15.C57/15.C571/6.C57/6.C571/IDS.C57: Optimization **Fall 2024**

Assignment 3: Integer optimization

Assigned: October 07; Due: October 28.

Model formulation [25 pts]

We consider the production of a product over T periods. If we decide to produce at period $t = 1, \cdots, T$, we incur a setup cost c_t. At each period $t = 1, \cdots, T$, let d_t be the demand, p_t be the unit production cost, and h_t be the unit storage cost.

a. Formulate an integer optimization model to minimize the total cost of production, storage and setup.

b. Suppose that we allow demand to be lost in every period except for period T, at a cost of b_t per unit of lost demand. Modify the model formulation to handle this option.

c. Suppose that production can occur in at most five periods, but cannot occur in two consecutive periods. Modify the model formulation to handle these requirements.

Branch-and-bound can take exponential time [25 pts]

Consider the following integer optimization problem, where n is an odd integer.

$$\begin{aligned} \min \quad & x_{n+1} \\ \text{s.t.} \quad & 2x_1 + 2x_2 + \cdots + 2x_n + x_{n+1} = n \\ & x_i \in \{0, 1\}, \ \forall i = 1, \cdots, n+1 \end{aligned}$$

We solve the problem via branch-and-bound algorithm, using the linear optimization relaxation to compute lower bounds and branching on any fractional variable to zero or one. Show that the algorithm will require the enumeration of an exponential number of subproblems.

Problem: Optimizing waste management [50 pts]

You are in charge of a public authority where waste management has grown into a significant issue. Past efforts have focused on waste collection by optimizing collection routes. However, waste is still disposed in open-air dumps, which is widely recognized to be the worst possible disposal technique!

The region can be represented as a square of 100 miles over 100 miles, with Euclidean distance. Waste is generated in 50 metropolitan centers. All waste needs to be disposed at a sanitary landfill.

Part A. Building landfills

You have identified 15 candidate landfills but, due to environmental concerns, you can only build 5 landfills. You have access to the following data and define the following optimization problem:

- centers.csv: A matrix of size 50×2 that indicates, for each of the 50 centers, (i) its x-coordinate, in miles (0 to 100), and (ii) its y-coordinate, in miles (0 to 100).
- landfills.csv: A matrix of size 15×2 that indicates, for each of the 15 centers, (i) its x-coordinate, in miles (0 to 100), and (ii) its y-coordinate, in miles (0 to 100).
- q.csv: a vector of size 50 that indicates the amount of waste (in tons) to collect every day from each of the 50 centers.

Parameters

$$\begin{aligned} q_i :& \quad \text{amount of waste generated at center } i = 1, \cdots, 50 \\ d_{ij} :& \quad \text{(Euclidean) distance between center } i \text{ and candidate landfill } j = 1, \cdots, 15 \end{aligned}$$

$$d_{ij} = \sqrt{(\text{x-axis of center } i \ - \ \text{x-axis of landfill } j)^2 + (\text{y-axis of center } i \ - \ \text{y-axis of landfill } j)^2}.$$

a. Write an optimization model that determines the 5 landfills to build. The objective is to minimize transportation costs, which we assume are proportional to the distance between centers and landfills. You should ensure that all the generated waste is collected and disposed at a landfill.

b. Implement the model. Which landfills do you build? Report the distance traveled by the waste (in miles-tons).

Part B. Building landfills and transfer stations

You can now build transfer stations for waste compaction. Each transfer station costs $10,000 per day to operate, and can treat up to 2,000 tons of waste per day. Waste can be transported directly to a landfill as uncompacted waste; alternatively, it can first go to a transfer station as uncompacted waste and then to a landfill as compacted waste. Transportation of uncompacted and compacted waste costs $1 and $0.50 per mile per ton, respectively. You are given the following data:

- stations.csv: A matrix of size 50×2 that indicates, for each of 50 candidate transfer stations, (i) its x-coordinate, in miles (0 to 100), and (ii) its y-coordinate, in miles (0 to 100).

c. Write an optimization model that determines the 5 landfills and the transfer stations to build. Your objective is to minimize total costs, comprising transportation costs and construction costs for transfer stations. You should still ensure that all the generated waste is collected and disposed at a landfill, and to capture the elements outlined above.

d. Implement the model. Which landfills and transfer stations do you build? Report the distance traveled by waste (in miles-tons), transportation costs, and daily costs.

Part C. A second region

Your success did not go unnoticed. You have been asked to apply your model to a neighboring region, also of 100 miles per 100 miles. It is located on the east side (with an x axis spanning 100 to 200 miles, and a y axis spanning 0 to 100 miles), and has 40 metropolitan centers. It is also planning to build 5 landfills and it is considering 15 candidate landfills and 50 candidate transfer stations. You obtain the equivalent data in centers2.csv, stations2.csv, landfills2.csv, and q2.csv.

e. Implement the model in the new region. Which landfills and transfer stations do you build? Report the distance traveled by waste, transportation costs, and daily costs.

You feel a sense of excitement from your leadership: “What if we planned the municipal waste management system together across the two regions? There has been a lot of talk to achieve greater collaboration between us. Plus, we might be able to find synergies. What do you think?”

f. Optimize the 10 landfills and the transfer stations to build across the two regions. What is the total daily cost? Write a one-paragraph recommendation to your leadership.