

**University College Dublin**  
**School of Electrical and Electronic**  
**Engineering:**

*OFDM-based Joint Radar and  
Communication System*

*By Dylan Boland*

*An interim report* submitted for the M.E. course in Electronic  
and Computer Engineering

**Supervised by: Dr Nam Tran**



**16th of January 2022**

## **Introduction and Project Specifications:**

This project investigates further the possibility of adapting the OFDM waveform - used in 4G and 5G communications - to be used for radar. With the advances that have been made in the IoT sector in recent years, it seems sensible to question if communication and radar functions can be combined into a single system. This would entail less hardware, reduce costs, and potentially allow for more efficient use of the frequency spectrum - something which is increasingly important, given the need for high data rates.

The project also aims to study the challenges in using the OFDM waveform as a radar with current IEEE communication standards. Furthermore, the project seeks to investigate different estimation algorithms that might be used to successfully adapt the OFDM communication system to perform as a radar. Parametric target estimation methods will also be studied and analysed in order to assess their feasibility with current standards.

## **Literature Review:**

Investigation of the use of OFDM waveforms for radar purposes has been undertaken before, such as in [1, 3].

In [2, 1], a matrix notation is introduced as a useful way to represent an OFDM frame mathematically:

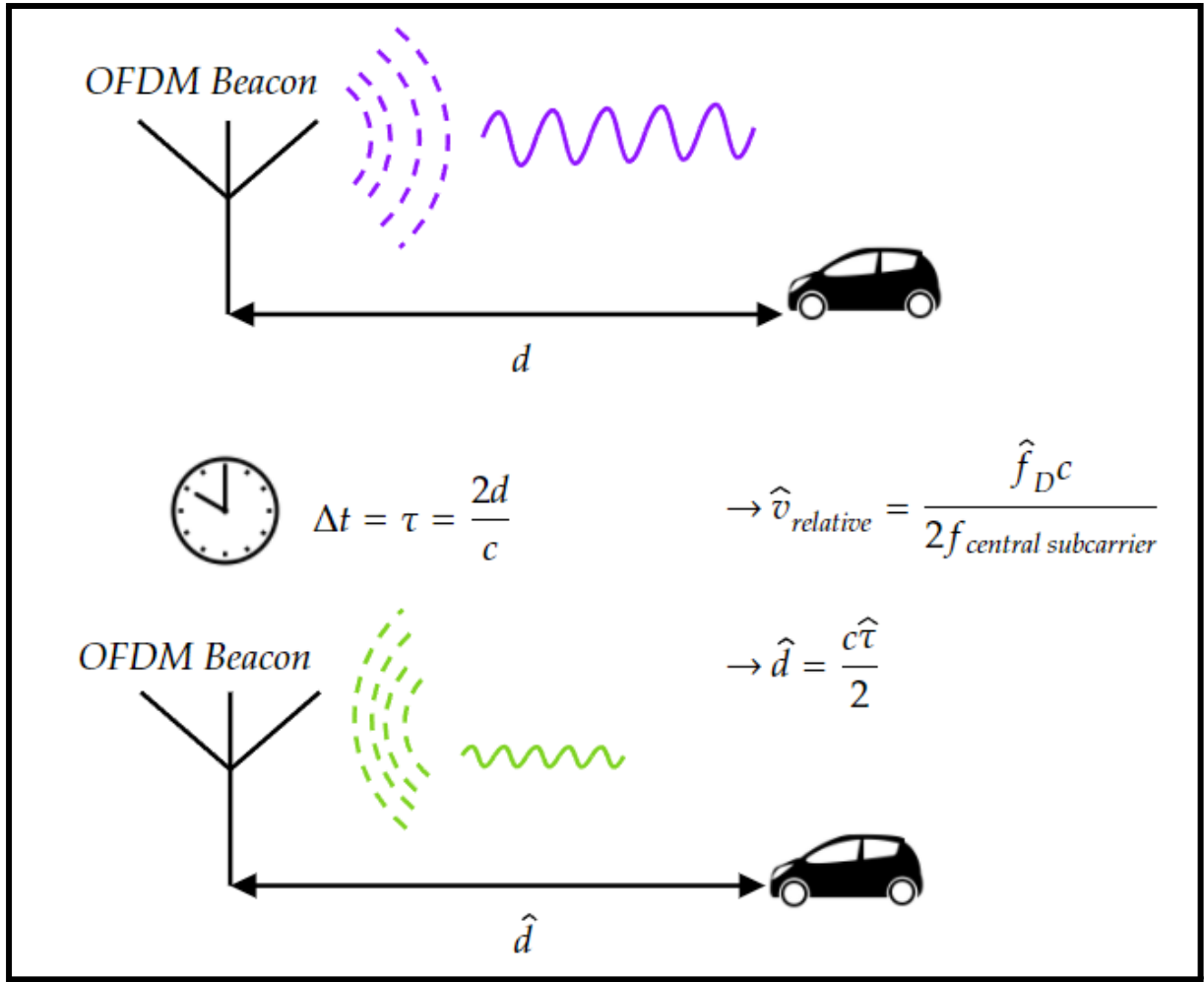
$$F_{T_x} = \begin{bmatrix} c_{0,0} & c_{0,1} & \cdot & \cdot & \cdot & c_{0,M-1} \\ c_{1,0} & c_{1,1} & \cdot & \cdot & \cdot & c_{1,M-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{N-1,0} & c_{N-1,1} & \cdot & \cdot & \cdot & c_{N-1,M-1} \end{bmatrix}$$

Each column represents an OFDM symbol, where each of the  $N$  rows correspond to a subcarrier. This is important, as it provides two avenues to further explore: (1) how does changing the number of subcarriers ( $N$ ), and the number of symbols ( $M$ ), affect the OFDM system's performance as a radar, and (2) which modulation schemes work best for the radar case, and can the symbols ( $c$ ) be adapted to enhance performance further.

In [2, 1], the matrix notation for the transmitted frame is used to express the received frame as:

$$(F_{Rx})_{k,l} = b(F_{Tx})_{k,l} \cdot e^{j2\pi T_o f_D l} \cdot e^{-j2\pi k \tau \Delta f} \cdot e^{j\varphi} + (\tilde{Z})_{k,l} \quad (1)$$

The above equation describes how each element ( $k, l$ ) of the received frame matrix is calculated in terms of each element of the transmitted frame matrix. The radar problem consists in estimating the time delay ( $\tau$ ) between the transmitted and received frame, as well as the Doppler shift ( $f_D$ ). With these two values, estimate values for the target's distance and velocity can be calculated:



In [1], the periodogram is investigated as a method of analysing the received frame in order to identify targets. Since the transmitted frame is known by the OFDM transmitter, the transmit symbols do not need to be estimated. As a result, the elements of the received frame ( $F_{Rx}$ ) can be divided by those of the transmitted frame ( $F_{Tx}$ ), giving:

$$(F)_{k,l} = \frac{(F_{Rx})_{k,l}}{(F_{Tx})_{k,l}} = b \cdot e^{j2\pi T_o f_D l} \cdot e^{-j2\pi k \tau \Delta f} \cdot e^{j\varphi} + \frac{(\tilde{Z})_{k,l}}{(F_{Tx})_{k,l}} \quad (2)$$

The equation above is valid for a single target, where there is a single time delay, and Doppler shift. Due to linearity, the received frame for the case where there are multiple targets can be expressed as the sum of the reflections from each target [1]:

$$(F)_{k,l} = \sum_{h=0}^{H-1} b_h \cdot e^{j2\pi T_o f_{D,h} l} \cdot e^{-j2\pi k \tau_h \Delta f} \cdot e^{j\varphi_h} + \frac{(\tilde{Z})_{k,l}}{(F_{Tx})_{k,l}} \quad (3)$$

This is important: each target at a different distance and travelling with a different velocity, leads to a different time delay ( $\tau$ ) and Doppler shift ( $f_D$ ). From [1], the Doppler shift appears as a modulation of each row of the transmit frame ( $F_{Tx}$ ), and the time delay causes a phase shift to each of the subcarriers. The two effects represent sinusoids in the matrix  $F$ , leading to peaks in the periodogram - the formula for which is given in [1], and shown below:

$$Per_F(n, m) = \frac{1}{NM} \left| \sum_{k=0}^{N_{per}-1} \left( \sum_{l=0}^{M_{per}-1} (F)_{k,l} e^{-j2\pi \frac{lm}{M_{per}}} \right) e^{j2\pi \frac{kn}{N_{per}}} \right|^2 \quad (4)$$

The above can be computed by means of FFT and IFFT computations, as discussed in [2, 1]. Indexes ( $n, m$ ) of potential targets can be translated into distance and velocity values, as mentioned in [2, 1], with the following equations:

$$\hat{d} = \frac{\hat{n}c}{2\Delta f N_{per}} \quad and \quad \hat{v} = \frac{\hat{m}c}{2f_c T_o M_{per}} \quad (5)$$

As mentioned in [1], the indexes  $n$  and  $m$  are chosen as follows:

$$\begin{aligned} n &= 0, 1, 2, \dots, N_{per} - 1 \\ m &= -\frac{M_{per}}{2}, \dots, \frac{M_{per}}{2} - 1 \end{aligned} \quad (6)$$

The indexes for  $m$  can be negative, so as to allow for the estimation of negative velocities, which correspond to targets moving away from the OFDM transmitter.

Another important point, as mentioned in [2, 1], are the dimensions of the periodogram. Making the dimensions of the periodogram ( $N_{per}$  and  $M_{per}$ ) bigger than those of  $F$  ( $N$  and  $M$ ) can improve the resolution with which a target's distance and velocity can be estimated. If the resolution is too low, then the radar might not be able to identify two or more targets which are located within a small distance of one another. However, increasing the periodogram dimensions too much leads to more and more computations needing to be done.

In [1], there are five assumptions made regarding the estimation process, two of which are:

1. "The CP duration is larger than the round-trip propagation time for the furthestmost target."
2. "The sub-carrier distance is at least one order of magnitude larger than the largest occurring Doppler shift."

These two assumptions are very important, and allow for the calculation of  $n_{\max}$  and  $m_{\max}$ , as alluded to in [2, p. 5]:

$$m_{\max} = D \cdot M_{\text{FFT}}$$

$$n_{\max} = G \cdot N_{\text{FFT}}$$

The quantity  $G$  is the fraction of the OFDM symbol taken up by the cyclic prefix.

Typical values for this quantity are  $1/4$  (as in the 802.11a standard), and  $1/8$ . In [2, 1], the following expression is given for the time delay associated with an index  $n$  of the matrix  $F$ :

$$\tau_n = \frac{n}{N_{\text{FFT}}\Delta f} = \frac{n}{N_{\text{FFT}}}T$$

(\*If  $N_{\text{per}} > N_{\text{FFT}}$ , then  $N_{\text{per}}$  is substituted in for  $N_{\text{FFT}}$ .)

Using this equation for the time delay, with assumption 1 above (i.e.,  $T_G > \tau_{\max}$ ), gives:

$$n_{\max} = \frac{T_G}{T}N_{\text{FFT}} = G \cdot N_{\text{FFT}}$$

The quantity  $D$  is the fraction of the subcarrier spacing that the maximum Doppler shift can occupy. A typical value for this quantity is 0.1, as mentioned in [2, p. 5].

These maximum index values help to define a search range within the periodogram. Any peaks outside the search range can be ignored, and as a result, the periodogram can be cropped to a size of  $n_{\max}(2m_{\max} + 1)$ .

The use of quadratic interpolation to reduce quantization errors in the estimates of the distance and velocity values is explored in [1], and shown to be effective. This offers an alternative to increasing the length of the zero padding in the FFT and IFFT calculations.

The ESPRIT algorithm is also investigated by Braun in [1]. The main advantages it holds over the periodogram-based approaches are:

- Its simplicity
- No target detection algorithms are needed
- It is not vulnerable to quantization errors

Nevertheless, it has some practical disadvantages when compared to the periodogram-based approaches, which are stated below:

- It requires a higher SNR than the periodogram-based methods.
- It requires prior knowledge of the number of targets present. This is not always known.
- The Doppler shift and distance values are not linked together, as with the periodogram estimation.

Since the ESPRIT algorithm requires prior knowledge of the number of targets present, its main practical use is in single-target tracking, as mentioned by Braun in [1]. In other situations, the drawbacks of the ESPRIT algorithm outweigh the benefits.



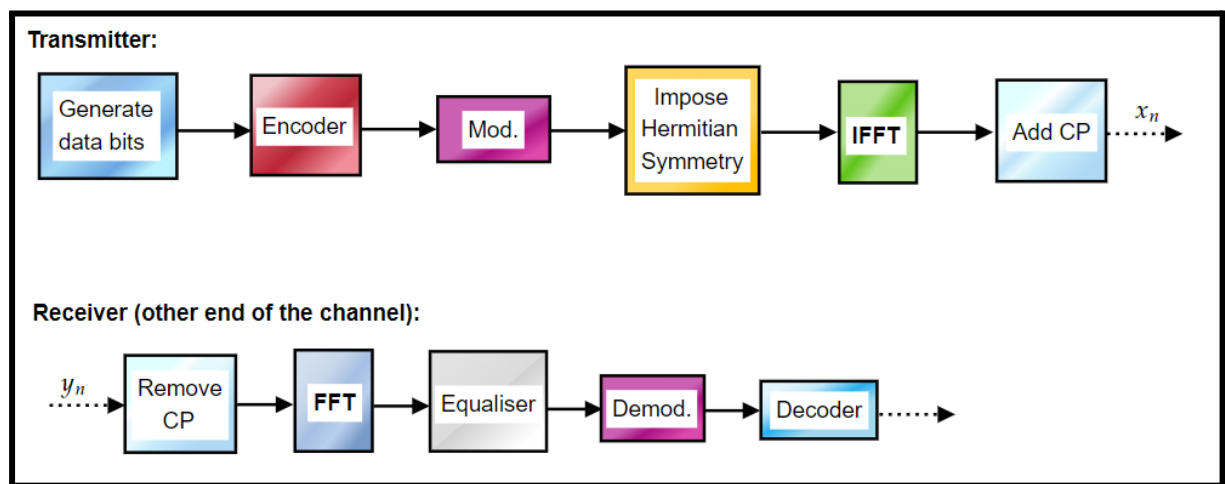
## Work Completed:

### (Communication-link Simulations):

First, simulations of an OFDM communication channel were carried out in MATLAB.

This entailed a study of OFDM transmitters and receivers. The figure below shows the high-level block diagram of the transmitter and receiver, as simulated in

MATLAB:



The channel was modelled by a 3-tap FIR filter, with additive White Gaussian noise:

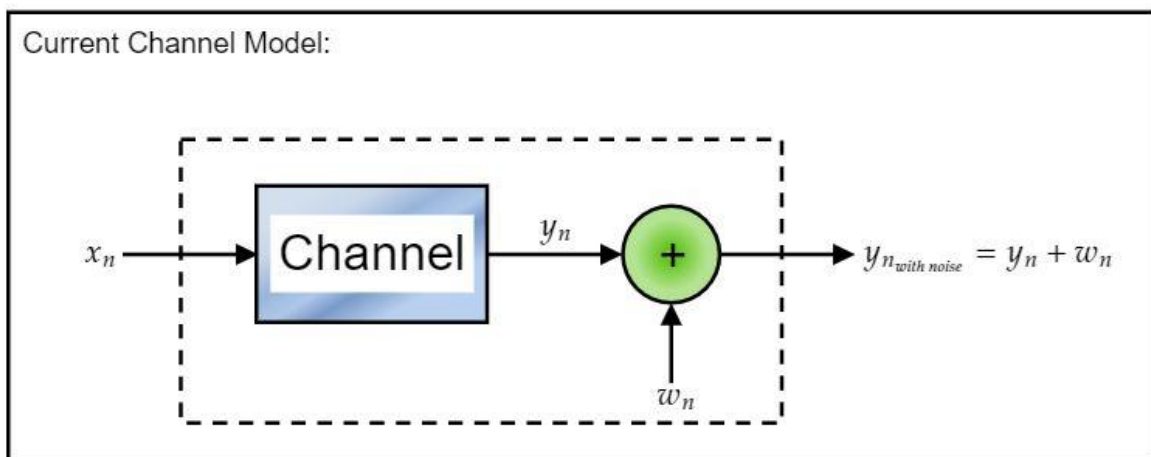


Figure 1: noise is added after the signal passes through the channel

The FIR coefficients were changed, and the correct behaviour of the equaliser block was verified in each simulation.

Below is the receiver constellation plot from one of the simulations. It shows that the transmitted data has been recovered correctly:

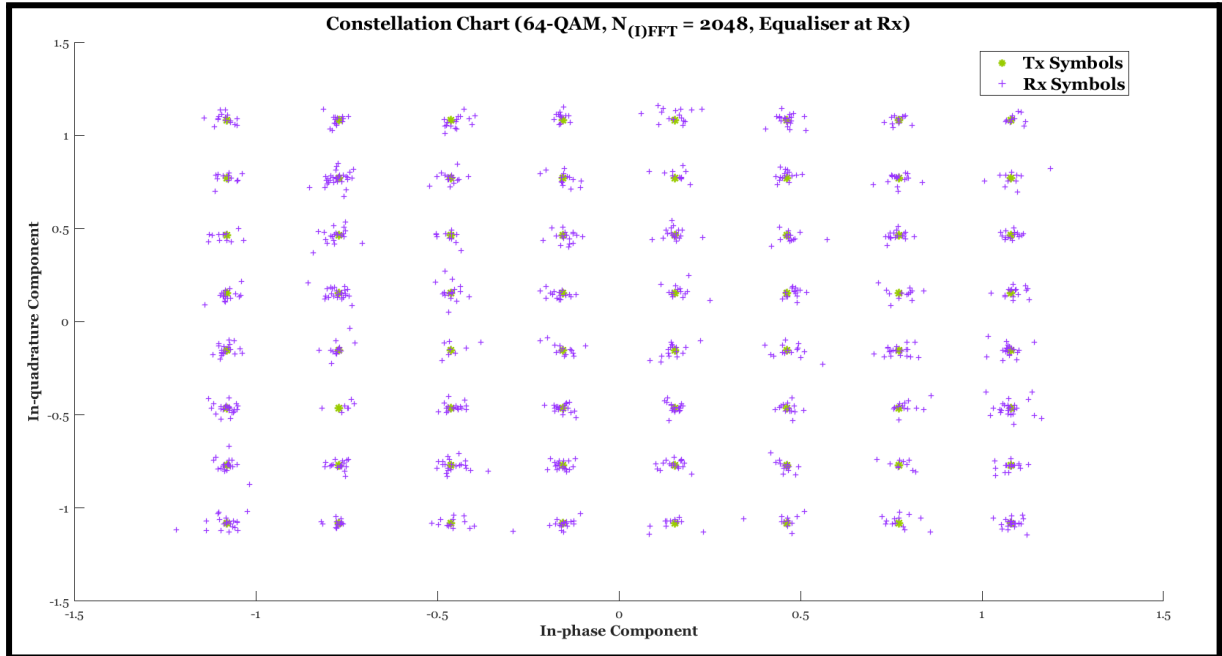


Figure 2: the constellation plot at the receive side of the OFDM communication link; obtained in MATLAB.

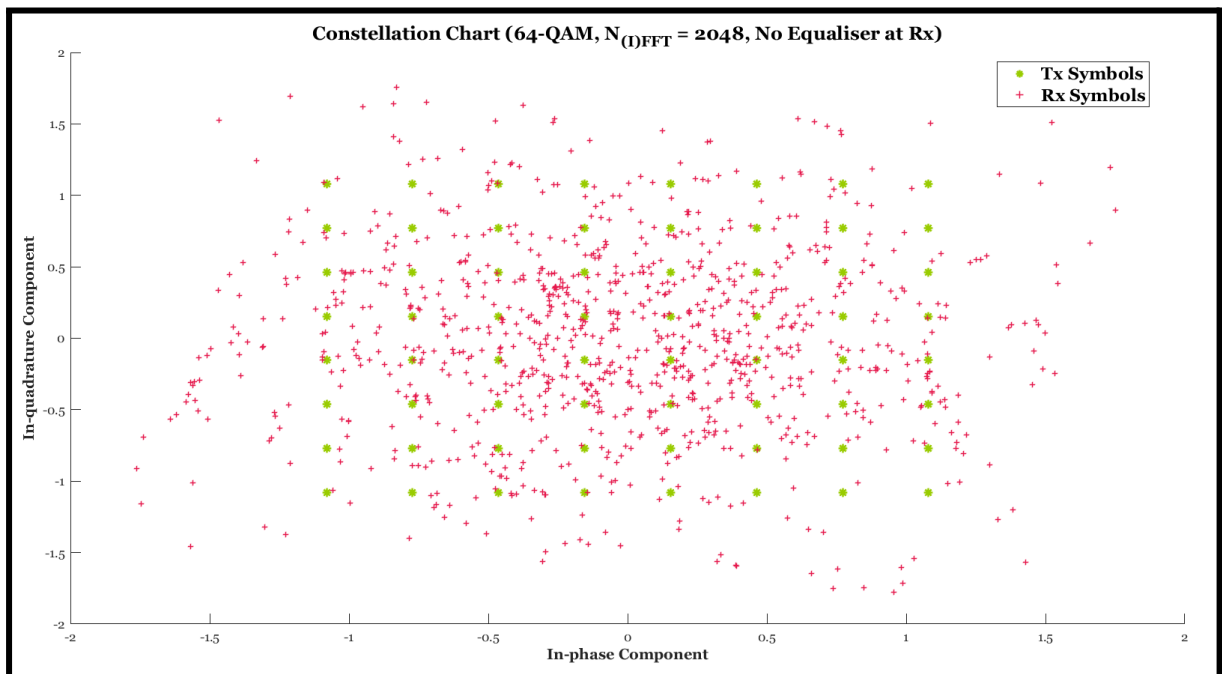


Figure 3: the constellation plot at the receive side of the OFDM communication link when **no equaliser is used**; obtained in MATLAB.

**(Radar Simulations):**

Next, a simulation of the OFDM communication system being used as a radar was completed. The periodogram-based approach was used in order to identify a single target. The table below shows the important parameters of the simulations:

Number of subcarriers ( $N$ )	64
Frame size ( $M$ )	256
$N_{\text{per}}$ (rows in the periodogram)	$4N = 256$
$M_{\text{per}}$ (columns in the periodogram)	$5M = 1280$
Modulation scheme	16-QPSK
OFDM symbol time ( $T_o$ )	$3.2 \mu\text{s}$
Centre frequency ( $f_c$ )	5.5 GHz
Subcarrier spacing ( $\Delta f$ )	312.5 kHz
$G$ (Fraction of OFDM symbol used as CP)	0.25
$D$ (Ratio of maximum Doppler shift to subcarrier spacing)	0.1
$N_{\text{max}} = GN_{\text{per}}$	64
$M_{\text{max}} = DM_{\text{per}}$	128

The values for  $G$  and  $D$  in the above table were based on the discussion in [2, p. 5].

The choices for the symbol time, centre frequency, and subcarrier spacing are based on the IEEE 802.11a standard.

The figures below show that the periodogram-based approach can identify the target when it is at different distances from the OFDM transmitter, and travelling with different velocities:

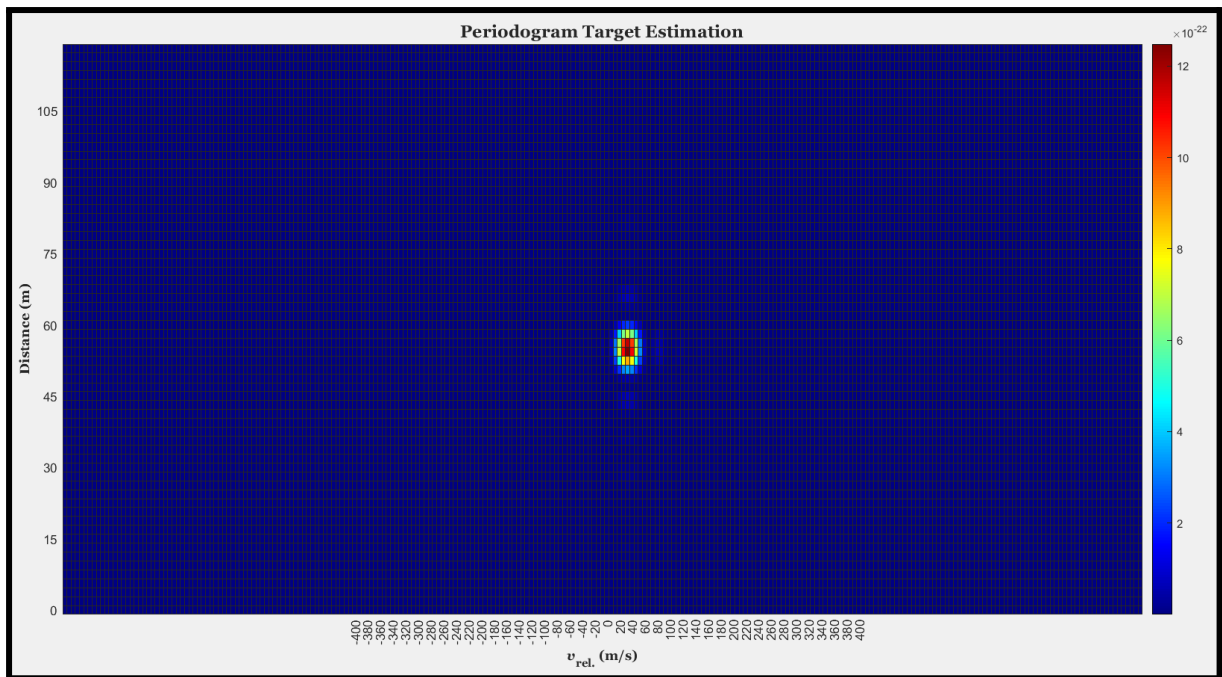


Figure 4: the target was placed 55 m from the transmitter, and it had a velocity of 44 m/s; results from MATLAB.

In the plot below, the value for  $N_{per}$  was reduced from 4N (or 256) to 2N (or 128). It can be seen that the resolution has reduced:

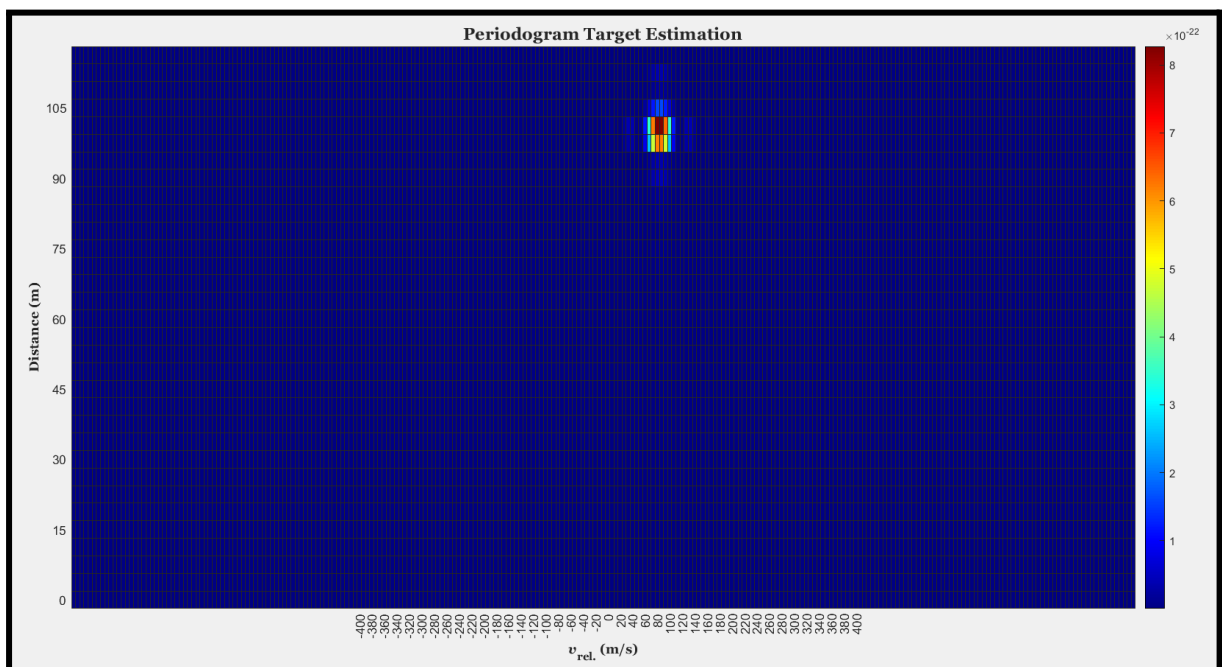


Figure 5: the target was placed 100 m from the transmitter, and it had a velocity of 90 m/s; results from MATLAB.

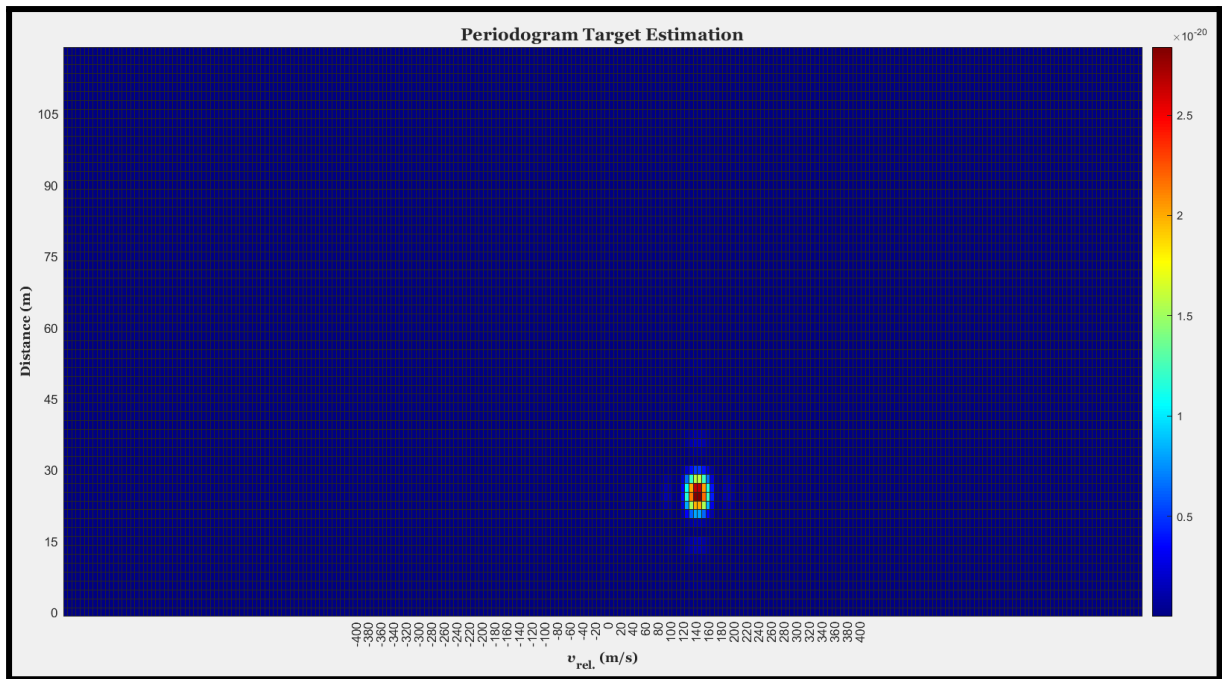


Figure 6: the target was placed 25 m from the transmitter, and it had a velocity of 150 m/s; results from MATLAB.

### (Radar performance using 16-QAM):

The figures below show the results of using the periodogram-based estimation with 16-QAM, which is another common modulation scheme used in OFDM systems:

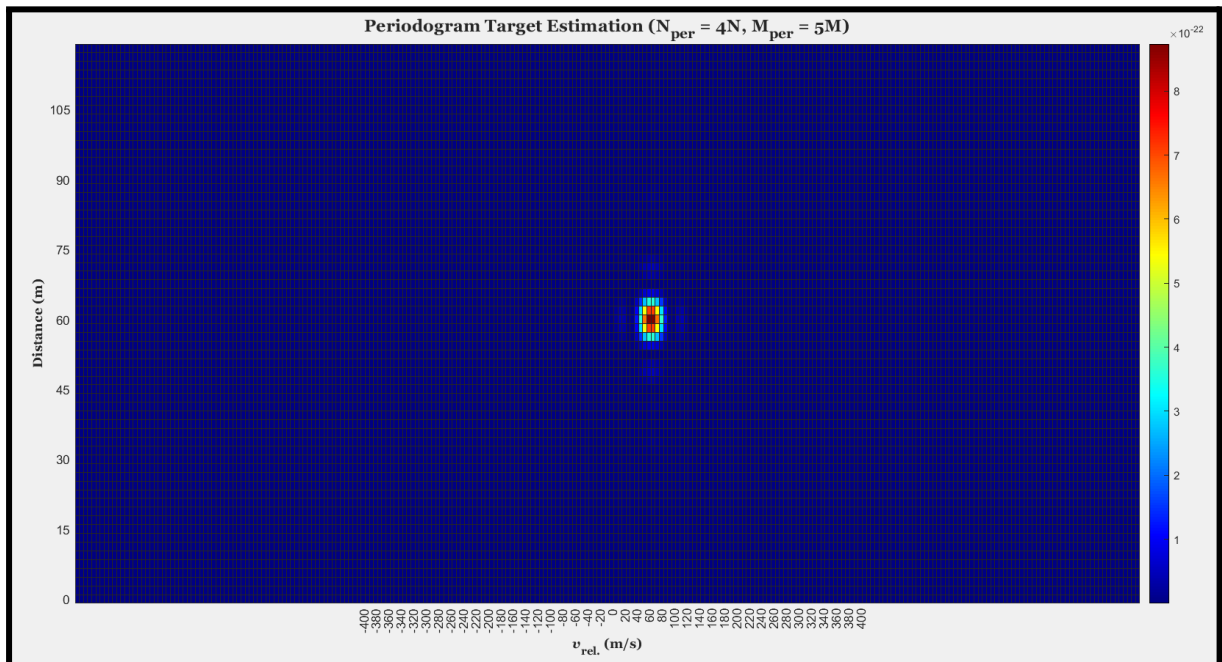


Figure 7: the target was placed 60 m from the transmitter, and it had a velocity of 70 m/s.

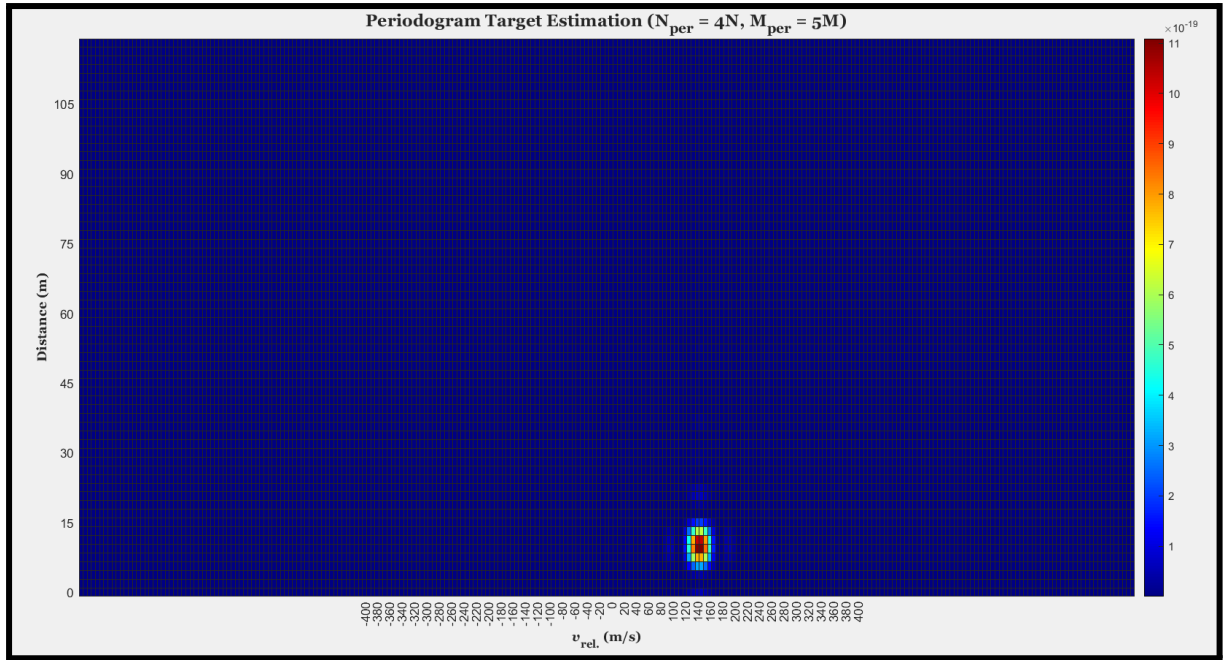


Figure 8: the target was placed 10 m from the transmitter, and it had a velocity of 150 m/s.

The figure below shows the estimation of the target's distance and velocity when the periodogram's dimensions have been reduced:

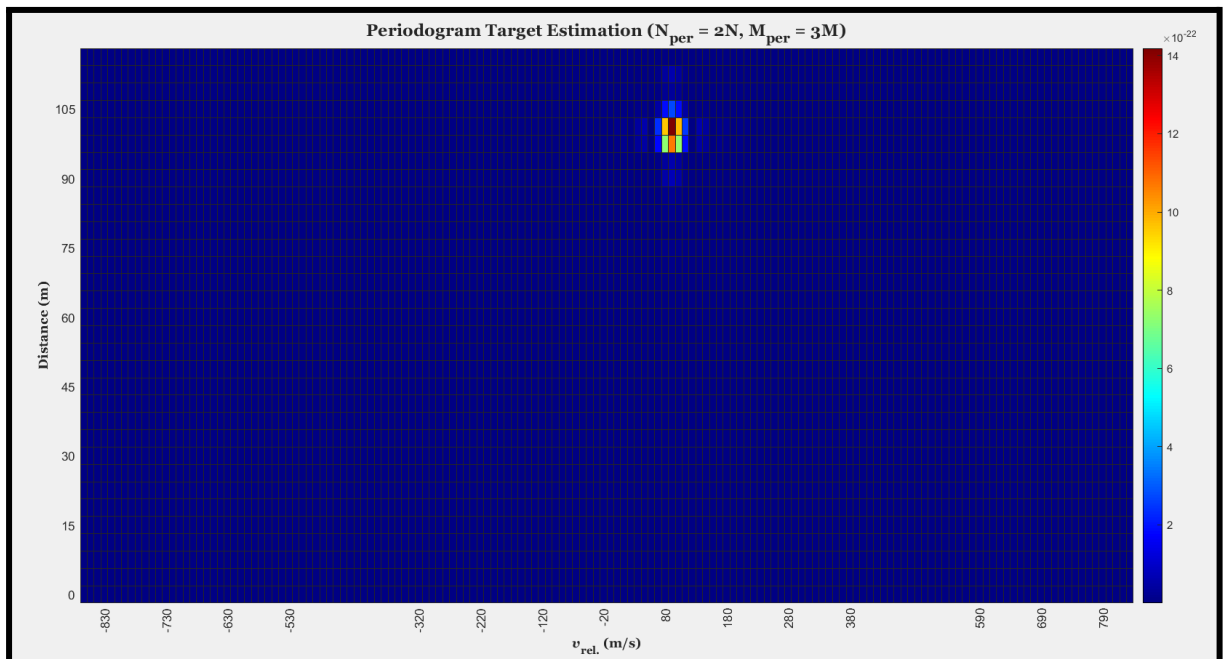


Figure 9: the target was placed 100 m from the transmitter, and it had a velocity of 100 m/s.

## **Ethics and Sustainability:**

Since one of the potential applications of OFDM radar is in vehicles, health and safety would be of utmost importance. Testing of any future OFDM-radar technology would need to be broad and thorough, so as to avoid accidents or disasters in real-world settings. Testing constitutes a significant cost in both time and money throughout the development of a product or technology. An ethical concern might be that the testing phase is neglected or insufficiently done by commercial companies, which might lead to bugs or errors in real-world settings such as roads - this is especially important as investment in the autonomous vehicle sector continues to increase. The hardware and software associated with a potential OFDM communication-and-radar system would need to be maintained in order to keep pace with the newer and newer autonomous vehicles.

## **Plan:**

The plan going forward is outlined more generally below:

- A further study of periodogram-based estimation algorithms. This will include simulating scenarios where there are multiple targets.
- Study of parametric target estimation methods, and the relevant key performance indicators.
- An analysis of parametric target estimation methods, and a simulation of their use for radar with current OFDM standards.

The Gantt chart below shows a more detailed timeline, with start and end dates, as well as the predicted duration of each task:

TASK	START	DAYS	END
<b>Phase 1</b>			
Task 1	22/01/2022	7	28/01/2022
Task 2	29/01/2022	10	08/02/2022
Task 3	09/02/2022	12	21/02/2022
Task 4	22/02/2022	14	08/03/2022

**Task 1:** Further investigation of periodogram-based algorithms

**Task 2:** Study parametric target estimation methods & their KPIs

**Task 3:** Analysis of parametric target estimation; simulation and results

**Task 4:** Investigation of feasibility of the above with the current standards

17/01/2022	24/01/2022	31/01/2022	07/02/2022	14/02/2022	21/02/2022	28/02/2022	07/03/2022
17 18 19 20 21 22 23	24 25 26 27 28 29 30	31 1 2 3 4 5 6	7 8 9 10 11 12 13	14 15 16 17 18 19 20	21 22 23 24 25 26 27	28 1 2 3 4 5 6	7 8 9 10 11 12 13
M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S



## **References:**

- [1] M. Braun, "OFDM Radar Algorithms in Mobile Communication Networks", 2014
- [2] M. Braun, C. Sturm, F. Jondral, "Maximum Likelihood Speed and Distance Estimation for OFDM Radar", 2010 IEEE Radar Conference
- [3] C. Sturm, T. Zwick, W. Wiesbeck, "An OFDM System Concept for Joint Radar and Communications Operations", 2009, VTC Spring 2009 - IEEE 69th Vehicular Technology Conference

