# HW1 Programming Problem 5 (30 points)

## Problem Description

Here, you will perform *weighted* KNN regression.

After you write your own code for weighted KNN regression, you will also try out sklearn's built-in KNN regressor.

Fill out the notebook as instructed, making the requested plots and printing necessary values.

### Summary of deliverables:

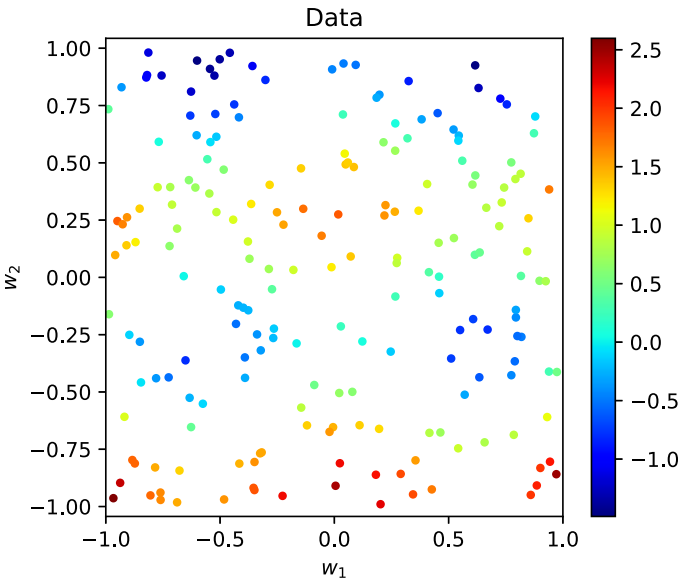
Functions:

- `weighted_knn(w1, w2, k)`

Plots:

- 3 plots of by-hand KNN results
- 3 plots of sklearn.

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.neighbors import KNeighborsRegressor

# Data generation -- don't change
np.random.seed(42)
N = 200
w1_data = np.random.uniform(-1,1,N)
w2_data = np.random.uniform(-1,1,N)
L_data = np.cos(4*w1_data) + np.sin(5*w2_data) + 2*w1_data**2 - w2_data/2
# (end of data generation)

plt.figure(figsize=(5,4.2),dpi=80)
plt.scatter(w1_data,w2_data,s=10,c=L_data,cmap="jet")
plt.colorbar()
plt.axis("equal")
plt.xlabel("$w_1$")
plt.ylabel("$w_2$")
plt.xlim(-1,1)
plt.ylim(-1,1)
plt.title("Data")
plt.show()
```




## Weighted KNN function

Here, define a function, `weighted_knn(w1, w2, k)`, which takes in a point at [`w1`, `w2`] and a `k` value, and returns the weighted KNN prediction.

- As in the lecture activity, data is in the variables `w1_data`, `w2_data`, and `L_data`.
- You can create as many helper functions as you want

- The key difference between unweighted and weighted KNN is summarized below:

**Unweighted KNN**

1. Find the $k$ data points closest to the target point $w$
2. Get the output values at each of these points
3. Average these values together: this is the prediction at $w$

**Weighted KNN**

1. Find the $k$ data points closest to the target point $w$
2. Compute the proximity of each of these points as $\text{prox}_i = 1/(\text{distance}(w, w_i) + 1e-9)$
3. For each $w_i$, multiply $\text{prox}_i$ by the output value at $w_i$, and divide by the sum of all $k$ proximities
4. Add all $k$ of these results together: this is the prediction at $w$

In [ ]:
```
def weighted_knn(w1, w2, k):
    distance = np.sqrt((w1_data - w1)**2 + (w2_data - w2)**2)
    sorted_index = np.argsort(distance)[:k]
    prox = 1 / (distance[sorted_index] + 1e-9)
    sum_prox = np.sum(prox, axis=0)
    assert sum_prox != 0
    assert sum_prox.shape == () #sum_prox should be a scaler
    weight = prox / sum_prox

    return np.sum(weight * L_data[sorted_index])
```

# Plotting

Now create 3 plots showing KNN regressor predictions for k values [1, 5, 25].

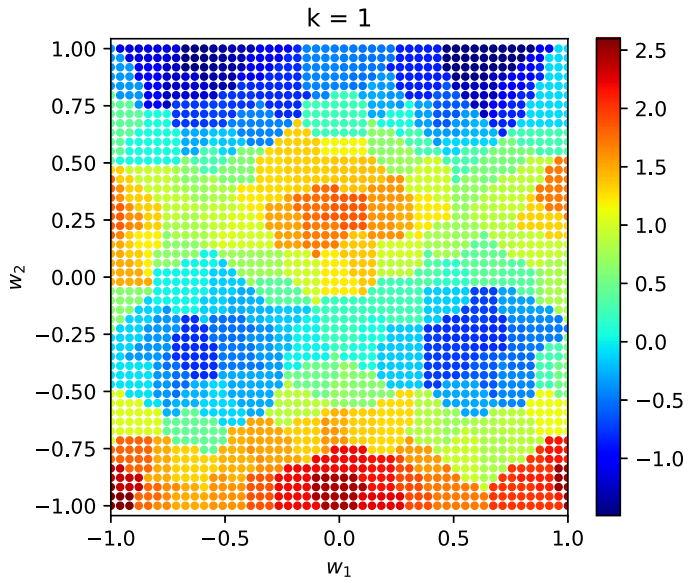
You should plot a 50x50 grid of points on a grid for `w1` and `w2` values between -1 and 1. Consult the lecture activity for how to do this.

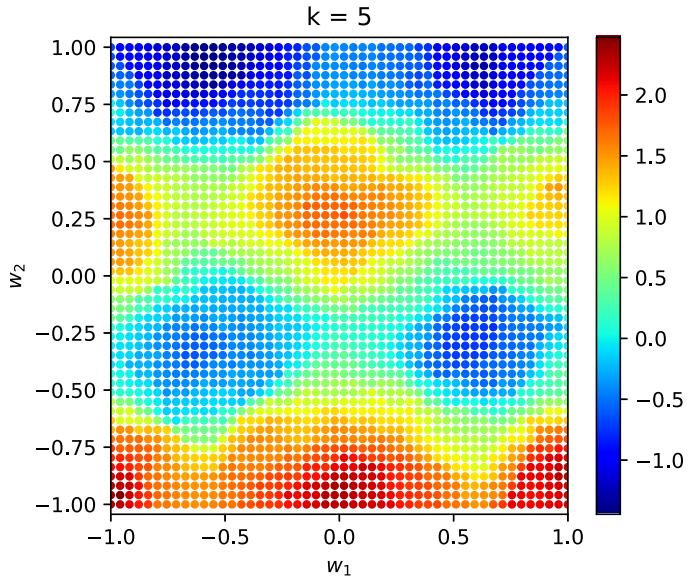
We recommend creating a function, e.g. `plot(k)`, so that you need to rewrite less code.

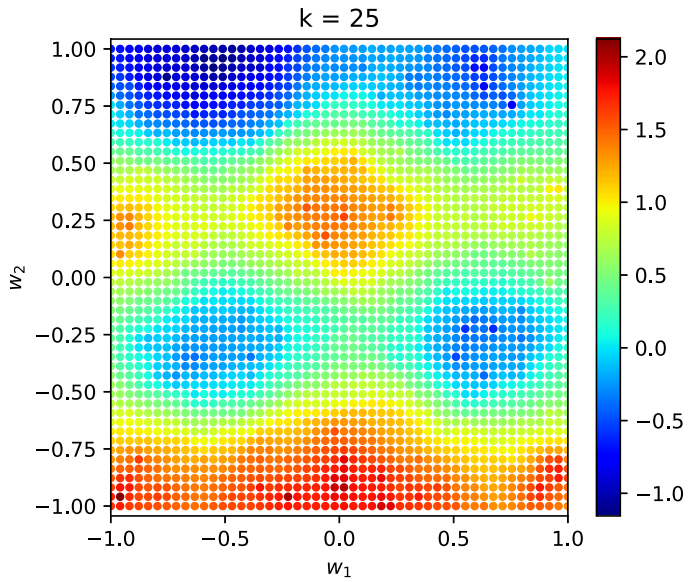
In [ ]:
```
w1 = np.linspace(-1, 1, 50)
print(w1.shape)
z1 = np.zeros((50, 50))
z2 = np.zeros_like(w1.flatten())
```

```
(50,)
```

In [ ]:
```
# YOUR CODE GOES HERE
# Visualize results for k = 1, 5, and 25

k = [1, 5, 25]

def plot_knn(k, knn_calculator = weighted_knn):
    i = k
    w1 = np.linspace(-1, 1, 50)
    w2 = np.linspace(-1, 1, 50)
    w1, w2 = np.meshgrid(w1, w2)
    z = np.zeros_like(w1.flatten())
    for j in range(len(z)):
        if knn_calculator != weighted_knn:
            z[j] = knn_calculator(np.vstack([w1.flatten()[j], w2.flatten()[j]]).T)
        else:
            z[j] = knn_calculator(w1.flatten()[j], w2.flatten()[j], i)
    plt.figure(figsize=(5,4.2),dpi=120)
    plt.scatter(w1.flatten(), w2.flatten(), s=10, c=z, cmap='jet')
    plt.colorbar()
    plt.axis("equal")
    plt.xlabel("$w_1$")
    plt.ylabel("$w_2$")
    plt.xlim(-1,1)
    plt.ylim(-1,1)
    plt.title("k = " + str(i))
    plt.show()


for i in k:
    plot_knn(i)
```




## Using SciKit-Learn

We can also use sklearn's `KNeighborsRegressor()`, which is a very efficient implementation of KNN regression.

The code to do this has been done for one case below. First, make note of how this is done.

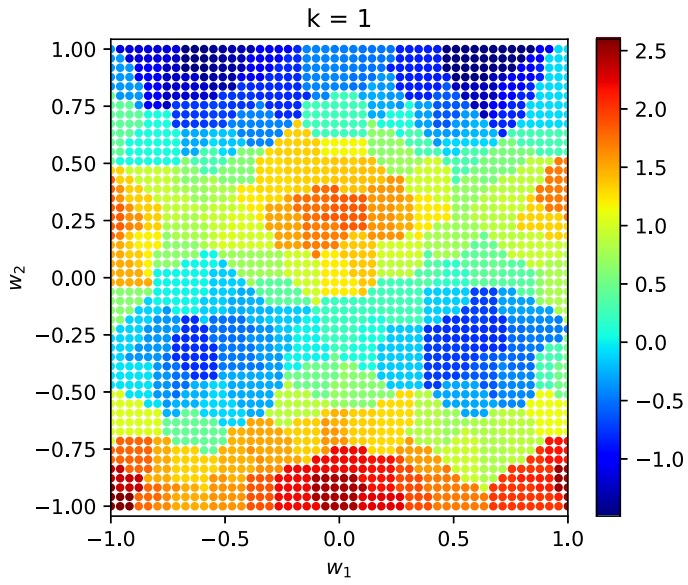
In [ ]:
```
model = KNeighborsRegressor(n_neighbors = 1, weights="distance")
X = np.vstack([w1_data,w2_data]).T
model.fit(X, L_data)

# Get a prediction at a point (0, 0):
```

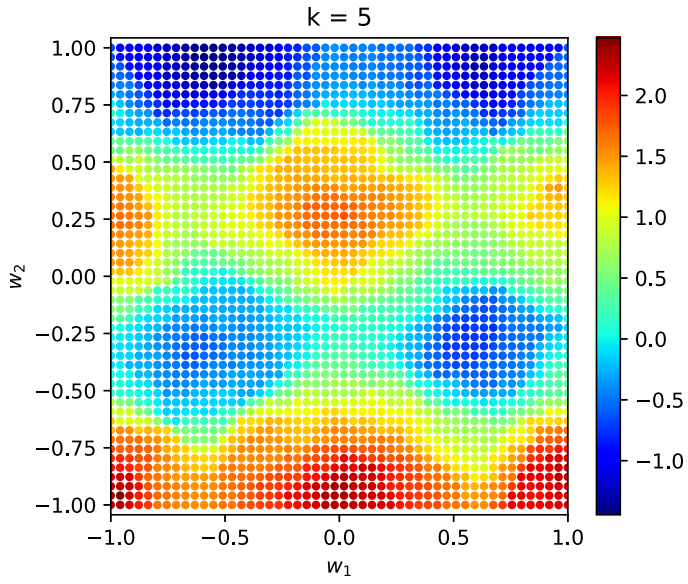
```
print(model.predict(np.array([[0,0]])))
```

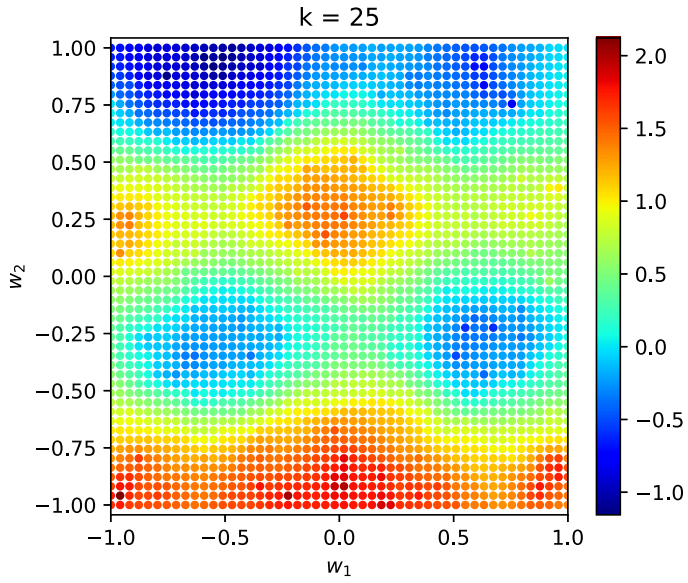
```
[1.19743607]
```

Now create 3 plots for the same values of k as before, using this KNN implementation instead. You can make sure these are visually the same as your from-scratch KNN regressor.

In [ ]:
```
# YOUR CODE GOES HERE
# Visualize sklearn results for k = 1, 5, and 25

for i in k:
    model = KNeighborsRegressor(n_neighbors = i, weights="distance")
    X = np.vstack([w1_data,w2_data]).T
    model.fit(X, L_data)
    plot_knn(i, model.predict)
```




k = 25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
$w_2$
−1.0
−0.5
0.0
0.5
1.0
$w_1$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0