

Tiempo de Ejecución (II)

Ejercicio

```
int c=1;
while (c < n) {
    algo_de_O(1);
    c=2*c;
}
```

1

Analizar

El bucle termina cuando
 $c \geq n$

2

Simular la ejecución del código

$$P_0 \quad c = 1$$

$$P_1 \quad c = 2*1$$

$$P_2 \quad c = 2*2*1$$

$$P_3 \quad c = 2*2*2*1$$

$$P_k \quad c = 2^k$$

3

El bucle termina en un paso **k-ésimo** donde la condición no se cumple

$$c = 2^k \Rightarrow 2^k = n - 1$$
$$k = \text{Log}_2(n-1)$$

3

El valor de **K** indicará la cantidad de veces que se ejecutó el bucle

$$T(n) = c_1 + \sum_{k=1}^{\log n - 1} (c_2)$$

Recurrencia

$$T(n) = \begin{cases} 1, n=1 \\ 8T\left(\frac{n}{2}\right) + n^3, n \geq 2 \end{cases}$$

1 $T(n) = 8T\left(\frac{n}{2}\right) + n^3, n \geq 2$

$$T(n) = 8 \left[8T\left(\frac{n}{2}\right) + \left(\frac{n}{2}\right)^3 \right] + n^3, n \geq 4$$

$$T(n) = 8 \left[8T\left(\frac{n}{4}\right) + \frac{n^3}{2^3} \right] + n^3, n \geq 4$$

$$T(n) = 8^2 T\left(\frac{n}{4}\right) + 8 \frac{n^3}{2^3} + n^3, n \geq 4$$

$$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + n^3 + n^3, n \geq 4$$

$$T(n) = \begin{cases} 1, & n = 1 \\ 8T\left(\frac{n}{2}\right) + n^3, & n \geq 2 \end{cases}$$

2 $T(n) = 8^2 T\left(\frac{n}{2^2}\right) + 2n^3, n \geq 2^2$

$$T(n) = 8^2 \left[8T\left(\frac{\frac{n}{2^2}}{2}\right) + \left(\frac{n}{2^2}\right)^3 \right] + 2n^3, n \geq 2^3$$

$$T(n) = 8^2 \left[8T\left(\frac{n}{2^3}\right) + \frac{n^3}{2^6} \right] + 2n^3, n \geq 2^3$$

$$T(n) = 8^3 T\left(\frac{n}{2^3}\right) + 8^2 \frac{n^3}{2^6} + 2n^3, n \geq 2^3$$

$$T(n) = \begin{cases} 1, & n = 1 \\ 8T\left(\frac{n}{2}\right) + n^3, & n \geq 2 \end{cases}$$

$$T(n) = 8^3 T\left(\frac{n}{2^3}\right) + n^3 + 2n^3, n \geq 2^3$$

3 $T(n) = 8^3 T\left(\frac{n}{2^3}\right) + 3n^3, n \geq 2^3$

$$T(n) = 8^i T\left(\frac{n}{2^i}\right) + in^3, n \geq 2^i$$

$$\frac{n}{2^i} = 1 \Rightarrow n = 2^i \Rightarrow i = \log_2(n)$$

$$a^{\text{Log}_b(c)} = c^{\text{Log}_b(a)}$$

$$8^{\text{Log}_2(n)} = n^{\text{Log}_2(8)}$$

$$T(n) = \begin{cases} 1, & n=1 \\ 8T\left(\frac{n}{2}\right) + n^3, & n \geq 2 \end{cases}$$

$$T(n) = 8^{\text{Log}_2(n)} T\left(\frac{n}{2^{\text{Log}_2(n)}}\right) + \text{Log}_2(n)n^3$$

$$T(n) = n^{\text{Log}_2(8)} T\left(\frac{n}{n}\right) + \text{Log}_2(n)n^3$$

$$T(n) = n^3 T(1) + \text{Log}_2(n)n^3$$

$$T(n) = n^3 + \text{Log}_2(n)n^3$$