Digital Communications (EEEN40060): Assignment 2

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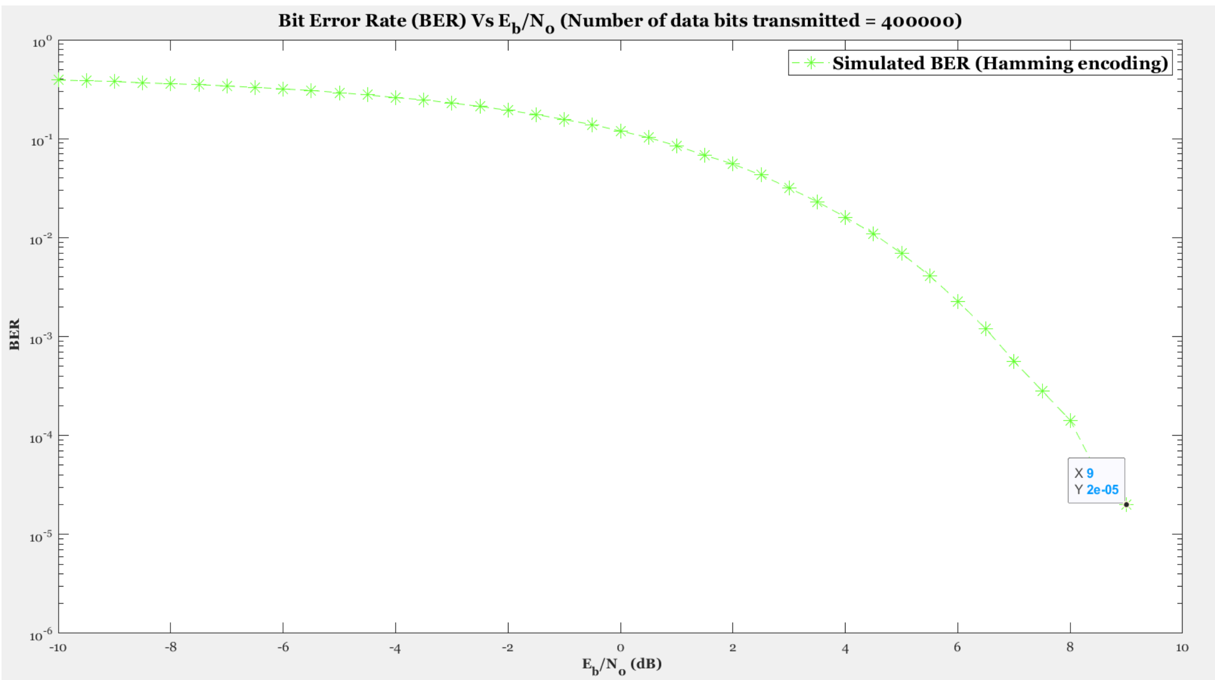
(Q.1) The graph below depicts the simulated bit error rate (BER) for the system under consideration - a diagram of which is included at the end of this report.



We can also plot the simulated BER against the theoretical expression. The theoretical expression for the probability of error of a system using binary phase-shift keying (BPSK) in a AWGN (Additive white Gaussian noise) channel is:

$$\rightarrow P_{error} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

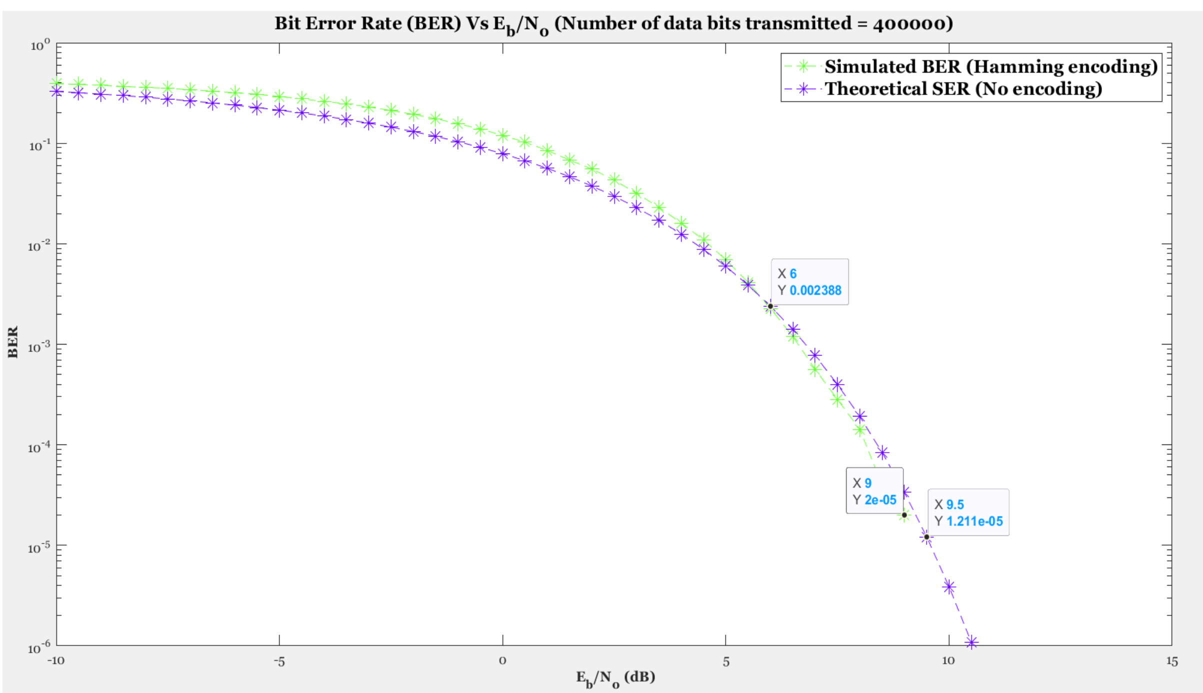
Where Q denotes the *Q-function*. Before plotting and comparing both curves, it is worth discussing what will likely be seen. The *minimum distance* of the Hamming code is 3. We can use this value to deduce the maximum number of errors that a Hamming code can correct as being:

$$\rightarrow N_{corrections} = \frac{d_{min} - 1}{2} = \frac{3 - 1}{2} = 1$$

This means that our Hamming code can correct, at most, one bit error in each transmitted codeword. At a very low signal-to-noise ratio (SNR), there will be many bit flips (errors), and in this case the Hamming code will be insufficient to completely correct each received word, as it can only correct, at most, one bit flip in each transmitted codeword. As a result, the Hamming code will likely offer no advantage at low SNR values when compared to the non-encoded BPSK system. As well as that, we require *more* energy when transmitting every group of four bits with the Hamming-encoded system. This is because we have to also transmit three parity bits alongside each group of four information or data bits. This means that the Hamming-encoded system will have a worse BER-vs-SNR characteristic at low SNR values.

At higher SNR values, however, there will be less and less bit flips (errors). When the average number of bit flips becomes one, the Hamming code will be able to correct the errors, thereby further reducing the BER. The non-encoded BPSK system will have no such capability, although there will be few bit flips at higher and higher SNR values, meaning the BER will still be low for this system.

(Q.2) Shown below is the graph with both curves plotted.

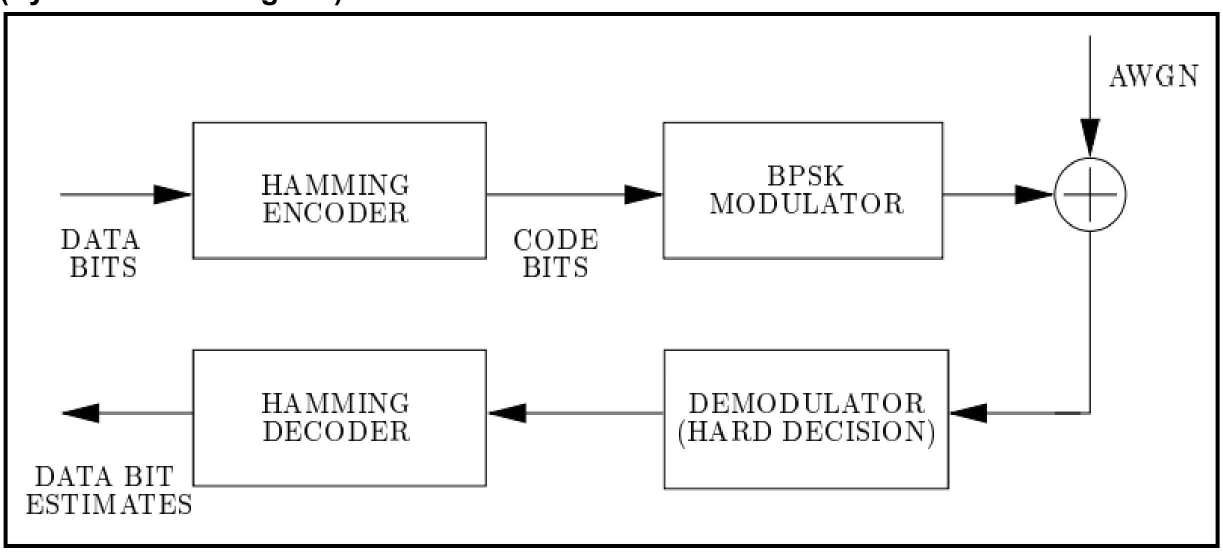


(Q.3) As can hopefully be seen in the graph above, the crossover point between the two curves occurs at approximately 6 dB.

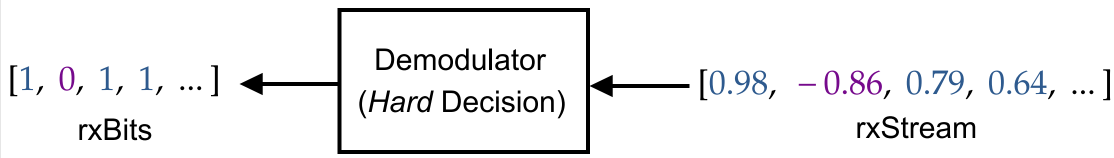
(Q.4) We can also get an approximate value for the coding gain that the Hamming-encoded system yields at a BER of 10^{-5} or 1 in 100000. From the graph, it looks to be approximately 0.5 dB. This means that the same BER can be achieved with a lower E_b/N_0 value when Hamming encoding is used. If one considers the Noise power spectral density (N_0) to be fixed, then a lower E_b/N_0 value would imply a lower E_b value. This might suggest that towards the right of the crossover point, the Hamming-encoded system requires less energy per bit in order to achieve the same bit error rate. This is due to its error-correcting capabilities. Another way to look at it might be that, for a given BER, the Hamming-encoded system requires a lower E_b/N_0 value, and can therefore deal with slightly harsher channel conditions.

Below is shown the system block diagram, as well as some illustrations that I did.

(System Block Diagram)



The demodulator uses a threshold of 0. If a received symbol is greater than 0 (to the right of 0) it becomes a 1. If a received symbol is less than 0 (to the left of 0) it becomes a 0.



$$rxBits = [\ \boxed{1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0}\ \boxed{0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1}\ \ldots]$$

$$rxBlock = \begin{bmatrix} 1 & 0 & . & . \\ 0 & 0 & . & . \\ 1 & 1 & . & . \\ 1 & 0 & . & . \\ 0 & 1 & . & . \\ 1 & 1 & . & . \\ 0 & 1 & . & . \end{bmatrix}$$

2nd received codeword

$$\underbrace{s}_{\textit{The Syndrome}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{received codeword 1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{\text{transpose of the } \textit{parity} \text{ check matrix } (H^T)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \longrightarrow \boxed{\text{Lookup Table (LUT)}}$$

$$\downarrow$$

$$e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Modulo 2 Addition:

$$\underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{received codeword 1}} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{error vector (pattern)}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{updated received word 1}}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{received } \textit{message} \text{ word}}$$

With systematic encoders, the message word appears either at the beginning or end of the codeword.