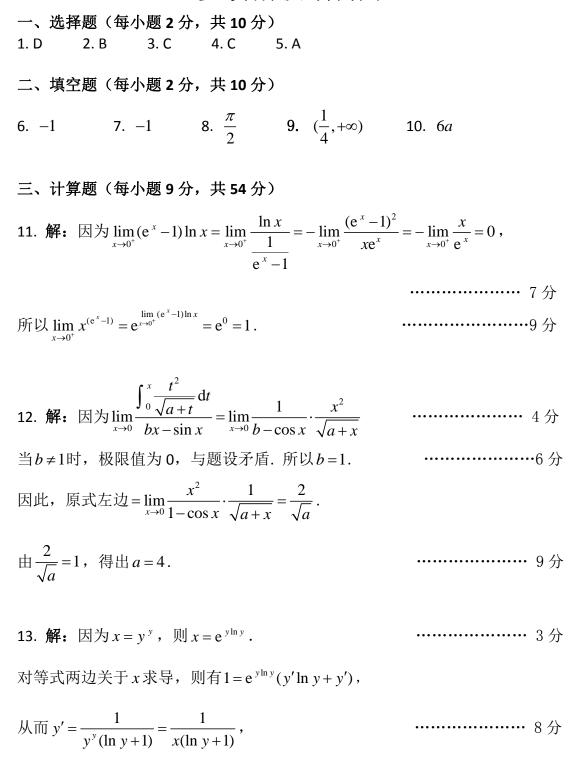
安徽大学 2020—2021 学年第一学期

《高等数学 A (一)》期末考试试题 (A 卷)

参考答案及评分标准



$$= \int \frac{1}{\sqrt{1 - \left(\frac{x+1}{\sqrt{2}}\right)^2}} d\left(\frac{x+1}{\sqrt{2}}\right) = \arcsin\left(\frac{\sqrt{2}(x+1)}{2}\right) + C.$$

$$= \frac{\ln(1+x)}{2-x} \Big|_{0}^{1} - \int_{0}^{1} \frac{1}{2-x} \cdot \frac{1}{1+x} dx = \ln 2 - \frac{1}{3} \int_{0}^{1} (\frac{1}{2-x} + \frac{1}{1+x}) dx \quad \cdots \quad 6 \text{ }$$

$$= \ln 2 - \frac{1}{3} \left[-\ln(2-x) \Big|_{0}^{1} + \ln(1+x) \Big|_{0}^{1} \right] = \frac{\ln 2}{3}.$$

$$= \lim_{b \to +\infty} \left[\ln x - \frac{1}{2} \ln(x^2 + 1) \right]_b^b \qquad \cdots \qquad 6 \,$$

$$= \lim_{b \to +\infty} \ln \frac{b}{\sqrt{b^2 + 1}} + \frac{1}{2} \ln 2 = \ln 1 + \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2.$$
 9 \(\frac{1}{2}\)

四、应用题(每小题8分,共16分)

17. **解**: 因为 y' = 2x, y'' = 2,

从而曲率半径 $\rho = \frac{1}{2}(1+4x^2)^{\frac{3}{2}}, \quad \rho' = 6x(1+4x^2)^{\frac{1}{2}}.$

令 $\rho'=0$, 得x=0. 当x<0时, $\rho'<0$; 当x>0时, $\rho'>0$. 所以在x=0时, ρ 取

$$V_{y} = \pi \int_{\frac{1}{2}}^{1} \left(\frac{a}{y}\right)^{2} dy + \pi \int_{0}^{\frac{1}{2}} 4a^{2} dy - \pi \int_{0}^{1} a^{2} dy = 2\pi a^{2}.$$
 \tag{7}

五、证明题(每小题10分,共10分)

从而对任意的 $x,y \in (0,+\infty)$, $x \neq y$, 恒有

$$f(\frac{x+y}{2}) < \frac{1}{2}[f(x) + f(y)],$$

$$\mathbb{I} \frac{x+y}{2} \ln \frac{x+y}{2} < \frac{1}{2} (x \ln x + y \ln y),$$