COT 2000
Foundations of Computing

Summer 2024

Lecture 4 – part 1

Homework 1 - Due:05/24/24
Lab 2
Exam 1 – 05/31/24

Lecture 4 – part 2

Review

Review

- Logic
- Statements or propositions
- Compound statements
- Compound statements notation for inequalities
- Negation (not), Conjunction (and), Disjunction (or)
- Truth tables
- Propositional forms, example: $(p \vee q) \wedge \sim(p \wedge q)$
- Exclusive OR

Statements or propositions

> **• Definition**
>
> A **statement** (or **proposition**) is a sentence that is true or false but not both.

Compound Statements

$$\sim p \wedge q = (\sim p) \wedge q$$

$$p \wedge q \vee r$$

$$(p \wedge q) \vee r \quad \text{or} \quad p \wedge (q \vee r)$$

The symbol $\sim$denotes *not,*
$\wedge$ denotes *and,*
and $\vee$ denotes *or.*

Notation for Inequalities

$x \leq a$	means	$x < a$	or	$x = a$
$a \leq x \leq b$	means	$a \leq x$	and	$x \leq b$.

Truth Values

Negation (~) (not)

• Definition

If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

Truth Table for $\sim p$

p	$\sim p$
T	F
F	T

Conjunction (∧) (and)

• Definition

If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

Truth Table for $p \wedge q$

p	q	$p \wedge q$
T	**T**	**T**
T	F	F
F	T	F
F	F	F

Disjunction (∨) (or)

• Definition

If p and q are statement variables, the **disjunction** of p and q is "p or q," denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

Truth Table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	**F**	**F**

Propositional forms

$(p \vee q) \wedge \sim(p \wedge q)$

• Definition

A **statement form** (or **propositional form**) is an expression made up of statement variables (such as p, q, and r) and logical connectives (such as $\sim$, $\wedge$, and $\vee$) that becomes a statement when actual statements are substituted for the component statement variables. The **truth table** for a given statement form displays the truth values that correspond to all possible combinations of truth values for its component statement variables.

Lecture 4 – part 3

More on Logic of Compound Statements

Example

Truth Table for $(p \wedge q) \vee \sim r$

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee \sim r$

Logical Equivalence

6 is greater than 2 and 2 is less than 6

(1) Dogs bark and cats meow and (2) Cats meow and dogs bark

p	q	$p \wedge q$	$q \wedge p$

Logical Equivalence

6 is greater than 2 and 2 is less than 6

(1) Dogs bark and cats meow and (2) Cats meow and dogs bark

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Example

- Are $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ logically equivalent ?

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$

Example

- Are $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ logically equivalent ?

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

De Morgan's Laws

"John is tall and Jim is redheaded"

"John is not tall or Jim is not redheaded."

$\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$

De Morgan's Laws

"John is tall and Jim is redheaded"

"John is not tall or Jim is not redheaded."

$\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\sim(p \wedge q)$ and $\sim p \vee \sim q$ always have the same truth values, so they are logically equivalent

De Morgan's Laws

Symbolically,

$$\sim(p \wedge q) \equiv \sim p \vee \sim q.$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q.$$

https://en.wikipedia.org/wiki/Augustus_De_Morgan

De Morgan's Laws

The negation of an *and* statement is logically equivalent to the *or* statement in which each component is negated.

The negation of an *or* statement is logically equivalent to the *and* statement in which each component is negated.

Example

Use De Morgan's laws to write the negation of $-1 < x \leq 4$.

$$-1 < x \quad \text{and} \quad x \leq 4.$$

The negation is:

$$-1 \not< x \quad \text{or} \quad x \not\leq 4,$$

Equivalent to:

$$-1 \geq x \quad \text{or} \quad x > 4.$$

Tautologies and Contradictions

Example:
The statement form $p \vee \sim p$ is a tautology and the statement form $p \wedge \sim p$ is a contradiction.

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

↑ all T's so $p \vee \sim p$ is a tautology

↑ all F's so $p \wedge \sim p$ is a contradiction

• Definition

A **tautology** is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a **tautological statement.**

A **contradication** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradication is a **contradictory statement.**

Example

If $\mathbf{t}$ is a tautology and $\mathbf{c}$ is a contradiction, show that $p \wedge \mathbf{t} \equiv p$ and $p \wedge \mathbf{c} \equiv \mathbf{c}$.

Solution

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p	$\mathbf{c}$	$p \wedge \mathbf{c}$

Example

If **t** is a tautology and **c** is a contradiction, show that $p \wedge \mathbf{t} \equiv p$ and $p \wedge \mathbf{c} \equiv \mathbf{c}$.

Solution

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p	$\mathbf{c}$	$p \wedge \mathbf{c}$
T	T	T	T	F	F
F	T	F	F	F	F

same truth values, so $p \wedge \mathbf{t} \equiv p$

same truth values, so $p \wedge \mathbf{c} \equiv \mathbf{c}$

Logical Equivalences

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* **t** *and* **c**:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Simplifying Statements Forms

Verify the logical equivalence of:

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p.$$

Solution:

$$\begin{aligned}
\sim(\sim p \wedge q) \wedge (p \vee q) &\equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q) && \text{by De Morgan's laws} \\
&\equiv (p \vee \sim q) \wedge (p \vee q) && \text{by the double negative law} \\
&\equiv p \vee (\sim q \wedge q) && \text{by the distributive law} \\
&\equiv p \vee (q \wedge \sim q) && \text{by the commutative law for } \wedge \\
&\equiv p \vee \mathbf{c} && \text{by the negation law} \\
&\equiv p && \text{by the identity law.}
\end{aligned}$$

Lecture 4 – part 4

Logic Exercises 1