

作业: 令 $A = \{a, b, c\}$ A 上所有(二元关系)关系.
有 $\geq |A| \times |A|$ 个.

$$A \times A = \{\langle a, a \rangle, \langle a, b \rangle, \langle a, c \rangle, \langle b, a \rangle, \langle b, b \rangle, \langle b, c \rangle, \langle c, a \rangle, \langle c, b \rangle, \langle c, c \rangle\}$$

$$\rho(A \times A) = \left\{ \begin{aligned} &\emptyset, \\ &\{\langle a, a \rangle\}, \{\langle a, b \rangle\}, \{\langle a, c \rangle\}, \dots, \{\langle c, a \rangle\}, \{\langle c, b \rangle\}, \{\langle c, c \rangle\} \quad G^0 \\ &\{\langle a, a \rangle, \langle a, b \rangle\}, \{\langle a, a \rangle, \langle a, c \rangle\}, \dots, \{\langle c, b \rangle, \langle c, c \rangle\} \quad G^1 \\ &\{\langle a, a \rangle, \langle a, b \rangle, \langle a, c \rangle\}, \dots, \{\langle c, a \rangle, \langle c, b \rangle, \langle c, c \rangle\} \quad G^2 \\ &\vdots \\ &\{\langle a, a \rangle, \langle a, b \rangle, \langle a, c \rangle, \langle b, a \rangle, \langle b, b \rangle, \langle b, c \rangle, \langle c, a \rangle, \langle c, b \rangle, \langle c, c \rangle\} \quad G^7 \end{aligned} \right\}$$

$$|\rho(A \times A)| = G^0 + G^1 + \dots + G^7 = 2^7 = 2^{|A| \times |A|}$$

$$R_1 = \emptyset \text{ (空关系)} \quad R_2 = \{\langle a, a \rangle\}, \dots \quad R_{2^7-1} = \{\langle a, a \rangle, \dots, \langle c, c \rangle\} = A \times A \text{ (全域关系)}$$

思考: R_i 中的等价关系有几种 分别对应的等价类 (划分块)
对应的等价类集合 (划分)

从划分角度找等价关系.

$$\pi_1 = \{\{a, b, c\}\} \quad \pi_2 = \{\{a\}, \{b, c\}\} \quad \pi_3 = \{\{b\}, \{a, c\}\}$$

① 求 A 的其它划分, 令 A 的所有划分构成集合 P , 画出偏序集 $\langle P, \text{细分} \rangle$ 的哈斯图.

② 求划分 π_2, π_3 诱导的等价关系 R'', R''' .

③ 求 $R'' \cap R'''$, $\pi_2 \cdot \pi_3$, $\pi_2 \cap \pi_3$ 及 $\{\pi_2, \pi_3\}$ 的最大下界.

④ 求 $R'' \cup R'''$, $\pi_2 \cup \pi_3$, $\pi_2 + \pi_3$ 及 $\{\pi_2, \pi_3\}$ 的最小上界.
划分积 $\pi_2 \cdot \pi_3$ 细分 π_2 和 π_3 .
 π_2 和 π_3 均 划分和 $\pi_2 + \pi_3$

(可参考 P130, 10, 20.)