COT2000 - Foundations of Computing
Exam 3

Instructions

Instruction page – please read very carefully.

Date: Friday, July 12, 2024
Time Window: 9:00 am - 9:00 pm

Format:

- **Location:** This test is administered remotely; there's no need to come to the classroom. The test will be accessible for download on Canvas during the designated time window.
- This is an open-book test. You may use: textbooks, lecture notes, personal notes, formulae pages, handouts, other supplementary materials prepared in advance. These materials can be either paper or electronic format.
- **Individual Work:** This test is meant to be completed independently. Collaboration is strictly prohibited. Do not discuss or share any details about the test or its solutions with anyone.

Submission:

- Download the test and print it to answer. If you cannot print it, write your answers clearly on separate sheets of paper. You can also use electronic form as long as you submit in PDF.
- **5 Bonus points if you submit a pdf compiled document using LaTeX.**
- Clearly show and explain your work for each question, where necessary.
- After completion, scan your test and submit it online via Canvas. Set aside at least 10 minutes for this process.
- Use a scanning app to convert your test into a single PDF. Ensure your submission is in the form of a single PDF file.
- Clearly write your name and Z number on your test.
- **File Naming Convention:** `[Your Name]_[Z Number].pdf`
- While Canvas does allow multiple submissions, only the last one will be considered for grading.
- **Do Not share any information about the test or its solutions with others.**
- Exclude the instruction page from your submission.
- Do not include your formula sheets in your test submission.

Please review these instructions thoroughly to ensure a smooth testing experience. Best of luck!

Note: Inquiries about test results, homework and extra credit grades must be sent to the instructors within 3 days after grades are published.

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Your Name:

Your zNumber:

Questions

1. **(15 points)** $a_n = 5 + \left(\frac{1}{2}\right)^n$, for all integers $n \geq 0$.
   (a) Write the first four terms of the sequence.
   (b) Show that the sequence converges to 5 as n approaches infinity.

2. **(15 points)** Find explicit formulas for sequences of the form $b_1, b_2, b_3, \ldots$ with the initial terms given in the following exercise:
   (a) $2, 4, 6, 8, 10, 12$
   (b) $3, -3, 3, -3, 3, -3$
   (c) $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$

3. **(15 points)** Compute the summations and products:
   (a) $\sum_{k=2}^{7}(k+3)$
   (b) $\prod_{k=2}^{4}(k+1)^2$

4. **(15 points)** Let $X = \{1, 3, 5, 7, 9\}$ and $Y = \{2, 4, 6, 8, 10\}$.
   (a) Define $f : X \to Y$ by specifying that

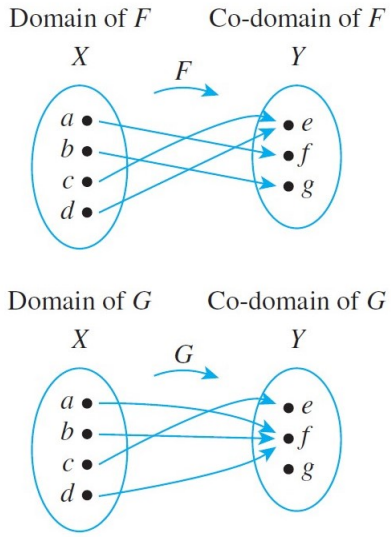
$$\begin{aligned} f(1) &= 8, \\ f(3) &= 6, \\ f(5) &= 4, \\ f(7) &= 8, \\ f(9) &= 10. \end{aligned}$$

   Is f one-to-one? Is f onto? Explain your answers.
   (b) Define $g : X \to Y$ by specifying that:

$$\begin{aligned} g(1) &= 2, \\ g(3) &= 4, \\ g(5) &= 6, \\ g(7) &= 8, \\ g(9) &= 10. \end{aligned}$$

   Is g one-to-one? Is g onto? Explain your answers.

5. **(20 points)** Let $X = \{a, b, c, d\}$ and $Y = \{e, f, g\}$. Define functions F and G by the arrow diagrams below.



(a) Is F one-to-one? Why or why not? Is it onto? Why or why not?

(b) Is G one-to-one? Why or why not? Is it onto? Why or why not?

(c) Redefine F and G to make them one-to-one correspondences. Explain your answer.

6. **(20 points)**

(a) Define $h : \mathbf{Z} \to \mathbf{Z}$ by the rule $h(x) = 2x + 1$, for all integers x.

i. Is h one-to-one? Prove or provide a counterexample.

ii. Is h onto? Prove or provide a counterexample.

(b) Define $g : \mathbf{R} \to \mathbf{R}$ by the rule $g(x) = x^2 - 1$ for all real numbers x.

i. Is g one-to-one? Prove or provide a counterexample.

ii. Is g onto? Prove or provide a counterexample.