

# COT2000 - Foundations of Computing

## Exam 2

### Instructions

Instruction page – please read very carefully.

**Date:** Friday, June 21, 2024

**Time Window:** 9:00 am - 9:00 pm

#### Format:

- **Location:** This test is administered remotely; there's no need to come to the classroom. The test will be accessible for download on Canvas during the designated time window.
- This is an open-book test. You may use: textbooks, lecture notes, personal notes, formulae pages, handouts, other supplementary materials prepared in advance. These materials can be either paper or electronic format.
- **Individual Work:** This test is meant to be completed independently. Collaboration is strictly prohibited. Do not discuss or share any details about the test or its solutions with anyone.

#### Submission:

- Download the test and print it to answer. If you cannot print it, write your answers clearly on separate sheets of paper. You can also use electronic form as long as you submit in PDF.
- **5 Bonus points if you submit a pdf compiled document using LaTeX.**
- Clearly show and explain your work for each question, where necessary.
- After completion, scan your test and submit it online via Canvas. Set aside at least 10 minutes for this process.
- Use a scanning app to convert your test into a single PDF. Ensure your submission is in the form of a single PDF file.
- Clearly write your name and Z number on your test.
- **File Naming Convention:** [Your Name].[Z Number].pdf
- While Canvas does allow multiple submissions, only the last one will be considered for grading.
- **Do Not share any information about the test or its solutions with others.**
- Exclude the instruction page from your submission.
- Do not include your formula sheets in your test submission.

Please review these instructions thoroughly to ensure a smooth testing experience. Best of luck!

Note: Inquiries about test results, homework and extra credit grades must be sent to the instructors within 3 days after grades are published.

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## Exam 2

Your Name:

Your zNumber:

### Questions

1. (10 points) Construct a truth table for the statement form.

$$(p \wedge r) \leftrightarrow (q \wedge r)$$

2. (10 points) In the following exercises: (a) use the logical equivalences  $p \rightarrow q \equiv \neg p \vee q$  and  $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$  to rewrite the given statement forms without using the symbol  $\rightarrow$  or  $\leftrightarrow$ , and (b) use the logical equivalence  $p \vee q \equiv \neg(\neg p \wedge \neg q)$  to rewrite each statement form using only  $\wedge$  and  $\neg$ .

$$p \vee \neg q \rightarrow r$$

3. (20 points) Write negations, contrapositives, converse and inverse for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

- (a) If  $P$  is a triangle, then  $P$  is an equilateral triangle.
- (b) If John is Mike's brother, then Sarah is his sister and Paul is his cousin.

4. (10 points) Which argument form exhibits the converse error?

- (a) If Alice finished her project, then she will present it. Alice presented her project. Therefore, Alice finished her project.
- (b) If I study, I will pass the exam. If I pass the exam, I will graduate. Therefore, if I study, I will graduate.
- (c) If either of the numbers is even, then their sum is even. Both numbers are odd. Therefore, their sum is odd.
- (d) Bob knows Python and Bob knows JavaScript. Therefore, Bob knows JavaScript.

5. (10 points) Use truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

(a)  $(p \vee q) \rightarrow \neg r$   
 $\neg p \wedge q$   
 $q \rightarrow p$   
 $\therefore \neg r$

(b)  $p \rightarrow q$   
 $r \rightarrow q$   
 $\therefore (p \vee r) \rightarrow q$

6. (10 points) Let  $Q(x)$  be the predicate " $x^2 < 2x$ ".

- (a) Write  $Q(1)$ ,  $Q(2)$ ,  $Q(0)$ ,  $Q(-1)$ , and  $Q(3)$ , and indicate which of these statements are true and which are false.

- (b) Find the truth set of  $Q(x)$  if the domain of  $x$  is  $\mathbb{R}$ , the set of all real numbers.
- (c) If the domain is the set  $\mathbb{R}^+$  of all positive real numbers, what is the truth set of  $Q(x)$ ?
7. (10 points) Which of the following is the truth set for the predicate  $R(x) : x^2 - 5x + 6 = 0$ , if the domain is  $\mathbb{R}$ ? Explain.
- (a)  $\{2, 3\}$
- (b)  $\{-2, 3\}$
- (c)  $\{3, 4\}$
- (d)  $\{1, 6\}$
8. (10 points) Write a formal negation for each of the following statements:
- (a)  $\forall$  birds  $x, x$  can fly.
- (b)  $\forall$  cars  $c, c$  has wheels.
- (c)  $\exists$  a building  $b$  such that  $b$  is over 100 stories tall.
- (d)  $\exists$  a tree  $t$  such that  $t$  is over 100 years old.
9. (10 points) Consider the following definitions: Let  $D$  be the set of all students at your school,  $M(s)$  represents "student  $s$  is an engineering major," and  $S(s)$  represents "student  $s$  is a physics major." What is the correct representation of the statement: "Every engineering major is also a physics major."?
- (a)  $\forall s \in D, M(s) \Rightarrow S(s)$
- (b)  $\exists s \in D, M(s) \wedge \neg S(s)$
- (c)  $\forall s \in D, S(s) \Rightarrow M(s)$
- (d)  $\exists s \in D, S(s) \vee M(s)$