Homework 2

Due Date: 06/07/2024

- Please submit your answers as a single consolidated PDF file.

- Upload this file to Canvas.

- You may submit multiple times, but only the last submission made before the due date will be considered for grading.

- Each question is 10 points.

- Bonus(10 points) if you submit a .pdf compiled document made in LaTeX.

Questions

1. **Write the following statements in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements.**

   Let h represent the statement "John is healthy," w represent the statement "John is wealthy," and s represent the statement "John is wise."

   (a) John is healthy and wealthy but not wise.
   (b) John is not wealthy but he is healthy and wise.
   (c) John is neither healthy, wealthy, nor wise.
   (d) John is neither wealthy nor wise, but he is healthy.
   (e) John is wealthy, but he is not both healthy and wise.

2. **Write truth tables for the statement forms**

   (a) $\sim (p \wedge q) \vee (p \vee q)$
   (b) $p \wedge (\sim q \vee r)$

3. **Determine whether the statement forms are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.**

(a) $p \vee (p \wedge q)$ and q

(b) $\sim (p \wedge q)$ and $\sim p \wedge \sim q$

(c) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

4. **Assume x is a particular real number and use De Morgan's laws to write negations for the statements**

   (a) $-2 < x < 6$

   (b) $-9 < x < 2$

   (c) $x < 2$ or $x > 6$

   (d) $x \leq -1$ or $x > 1$

   (e) $0 > x \geq -4$

5. **Use truth tables to establish which of the statement forms are tautologies and which are contradictions.**

   (a) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

   (b) $(p \wedge \sim q) \wedge (\sim p \vee q)$

   (c) $((\sim p \wedge q) \wedge (q \wedge r)) \vee \sim q$

6. **Determine whether the statements in (a) and (b) are logically equivalent.**

   (a) Bob is a double math and computer science major and Ann is a math major, but Ann is not a double math and computer science major.

   (b) It is not the case that both Bob and Ann are double math and computer science majors, but it is the case that Ann is a math major and Bob is a double math and computer science major.

7. **Using Exclusive OR**
   Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? Justify your answer.

8. **What logical equivalence is used for each step.**

$$
\begin{aligned}
(p \wedge \sim q) \vee (p \wedge q) &\equiv p \wedge (\sim q \vee q) && \text{by (a)} \\
&\equiv p \wedge (q \vee \sim q) && \text{by (b)} \\
&\equiv p \wedge t && \text{by (c)} \\
&\equiv p && \text{by (d)}
\end{aligned}
$$

   Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

   Verify the logical equivalences. Supply a reason for each step.

9. $(p \wedge \sim q) \vee p \equiv p$

10. $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$