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COT2000 - Foundations of Computing
Exam 1 - Solution Key

Questions

1. **(15 points) Given the sets:**

$A = \{2, 3, \{4\}\}$
$B = \{3, 2, \{4\}\}$
$C = \{\{4\}, +\sqrt{9}, 2\}$
$D = \{3, 2, 4\}$
$E = \{\text{"apple"}, \text{"banana"}, \{\text{"cherry"}\}\}$
$F = \{x \in \mathbb{Z} \mid 1 < x < 5\}$

A = B = C because the +sqrt(9) =3 and order does not matter.
D=F because 1<x<5 is 2,3,4

(a) Specify which of the given sets are equal to each other.

(b) For each set, determine the number of elements it contains. All sets contain 3 elements

(c) Determine if the following statements are true or false:

i. $\{4\} \subseteq A$ False, true would be, {{4}} ⊆ A

ii. $\{4\} \subseteq D$ True

iii. $\{\text{"cherry"}\} \subseteq E$ False, true would be, {{ “cherry” }} ⊆ E

iv. $\{\{4\}\} \subseteq C$ True

v. $\{2, 3\} \subseteq B$ True

2. **(15 points) Use the set-roster notation to write the following sets, and indicate the number of elements:** ($\mathbb{Z}$ means the integers)

(a) $V = \{t \in \mathbb{Z} \mid t > -3 \text{ and } t < 7\}$. V={−2,−1,0,1,2,3,4,5,6} , elements 9

(b) $V = \{t \in \mathbb{Z} \mid t < -3 \text{ or } t > 7\}$. V={ - infinity to -4, 8 to infinity}, infinite elements

(c) Let $A = \{p, q, r\}$ and $B = \{x, y\}$, Find $B \times A$ B×A={(x,p),(x,q),(x,r),(y,p),(y,q),(y,r)} , 6 elements

3. **(15 points) Let $A = \{m, n, o, p\}$ and $B = \{g, h\}$.**
Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

(a) $A \times B$ A×B={(m,g),(m,h),(n,g),(n,h),(o,g),(o,h),(p,g),(p,h)} , 8 elements

(b) $B \times A$ B×A={(g,m),(g,n),(g,o),(g,p),(h,m),(h,n),(h,o),(h,p)} , 8 elements

(c) $A \times A$ A×A={(m,m),(m,n),(m,o),(m,p),(n,m),(n,n),(n,o),(n,p),(o,m),
(o,n),(o,o),(o,p),(p,m),(p,n),(p,o),(p,p)} 16 elements

(d) $B \times B$

B×B={(g,g),(g,h),(h,g),(h,h)} , 4 elements

4. **(20 points) Answer each of the following:**

Let $A = \{10, 11, 12\}$ and $B = \{2, 3, 4\}$. Define a relation R from A to B as follows:

$$\text{For all } (x, y) \in A \times B, (x, y) \in R \text{ if and only if } \frac{x}{y+1} \text{ is an integer.}$$

(a) Write $A \times B$ in set-roster notation A×B={(10,2),(10,3),(10,4),(11,2),(11,3),(11,4),(12,2),(12,3),(12,4)}

(b) Determine the validity of the following:

i. Is $12 \, R \, 3$? Yes, 12/3+1 = 3, 3 is an integer

ii. Is $(11, 4) \in R$? no, 11/4+1= 2.2 not an integer

(c) Write R as a set of ordered pairs.

R={(10,4),(12,2),(12,3)}

(d) Identify the domain and co-domain of R. Domain: {10,12} Co-Domain: {2,3,4}

(e) Is R a function, explain. Yes, each element in the domain is related to one element in the co-domain

5. **(15 points) Write each statement in symbolic form.**

Let p = "I enjoy programming", q = "I will graduate", and r = "I will complete my project." Express each of the following propositions in symbolic form:

(a) i. I enjoy programming and I will graduate. p ^ q

ii. I will complete my project or I will not graduate. r v ~p

iii. It is not true that I both enjoy programming and will complete my project. ~(p ^ r)

iv. I will not complete my project and I will not graduate. ~r ^ ~q

(b) For each of the following propositions, identify simple propositions, express the compound proposition in symbolic form, and determine whether it is true or false:

i. The moon is made of cheese or one plus one equals two. p v q, false neither statement is true, need one

ii. If 1,000,000 is a multiple of 5, then 1,000,000 is even. p -> q, True, both are true

iii. 7 is a prime number and 9 is not divisible by 3. p ^ q, False, p=true q=false, both need to be true to = true

iv. $4 \in \mathbb{Z}$ and $4 \in \mathbb{Q}$. p ^ q, True, both are true

v. $\frac{3}{4} \in \mathbb{Z}$ and $\frac{3}{4} \in \mathbb{Q}$. (Note: Q is the rational numbers and Z is the integer numbers.) p ^ q , Flase, p=false q=true

vi. The product of two odd integers is odd and the product of two even integers is even. p ^ q, True both are true

6. **(20 points) Write the truth table for the following statement forms:**

(a) $p \vee (\neg q \wedge r)$

(b) $(p \vee q) \wedge (\neg p \vee (q \wedge \neg r))$

(c) **Determine whether the statement forms are logically equivalent. Explain.**

$p \wedge (p \vee q)$ equivalent to the statement p

7. **(Bonus 5 points)** Expand the compound statement $(p \oplus q) \wedge r$ using the definition of Exclusive OR? Explain.

p	q	r	~q	~q ∧ r	p V(~q ∧ r)
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	F	F

p	q	r	~p	~r	p V q	q ∧ ~r	(~p V (q ∧ ~r))	(p V q) ∧ (~p V (q ∧ ~r))
T	T	T	F	F	T	F	F	F
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	F	F	F
T	F	F	F	T	T	F	F	F
F	T	T	T	F	T	F	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	F	T	F
F	F	F	T	T	F	F	T	F

6. C) No truth tables do not match