What is Fisher information?

$$I(\theta) \;=\; \mathbb{E}\Bigg[\Big(\tfrac{\partial}{\partial\theta}\ln f(Y;\theta)\Big)^2\Bigg].$$

Example: $Y = \theta + W$**,** $W \sim \mathcal{N}(0,\sigma^2)$

$$f(y;\theta) \;=\; \frac{1}{\sqrt{2\pi}\,\sigma}\exp\!\Big(-\tfrac{(y-\theta)^2}{2\sigma^2}\Big).$$

The Fisher information is then found by

$$I(\theta) \;=\; \int_{-\infty}^{\infty} f(y;\theta)\left(\tfrac{\partial}{\partial\theta}\ln f(y;\theta)\right)^2 dy \;=\; \frac{1}{\sigma^2}.$$

Block diagram (schematic)