Background + Physical Model: Consider two types of unstable nuclei, A and B. Type A decay with a time constant of τ_A, producing a type B nucleus after each decay. Type B nuclei are *also* unstable, so decay with their own time constant of τ_B, into products that we won't try to track. This system can then be described with the differential equations

$$\frac{dN_A}{dt} = -\frac{N_A}{\tau_A}, \tag{1}$$

$$\frac{dN_B}{dt} = \frac{N_A}{\tau_A} - \frac{N_B}{\tau_B}, \tag{2}$$

Given an appropriate set of initial conditions, one could then calculate the absolute number of each nuclei as $N_A(t)$ and $N_B(t)$. We will also assume that nuclei of type A are naturally occurring, but nuclei of type B are not: that is, type B nuclei only appear as the product of a prior decay of a type A nucleus, so $N_B(t = 0) = 0$.

Motivation + Goal: Now imagine that you have been asked to determine the age of an object – say, a meteorite– by measuring the amount of type A and B nuclei within the object. Assume that you know the time constants of the two species, but that you can only measure the ratio of the two types of nuclei (i.e., $\frac{N_B}{N_A}$), not their individual populations (i.e., you cannot measure $N_A(t)$ or $N_B(t)$ alone). **How well can you measure the age of this object if your measurement of $\frac{N_B(t)}{N_A(t)}$ has an accuracy of 0.5%? Does the precision of your age measurement depend on the age of the object and/or the ratio of the time constants of the two species ($\gamma = \frac{\tau_A}{\tau_B}$)?**

You must solve this problem numerically, not analytically; the analytic solutions are

$$\frac{N_A(t)}{N_A(0)} = e^{-T}, \tag{3}$$

$$\frac{N_B(t)}{N_A(0)} = \begin{cases} \frac{N_B(0)}{N_A(0)}\, e^{-\gamma T} + \frac{e^{-T} - e^{-\gamma T}}{\gamma - 1} & \gamma \neq 1, \\ \frac{N_B(0)}{N_A(0)}\, e^{-T} + T e^{-T} & \gamma = 1 \end{cases} \tag{4}$$

where $\gamma = \tau_A/\tau_B$ and T is a scaled time coordinate: $T = t/\tau_A$.

Accuracy Check: For future assignments, you will need to develop tests that do not rely on the availability of an analytic solution to demonstrate that your numerical results are sound. For this problem, however, you can use the analytic solutions to verify that your numerical algorithm is working, neglecting the first term in $\frac{N_B(t)}{N_A(0)}$ since we have specified that $N_B(0) = 0$.

Deliverable: Since this is project zero, all you will need to deliver for this project is an updated copy of the Project Zero template notebook, edited to include:

- accurate and correct calculate_derivatives() and update_populations() functions;

- updated versions of the 'ProjectZero_NB_NAvsGamma0.1.png' and 'ProjectZero_NB_NAvsGamma5.png' plots showing the agreement between the analytic and numerical solutions for γ=0.1 and γ=5.

The updated notebook file, and the two plots, should be committed back to your public GitHub repo by 11pm on Sunday, April 10th to receive full credit for this out-of-class activity. You will also receive in-class credit on Wednesday for committing an updated version of the notebook to your repo; it need not be correct, you will earn the credit simply by demonstrating the work done and the ability to save/commit files to your public repo.