4363/0.4711</b>	<b>0.7533/0.7806</b>

Table 3: Off-task utility of embeddings produced by PinSage and MULTISAGE in shopping recommendation.

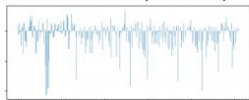
# Context impact

Query Pin

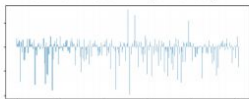
*Fashion  
Model*



Board Mask I (*Fashion*)



Board Mask II (*Crafts*)

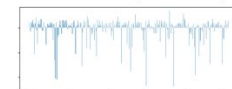


Query Pin

*Decorative  
Accessory*



Board Mask I (*Christmas*)



Board Mask II (*Decoration*)

