Home Work 2

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1 Question

Write the following statements in symbolic form using the symbols $\sim$, $\wedge$, and $\vee$ and the indicated letters to represent component statements.

(a) $h \wedge w \wedge \sim s$

(b) $\sim w \wedge h \wedge s$

(c) $\sim h \wedge \sim w \wedge \sim s$

(d) $\sim w \wedge \sim s \wedge h$

(e) $w \wedge \sim h \wedge \sim s$

2 Question

(a)

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee q$	$\sim (p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

(b)

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

3 Question

Determine whether the statement forms are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

(a)

p	q	$p \wedge q$	$p \vee (p \wedge q)$	q
T	T	T	T	T
T	F	F	T	F
F	T	F	F	T
F	F	F	F	F

The columns for $p \vee (p \wedge q)$ and q are not the same, so they can not be logically equivalent

(b)

p	q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

The columns for $\sim (p \wedge q)$ and $\sim p \wedge \sim q$ are not the same, so they can not be logically equivalent

(c)

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

The columns for $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are no the same, so they are logically equivalent

4 Question

Assume x is a particular real number and use De Morgan's laws to write negations for the statements

(a) $x \leq -2$ or $x \geq 6$

(b) $x \leq -9$ or $x \geq 2$

(c) $x \geq 2$ and $x \leq 6$

(d) $x > -1$ and $x \leq 1$

(e) $x \geq 0$ or $x < -4$

5 Question

Use truth tables to establish which of the statement forms are tautologies and which are contradictions.

(a)

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee q$	$\sim (p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

(b)

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

6 Question

Determine whether the statement forms are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

(a)

p	q	$p \wedge q$	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg p \vee (p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$
T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	F	T	T

This is a tautology

(b)

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg p \vee q$	$(p \wedge \neg q) \wedge (\neg p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

This is a contradiction

(c)

p	q	r	$\neg p$	$\neg q$	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$((\neg p \wedge q) \wedge (q \wedge r)) \vee \neg q$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	T
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	T
F	F	F	T	T	F	F	F	T

This is a neither

7 Question

Using Exclusive OR

Is ((p q) r p (q r))? Justify your answer.

They are logically equal. They both evaluate to true if either only one of (p), (q), or (r) is true, but not more than one. But, also they both evaluate to false if all three are true or all three are false.

8 Question

What logical equivalence is used for each step?

(a) Distributive

(b) Commutative Law

(c) Negation Law

(d) Identity Law

9 Question

Verify the logical equivalences. Supply a reason for each step.

((p ¬q) p p)

(a) Distributive: $(p \wedge \neg q) \vee p \equiv (p \vee p) \wedge (p \vee \neg q)$
Distributing p across the OR operation.

(b) Idempotent Law: $(p \vee p) \equiv p$
Any proposition OR-ed with itself is logically equivalent to itself.

(c) Absorption Law: $(p \vee \neg q) \wedge (p \vee q) \equiv p$
Whenever p is true, the whole expression evaluates to true.

Yes, ((p ¬q) p p).