Home Work 7

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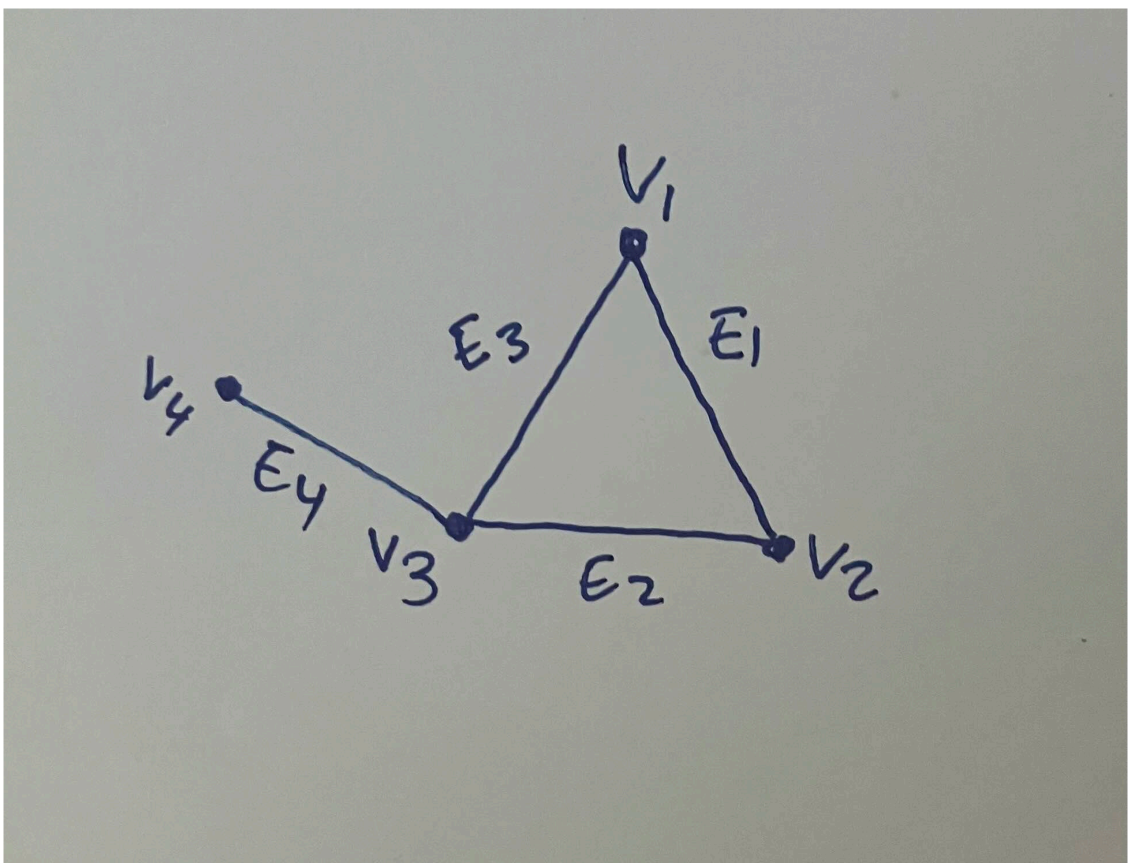
1 Graph Theory

1.1

Consider a graph G consisting of vertices V = {v1, v2, v3, v4} and edges E = {e1 = (v1, v2), e2 = (v2, v3), e3 = (v3, v1), e4 = (v3, v4)}.

(a) Undirected, because there are no indications of a direction for the edges, each edge connects to 2 vertices.



(b)

(c) v1 = 2, v2 = 2, v3 = 3, v4 =1

(d) Yes, there is a path to any of the vertices

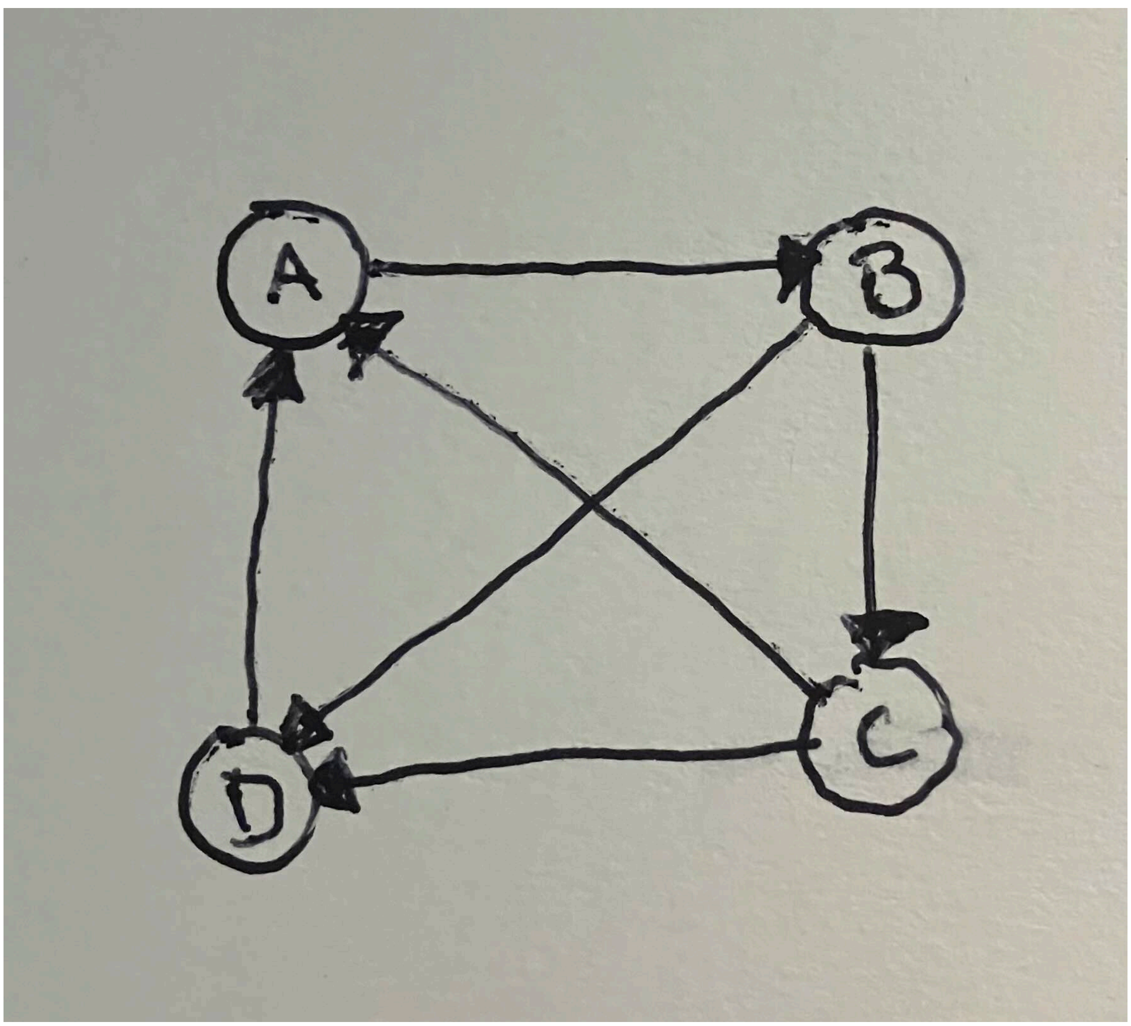
1.2

Define a multigraph and explain how it differs from a simple graph. Provide an example of a multigraph that includes at least one loop and multiple edges between two vertices. In your example, describe a possible real-world scenario that this multigraph could represent

Answer: A multigraph is a type of graph where multiple edges or loops can have the same vertices. Example V = {A,B}, E = {(A,A), (A,B), (A,B)}. A real world example could be roadways, ie: you could take two different routes on different roads, from the same start point and arrive at the same end point

1.3

Given the following list of edges in a directed graph: {(A, B), (B, C), (C, D), (D, A), (B, D), (C, A)},



(a)

(b)

- A → B → C → A; Simple
- A → B → D → A; Simple
- A → B → C → D → A; Simple

(c) No, there is no euler path. A and D both have 2 ins and 1 out, while B and C have 1 in and 2 outs. There is no path that could use each edge once.

1.4

A tournament graph is a directed graph D on n vertices such that there is exactly one directed edge between each pair of distinct vertices. Consider a tournament graph with 5 vertices.

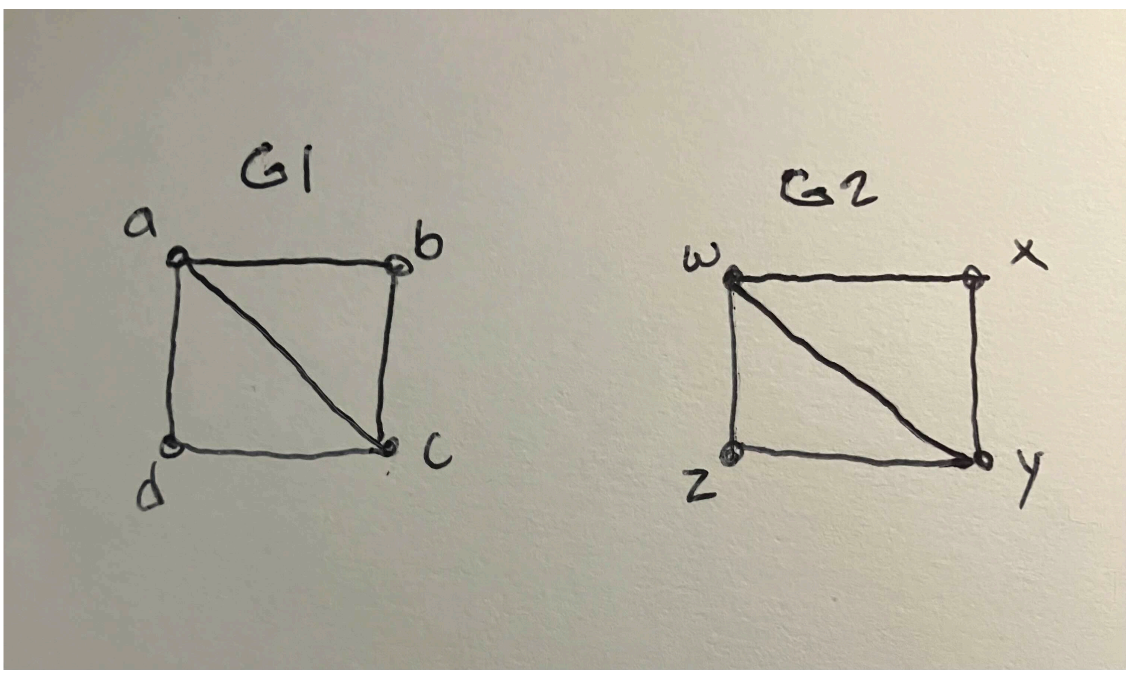
(a) 10 Unique edges

(b) No, Since a tournament graph is directed and must have 1 unique edge between each vertex, so each vertex will have a different number of degrees of in and outs

(c) No, simple graphs are graphs with only one edge between each vertex, but tournament graphs must have to have a unique edge between all vertices.

1.5

Analyze the following graphs: - G1 with vertices {a, b, c, d} and edges {(a, b), (b, c), (c, d), (d, a), (a, c)}, - G2 with vertices {w, x, y, z} and edges {(w, x), (x, y), (y, z), (z, w), (w, y)}.



(a)

(b)

(c) $V(G_1) \to v(G_2)$

$f(a) = w$

$f(b) = x$

$f(c) = y$

$f(d) = z$

(ii)

$E(G_1) \to E(G_2)$

$h((a, b)) = (w, x)$

$h((b, c)) = (x, y)$

$h((c, d)) = (y, z)$

$h((d, a)) = (z, w)$

$h((a, c)) = (w, y)$

(iii) Both edge and vertex matches one to one, so the graphs are isomorphic.