STU22004 Applied Probability I

Group assignment

See instructions at the end of this document.

Question 1 (30 marks)

Consider a scenario where the temperature $X(t)$ varies randomly over a continuous time interval t, where t is in the range from 0 to 1. We begin with the assumption that $X(0) = 0$, which means that the temperature at time 0 is 0. Now, if we choose a small time increment represented by Δt, we can make the assumption that the change in temperature from time t to $t + \Delta t$, denoted as $X(t + \Delta t) - X(t)$, follows a normal distribution. This normal distribution is characterized by a mean of 0 and a variance of Δt.

1. Let P be the random variable denoting the proportion of time in $[0, 1]$ such that the temperature is positive. Estimate the distribution of P by Monte Carlo simulation and experimenting with various values of Δt (e.g. $\Delta t = 0.01, 0.001, 0.0001, \cdots$.)

2. Let $T_{\max}$ be the random variable denoting the time in $[0, 1]$ such that the temperature is at its maximum. Estimate the distribution of $T_{\max}$ by Monte Carlo simulation and experimenting with various values of Δt (e.g. $\Delta t = 0.01, 0.001, 0.0001, \cdots$.)

Question 2 (20 marks)

Suppose you play a game where you simulate n uniform random numbers on $[0, 1]$. You win €10 if all n numbers are between $\frac{1}{n}$ and $1 - \frac{1}{n}$. Otherwise you lose €1.

1. Estimate the expected net payoff of this game using Monte Carlo simulation for $n = 10, 100, 1000$. Which game has the highest expected net payoff?

Question 3 (30 marks)

Suppose you have a collection of n pieces of paper, and each piece bears a positive random number. The underlying distribution of these random numbers is unknown. Importantly, each number is unique, with no repetitions. Your task is to select one paper at a time from this set without replacement. Your ultimate objective is to identify and select the paper with the highest number.

At each step of the process, you must decide whether the current paper has the highest number among the remaining papers. If you decide that the number is not the largest, you choose the next one, and so on until the papers are exhausted. If you make the correct decision and select the paper with the highest number, you win a prize of €1000. However, if your decision is incorrect and you do not choose the paper with the highest number, you receive no prize.

1. Design a few strategies and evaluate the probability of winning for each strategies.

Question 4 (20 marks)

A deck of 100 cards - numbered $1, 2, \ldots, 100$ - is shuffled and then turned over one card at a time. We say that a "hit" occurs whenever card i is the ith card to be turned over, $i = 1, \ldots, 100$. Simulate $10\,000$ repetitions of the game to estimate the expectation and variance of the total number of hits.

Instructions

Submission

Submission date: **Friday 1st of December 2023**. Late submissions will not be accepted. You can submit your project by uploading to Blackboard.

Assessment

The project will carry 20% of the course marks. 80% of the marks will be awarded for doing a competent job; an extra 20% will be awarded for flair, imagination, thoroughness (good projects will go well beyond the lectures).

Project and report

To complete this project you can any software or resource that you think might help. The report should comprise no more than 10 pages[1], including tables, plots and diagrams, a good use of which is encouraged and will be rewarded, as long as they provide useful information. Details on the computations should be provided concisely. The report should be prepared **professionally** using any appropriate software (e.g. Word or Latex).

Teamwork

A group project is proposed mainly because group learning can be valuable, but also because team-working is a useful skill. **All members of each team must make significant contribution** to the group project. Clearly the project work does naturally break into sections which can be conducted in parallel before being brought together. The team will receive a mark and all members will receive by default the same mark. In case particular situations arise I might decide to allocate the group mark to the individual team members in a different way.

Plagiarism and sources

While learning from each other and from other groups is encouraged, plagiarism is remarkably easy to spot, especially with online resources. This includes projects that were submitted for the same course in the last years. The same work submitted by two teams will receive a single mark which will be then divided between the teams involved, following discussion. Sources must be cited.

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[1]this page limit is only indicative but reports that are too long will be penalised