COT 2000
Foundations of Computing

Summer 2024

Lecture 12 – part 1

Lab 6

Exam 2 – 06/21/24

Homework 4 – 06/28/24

Lecture 12 – part 2

Review

Review

- Bash/Terminal
  - Commands: ls, cd, pwd, cp, mv, rm, touch, cat, man, find, grep...
- Sequences
- Summation Notation

SOME COMMANDS

ls - List files/directories.
Example: ls -l (Lists files in detailed view)

cd <directory> - Change to <directory>.
Example: cd Documents/ (Changes the directory to 'Documents')

pwd - Show current directory.
Example: pwd (Displays the path of the current directory)

touch <filename> - Create an empty file.
Example: touch newfile.txt (Creates a new file named newfile.txt)

cat <filename> - Display file content.
Example: cat myfile.txt (Displays the content of myfile.txt)

cp <source> <destination> - Copy file.
Example: cp file1.txt file2.txt (Copies file1.txt to file2.txt)

mv <source> <destination> - Move/rename file.
Example: mv oldname.txt newname.txt (Renames oldname.txt to newname.txt)

rm <filename> - Delete file.
Example: rm unwantedfile.txt (Deletes the file named unwantedfile.txt)

grep "pattern" <filename> - Search for a pattern in a file.
Example: grep "hello" myfile.txt (Searches for the word "hello" in myfile.txt)

find . -name "filename" - Search for a file in current and sub-directories.
Example: find . -name "notes.txt" (Searches for notes.txt in the current directory and all sub-directories)

man <command> - Display the manual for a command.
Example: man ls (Displays the manual for the 'ls' command)

<command> --help - Get a quick help for a command.
Example: ls --help (Provides a brief help documentation for the 'ls' command)

Introduction to Sequences

An ordered list of numbers (or elements).

Example: Imagine that a person decides to count his ancestors.

2, 4, 8, 16, 32, 64, 128, . . .

Position in the row	1	2	3	4	5	6	7 . . .
Number of ancestors	2	4	8	16	32	64	128 . . .

$$n = 1, 2, 3, 4, \ldots$$

$$2^n = 2, 4, 8, 16, \ldots$$

$$A_k = 2^k$$

• Definition

A **sequence** is a function whose domain is either all the integers between two given integers or all the integers greater than or equal to a given integer.

$$a_m, a_{m+1}, \ldots, a_n$$

Finite Sequence

$$a_m, a_{m+1}, a_{m+2}, \ldots$$

Infinite Sequence

a_k is called a term.

k is the subscript or index.

Summation Notation

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 126.$$

• Definition

If m and n are integers and $m \leq n$, the symbol $\sum_{k=m}^{n} a_k$, read the **summation from *k* equals *m* to *n* of *a*-sub-*k***, is the sum of all the terms $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$. We say that $a_m + a_{m+1} + a_{m+2} + \ldots + a_n$ is the **expanded form** of the sum, and we write

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \cdots + a_n.$$

We call k the **index** of the summation, m the **lower limit** of the summation, and n the **upper limit** of the summation.

Lecture 12 – part 3

More on Sequences

Product Notation

$$\prod_{k=1}^{5} a_k = a_1a_2a_3a_4a_5.$$

- **Definition**

If m and n are integers and $m \le n$, the symbol $\prod_{k=m}^{n} a_k$, read the **product from k equals m to n of a-sub-k**, is the product of all the terms $a_m,\ a_{m+1},\ a_{m+2},\ \ldots,\ a_n$. We write

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdots a_n.$$

Example:

Compute the following products:

a. $\prod_{k=1}^{5} k$

b. $\prod_{k=1}^{1} \frac{k}{k+1}$

Solution:

a. $\prod_{k=1}^{5} k = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

b. $\prod_{k=1}^{1} \frac{k}{k+1} = \frac{1}{1+1} = \frac{1}{2}$

Properties

If $a_m, a_{m+1}, a_{m+2}, \ldots$ and $b_m, b_{m+1}, b_{m+2}, \ldots$ are sequences of real numbers and c is any real number, then the following equations hold for any integer $n \geq m$:

1. $\displaystyle\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$

2. $\displaystyle c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k$ generalized distributive law

3. $\displaystyle\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \prod_{k=m}^{n} (a_k \cdot b_k).$

Example:

Let $a_k = k + 1$ and $b_k = k - 1$ for all integers k. Write each of the following expressions as a single summation or product:

a. $\displaystyle\sum_{k=m}^{n} a_k + 2 \cdot \sum_{k=m}^{n} b_k$ b. $\displaystyle\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right)$

Solution:

a.
$$\begin{aligned}
\sum_{k=m}^{n} a_k + 2 \cdot \sum_{k=m}^{n} b_k &= \sum_{k=m}^{n} (k+1) + 2 \cdot \sum_{k=m}^{n} (k-1) && \text{by substitution} \\
&= \sum_{k=m}^{n} (k+1) + \sum_{k=m}^{n} 2 \cdot (k-1) && \text{by Theorem 5.1.1 (2)} \\
&= \sum_{k=m}^{n} ((k+1) + 2 \cdot (k-1)) && \text{by Theorem 5.1.1 (1)} \\
&= \sum_{k=m}^{n} (3k-1) && \text{by algebraic simplification}
\end{aligned}$$

b.

$$\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \left(\prod_{k=m}^{n} (k+1)\right) \cdot \left(\prod_{k=m}^{n} (k-1)\right) \qquad \text{by substitution}$$

$$= \prod_{k=m}^{n} (k+1) \cdot (k-1) \qquad \text{by Theorem 5.1.1 (3)}$$

$$= \prod_{k=m}^{n} (k^2 - 1) \qquad \text{by algebraic simplification}$$

Transforming a Sum by a Change of Variable

Example: Transform the following summation by making the specified change of variable

$$\textit{summation: } \sum_{k=0}^{6} \frac{1}{k+1} \qquad \textit{change of variable: } j = k+1$$

Solution: First calculate the lower and upper limits of the new summation:

$$\text{When } k = 0, \quad j = k + 1 = 0 + 1 = 1.$$
$$\text{When } k = 6, \quad j = k + 1 = 6 + 1 = 7.$$

Since $j = k + 1$, then $k = j - 1$.

Hence $\dfrac{1}{k+1} = \dfrac{1}{(j-1)+1} = \dfrac{1}{j}$.

$$\sum_{k=0}^{6} \frac{1}{k+1} = \sum_{j=1}^{7} \frac{1}{j}.$$

Upper Limit change

Example: a. Transform the following summation by making the specified change of variable.

$$\textit{summation: } \sum_{k=1}^{n+1}\left(\frac{k}{n+k}\right) \qquad \textit{change of variable: } j = k - 1$$

b. Transform the summation obtained in part (a) by changing all j's to k's.

Solution:

a) $$\frac{k}{n+k} = \frac{j+1}{n+(j+1)}$$

$$\sum_{k=1}^{n+1}\frac{k}{n+k} = \sum_{j=0}^{n}\frac{j+1}{n+(j+1)}.$$

b) $$\sum_{j=0}^{n}\frac{j+1}{n+(j+1)} = \sum_{k=0}^{n}\frac{k+1}{n+(k+1)}$$

Factorial Notation

• Definition

For each positive integer n, the quantity ***n* factorial** denoted ***n*!,** is defined to be the product of all the integers from 1 to n:

$$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1.$$

Zero factorial, denoted $0!$, is defined to be 1:

$$0! = 1.$$

Recursive definition

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n \geq 1. \end{cases}$$

Example: Fist 10 factorials

$$\begin{aligned} 0! &= 1 & 1! &= 1 \\ 2! &= 2 \cdot 1 = 2 & 3! &= 3 \cdot 2 \cdot 1 = 6 \\ 4! &= 4 \cdot 3 \cdot 2 \cdot 1 = 24 & 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \\ 6! &= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 & 7! &= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040 \\ 8! &= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 & 9! &= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 40{,}320 & &= 362{,}880 \end{aligned}$$

Example: Simplify the following expressions:

a. $\dfrac{8!}{7!}$ b. $\dfrac{5!}{2!\cdot 3!}$ c. $\dfrac{1}{2!\cdot 4!}+\dfrac{1}{3!\cdot 3!}$ d. $\dfrac{(n+1)!}{n!}$ e. $\dfrac{n!}{(n-3)!}$

Solution:

a. $\dfrac{8!}{7!}=\dfrac{8\cdot \cancel{7!}}{\cancel{7!}}=8$

b. $\dfrac{5!}{2!\cdot 3!}=\dfrac{5\cdot 4\cdot \cancel{3!}}{2!\cdot \cancel{3!}}=\dfrac{5\cdot 4}{2\cdot 1}=10$

c. $\dfrac{1}{2!\cdot 4!}+\dfrac{1}{3!\cdot 3!}=\dfrac{1}{2!\cdot 4!}\cdot\dfrac{3}{3}+\dfrac{1}{3!\cdot 3!}\cdot\dfrac{4}{4}$ by multiplying each numerator and denominator by just what is necessary to obtain a common denominator

$=\dfrac{3}{3\cdot 2!\cdot 4!}+\dfrac{4}{3!\cdot 4\cdot 3!}$ by rearranging factors

$=\dfrac{3}{3!\cdot 4!}+\dfrac{4}{3!\cdot 4!}$ because $3\cdot 2!=3!$ and $4\cdot 3!=4!$

$=\dfrac{7}{3!\cdot 4!}$ by the rule for adding fractions with a common denominator

$=\dfrac{7}{144}$

d. $\dfrac{(n+1)!}{n!}=\dfrac{(n+1)\cdot \cancel{n!}}{\cancel{n!}}=n+1$

e. $\dfrac{n!}{(n-3)!}=\dfrac{n\cdot(n-1)\cdot(n-2)\cdot\cancel{(n-3)!}}{\cancel{(n-3)!}}$

$=n\cdot(n-1)\cdot(n-2)$

$=n^3-3n^2+2n$

(n choose r) Notation

- Binomial coefficient $\binom{n}{r}$

• Definition

Let n and r be integers with $0 \le r \le n$. The symbol

$$\binom{n}{r}$$

is read "***n* choose *r***" and represents the number of subsets of size r that can be chosen from a set with n elements.

• Formula for Computing $\binom{n}{r}$

For all integers n and r with $0 \le r \le n$,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

also called *combinations*

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example: Use the formula for computing $\binom{n}{r}$ to evaluate the following expressions:

a. $\binom{8}{5}$ b. $\binom{4}{0}$ c. $\binom{n+1}{n}$

Solution:

a. $$\binom{8}{5} = \frac{8!}{5!(8-5)!}$$
$$= \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3\cdot 2\cdot 1}}{(\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1)\cdot(\cancel{\cdot 3\cdot 2\cdot 1})}$$
$$= 56.$$

b. $$\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!0!} = \frac{\cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{(4\cdot 3\cdot 2\cdot 1)}(1)} = 1$$

c. $$\binom{n+1}{n} = \frac{(n+1)!}{n!((n+1)-n)!} = \frac{(n+1)!}{n!1!} = \frac{(n+1)\cdot \cancel{n!}}{\cancel{n!}} = n+1$$

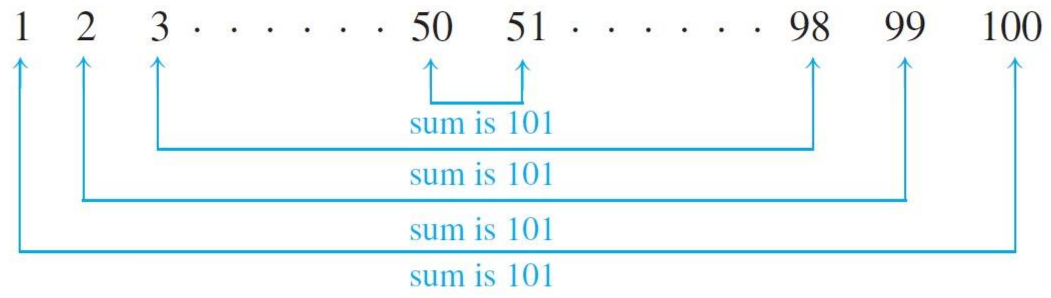
Lecture 12 – part 4

Sequences Exercises

Lecture 12 – part 5

Mathematical Induction

The story is told that one of the greatest mathematicians of all time, Carl Friedrich Gauss (1777–1855), was given the problem of adding the numbers from 1 to 100 by his teacher when he was a young child. The teacher had asked his students to compute the sum, supposedly to gain himself some time to grade papers. But after just a few moments, Gauss produced the correct answer. Needless to say, the teacher was dumbfounded. How could young Gauss have calculated the quantity so rapidly? In his later years, Gauss explained that he had imagined the numbers paired according to the following schema.



The sum of the numbers in each pair is 101, and there are 50 pairs in all; hence the total sum is $50 \cdot 101 = 5{,}050$.

Mathematical Induction

Principle of Mathematical Induction

Let $P(n)$ be a property that is defined for integers n, and let a be a fixed integer. Suppose the following two statements are true:

1. $P(a)$ is true.

2. For all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true.

Then the statement

$$\text{for all integers } n \geq a,\ P(n)$$

is true.

Method of Proof by Mathematical Induction

Consider a statement of the form, "For all integers $n \geq a$, a property $P(n)$ is true." To prove such a statement, perform the following two steps:

Step 1 (basis step): Show that $P(a)$ is true.

Step 2 (inductive step): Show that for all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true. To perform this step,

suppose that $P(k)$ is true, where k is any particular but arbitrarily chosen integer with $k \geq a$.

[This supposition is called the **inductive hypothesis.***]*

Then

show that $P(k+1)$ is true.

Example

Let $P(n)$ be the proposition that the sum of the first n odd numbers is n^2.

That is,

$$P(n) : 1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

The kth odd number is $2k - 1$, and the next odd number is $2k + 1$.

STEP 1: Observe that $P(1)$ is true: $1 = 1^2$.

STEP 2: Assuming $P(k)$ is true, we add $2k + 1$ to both sides, obtaining:

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = k^2 + (2k + 1).$$

This simplifies to:

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = (k + 1)^2.$$

By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

$$P(1) : 1 = 1^2$$

$$P(2) : 1 + 3 = 2^2$$

$$P(3) : 1 + 3 + 5 = 3^2$$

$$P(4) : 1 + 3 + 5 + 7 = 4^2$$

Example

Any whole number of cents of at least 8¢. I can be obtained using 3¢ and 5¢ coins.

Number of Cents	How to Obtain It
8¢	3¢ + 5¢
9¢	3¢ + 3¢ + 3¢
10¢	5¢ + 5¢
11¢	3¢ + 3¢ + 5¢
12¢	3¢ + 3¢ + 3¢ + 3¢
13¢	3¢ + 5¢ + 5¢
14¢	3¢ + 3¢ + 3¢ + 5¢
15¢	5¢ + 5¢ + 5¢
16¢	3¢ + 3¢ + 5¢ + 5¢
17¢	3¢ + 3¢ + 3¢ + 3¢ + 5¢

For all integers $n \geq 8$, n¢ can be obtained using 3¢ and 5¢ coins.

Proof (by mathematical induction):

Let the property $P(n)$ be the sentence

n¢ can be obtained using 3¢ and 5¢ coins. $\leftarrow P(n)$

Show that P(8) is true:
$P(8)$ is true because 8¢ can be obtained using one 3¢ coin and one 5¢ coin.

Show that for all integers $k \geq 8$, if P(k) is true then P(k+1) is also true:

[Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 8$. That is:] Suppose that k is any integer with $k \geq 8$ such that

k¢ can be obtained using 3¢ and 5¢ coins. $\leftarrow P(k)$ inductive hypothesis

[We must show that $P(k+1)$ is true. That is:] We must show that

$(k+1)$¢ can be obtained using 3¢ and 5¢ coins. $\leftarrow P(k+1)$

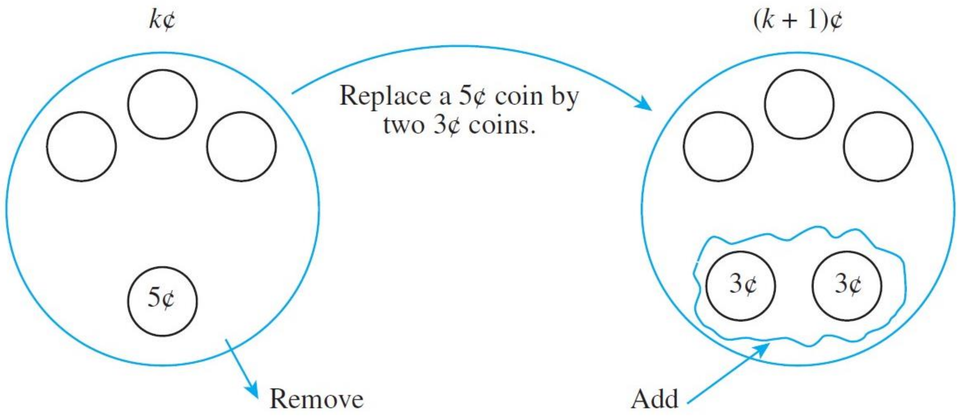
Case 1 (There is a 5¢ coin among those used to make up the k¢.): In this case replace the 5¢ coin by two 3¢ coins; the result will be $(k+1)$¢.

Case 2 (There is not a 5¢ coin among those used to make up the k¢.): In this case, because $k \geq 8$, at least three 3¢ coins must have been used. So remove three 3¢ coins and replace them by two 5¢ coins; the result will be $(k+1)$¢.

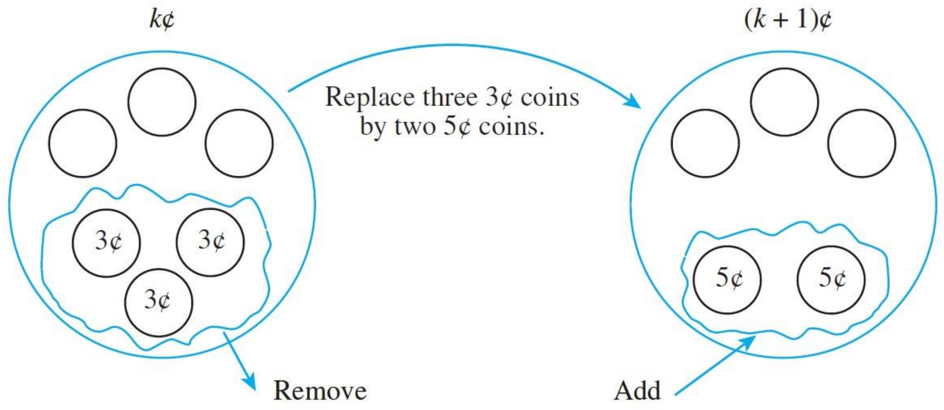
Thus in either case $(k+1)$¢ can be obtained using 3¢ and 5¢ coins *[as was to be shown]*.

[Since we have proved the basis step and the inductive step, we conclude that the proposition is true.]

Case 1



Case 2



Closed Form

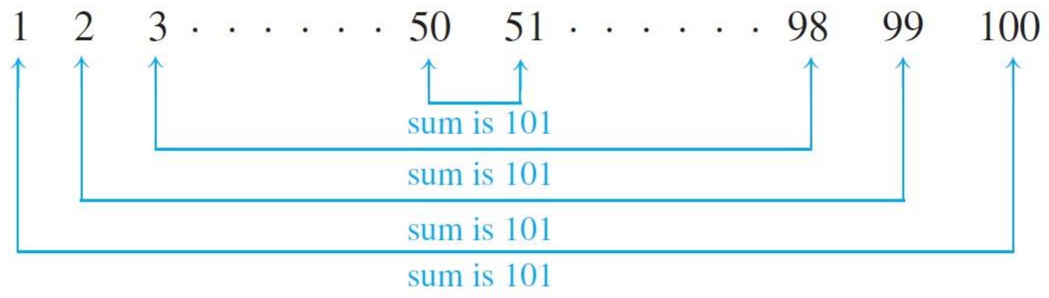
• Definition Closed Form

If a sum with a variable number of terms is shown to be equal to a formula that does not contain either an ellipsis or a summation symbol, we say that it is written **in closed form**.

For example, writing $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$

expresses the sum $1 + 2 + 3 + \cdots + n$ in closed form.

The story is told that one of the greatest mathematicians of all time, Carl Friedrich Gauss (1777–1855), was given the problem of adding the numbers from 1 to 100 by his teacher when he was a young child. The teacher had asked his students to compute the sum, supposedly to gain himself some time to grade papers. But after just a few moments, Gauss produced the correct answer. Needless to say, the teacher was dumbfounded. How could young Gauss have calculated the quantity so rapidly? In his later years, Gauss explained that he had imagined the numbers paired according to the following schema.



The sum of the numbers in each pair is 101, and there are 50 pairs in all; hence the total sum is $50 \cdot 101 = 5{,}050$.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(n+1) \text{ in closed form.}$$

$$1+2+3+\cdots+n=\frac{n}{2}(n+1)$$

Example:

a. Evaluate $2+4+6+\cdots+500$.

b. Evaluate $5+6+7+8+\cdots+50$.

c. For an integer $h \geq 2$, write $1+2+3+\cdots+(h-1)$ in closed form.

Solution:

a.
$$\begin{aligned}2+4+6+\cdots+500 &= 2\cdot(1+2+3+\cdots+250)\\ &= 2\cdot\left(\frac{250\cdot 251}{2}\right) && \text{by applying the formula for the sum of the first } n \text{ integers with } n=250\\ &= 62{,}750.\end{aligned}$$

b.
$$\begin{aligned}5+6+7+8+\cdots+50 &= (1+2+3+\cdots+50)-(1+2+3+4)\\ &= \frac{50\cdot 51}{2}-10 && \text{by applying the formula for the sum of the first } n \text{ integers with } n=50\\ &= 1{,}265\end{aligned}$$

c.
$$\begin{aligned}1+2+3+\cdots+(h-1) &= \frac{(h-1)\cdot[(h-1)+1]}{2} && \text{by applying the formula for the sum of the first } n \text{ integers with } n=h-1\\ &= \frac{(h-1)\cdot h}{2} && \text{since } (h-1)+1=h.\end{aligned}$$

Sum of a geometric sequence

- In a geometric sequence, each term is obtained from the preceding one by multiplying by a constant factor.
- If the first term is 1 and the constant factor is *r* , then the sequence is 1, $r, r^2, r^3, \ldots, r^n, \ldots$.
- For all integers n ≥ 0 and real numbers r not equal to 1, the sum is given by:

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}$$

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Proof (by mathematical induction):

Suppose r is a particular but arbitrarily chosen real number that is not equal to 1, and let the property $P(n)$ be the equation

$$\sum_{i=0}^{n} r^i = \frac{r^{i+1} - 1}{r - 1} \qquad \leftarrow P(n)$$

We must show that $P(n)$ is true for all integers $n \geq 0$. We do this by mathematical induction on n.

Step 1

Show that P(0) is true:

To establish $P(0)$, we must show that

$$\sum_{i=0}^{0} r^i = \frac{r^{0+1} - 1}{r - 1} \qquad \leftarrow P(0)$$

The left-hand side of this equation is $r^0 = 1$ and the right-hand side is

$$\frac{r^{0+1} - 1}{r - 1} = \frac{r - 1}{r - 1} = 1$$

also because $r^1 = r$ and $r \neq 1$. Hence $P(0)$ is true.

Step 2

Show that for all integers $k \geq 0$, if P(k) is true then P(k + 1) is also true:

[Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 0$. That is:]

Let k be any integer with $k \geq 0$, and suppose that

$$\sum_{i=0}^{k} r^i = \frac{r^{k+1} - 1}{r - 1} \qquad \leftarrow P(k)$$

inductive hypothesis

[We must show that $P(k+1)$ is true. That is:] We must show that

$$\sum_{i=0}^{k+1} r^i = \frac{r^{(k+1)+1} - 1}{r - 1},$$

or, equivalently, that

$$\sum_{i=0}^{k+1} r^i = \frac{r^{k+2} - 1}{r - 1}. \qquad \leftarrow P(k+1)$$

[We will show that the left-hand side of $P(k+1)$ equals the right-hand side.]

[We will show that the left-hand side of $P(k+1)$ equals the right-hand side.]
The left-hand side of $P(k+1)$ is

$$\sum_{i=0}^{k+1} r^i = \sum_{i=0}^{k} r^i + r^{k+1}$$ by writing the $(k+1)$st term separately from the first k terms

$$= \frac{r^{k+1}-1}{r-1} + r^{k+1}$$ by substitution from the inductive hypothesis

$$= \frac{r^{k+1}-1}{r-1} + \frac{r^{k+1}(r-1)}{r-1}$$ by multiplying the numerator and denominator of the second term by $(r-1)$ to obtain a common denominator

$$= \frac{(r^{k+1}-1) + r^{k+1}(r-1)}{r-1}$$ by adding fractions

$$= \frac{r^{k+1}-1+r^{k+2}-r^{k+1}}{r-1}$$ by multiplying out and using the fact that $r^{k+1} \cdot r = r^{k+1} \cdot r^1 = r^{k+2}$

$$= \frac{r^{k+2}-1}{r-1}$$ by canceling the r^{k+1}'s.

which is the right-hand side of $P(k+1)$ *[as was to be shown.]*
[Since we have proved the basis step and the inductive step, we conclude that the theorem is true.]

Note: Two different ways to show that an equation is true:
(1) Transforming the left-hand side and the right-hand side independently until they are seen to be equal, and
(2) Transforming one side of the equation until it is seen to be the same as the other side of the equation.

Example: In each of (a) and (b) below, assume that m is an integer that is greater than or equal to 3. Write each of the sums in closed form.

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}$$

a. $1 + 3 + 3^2 + \cdots + 3^{m-2}$

b. $3^2 + 3^3 + 3^4 + \cdots + 3^m$

Solution:

a. $1 + 3 + 3^2 + \cdots + 3^{m-2} = \dfrac{3^{(m-2)+1} - 1}{3 - 1}$ by applying the formula for the sum of a geometric sequence with $r = 3$ and $n = m - 2$

$= \dfrac{3^{m-1} - 1}{2}.$

b. $3^2 + 3^3 + 3^4 + \cdots + 3^m = 3^2 \cdot (1 + 3 + 3^2 + \cdots + 3^{m-2})$ by factoring out 3^2

$= 9 \cdot \left(\dfrac{3^{m-1} - 1}{2}\right)$ by part (a).