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BSc/BA, SECOND SEMESTER SUPPLEMENTARY EXAMINATIONS - 2020/2021

DEPARTMENT OF MATHEMATICS

MATH 223: Calculus II (3 credits)

INSTRUCTION:

Answer all questions in the answer booklet. For section A, write the LETTER for the correct answer only for each question in the FIRST PAGE of the answer booklet.

TIME ALLOWED:

TWO (2) HOURS

Section A: Clearly write the letter (a, b, c or d) of the correct answer only for each question in the FIRST PAGE of the answer booklet. Use the available space on this question paper for scratch work. (50 marks)

1. Find the limit:

$$\lim_{y \to 0} \frac{\sqrt{5} - \sqrt{5+y}}{y}$$

   (a) $\frac{1}{\sqrt{5}}$
   (b) 0
   (c) $-\frac{\sqrt{5}}{10}$
   (d) ∞

2. Consider the function

$$f(x) = \begin{cases} x^2, & \text{if } \quad x < 2, \\ 6 - x, & \text{if } \quad x \geq 2. \end{cases}$$

   On which interval does $f(x)$ NOT satisfy the hypothesis of the Mean Value Theorem?

   (a) $[-\infty, 2)$
   (b) $[0, 4]$
   (c) $[0, 2)$
   (d) $[2, \infty)$

3. Let $f(x)$ and $g(x)$ be inverse functions. Use the following table to determine $g'(0)$.

x	$f(x)$	$g(x)$	$f'(x)$
-1	5	9	$1/3$
0	3	-1	0

   (a) 1
   (b) $\frac{1}{3}$
   (c) 3
   (d) -1

4. Suppose we know that $f(x)$ is continuous and differentiable on the interval $[-7, 0]$, that $f(-7) = -3$ and that $f'(x) \leq 2$. What is the largest possible value for $f(0)$?

   (a) 17
   (b) 14
   (c) 11
   (d) 2

5. Find the value of c that satisfies the Mean Value Theorem on the interval [0,5] for the function

$$f(x) = x^3 - 6x$$

   (a) $-\frac{5}{\sqrt{3}}$
   (b) $\frac{25}{3}$
   (c) 1
   (d) $\frac{5}{\sqrt{3}}$

6. Express the following function in exponential form:

$$y = \tanh(2x)$$

   (a) $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$
   (b) $\frac{e^{2x} + e^{-2x}}{e^{2x} + e^{-2x}}$
   (c) $\frac{e^{2x} - e^{-2x}}{e^{2x} - e^{-2x}}$
   (d) $\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$

7. A sequence is called **monotonic** if
   (a) it is neither increasing nor decreasing
   (b) it is either increasing or decreasing
   (c) it is eventually increasing or eventually decreasing
   (d) it is both increasing and decreasing

8. Solve the inequality
$$\log_3(x-9)+\log_3(x-7)<1$$
   (a) $9<x<10$
   (b) $6<x<10$
   (c) $7<x<9$
   (d) $6<x<9$

9. Find the derivative of
$$f(x)=\ln\left(e^{3x}+e^{-3x}\right)$$
   (a) $\dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$
   (b) $3\left(\dfrac{e^{3x}+e^{-3x}}{e^{3x}-e^{-3x}}\right)$
   (c) $3\left(\dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}\right)$
   (d) $\dfrac{e^{3x}-e^{-3x}}{e^{3x}-e^{-3x}}$

10. Evaluate
$$\lim_{x\to 1^+}\frac{\sin(2\pi x)}{\sqrt{x-1}}$$
   (a) 2π
   (b) -2π
   (c) 1
   (d) 0

11. Find
$$\lim_{n\to\infty} e^{\cos(\pi/n)}$$
   (a) 1
   (b) e
   (c) 0
   (d) ∞

12. In application of the integral test to the improper integral $\int_1^\infty f(x)dx$, the function $f(x)$ must be

(a) continuous, positive and increasing on $[1, \infty)$

(b) discontinuous, positive and decreasing on $[1, \infty)$

(c) continuous, positive and decreasing on $[1, \infty)$

(d) piece-wise continuous, negative and decreasing on $[1, \infty)$

13. Which value does the following sequence converge to?

$$\left\{ \frac{2n^2 + 3}{3n - 7n^2} \right\}_{n=2}^{\infty}$$

(a) $\frac{2}{3}$

(b) $-\frac{2}{7}$

(c) $-\frac{3}{7}$

(d) $\frac{2}{7}$

14. Evaluate the integral

$$\int_0^3 \frac{4^x}{9} dx$$

(a) $\frac{27}{\ln 4}$

(b) $\frac{63}{4 \ln 9}$

(c) $\frac{7}{\ln 3}$

(d) $\frac{7}{2 \ln 2}$

15. Which of the following is true about the series

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

(a) It is convergent.

(b) It is a convergent p-series.

(c) It is divergent

(d) It is exponential series.

16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{5^n}$

(a) $\frac{4}{5}$

(b) $\frac{1}{4}$

(c) 4

(d) 1

17. Differentiate the function

$$y = 4^{\sqrt{x+1}}$$

(a) $\left(\frac{\ln 2}{\sqrt{x+1}}\right) 4^{\sqrt{x+1}}$

(b) $\left(\frac{\ln 2}{\sqrt{x+1}}\right)$

(c) $\left(\frac{\ln 4}{\sqrt{x+1}}\right) 4^{\sqrt{x+1}}$

(d) $\left(\frac{\ln 4}{4\sqrt{x+1}}\right)$

18. Solve

$$x - xe^{5x+2} = 0$$

(a) $x = -\frac{2}{5}$

(b) $x = -\frac{1}{5}$ or $x = 0$

(c) $x = -\frac{2}{5}$ or $x = 0$

(d) $x = 0$ or $x = \frac{2}{5}$

19. Find the derivative of

$$f(x) = \frac{\sinh x}{2x}$$

(a) $\frac{2x \sinh x - \cosh x}{x^2}$

(b) $\frac{x \cosh x + \sinh x}{2x^2}$

(c) $\frac{x \cosh x - \sinh x}{2x^2}$

(d) $\frac{x \cosh x - 2x \sinh x}{4x^2}$

20. Solve the logarithmic inequality

$$2\log_{16}(2x+1) - 1 \leq 0.$$

(a) $x \leq \frac{3}{2}$

(b) $x > -\frac{1}{2}$

(c) $0 \leq x \leq \frac{3}{2}$

(d) $-\frac{1}{2} < x \leq \frac{3}{2}$

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SECTION B: Answer all questions. Solve the questions in the answer booklet. [50 Marks]

1. (a) [**3 Marks**] State the Mean Value Theorem (in your answer booklet) by filling in the correct words and expressions in the following statement for a function f on the interval $[m, n]$:

   Let f be continuous on the interval $\cdots\cdots\cdots$ and $\cdots\cdots\cdots$ on the interval $\cdots\cdots\cdots$. Then there exists at least one number c in $\cdots\cdots\cdots$ such that

   $$f'(c) = \cdots\cdots\cdots$$

   (b) [**7 Marks**] Use the Mean Value Theorem to prove the inequality

   $$\tan x > x \text{ for } \quad 0 < x < \frac{\pi}{2}$$

2. [**10 Marks**] Find the derivative of

   $$f(x) = \int_{1}^{2x^2} \sqrt{t^2 + 3}\, dt$$

3. [**10 Marks**] Given that $\sinh x = \dfrac{5}{12}$, find the value of

   (a) $\cosh x$, and hence

   (b) $\sinh(2x)$

4. [**10 Marks**] Evaluate the limit

   $$\lim_{x \to \frac{\pi}{2}^-} \frac{1 + \tan x}{\sec x}$$

5. [**10 Marks**] Determine whether the given series is convergent or divergent. **Justify your answer. Only one point will be awarded if no correct justification is provided.**

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n + 2}$