Homo Satiabilis: The Effect of Changing Income Inequality on Markups

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Abstract

A firm behaving optimally will set its markup – the ratio of price to marginal cost – based on the price elasticity for its product. One way that price elasticities might differ across time or across products is the ready availability, or not, of close substitutes – what we might think of as competition or market power. However, it might equally be that consumers differ in their individual price elasticities, at which point the composition of demand matters. In this spirit, this paper examines how the increase in income inequality since 1980 can help to explain the increase in the average and in the variance of markups. Using a rich dataset on retail markups, I first show that income and markups are related, as rich consumers tend to pay higher markups. In fact, as a consumer's incomes increases, they do not necessarily increase the physical quantity of products purchased, but rather trade up to higher price, higher markup products. To match these facts, I create a novel model of satiable preferences. I show that these preferences can be aggregated and resemble a discrete choice model extended to a macroeconomic environment. In the model, an increase in income inequality leads to a change in the composition of demand across firms, leading low quality, low markup firms to lower their markups while high quality, high markup firms increase their markups. Calibrating the model to match facts about income, consumption and markups in 2016 and changing the income distribution to that prevailing in 1980, the model generates 15% of the empirical change in the average markup, and a little over 100% of the growth in the variance.

*University of Toronto, Dylan.Gowans@mail.utoronto.ca My greatest thanks is extended Serdar Ozkan, Murat Alp Çelik, and Victor Aguirregabiria for their continuous support, encouragement and supervision. The analysis in this paper is my own, calculated based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at the University of Chicago Booth School of Business. The conclusions drawn from the NielsenIQ data are my own and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

1 Introduction

As a man's riches increase, his food and drink become more various and costly, but his appetite is limited by nature.
-Alfred Marshall

Several recent papers have found that markups have changed significantly since the 1980s (De Loecker et al. 2020, Hall 2018, Barkai 2020, Autor et al. 2020). Not only has the average increased, but so has the variance: there is now a larger mass of firms with both relatively high and relatively low markups (De Loecker et al., 2020). A firm behaving optimally will set its markup based on the price elasticity of demand. Thus, explanations of changing markups can equally be phrased as explanations of changing price elasticities. One determinant of price elasticities is the ready availability of, or threat of, close substitutes – what we might think of as competition or market power (e.g. Autor et al. 2020, Barkai 2020, De Loecker et al. 2021, Grullon et al. 2019). However, it is also possible that consumers differ in their individual price elasticities, at which point changes in the composition of demand are important for determining aggregate price elasticities (this underlies the arguments of Bornstein (2021) and Sangani (2022)).

This paper follows this latter approach, focusing on the distribution of income. I develop a model in which the distribution of income can have important effects on the level and variance of markups in the aggregate economy and suggest that this helps to explain the changes in the distribution of markups since the 1980s. I introduce a new set of preferences which are satiable. These preferences are able to match important facts about consumption, markups, and incomes. Unlike standard preferences used in macro models, these preferences are able to genereate inferior goods, which are a ubiquitous in the data. They are also able to match the relative unimportance of differences in physical quantities to explain differences in expenditure across income groups. Finally, they generate price elasticities which are decreasing in income, without relying on differences in search behaviour.

I calibrate this model to match data on consumption and the markup distribution in 2016, and then shock the model by changing the level of income inequality to that in 1983, while leaving the median income unchanged. In the model, faced with a hollowing-out of the income distribution, low-quality, low-markup firms lower their prices to attract the now-larger mass of poor consumers. Conversely, faced with a lower mass of median income consumers, high-markup luxury firms find there is less benefit to keeping their prices low to attract such customers. These luxury firms then raise their prices and, consequently, their markups.

To understand the paper's argument, note that a product's price elasticity is simply the weighted average price elasticity of its customers:

$$\eta_i = \int \eta_i(y) \frac{q_i(y)}{Q_i} \; dF(y)$$

where $q_i(y)$ and $\eta_i(y)$ are, respectively, the quantity and price elasticity for good i by consumers with income y, and Q_i is the total quantity of good i purchased by all consumers. Several authors have found empirically that poor consumers tend to have lower price elasticities than rich consumers (c.f. Stroebel and Vavra 2019, DellaVigna and Gentzkow 2019, Auer et al. 2022). Given this, we should expect the customer base of a firm to be correlated with its markup, and that rich consumers should pay higher markups on average. Using a rich dataset of retail markups, I show in Section 2 that this is the case and that rich consumers tend to pay higher markups than poor consumers. Furthermore, this is the result of rich consumers purchasing different products which have higher markups; it is not the result of rich consumers purchasing identical products at higher prices.[1]

If the price elasticity faced by a firm depends on the weighted average of the elasticities of its customers, then changes in the distribution of these customers will have impacts on a firm's elasticity, and therefore its markup. Taking the total differential of elasticity with respect to the distribution of income yields:

[1] Anderson et al. (2020) come to the same conclusion using variation in markups at the regional level. Using a very similar dataset, Sangani (2022) also shows that average markups paid increase in the income of customers, although he attributes this to higher prices paid for identical products.

$$d\eta_i = \int \underbrace{(\eta_i(y) - \eta_i)}_{\text{Relative elasticity}} \quad \underbrace{\frac{q_i(y)}{Q_i}}_{\text{Engel curve}} \quad df(y)\ dy$$

To a first order, a firm's elasticity will change depending on how elasticities differ across income groups, and the shares of these income groups in the firm's customer base. This latter depends on the shape of a product's Engel curves. It therefore becomes important to understand how consumption and elasticities vary across goods and across the income distribution. Therefore, in Section 3, I examine consumption patterns using the Nielsen Homescan Consumer Panel dataset. Here, I find two important facts that a model of consumption should be able to replicate. First, I find that inferiority is very common, with around 3/4 of Engel curves in the dataset having a significant downward sloping component. Not surprisingly, inferiority is correlated with relative prices. Looking within a given category of products – for example, milk – more expensive goods are more likely be normal, i.e. to have strictly increasing Engel curves. Second, I show that as incomes increase, average expenditures increase; however, this increase is largely the result of increases in average prices paid, rather than increases in total physical quantities consumed. Taken with the first fact, this suggests a process for consumption in which, as income increases, consumers trade up to higher-priced, higher-quality goods, rather than consuming more goods of a given quality.

I suggest that this consumption process makes sense as the result of satiable preferences. By this, I mean that goods fulfil needs, and that once these needs are satisfied, consuming additional units of similar goods will yield no additional utility. To use the example given in the quote above from Alfred Marshall, food and drink fulfil a need – calories – which is satiable. Past some income level households no longer increase their consumption of calories. Despite this, the grocery bag of the rich household will differ from that of the poor: name brands will replace store brands, luxury wines will replace cheap beer, organic will replace non-organic, fresh will replaced canned, etc. Because consumers cannot increase their utility by increasing the quantity of a given good, they increase utility by increasing the average quality consumed. Thus, consumers trade up to higher quality products as incomes increase.

In Section 4, I create a novel model of satiable consumer preferences. Satiability means that at the individual level, consumers form a hierarchy of purchases – they first purchase those goods with the highest utility per dollar spent – necessities – and only at higher incomes do they trade up to lower priority goods – luxuries. When these preferences are aggregated, they resemble a discrete choice demand system, modified to account for the macroeconomic nature of the model.

This hierarchical model of consumption has important implications for the markup decisions of firms. In the model, the decision to trade up is a function of both income and price. A firm lowering its price a little bit is apt to attract consumers for whom that product is next in their hierarchy. To give an example, if Lamborghini dropped their price by $5,000, they might expect to attract purchases from middle-to-high income consumers, but not low-income consumers. For poor consumers, a luxury car is not enough of a priority, and would mean giving up purchases of necessities, which yield a much higher utility per dollar spent. Conversely, dropping the price of a basic car may attract purchases from low-income consumers, who now find it preferable to other means of transportation, but will do little to attract purchases for high-income consumers, who are already consuming cars of higher quality.[2]

Thus, firms with different customer bases will react react differently to the same change in the income distribution. Each considers the size of potential markets it can gain by lowering its price, and the size of these markets are shaped by the distribution of income and by the patterns of consumption across the income distribution.

This means that the pricing of basic and luxury products will respond differently to a hollowing-out of the income distribution. In the calibrated model, low-markup, basic firms lower their prices to attract the now-larger mass of poor consumers. Conversely, luxury firms now see fewer median income consumers who, for a sufficiently low price, would purchase

[2]Here also, Alfred Marshall beats us to the punch. In his *Principles*, he writes: "The current prices of wall-fruit, of the better kinds of fish and other moderately expensive luxuries are such as to make the consumption of them by the middle classes increase much with every fall in price... While the demand on part of the rich and on the part of the working class is less elastic, the former because it is already nearly satiated, the latter because the price is still too high."

their products. They therefore raise their prices and their markups. The mass of poor consumers is inconsequential for the luxury firm: its higher cost means that it would have to lower its price to uneconomical levels to attract these consumers. The overall effect of an increase in income inequality is an increase in the variance of markups, but also an increase in the average markup. The increasing average is driven by the fact that low-markup firms face a bound on how low their markups can fall, while a similar upper bound does not exist for high-markup firms whose markups are rising.

With this, it is now possible to state the structure of the argument as it appears in this paper. In Section 2, I first show that rich households purchase different, higher-markup products than poor households. In Section 3, I examine empirically how consumption varies with income. In Section 4, I suggest that preferences which are satiable are able to generate the facts of Section 3. In this section, I build a novel model of consumers preferences with this feature, first at the individual and then at the aggregate level. Aggregate consumption takes a form similar to the discrete choice demand systems found in the IO literature, although it is extended to a macroeconomic environment.

Next, to examine the aggregate equilibrium effects on markups of a change in the distribution of income, in Section 5, I take the model of consumer preferences to the data and calibrates it to match facts about consumption in the Nielsen Homescan dataset. Then, I change the distribution of income to that prevailing in 1983 and observe the equilibrium effects on the distribution of markups. The model generates 15% of the increase in the average markup, and a little over 100% of the increase in the variance.

Related Literature When it comes to the distribution of markups, authors have tended to concentrate on the increasing average markup. Explanations rely on the fact that rational firms will set markups based on price elasticities – the more sensitive one's customers are to price increases, the less one can get away with high prices. Consequently, seeking to explain rising markups could equally be phrased as seeking to explain falling aggregate price elasticities. Explanations have tended to rely on supply-side arguments, often about market power and competition (e.g. Autor et al. 2020, Barkai 2020, De Loecker et al. 2021); the fewer competitors firms face, the harder it is for consumers to switch away from the products of a price-increasing firm.

However, consumers may differ in their price elasticities for reasons other than the competitive structure of an industry. Indeed, to the extent that customers price elasticites may differ for identical products, anything that alters the composition of consumers has the capability of altering average price elasticities. Thus, a few recent papers examine how the changing distribution of consumers might impact aggregate elasticities, and therefore markups: Bornstein (2021) looks at the changing age profile of the economy, while Sangani (2022) looks at changes in the income distribution.

Sangani (2022) argues that rich consumers have a low price elasticity which is the result of low search intensity. Consumers purchase goods at the lowest price available. However, rich consumers have a relatively high search cost, and therefore search less and are more likely to purchase high-price, high-markup goods. However, as shown in Section 2, changes in prices paid for identical products explains only a very small fraction of the higher markups paid by rich consumers. Rather, markups are higher for rich consumers because they purchase distinct, higher-quality, products.[3] Thus, like Sangani (2022), this paper considers changes in the distribution of income. However, unlike Sangani, this paper expressly models the non-homotheticity of consumer choice: as incomes grow, rich consumers will purchase higher-quality, higher-markup goods. Because of this non-homotheticity, producers of luxury goods will behave differently in response to an increase in income inequality than will producers of inferior goods.

Several papers have shown empirically that income matters for price elasticities and pricing. Stroebel and Vavra (2019) find that markups vary positively with housing wealth. Using an instrument for price changes, DellaVigna and Gentzkow (2019) find that price elaticities are larger in poorer zip codes. Using data on the appreciation of the Swiss franc, Auer et al. (2022) show that the consumption of low-income consumers is more price-elastic.

In terms of preferences, this paper suggests that satiability may be an important feature

[3] Anderson et al. (2020) find that markups differ in richer zip codes because of variation in products purchased, rather than a variation in prices charged for identical products.

of consumption behaviour. Satiability itself is not novel. As already mentioned, Marshall (1920) is replete with references to satiability as well as its effect on price elasticities of demand (see Footnote 2). The view is present earlier, however, being found also among the early marginalists, who noted that falling marginal utility could eventually reach zero for at least some goods.[4]

As I will show, satiability also gives rise to consumption which is hierarchical: a consumer's most important needs are satisfied first (say, food or shelter), while less important needs are satisfied last (say, luxuries). Again, this is very similar to the view of the early marginalists. This is particularly evident in Menger (1871).[5] The view that consumption proceeded in a hierarchical fashion was elaborated in greater detail by Roy (1943), whose goal was to answer questions regarding aggregate price elasticities[6] (for an english translation see Roy (2005)). It has also been popular with some in the Cambridge tradition (c.f. Pasinetti 1981 and Lavoie 2014). More recently, Foellmi and Zweimüller (2006), Foellmi et al. (2014) and Foellmi and Zweimüller (2017) adopt hierarchical preferences to examine the effects of inequality on economic growth.

2 Markups and Income

I start by showing that the average markup paid by a consumer varies positively with income. To do so, I construct a dataset of retail markups by matching the Nielsen Homescan

[4]Marshall (1920) uses the "law of satiable wants" and the "law of diminishing utility" interchangeably. In his chapter on the theory of value, Menger (1871) notes that abundant goods, like water in a stream, become non-economic (having no-value) once consumers reach a point of satiation. He makes a similar point later regarding satiation in terms of food. With regards to economic necessities, Jevons (1871) writes: "The necessaries of life are so few and simple that man is soon satisfied in regard to these, and desires to extend his range of enjoyment."

[5]Menger writes: "The maintenance of our lives depends on the satisfaction of our need for food, and also, in our climate, on clothing our bodies and having shelter at our disposal. But merely a higher degree of well-being depends on our having a coach, a chessboard, etc. Thus we observe that men fear the lack of food, clothing, and shelter much more than the lack of a coach, a chessboard, etc."

[6]It's interesting that, despite the close similarity to the views of Maslow (1943), Roy (1943) seems to have arrived at this idea independently, and possibly earlier. Due to delays brought on by World War II, Roy's paper was submitted several years before 1943. Furthermore, Roy suggests that he arrived at these ideas in the early 1930s in response to criticism from Marschak.

Consumer Panel and Price Trak Wholesale datasets. Nielsen Homescan Consumer Panel is a panel dataset of 40,000-60,000 U.S. households. It asks respondents to record purchases of all goods meant for personal, in-home use. Purchases are recorded with in-home scanners or mobile apps. For each purchase, we are able to observe the product's unique UPC barcode. Respondents also provide demographic information, including binned household income. The Price Trak Wholesale database is collected through a weekly monitoring service of 12 grocery wholesalers. Wholesale costs are also recorded at the UPC level. Because the Homescan dataset contains details on prices paid by consumers, while Price Trak contains information on wholesale costs, matching the two datasets allows us to construct product-level retail markups. I do so using 2018 data, which yields a matched dataset of 414,256 UPCs. I assume that all retailers in the Nielsen dataset face the same wholesale cost for a given product, which I set equal to the average wholesale cost of that product in the Price Trak dataset. Despite common wholesale costs, markups may differ across individuals as different consumers may face different prices. Because consumers input prices themselves, there is room for measurement error. Doing a cursory look reveals several obvious instances of this.[7] To account for this, I remove the top and bottom 5% of recorded prices for each product.[8]

For each income group, y, and each product, i, I calculate the average markup paid as:

$$\mathbb{E}(\mu_i|y) = \sum_n \frac{p_{i,n}q_{i,n}}{\sum_m p_{i,m}q_{i,m}} \frac{p_{i,n}}{\phi_i} \quad \forall n, m \in y \tag{1}$$

where n and m index individuals. Meanwhile, the average markup across products for consumers in income group y is:

[7] Nielsen asks for "total price paid" ($p_i q_i$) and number of units purchased (q_i). One common mistake seems to be treating "total price paid" as the price-per-unit. Another is to treat the number of items in a pack as number of units purchased – for example, a six-pack of Coca-Cola is one unit, but some households record this as six units.

[8] This seems to explain why the markups I find are lower than in Sangani (2022).

$$\mathbb{E}(\mu|y) = \sum_i \frac{\sum_n p_{i,n} q_{i,n}}{\sum_n \sum_j p_{j,n} q_{j,n}} \mathbb{E}(\mu_i|y) \quad \forall n \in y$$
$$= \sum_i S_i(y) \mathbb{E}(\mu_i|y) \tag{2}$$

where $S_i(y)$ is the expenditure share by income group y on good i.

The blue solid line in Figure 1 plots average markups as defined by eq. (2). It shows that average markups paid are increasing in income, from about 1.18 for those making between $12,000-$15,000 to over 1.22 for those making $100,000+. This is relatively low compared to the average of about 1.6 found by De Loecker et al. (2020). However, it is very similar to the average they find for the retail sector, with estimates ranging from about 1.1 to 1.3. The average in this case is less important than the trend, i.e. that markups paid tend to increase with income.[9]

Sangani (2022) shows a figure similar to Figure 1. He attributes the difference in markups to differences in search behaviour across incomes: the rich have a higher cost of shopping time, and therefore spend less time searching for low prices. The result is that, *for identical products*, the rich will pay higher prices and therefore higher markups. This seems empirically plausible, given that Broda et al. (2009) find that, in the Nielsen Homescan Dataset, the average price paid for identical products increases by about 0.1% for every 10% increase in income.

To test this hypothesis, I consider the counterfactual average markup where expenditure shares are held constant at the level of individuals making between $12,000-$15,000, but allow markups paid to vary with income. In other words:

$$\mathbb{E}(\mu|y)_{cf} = \sum_i S_i(y = 12000) \mathbb{E}(\mu_i|y) \tag{3}$$

[9]Sangani (2022) shows a similar figure but using data from 2007. Because Nielsen data in 2007 is top-coded at $200,000 rather than $100,000, he is able to observe this pattern over a larger domain. The general trend continues past $100,000, increasing approximately linearly.

If the rich are paying higher markups because they are paying higher prices for identical products, then we would expect this counterfactual to explain a large share of the increase seen in the average markup. I plot this counterfactual in Figure 1. Keeping expenditure shares fixed, the average markup rises very little.

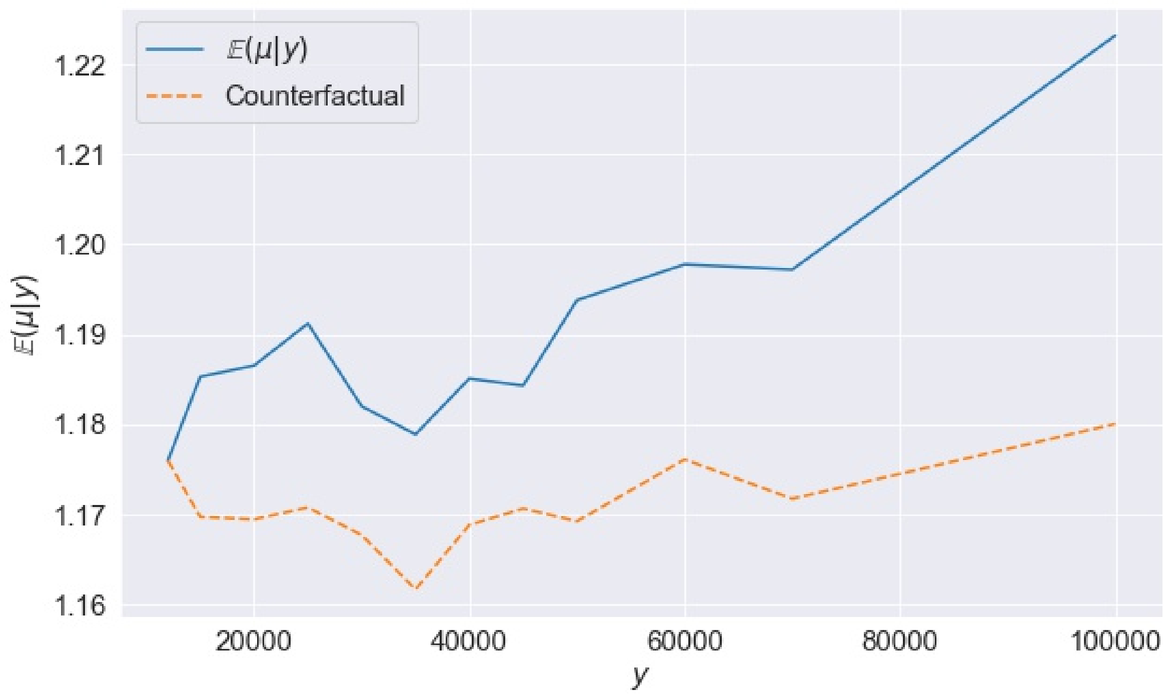


Figure 1: Average markup paid by different income groups, and counterfactual keeping expenditure shares fixed at the lowest income level

This suggests that positive relationship between household income and average markups is explained not by higher markups for identical products, but rather by consumers switching their consumption bundles to products with higher markups. For robustness, Figure 2 reproduces Figure 1 but calculates markups using the price paid less any coupons customers applied. The results trends are largely unchanged, although the average markup falls.

Discussion One explanation for the higher markups paid by rich consumers is that rich consumers have lower price elasticities, as found by DellaVigna and Gentzkow (2019) and

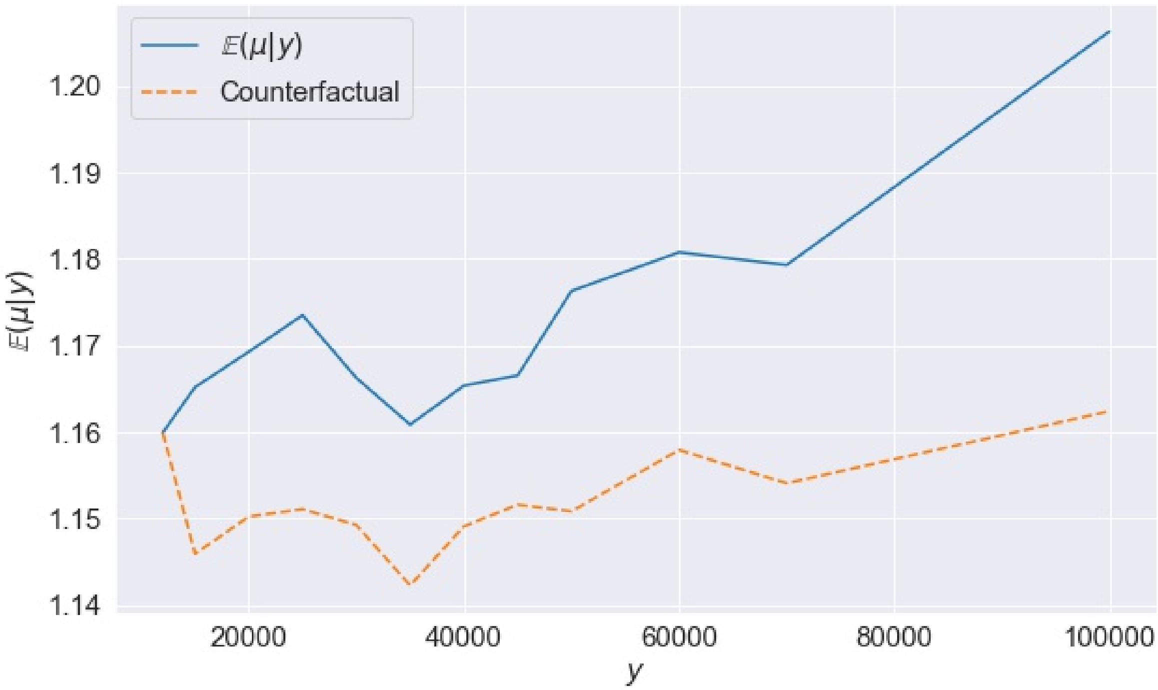


Figure 2: Reproduction of Figure 1 subtracting coupon values from prices paid.

Auer et al. (2022). We can represent this mathematically, by noting that a product's overall price elasticity is given by:

$$\eta_i = \int \eta_i(y) \frac{q_i(y)}{Q_i} dF(y) \tag{4}$$

where $\eta_i(y)$ is the price elasticity of consumers of income group y for good i, $q_i(y)$ is the average quantity of good i consumed by consumers of income group y, and Q_i is total quantity of good i, i.e.

$$Q_i = \int q_i(y) dF(y)$$

A product's overall price elasticity is simply the weighted-average of the price elasticities of all of its customers. If, on average, price elasticities are falling in income, we would expect products whose customer base is dominated by the rich to charge higher markups. Equally importantly, changes in the composition of income will change the weights on different income groups, and therefore change the aggregate elasticity. To be precise, taking the total differential of eq. (4) with respect to changes in the distribution of income:

$$d\eta_i = \int (\eta_i(y) - \eta_i) \frac{q_i(y)}{Q_i} \ \ df(y) \ \ dy \tag{5}$$

To a first order, changes in price elasticities, and therefore markups, are determined by the relative elasticities of different income groups, and their share of a product's total sales. Importantly, note that an increase in the number of rich consumers will have a different impact on the elasticity of a luxury product and a basic product. Because the luxury product sells mainly to rich consumers, the luxury firm will put a higher weight on changes in the number of rich consumers. Intuitively, this makes sense: an increase in the number of poor consumers will matter very little to the pricing decision of Ferrari, since poor consumers make up such a small share of Ferrari's total customer base.

Equation (5) also suggests what will be important to determine the effects of a change in the distribution of income on the distribution of markups. This equation is made up of two parts: (1) relative elasticities, $\eta_i(y) - \eta_i$, and (2) Engel curves, $q_i(y)/Q_i$. The second feature means that to find the effects of a change in the distribution of income on markups, it will be important to get the facts about the consumption process correct. In the next section, then, I examine what we can say about empirical Engel curves. I will suggest that these are well rationalized by a model of consumption in which preferences are satiable.

3 Consumption and Income

The previous section showed that markups paid vary positively with income, and suggested that this was due to differences in price elasticities across the income distribution. Once we allow for this, changes in the distribution of income can play an important part in determining changes in the distribution of markups. It also showed that one determinant of this effect will be the shape of Engel curves and, more broadly, the process of consumption across income groups. In this section, I observe these features empirically. I then argue that product-level consumption data can be well understood as the result of satiable preferences. There are two features of the data that satiable preferences allow us to match. First, I show that inferior products are ubiquitous in the data. Perhaps not surprisingly, low-price products tend to be inferior while high-price products tend to be normal. Taken together, this suggests that consumers are trading-up from low-price, low-quality goods to high-price, high-quality goods as their incomes increase.

Second, I give evidence that as incomes increase, expenditure increases, but this increase is almost entirely due to an increase in the average price paid for goods, rather than an increase in the number of physical units purchased. I suggest that both of these facts can be rationalized by the assumption that consumption is satiable: goods fulfil needs, and once these needs are met, additional units of similar goods will yield no additional utility. Instead, as incomes increase, consumers trade up to higher-priced, higher-quality goods.

To show these facts, I rely again on the Nielsen Homescan Database, although here I use the years 2006-2009. In these years, household income is top-coded at \$200,000 rather than at \$100,000.

I first show that inferior products are common in this dataset. To do so, I construct Engel curves at the brand level. Although goods are registered at the UPC level, the dataset contains brand information. Products which are otherwise equivalent, but sold in different sizes will have different UPCs. For example, a 6-pack of Coca-Cola will have a different UPC than a 2-litre bottle. Since these are simply different sizes of the same product, I aggregate to the brand level, adjusting for difference in product size.[10] Some care must be taken here, however, as not all products of a given brand have common units of measurement. Thus, for example, tea is sold both in bags (measured with unit "count") and in liquid form (measured with unit "ml"). Therefore, a given brand, like Lipton, may have UPCs which correspond to goods measured in both ways. I treat goods measured in different

[10] In the Coca-Cola example, a household who bought a 2-litre bottle of Coca-Cola would be recorded as having purchased 2000 ml, while a household who bought a 6-pack of 500 ml bottles would be recorded as having purchased 3000 ml.

units as separate goods. Thus, Lipton tea bags are a different good than Lipton liquid tea. Finally, the Nielsen Homescan dataset also records the "group" in which a UPC is classified. Examples of groups include beer, candy, coffee, and sunglasses. To account for the fact that a given brand may sell multiple kinds of products (e.g. Arm & Hammer toothpaste and Arm & Hammer baking soda), a product is defined as a triple of {brand, product group, measurement units}. Finally, it should be noted that the description of brands is fairly granular. For example, Coca-Cola, Coca-Cola Vanilla and Coca-Cola Diet Vanilla all appear as separate products in our dataset.

Before constructing Engel curves, I keep only products that have sales in at least 8 out of 17 income groups. I construct Engel curves as the average amount of a product that is purchased by consumers within a given income category. To control for differences in household size, I only consider 3 or 4 person households. I then classify each Engel curve into either inferior, hump-shaped or normal. The classification is achieved by fitting each Engel curve with a quadratic function. The categorization rule is given in table Table 1. Categorization is based on the slope of the Engel curve at $y = 0$ and the location of the turning point. Intuitively, if the fitted function is mostly increasing over the domain, it is classified as normal. Conversely, if it decreases for a large part of the domain, it is classified as either hump-shaped or inferior.

The results of the categorization are given in Table 2. Only a minority of products are classified as normal. That is, the average quantity purchased of a products is monotonically increasing in income in only a minority of cases. Instead, over 70% of products have Engel curves which slope downwards after some income threshold.

Perhaps not surprisingly, inferior goods also tend to have the cheapest prices. To demonstrate, I first calculate each brand's price as the sales-weighted price of all UPCs which make up a product. I calculate the price decile to which each product belongs within its category, where category is defined as the pair {product group, units of measurement}. Figure 3 shows that lower priced goods tend to have inferior Engel curves, while higher priced goods tend to have normal Engel curves.

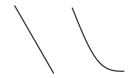
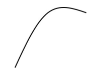
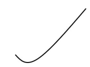
Classification	Slope at $y = 0$	Turning Point ($ thousands)	Shape
Inferior	-	> 100	
Inferior	-	< 0	
Inferior	+	$[0, 25]$	
Normal	+	< 0	
Normal	+	> 150	
Normal	-	$[0, 100]$	
Hump-Shaped	+	$[25, 150]$	

Table 1: Categorization rules for Engel curves

Classification	**Share**
Inferior	33.6%
Hump	40.0%
Normal	26.4 %

Table 2: Categorization of Engel curves in Nielsen Homescan

Finally, I examine aggregate expenditures. One minor puzzle here is that, despite the apparent ubiquity of inferior products, inferiority appears absent when considering broad categories of goods, for example in the Consumer Expenditure Survey. Although this is usually treated as evidence that physical purchases are increasing, it could equally be the result of consumers shifting to more expensive goods. Thus, I next decompose aggregate expenditures in the Nielsen database into changes in physical quantities and average prices. To do, notice that average expenditure by income group y is the sum of expenditures on each good, i, in each category, c:

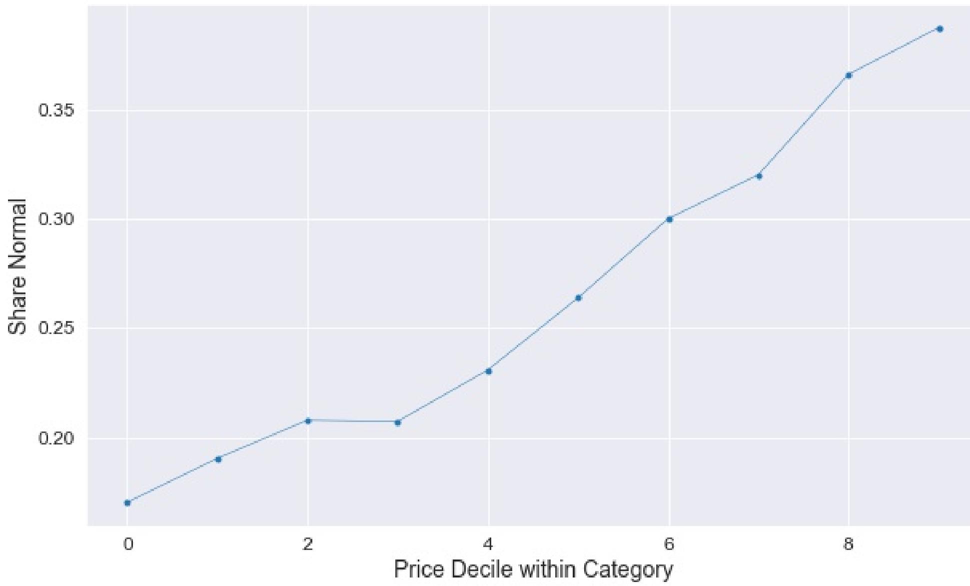


Figure 3: Share of Engel curves which are normal by price decile

$$X(y) = \sum_c \sum_{i \in c} p_{ic} q_{ic}(y)$$

Defining $Q_c(y) = \sum_{i \in c} q_{ic}(y)$:

$$X(y) = \sum_c Q_c(y) \frac{\sum_i p_{ic} q_{ic}(y)}{Q_c(y)}$$

$$X(y) = \sum_c Q_c(y) \mathbb{E}_c(p_{ic}|y)$$

In other words, total expenditure in a given category is total physical units purchased in that category multiplied by the average price paid. Then, the percentage difference in expenditures between any income group y and y' is given by:

$$\%\Delta X^{y,y'} = \sum_c \left(\%\Delta Q_c^{y,y'} + \%\Delta \mathbb{E}(p_{ic})^{y,y'} \right) \frac{X_c(y)}{X(y)} \tag{6}$$

where X_c is the expenditure on category c. This can then be divided into two parts:

$$\%\Delta X^{y,y'} = \underbrace{\sum_c \left(\%\Delta Q_c^{y,y'}\right)\frac{X_c(y)}{X(y)}}_{\text{Changes in quantity}} + \underbrace{\sum_c \left(\%\Delta \mathbb{E}(p_{ic})^{y,y'}\right)\frac{X_c(y)}{X(y)}}_{\text{Changes in average price}} \tag{7}$$

I perform this decomposition in Figure 4 for all expenditures in the Nielsen Homescan Database. Given that I define a category as the pair {product group, units of measurement}, Q_c makes sense as a physical aggregation since we are adding up like goods. The result is that greater than 100% of the total change in expenditure between the highest and lowest income groups is coming from changes in average price, rather than in physical units consumed. Of course, this does not mean that the number of physical units consumed by households is decreasing in income. The Nielsen database does not contain the universe of products purchased by consumers. Consequently, it is possible (and likely) that some of the decline in physical quantities is driven by rich consumers switching to products not contained in the Nielsen database. For example, rich consumers are more likely to eat at restaurants and less likely to purchase food to be made at home. Given that the former is not captured in Nielsen, this would lead to an apparent decline in the total quantity consumed.

3.1 Satiability as an Explanation

Taken together, the above facts suggest that as incomes increase, consumers don't necessarily purchase more goods, but rather purchase more expensive products. In particular, they trade up from low-price, low-quality products to high-price, high-quality products. I suggest that both facts are well explained by consumption being satiable. By this, I mean that goods fulfil some needs, and once these needs are met, additional consumption of similar goods yields no additional utility.

Of course, the fact that physical quantities consumed are not increasing in income is well explained by satiability: consumers need only so many units of a good, say food, to fulfil their needs regardless of their income.

The presence of inferior goods as evidence for satiability is perhaps more complex. Inferior

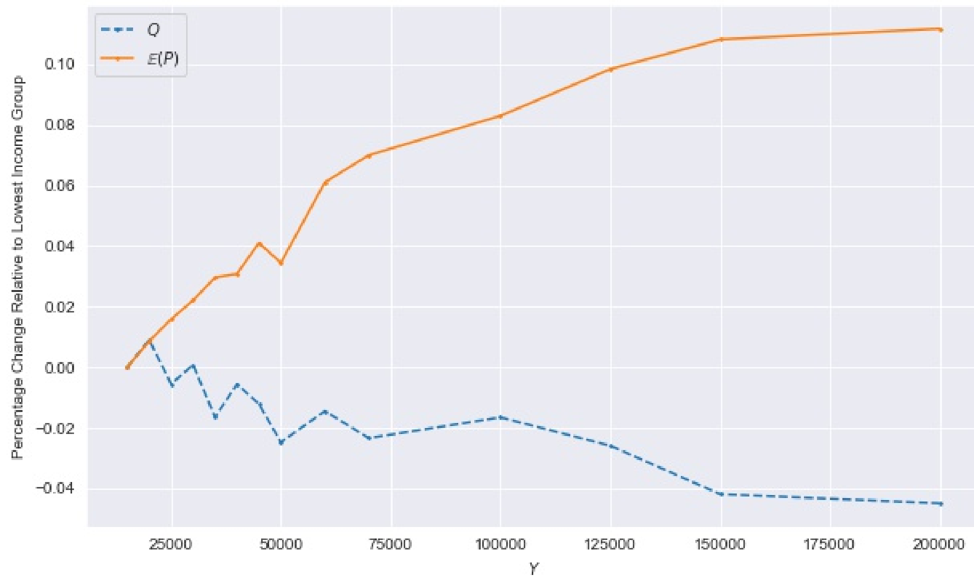


Figure 4: Decomposition of percent changes in expenditure relative to lowest income group

goods are difficult to generate with standard preferences. Past attempts at generating them have often relied on abandoning concavity in the utility function (c.f. Liebhafsky 1969).[11] To see the difficulty, consider a simple Cobb-Douglas utility function:

$$U(x, y) = x^{\alpha} y^{1-\alpha}$$

The reason this utility function cannot generate inferior goods is that $dU/dx > 0$, $dU^2/dx^2 < 0$ and $dU^2/dxdy \geq 0$. The last two features mean that as I consume more of good x, the relative benefit of consuming y increases, and therefore I also want to consume more y. Given that both goods yield positive utilities, as income increases I will consume more of both. As mentioned above, one solution is to do away with quasi-concavity. The

[11] Liebhafsky (1969) uses the utility function $U(x, y) = \alpha \ln x + \frac{y^2}{2}$. The implication is that the marginal utility of consuming good y is increasing in its consumption. It seems difficult to identify such goods. Moffatt (2002) makes the similar point that past attempts at generating Giffen goods – a family of inferior products – tend to rely on abandoning quasi-concavity. Moffatt (2002) proves that a quasi-concave utility funtion can generate Giffen behaviour, although he doesn't give an example of such a utility function.

result would be that at least one good becomes increasingly desirable the more I consume it. However, it's difficult to think of common examples of such goods.

Another solution might be to allow $dU^2/dxdy < 0$.[12] Note that this means that, as consumption of one good increases, the marginal utility of the other falls, leading the consumer to potentially want to switch from consuming one good to the other. Note also that $dU^2/dxdy < 0$ describes well two products which fulfil the same satiable need. Although the marginal utility of consuming only a hamburger is positive, it falls to 0 (or perhaps negative) if I first consume a lasagna.

4 A model of satiable preferences and inferior goods

This section develops a model of consumer preferences that are satiable and are capable of generating the consumption patterns seen in Section 3. In this section I write down a very general version of the model, although in Section 5 I simplify it for the sake of calibration.

4.1 Hierarchical Consumption

Each good, i, is part of a category, c. Categories are defined by the needs which they fulfil. For example, Arm & Hammer toothpaste might be a good in the category "oral hygiene". Because preferences are satiable, consumers can consume at most 1 unit of a good in a given category.[13] Each good is defined by a utility parameter u_{ic} and a price p_{ic}. An individual who consumes good i in category c receives the common utility component u_{ic} as well as an individual-good specific utility shock, ε_{icn}. Thus, utility for consumer n of consuming good i in category c is:

$$U_{icn} = u_{ic} + \varepsilon_{icn}$$

[12] This is the case in the model of Onuma (2020), who generates inferior goods by assuming that consumers make both quality and quantity choices across goods. This will be similar to the model presented in Section 4.

[13] Implicit in this formulation is that any amount of the good consumed above 1 unit yields no additional utility as the need has been satiated.

Consumer n, with income y_n solves the problem:

$$\max_{\{q_{icn}\}} \sum_c \sum_{i \in c} q_{icn} U_{icn}$$

$$\text{s.t.} \quad \sum_c \sum_{i \in c} p_{icn} q_{icn} \leq y_n$$

$$\sum_{i \in c} q_{icn} \in [0,1] \quad \forall c$$

Where the first constraint is the budget constraint, and the second constraint is the satiability constraint.

The solution has no closed-form expression, however it is solvable by an intuitive algorithm. Consider a consumer's choice of where to spend their first dollar of income. Obviously, the choice is the good which will yield the highest utility per dollar spent, U_{icn}/p_{ic}. Call this first good i^* in category c'. Because utility is linear in the consumption of this good, the consumer will devote each additional unit of income to purchasing this good until income reaches $p_{i^*c'}$, at which point they will have consumed the maximum amount possible, i.e. 1 unit. When income exceeds this level, what will their next choice be? The consumer may consume a good in another category, or they may switch the good they are consuming in category c' by trading up to a higher-price, higher-utility product. In particular, they must now consider the additional utility per additional dollar spent from trading up, or

$$\frac{U_{ic'n} - U_{i^*c'n}}{p_{ic'n} - p_{i*c'n}}$$

Note that this value must be positive, and therefore the new good must have higher utility and higher price.[14]

Why does the consumer choose a low-utility (inferior) product initially and then switch to a higher-utility (luxury) product when income is sufficiently high? Note that the inferior product is a more *efficient* means to gain utility, i.e. it has the highest U/p ratio. Meanwhile,

[14] Note that the consumer will never switch to a good with a lower utility and higher price; such a good is strictly worse.

although the luxury product has a lower U/p ratio, it yields greater utility overall. This behaviour is the result of satiability. If preferences weren't satiable, the consumer would simply consume more of whichever good has the highest U/p ratio. However, because of satiability, after the consumer has consumed their maximum of 1 unit, the only way to increase utility is to trade up to higher-utility, higher-priced goods. Thus, a low-income consumer might find that the basic car is the most efficient way to receive utility in the transportation category. However, a high-income consumer will find that the car with heated seats and sunroof yields greater utility overall, and therefore is preferable once their basic transportation needs have already been met.[15]

Importantly, this model leads to solution which is hierarchical. By this, I mean that consumers rank their purchases from most important to least important and then allocate their limited incomes accordingly. Hierarchical consumption models have often been derived from the premise that consumers have needs which vary in importance.[16] In these models, there exists some income threshold after which a consumer will purchase a given good. Because of trading up, the model presented here has the added feature that there may also exist also a higher income threshold past which the consumer no longer purchases this good.

Lastly, note that because the consumer prioritizes those purchases with the greatest ratio of additional utility per dollar spent and there is no complementarity in preferences, consumers will have a falling marginal utility of income.

Example As a brief example, consider a consumer faced with the following utilities and prices:

[15] Another way to think about this is that consumers are purchasing features which are bundled together as goods (Lancaster, 1971). Basic products, like the base model car, contain the most important features – the ability to drive between two points – while luxury products contain this feature as well as additional features – the comfort of heated seats, for example. To the extent that the primary features outweigh the secondary features in terms of utility per dollar spent, the consumer first purchases the basic model, and only at higher income levels do they purchase the luxury model.

[16] See the discussion of this type of model in the related literature section.

	Transportation			Smart Phone	
	Bus Pass	Hatchback	Lamborghini	Motorolla	iPhone
U	1	1.5	1.75	0.6	0.9
p	1	2	3	1	2

Table 3: Utilities and prices in two categories

Using the solution algorithm described above, it's evident that the consumer's first purchase will be of the bus pass, as this yields the highest U/p ratio of 1. Looking at the consumer's next consumption choice, the utility per dollar spent of trading up to the hatchback, $(1.5-1)/(2-1) = 0.5$, the Lamborghini, $(1.75-1)/(3-1) = 0.375$, or of consuming the iPhone, $(0.9/2 = 0.45)$, are strictly worse than the utility of consuming the Motorolla $0.6/1 = 0.6$. Consequently, this latter will be next in the consumer's hierarchy. We can continue in this way, which yields the following ranking of bundles:

	Bundle	**y**	**U**
1.	(∅,∅)	0	0.0
2.	(Bus, ∅)	1	1.0
3.	(Bus, Motorolla)	2	1.6
4.	(Hatchback, Motorolla)	3	2.1
5.	(Hatchback, iPhone)	4	2.4
6.	(Lamborghini, iPhone)	5	2.65

Table 4: Ranking of bundles

Note that the hierarchy allows us to draw Engel curves for individual goods. For example, at $y = 2$ the consumer purchases the bundle (Bus Pass, Motorolla). If the consumer's income is incrementally higher, however, they begin to trade up from the bus pass to the Hatchback. At $y = 3$, this process is complete. Then for income incrementally greater than $y = 4$, the consumer begins to trade up to the Lamborghini. Consequently, we can draw the resulting Engel curve for the Hatchback, shown in Figure 5.

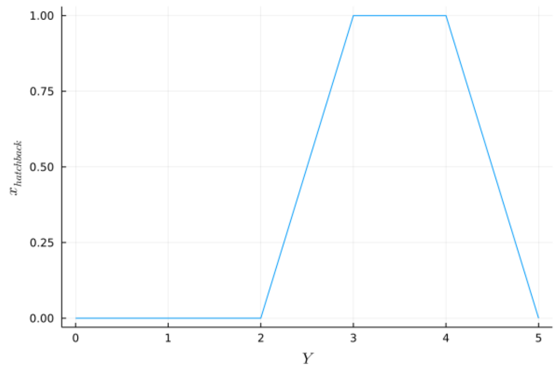


Figure 5: Individual's Engel curve for Hatchback

4.2 Effect of price changes on individual demand

Changes in price can change the location of a good in a consumer's hierarchy. Recall that the benefit of trading up from good 1 to good 2 is given by $(U_2 - U_1)/(p_2 - p_1)$. A decrease in the price of good 2 makes the benefit of trading up higher. Similarly, the benefit of trading up from good 2 to another good falls. The consequence is an expansion of the consumer's Engel curve for this good. Assuming a sufficiently large change in the price, the additional benefit increases the priority of consuming this good at lower levels of income and decreases the priority of trading up to higher utility goods at higher income levels.

Note, however, that the effect on a consumer's actual consumption choice depends on their level of income. For example, imagine that prior to a price decrease, a consumer will consume $q_A > 0$ in the income range (Y_l, Y_h) and consume $q_A = 1$ in the range (Y_l^*, Y_h^*). A price decrease makes the consumer more likely to trade up to good q_A at an earlier level of income, and to trade away from it at a higher level of income. Consequently, without loss of generality, following a price decrease, the income range over which the individual chooses $q_A > 0$ will become

$$(Y_l - \epsilon_l, Y_h + \epsilon_h) \qquad \epsilon_h, \epsilon_l \geq 0$$

Therefore, effective consumption of the good will only change if the consumer has income in the range $(Y_l - \epsilon_l, Y_l^*)$ or in the range $(Y_h^*, Y_h + \epsilon_h)$. This is illustrated in Figure 6.

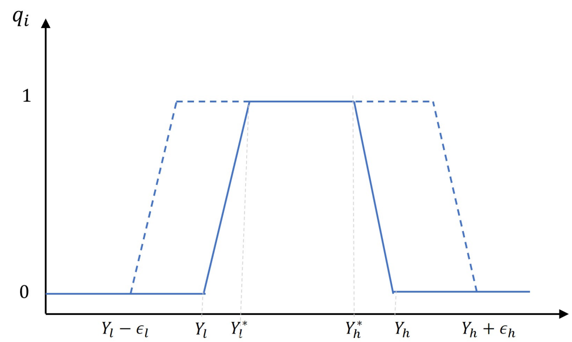


Figure 6: Effect of a price decrease

Consider the intuition for this result. There will be some luxury products which, even after a price decrease, low-income consumers will not want to purchase. For example, dropping the price of a Mercedes by $2,000 likely won't change the buying decision of many low-income households. These low-income consumers have higher consumption priorities, like purchasing basic necessities, and even at lower prices, luxuries still are not worthwhile. Similarly, there are some low-quality products which, even given a drop in price, rich consumers will not buy. Even given a low price for a bus pass, many rich consumers will still choose the comfort and convenience of driving, which yields greater utility.

4.3 The number and size of categories

To make it easier to aggregate the demands of many consumers, it is useful to first develop the idea of categories.

Categories are defined by a given need. However, the same set of goods may be available to meet many different needs. Indeed, this is obviously the case when we realize that needs are temporally specific. For example, the need for calories arises multiple times in a day, and each day of the week. Consequently, we can think of the need "calories for breakfast today", which is a distinct need from "calories for lunch tomorrow".[17] Note that although the same set of goods is available to fulfil these needs, a consumer may choose different products:

[17] This is similar to the definition of goods given by Arrow and Debreu (1954), albeit we will assume that needs and not goods are temporally specific.

pancakes for breakfast today, egg salad for lunch tomorrow, etc.

To account for this, I introduce the concept of "subcategories". Subcategories are defined by a temporally specific need – for example "breakfast today". We say that two subcategories belong to the same category if the subcategories contain the same set of goods. Therefore two subcategories, "breakfast today" and "lunch tomorrow" define two needs which are satisfied by the same sets of goods: food. Therefore, we say that both of these subcategories belong to the same category: "food".

With this in mind, I assume that there exists a countably infinite number of subcategories on the interval $(0, C]$. Define subcategories within the subinterval $(C_{j-1}, C_j]$ as being part of category j. Any two subcategories within a category contain the same set of goods, with the same u and p values. For simplicity, I assume that goods belong to only one category. Importantly, however, the consumer's utility shock, ε_{icn} is allowed to differ between subcategories. Thus, it is possible that a consumer will choose a different product in two subcategories of the same category.

However, since there is a countably infinite number of subcategories on $(C_{j-1}, C_j]$, by the law of large numbers all consumers will have the same distribution of ε shocks across categories of a given type. Because of this, all consumers will face the same hierarchy in terms of goods – although these goods may exist in different subcategories. To give an example, consumer A's and consumer B's hierarchies will both have the consumer trade up to "eggs" at an income level of $\bar{y}$, although for consumer A this will be for breakfast today, while for consumer B it will be for breakfast on Thursday. Therefore, consumers with the same income will consume the same set of goods within a given category. Consequently, consumers with the same income will have the same demand for a given good, albeit this demand will occur in different subcategories.

4.4 Aggregate demand for a good

Define $V_n(y_n)$ as the maximized utility of consumer n given income and define

$$\lambda_n(y_n) = dV_n/dy_n$$

which is the marginal utility of money. Note that, given the the hierarchical nature of consumption described in Section 4.1, $\lambda_n(Y_n)$ is simply the additional utility per dollar spent for the consumer at the current level of income in their hierarchy. Furthermore, recall that, because consumers prioritize those purchases with the greatest ratio of additional utility per dollar spent, we have that $\lambda_n'(y_n) < 0$.

Importantly, because of the restrictions we made on categories in Section 4.3, we have that

$$\lambda_n(y) = \lambda_m(y) = \lambda(y)$$

The countably infinite number of identical categories and the law of large numbers ensures that the marginal utility of money is not individually specific, but depends only on income.

We can now redefine the consumers problem within a category as:

$$\max_{i \in c}\{u_{ic} + \varepsilon_{icn} - \lambda(y_n)p_{ic}\} \tag{8}$$

The consumer chooses the good within a category which maximizes indirect utility. Note that, higher income consumers have a lower marginal utility of money, and therefore put lower weight on prices in their decision. For this reason, as incomes increase, consumers will tend to choose higher utility, higher priced products.

Finally, assume that the ε shocks are i.i.d. with cdf and pdf G and g respectively. Noting that a consumer will purchase good i in category c iff

$$u_{ic} - \lambda(y_n)p_{ic} + \varepsilon_{icn} \geq u_{jc} - \lambda(y_n)p_{jc} + \varepsilon_{jcn} \qquad \forall j \in c$$

or

$$\varepsilon_{jnc} \leq u_{ic} - u_{jc} - \lambda(y_n)(p_{ic} - p_{jc}) + \varepsilon_{icn} \quad \forall j \in c$$

Then, keeping in mind that good i is a good available in all categories of type k, demand for good i by a consumer with income y is given by:

$$q_i(y) = (C_k - C_{k-1}) \int \prod_{j \neq i} G\big(u_i - u_j - \lambda(y)(p_i - p_j) + \varepsilon\big) g\big(\varepsilon\big) d\varepsilon \ ; \quad i, j \in k \tag{9}$$

where $(C_k - C_{k-1})$ is the size of the market for good i. Finally, aggregate demand for good i is given by:

$$Q_i = N \int q_i(y) dF(y) \tag{10}$$

where N is the number of consumers, and F is the cdf of the income distribution.

Of course (8) and (9) will be recognizable for those familar with the literature in discrete choice models (c.f. Train 2009). For example, adopting the assumption that ε takes an extreme value type I distribution would change (9) into a logit demand equation of the form:

$$q_i(y) = (C_k - C_{k-1}) \frac{e^{u_i - \lambda(y) p_i}}{\sum_{j \in k} e^{u_j - \lambda(y) p_j}}$$

4.5 $\lambda(y)$ and relation to the single-category discrete choice model

Up until now we have defined $\lambda(y)$ implicitly: it is the derivative of maximized utility with respect to income. In the language of our hierarchical model, it is the additional utility per dollar spent on the next good in the consumer's hierarchy.

Note that, $\lambda(y)$ can be easily solved given (9) and the consumer's budget constraint. In particular, define total expenditure as $\chi(\lambda; u, p)$. Then, from the budget constraint we have

$$\chi = \sum_i q_i(\lambda; u, p) p_i$$

since expenditure must be equal to income,

$$y = \chi(\lambda; u, p)$$

Which we can invert to get an expression for $\lambda(y)$:

$$\lambda(y) = \chi^{-1}(y; u, p) \tag{11}$$

In other words, λ is simply the value of the marginal utility of money which equates the budget constraint.

In some sense, the model presented here is a macroeconomic extension of the traditional discrete-choice model. It extends the reasoning from consumption in a single market to consumption of all goods, and (11) is the necessary modification for this extension. Consider how this model differs from the usual single-industry discrete choice model (e.g. Nevo 2001 or Berry et al. 1995). In the latter, λ either does not depend on y or else depends in a simple way, (e.g. price enters as $\alpha \log(y - p_i)$). Importantly, however, it does not depend, or depends in a limited way, on the u and p parameters of the modeled products.

Partly, this is due to the fact that λ represents the value of the outside option – the additional utility per dollar spent on goods *outside* of the modeled industry. Evidently, since here we are modeling all industries, the "outside good" of any one category is another modeled category and therefore depends on the chosen u and p values.

However, in our model, $\lambda(y)$ depends also on the u and p values *within* a given category – note that (11) depends on the entire vector of u and p. The reason is that $\lambda(y)$ depends on the consumer's hierarchy, which in turn depends on all u and p values. Imagine, for example, that the prices of all cars fall by 50%. One effect will be that cars will be given higher priority in the consumer's hierarchy – consumers will purchase cars at a lower income. However, this change in the consumer's hierarchy will also have direct effects on $\lambda(y)$. For one, consumers with lower incomes are now receiving a larger utility per dollar spent.[18] Meanwhile, any

[18]This is evident from a revealed preference argument: households will only switch consumption following a price decrease if the change leads to higher utility per dollar spent.

consumer already consuming a car receives a pure income effect: it now costs them less to buy the same bundle of goods, and thus for a given level of income, they are able to move further up their hierarchy to lower priority goods. Since λ declines the further a consumer moves up their hierarchy, this will tend to depress $\lambda(y)$ at higher levels of income. See A.1 and A.2 in the appendix for simple numerical examples.

Note that a similar argument can be employed for a change in the average u of cars. Here too, the consumer's entire hierarchy is changed, and thus so too is $\lambda(y)$.

Of course, the extent to which the feedback of u and p values on $\lambda(y)$ is important depends on the size of our modeled industry (or industries) relative to consumers' total purchases. Although for a single small industry it may be valid to treat λ as exogenous, this is not the case when dealing with the macroeconomy.

4.6 Firms

Firms are single-good firms, although their good may be purchased in multiple categories. As shown in Section 2, price discrimination is not a significant factor in differences in average markups paid across income groups. Therefore I assume that firms cannot price discriminate and set a single price for their product. Each firm's product has a utility parameter u_i and a constant cost parameter ϕ_i.

Firms choose a price to maximize their profits, subject to demand, given by (9) and (10). In other words, firm i solves:

$$\max_{p_i} Q_i(p_i - \phi_i)$$

$$\text{s.t. } Q_i = N(C_k - C_{k-1}) \int\int \prod_{j \neq i \in k} G\big(u_i - u_j - \lambda(y)(p_i - p_j) + \varepsilon\big) g\big(\varepsilon\big) d\varepsilon \quad dF(y)$$

$$\lambda(y) = \zeta^{-1}(y; u, p)$$

The firm's choice depends explicitly on the prices of other firms of the same type. Not surprisingly, goods which fulfil the same need are in direct competition for a consumer's purchase. Thus demand for good i depends on the price of good j, where i and j are of the same type.

However, note that demand for product i also depends implicitly on the prices of goods of *different* types through their effect on $\lambda(y)$. Thinking about the consumer's hierarchy, this is not surprising. A decrease in the price of a good of type k can lead a consumer to delay trading up to the next good in their hierarchy of type m. For example, a drop in the price of a Mercedes may increase the probability that certain consumers purchase this car, but this will mean switching consumption away from other goods – whether they are cars or not! Some consumers may choose to trade up to a Mercedes at lower levels of income, and forego trading up to a larger house, for example. In this sense, all goods are in competition one with the other; competition is not limited to goods of the same category.

Firms set prices optimally, which results in equilibrium markups given by eq. (12):

$$\mu_i = \frac{p_i}{\phi_i} = \frac{\eta_i}{\eta_i - 1} \tag{12}$$

where $\eta_i = \int \eta_i(y) \frac{q_i(y)}{Q_i} f(y) \quad dy$.

5 Experiment

This section calibrates the model from Section 4 to match facts about consumption in 2006-2009, and then markups in 2016. The model is calibrated using the empirical distribution of income in 2016. Then, the calibrated model is shocked by changing the distribution of income to that prevailing in 1983, keeping the median fixed. In this way, I use the model to examine the impact of a change in income inequality on the level and distribution of markups.

Note that I abstract from changes in the median income between these two periods. However, to include this income growth would require a richer model. For example, we would have to make assumptions about the source of growth. Note that growth is not simply a movement up a fixed hierarchy. Put another way, the consumption of the poor today is not simply the consumption of the rich a century ago. Rather, consumers hierarchies are changing. This is true not simply because of of decreases in costs for a fixed set of goods, but more importantly because of the introduction of new goods. Income growth is not just that all individuals can now afford a Model T, but more importantly that the line of cars available to consumers have increased in quality.

Thus, the experiment here is to assume that the utility and cost relationships between goods of differing relative qualities are unchanged between these periods. The only exogenous change, then, is the increase in income inequality.

5.1 Model Assumptions

For simplicity, I assume there is a countably infinite number of subcategories belonging to a single category of goods. I normalize the size of the category, $C = 1$. There are n firms in the model. Firms are indexed by i. There is an outside good with utility and price $u_0 = p_0 = 0$.

In a sense, the task of calibration is to pick the values of u_i and ϕ_i for each firm such that, in equilibrium, the resulting distribution of markups and of consumption choices is able to match moments in the data.

In the calibration, I allow for an indefinite number of firms, n, thus making the total number of values to be chosen $2n$. To simplify and reduce the number of chosen parameters, then, I make assumptions about the distribution of ϕ_i and the relationship of u_i and ϕ_i.

In the baseline calibration, I assume that marginal costs are given by:

$$\phi_i = \begin{cases} \frac{\sqrt{((h-l)(m-l)}i+l}{\mu_i(\mathbf{c},\mathbf{u})}, & \text{for } i \leq (m-l)/(h-l) \\ \frac{h-\sqrt{(1-i)(h-l)(h-m)}}{\mu_i(\mathbf{c},\mathbf{u})}, & \text{else} \end{cases}$$

where $\mu_i(\mathbf{u},\mathbf{c})$ is the *equilibrium* markup for good i, given the vectors of all utilities and costs.[19] The result is simply that prices follow a triangular distribution in the baseline equilibrium. The values of h and l set the maximum prices in the baseline, while m controls

[19]Note that in equilibrium, for vectors $\mathbf{u}$ and $\mathbf{c}$ there is a unique vector of prices $\mathbf{p}$. Consequently, it is equivalent to choose $\mathbf{u}$ and $\mathbf{c}$ and to solve for equilibrium prices or to choose $\mathbf{u}$ and $\mathbf{p}$ and back out the cost vector $\mathbf{c}$. For ease of calibration, I do the latter.

the relative mass of luxury and basic firms.

The value of u_i is set to be an increasing function of ϕ_i. In particular, I assume that

$$\frac{u_i - u_{i-1}}{\phi_i \mu_i - \phi_{i-1}\mu_{i-1}} = \left(\zeta_h - \frac{\zeta_h - \zeta_l}{n} i\right)(1-i)^\sigma \tag{13}$$

The values of ζ_h, ζ_l, and σ allow me to control the shape of the $\lambda(y)$ function. Notice that the values of eq. (13) would be the values of λ at each step in the hierarchy of an individual with $\varepsilon_i = 0 \ \ \forall i$. The values of ζ_h and ζ_l control the maximum and minimum values of λ for this individual, while σ controls the curvature. Note that in this way, σ performs a similar function to the curvature parameter for CRRA preferences.

For the experiment, I set the variance of ε_i to be proportional to u_i. That is, the distribution of ε is given by:

$$\varepsilon_i \sim \text{Gumbel}(0, \gamma u_i) \tag{14}$$

I do this for two reasons. The first reason is logical. We would expect that the utility for a high utility good, say a car, varies much more across individuals than does the utility for a low utility good, say a pencil. The second reason is technical. In a traditional logit framework, with a constant variance of ε, price elasticities are given by:

$$\eta_i(y) = (1 - q_i)\lambda(y)p_i$$

Thus, although price elasticities are proportional to λ, which falls with income, they are also increasing in price. To the extent that rich consumers purchase higher cost, higher price products, one can easily end up in a situation in which elasticities of luxury products are *higher* than those of basic products. This can lead to the counterfactual result that average markups paid are decreasing in income. Employing the assumption in eq. (14) helps to mitigate this issue (see Bhat 1995).

Thus, demand for good i by households in income group y is given by:

$$q_i(y) = \int \prod_{j \neq i} e^{-e^{-\left(u_i - u_j - \lambda(y)(p_i - p_j) + \varepsilon \gamma u_i\right)/\gamma u_j}} e^{-e^{-\varepsilon}} e^{-\varepsilon} d\varepsilon \tag{15}$$

which is easily solvable by quadrature.

5.2 Empirical Moments

Recall eq. (5):

$$d\eta_i = \int (\eta_i(y) - \eta_i) \frac{q_i(y)}{Q_i} \; df(y) \; dy$$

The effects of the changing distribution of income depend on price elasticities – and therefore markups – depends on relative differences in elasticities across income groups, and on how consumption for different varieties of goods varies with income.

To match the latter, I rely in part on a measure taken from the Nielsen Homescan Database: the Euclidean distance measured between the bundles purchased by different income groups. The Euclidean distance for a given category is calculated as:

$$\text{Euc}_c(y) = \left[\sum_i (s_i(y) - s_i(12000))^2 \right]^{\frac{1}{2}} \tag{16}$$

where $s_i(y)$ is the market share of good i among consumers in income group y. It is a measure of how different the bundles purchased by a given income group are relative to the lowest income group, those making between \$12,000-\$15,000. This measure helps to capture the rate at which consumers trade up from basic products to luxury products as their incomes increase. I use the years 2006-2009 given the higher income top-coding and take the expenditure-weighted average across categories. This statistic is shown in Figure 7.

To match the number and distribution of firms, I rely on the Herfindahl-Hirschman Index (HHI) measured for each income group. For a given category, it is calculated as:

$$\text{HHI}_c(y) = \sum_i s_i(y)^2 \tag{17}$$

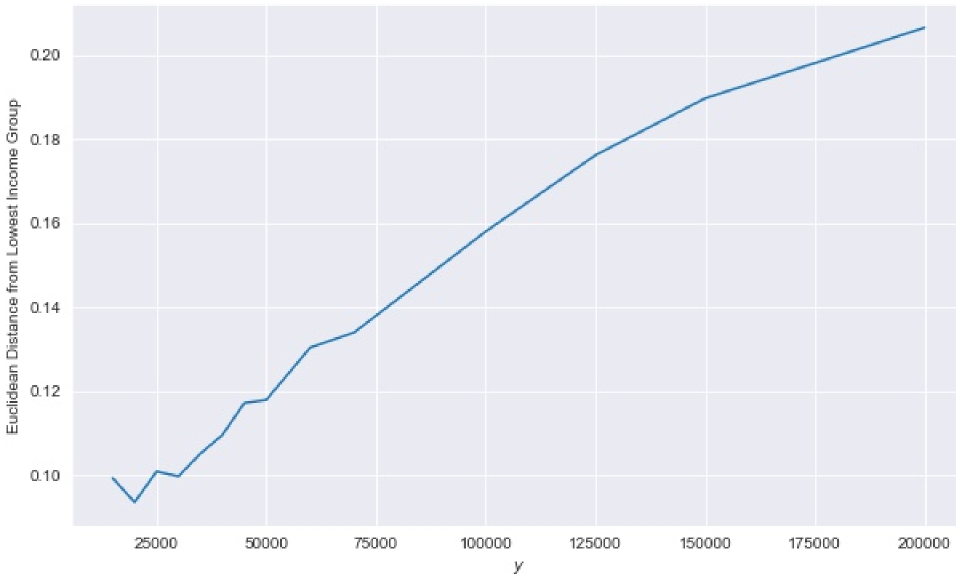


Figure 7: Euclidean distance from lowest income group

I show the sales weighted average HHI in Figure 8, again using the years 2006-2009. Interestingly, the HHI is falling in income. This can partly be explained by the fact that rich consumers are likely to purchase basic products as well as luxury products, while poor consumers buy mainly basic products.

Given the shape of Engel curves, values of relative elasticities are determined in equilibrium to match moments about the markup distribution. In particular, I match the values of the sales-weighted average sales-weighted variance of markups in 2016, as reported by De Loecker et al. (2020).

I use the empirical income distribution from the 2016 Survey of Consumer finances.

Table 5 summarizes the targeted moments for the baseline equilibrium.

5.3 Calibration Results

Table 6 displays the resulting parameter values for the calibration. Figure 9 displays the calibrated HHI for the model, while Figure 10 displays the calibrated Euclidean distance.

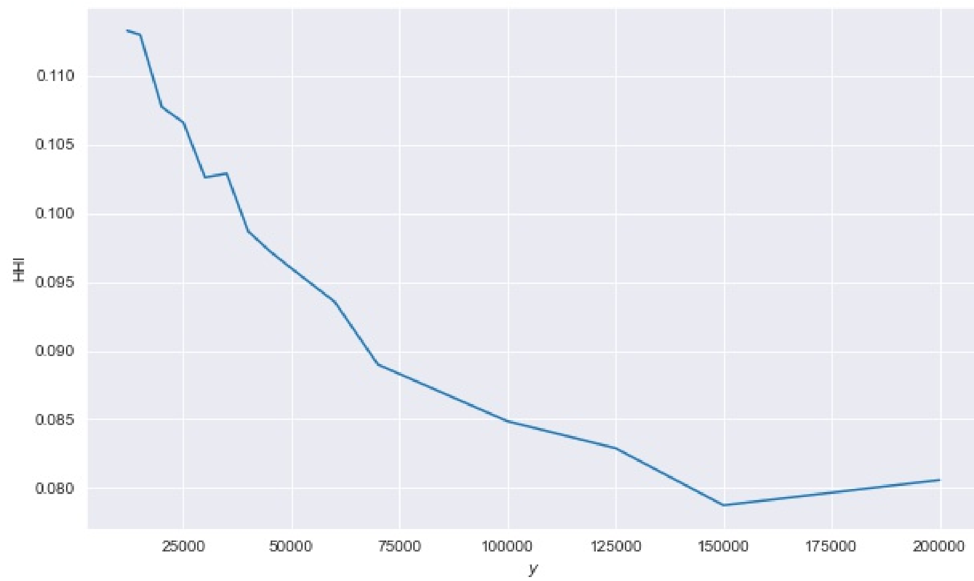


Figure 8: The Herfindahl-Hirschman Index measured for different income groups

Description	Target Value
HHI(y)	Figure 8
Euc(y)	Figure 7
$\mathbb{E}(\mu)$	1.6
var(μ)	0.3

Table 5: Calibration Targets

The model matches well the sales-weighted average and variance of markups. Moreover, just as in the data, Figure 11 markups are an increasing function of incomes.

5.4 Experiment Results

With calibrated model in hand, I ask how a change in the level of income inequality would impact the distribution of markups. Therefore, I change the distribution of income to that prevailing in 1983, while keeping the median the same. I once again use the empirical

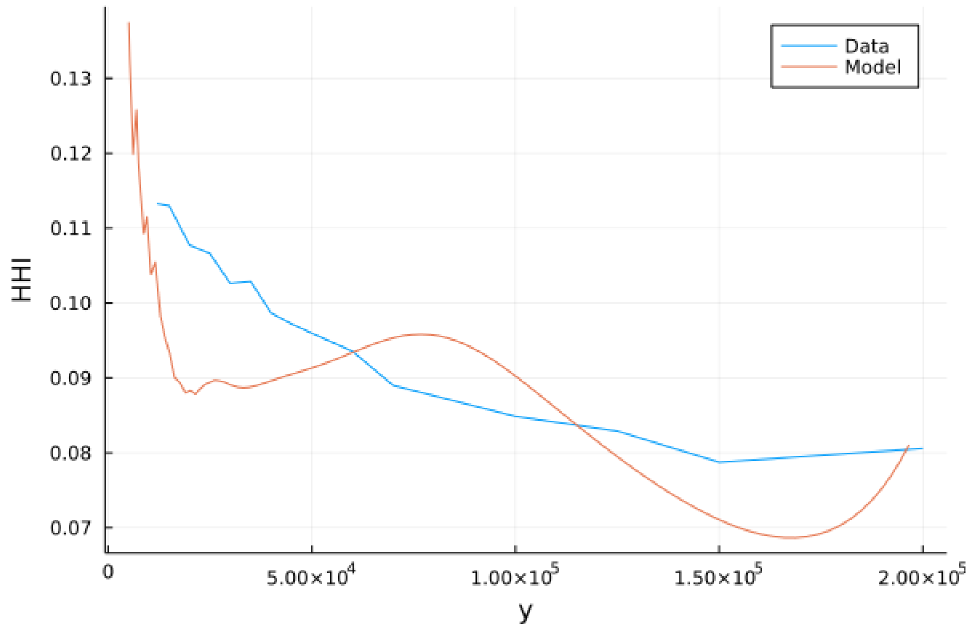


Figure 9: Calibrated HHI values

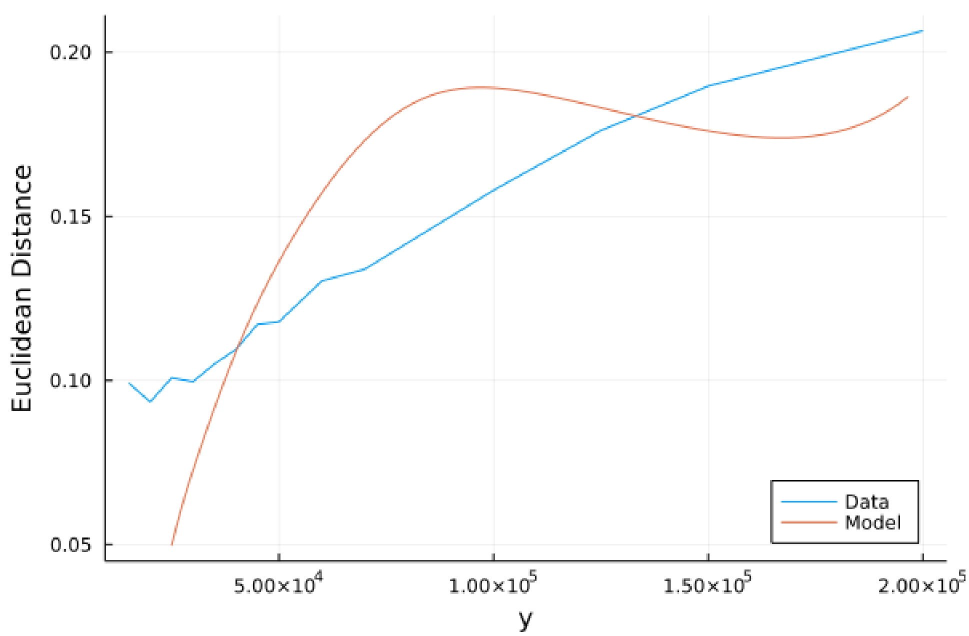


Figure 10: Calibrated Euclidean distance

Variable	Description	Value
N	Number of Firms	33
l	Lowest price	0.01
h	Highest price	2.5
m	Mode of prices	0.02
σ	Curvature of $\lambda(y)$	2
ζ_h	Height of $\lambda(y)$	2.1
ζ_l	Base of $\lambda(y)$	2
γ	Variance shifter for ε	0.01
$\mathbb{E}(\mu)$	Sales-weighted average markup	1.60
$\text{var}(\mu)$	Sales-weighted variance of markups	0.31

Table 6: Calibration Results

income distribution from the Survey of Consumer Finances. Figure 12 shows the two income distributions. The median-adjusted distribution of income in 1983 has greater mass around the middle, and less mass in the two tails.

Table 7 shows the results of the experiment for the sales-weighted average and sales-weighted variance of markups. Both statistics move in the direction they do empirically. However, the model generates a large change in the variance of markups (over 100% of the empirical change!) and a smaller change in the average markup (15%).

	Model		Data		
	2016	1980	2016	1980	$\Delta Model/\Delta Data$
$\mathbb{E}(\mu)$	1.6	1.54	1.6	1.21	15%
$\text{var}(\mu)$	0.31	0.09	0.3	0.1	110%

Table 7: Experiment Results

To lend intuition, first consider Figure 13, which shows the percentage change in markups generated by the model between 1983 and 2016, plotted against their initial values. Clearly, the decline in variance is coming from low markup firms increasing their markups, while high markup firms are decreasing their markups. Why do high-markup luxury firms behave differently than low-markup basic firms in response to the same income distribution shock?

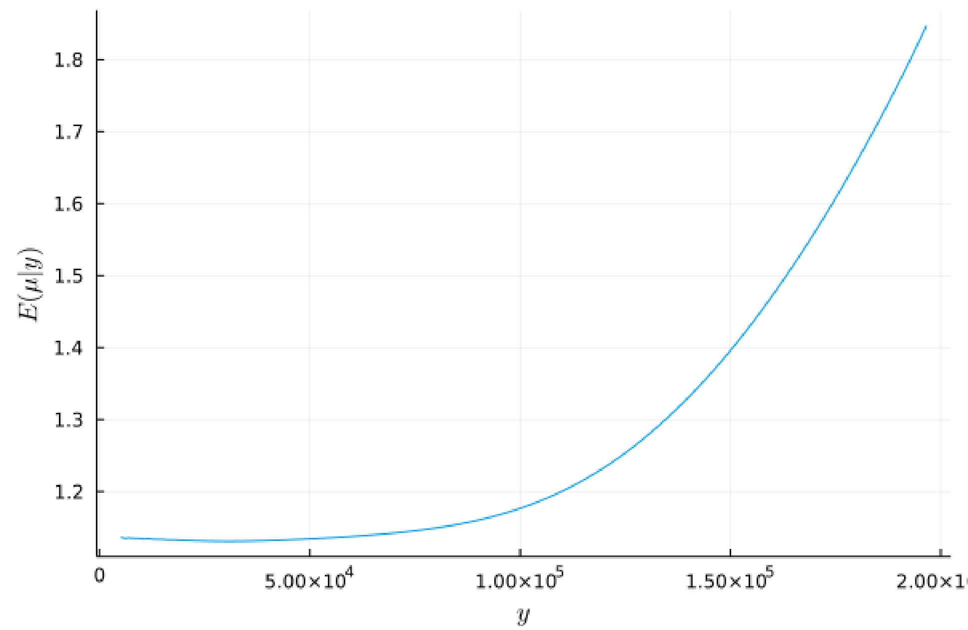


Figure 11: The relationship between markups and income in the calibrated equilibrium

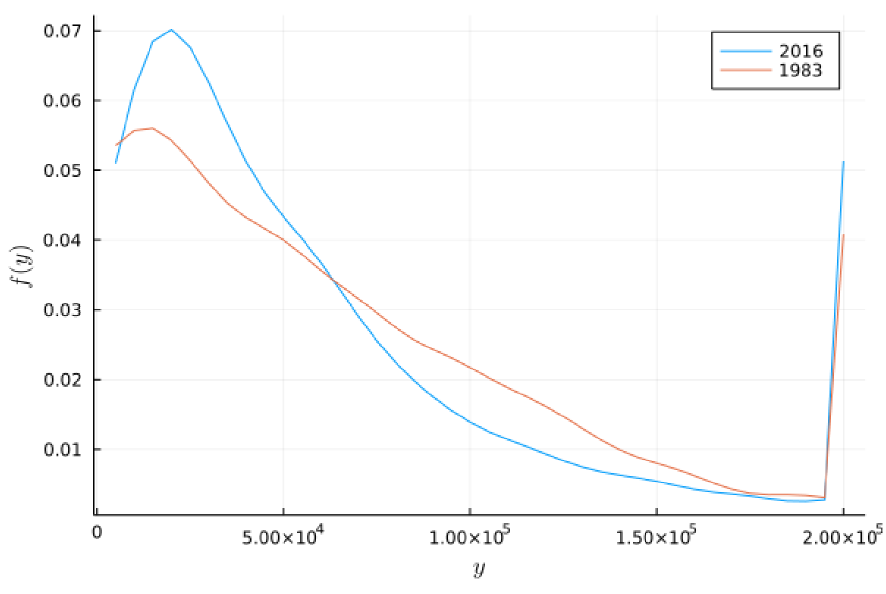


Figure 12: The distribution of income in 1983 and 2016

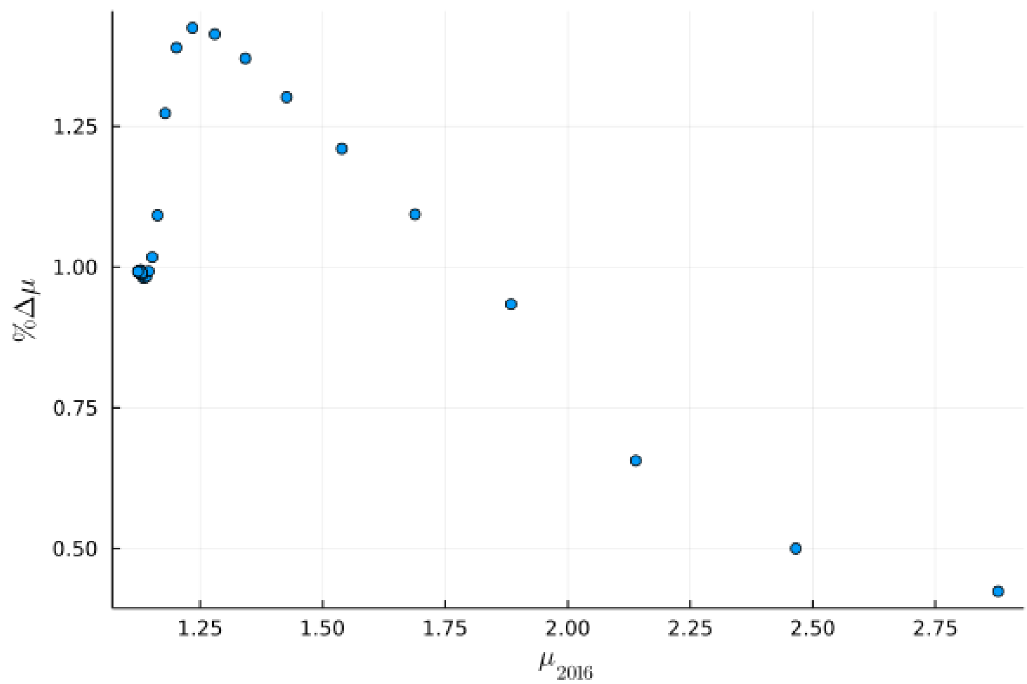


Figure 13: Model-generated changes in markups between 1983 and 2016

Recall once more eq. (5):

$$d\eta_i = \int (\eta_i(y) - \eta_i)\frac{q_i(y)}{Q_i} \quad df(y) \quad dy$$

Changes in the price elasticity vary depending on the covariance of $(\eta_i(y) - \eta_i)\frac{q_i(y)}{Q_i}$ and the change in the income distribution. To consider how this plays out, consider two firms in the model: one basic firm and one luxury firm. The Engel curves for both firms are given in Figure 14. In the baseline equilibrium, the basic firm has a markup of 1.12 while the luxury firm has a markup of 1.54. Next consider how the price elasticities of each firm vary with income. Figure 15 plots $\eta_i(y)$ for both goods. Because η tends to fall in income, both curves are downward sloping.

Note that because a greater share of the luxury product's market comes from rich consumers, its overall elasticity is lower, which explains its higher markup.[20]. Figure 14 and

[20] Recall that $\eta_i = \int \eta_i(y)\frac{q_i(y)}{Q_i} f(y) \quad dy$

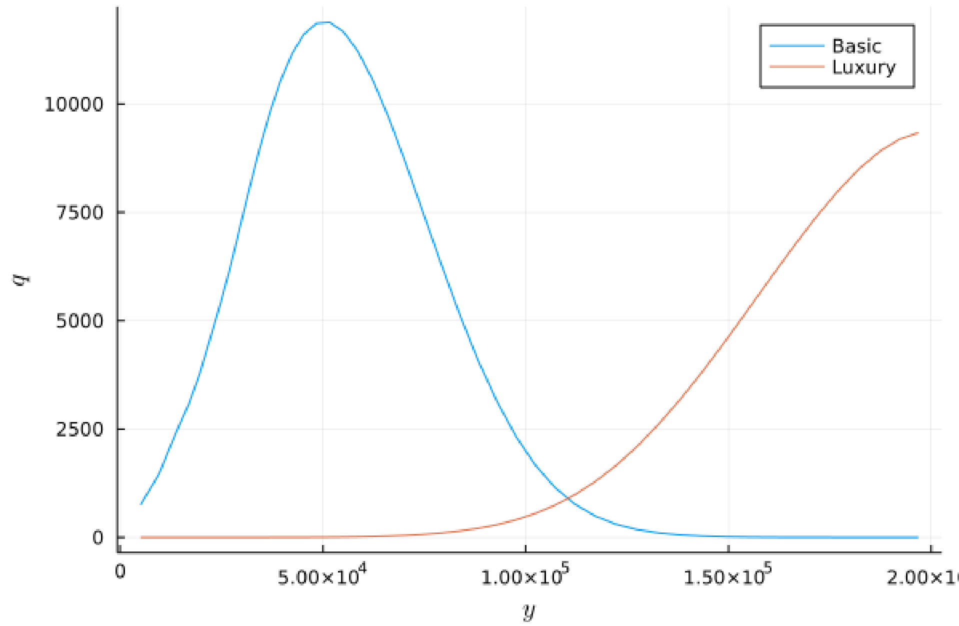


Figure 14: The Engel curves for a basic and a luxury product in the model

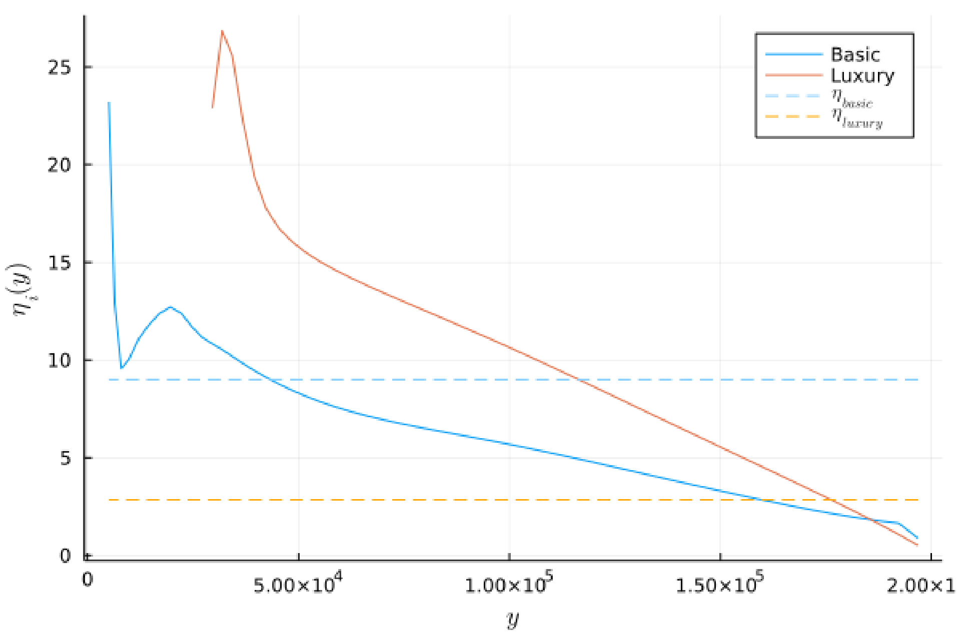


Figure 15: The relationship of price elasticities to income for two goods

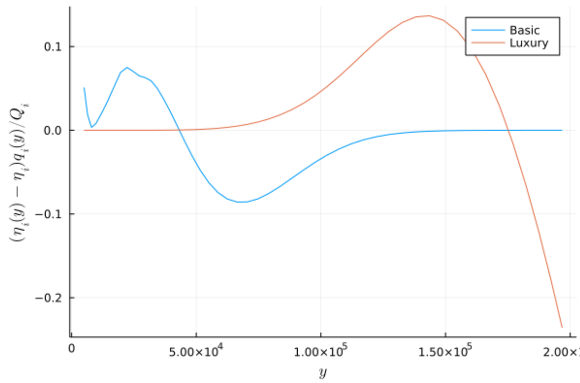


Figure 16: Equation (5)

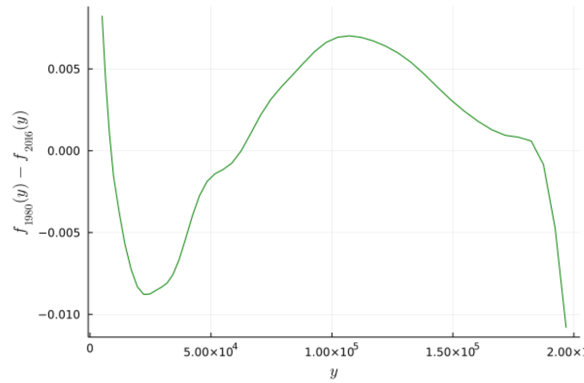


Figure 17: Change in the distribution of y

Figure 15 give us both of the ingredients necessary to construct eq. (5), which is shown in Figure 16 displayed next to the change in the distribution of income.

Consider first the luxury good. When the distribution of income changes between 2016 and 1983, there are fewer very rich consumers. These consumers are a large part of the luxury firm's customer base. They also have an elasticity lower than the overall price elasticity for the luxury product. Removing these very rich consumers serves to increase the overall elasticity of the luxury firm. There is also a growing mass of median income consumers, who make up a sizeable portion of the luxury firm's customer base. These median income consumers have elasticities higher than the overall price elasticity for the luxury product, and therefore increasing the number of such customers also serves to increase the overall elasticity. Finally, there is also a fall in the mass of very poor consumers. However, poor consumers make up such a small part of the luxury product's customer base, that this has no first-order effect on its elasticity. The overall result then is to take away rich consumers, who have a relatively low elasticity of demand, and replace them with median income consumers, who have a relatively high price elasticity of demand. The net result is an increase in the luxury firm's elasticity, and therefore a fall in its markup.

The inverse story is true for the basic product. The change in the distribution of income from 2016 to 1983 decreases the mass of poor households who have a relatively high elasticity of demand for the basic product, and increases the mass of median income consumers, who have a relatively low elasticity of demand for the basic product. There is also a decrease in the mass of rich consumers, but they make up negligible share of the basic firm's overall customer base. The overall result is a fall in the basic product's price elasticity, and an increase in its markup.

6 Conclusion

Previous studies on changing markups have focused on the increase in the average, and have relied on changes in competition and market power as explanations. This paper shows that changes in the distribution of income may also play a role in changing markups, and particularly may offer some explaination for changes in their variance.

I find that as consumers grow richer, they do not necessarily increase the quantity of physical units consumed, but rather change their consumption bundle to different higher-priced, higher-markup products. I show that this consumption process is well explained by preferences which are satiable.

Fitting this model to the data, I find that in response to an increase in income inequality, basic, low-markup firms will lower their markups to attract the larger mass of low-income consumers. Conversely, the lower mass of median-income consumers means that luxury, high-markup firms will increase their markups. The calibrated model is able to explain 15% of the change in the average markup since 1983, and around 100% of the change in variance.

This model has implications for the analysis of rising income inequality. First it suggests that increasing inequality may beget further increases in income inequality. As income inequality increases, it leads to higher average markups, and higher average markups tend to flow towards high-income earners. Changes in income inequality due to other factors therefore may have a multiplier effect.

Conversely, the model also suggests that changes in markups may serve to dampen the *real* effects of rising income inequality. If rising inequality leads to falling prices for basic products, and rising prices for luxury products, then in real terms, inequality has increased by less.

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A Examples

A.1 Effect of a price decrease on $\lambda(y)$ with two categories

Imagine a single individual consuming goods in two categories, each with two goods, and the following prices and utilities:

	Cars		Food	
	Hatchback	Lamborghini	Lentils	Steak
U	1	2	1	2
p	1	3	1.5	4.5

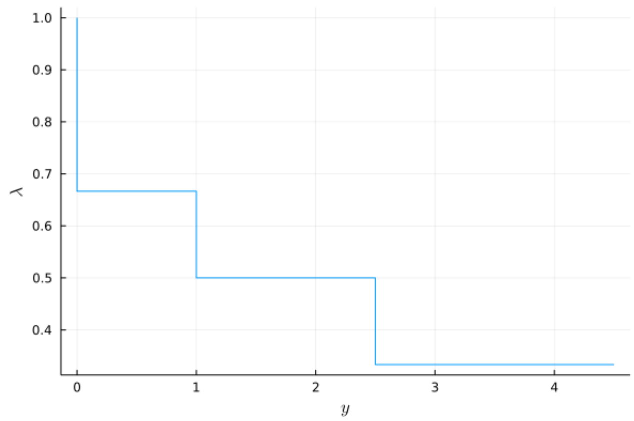
Table 8: Two categories, two goods

The consumer has the following hierarchy over purchases:

	Good	λ	y
1.	Hatchback	1	1
2.	Lentils	2/3	2.5
3.	Lamborghini (trade-up)	1/2	4.5
4.	Steak (trade-up)	1/3	7.5

Table 9: Ranking of bundles

Which yields the following for $\lambda(y)$



If the price of both cars are halved, we instead get the following hierarchy:

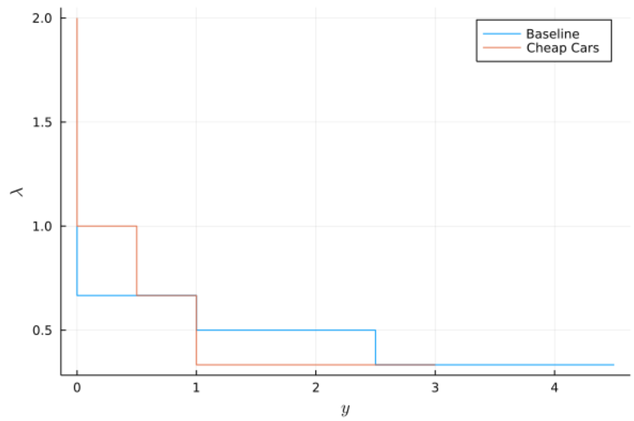
	Good	λ	y
1.	Hatchback	2	0.5
2.	Lamborghini (trade-up)	1	1
3.	Lentils	2/3	3
4.	Steak (trade-up)	1/3	6

Table 10: Ranking of bundles

The consumer receives a higher utility per dollar spent on cars, and therefore at lower levels of income, $\lambda(y)$ is higher. Conversely, due to the income effect, the consumer can now consume both food items at lower levels of income. Given that the utility per dollar spent on these goods hasn't changed, this means that $\lambda(y)$ is lower at higher levels of income. Comparing the new and old $\lambda(y)$ graphically:

A.2 Effect of a price decrease on $\lambda(y)$ with a continuum of categories and an infinite number of consumers

Imagine the same goods in Table 8, but now imagine they exist over a continuum of categories. That is there are a countably infinite number of identical categories of type "Cars" on the interval $[0, 1]$ and a countably infinite number of identical categories of type "Food"



on the interval $[1, 2]$.

Imagine that utility for consumer n of consuming good i in category c is given by $U_{icn} = u_{ic} + \varepsilon_{icn}$ where ε is i.i.d. extreme value type I.

Then, from (9) we have that the demand for good i of type k by a consumer with income y is:

$$q_i(y) = \frac{e^{u_i - \lambda(y)p_i}}{\sum_{j \in k} e^{u_j - \lambda(y)p_j}}$$

where $k \in \{\text{Cars}\ ,\ \text{Food}\}$ and each type also contains an outside good with $u_0 = p_0 = 0$.

Then using (11), we can solve for $\lambda(y)$ in the baseline case, and for the case where the price of cars drops by 50%. This yields the following: