Home Work 5

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Question 1

Fill in each blank with the word most or least.

(a) A function F is one-to-one if, and only if, each element in the co- domain of F is the image of at $\mathbf{MOST}$ one element in the domain of F .

(b) A function F is onto if, and only if, each element in the co-domain of F is the image of at $\mathbf{LEAST}$ one element in the domain of F .

Question 2

When asked to state the definition of one-to-one, a student replies, "A function f is one-to-one if, and only if, every element of X is sent by f to exactly one element of Y ." Give a counterexample to show that the student's reply is incorrect.

Counter:

Let $X = \{1, 2\}$ and $Y = \{3\}$.

Define $f : X \to Y$ by:

- $f(1) = 3$
- $f(2) = 3$

Question 3

Let $X = \{1, 5, 9\}$ and $Y = \{3, 4, 7\}$.

$\mathbf{(a)}$ Define $f : X \to Y$ by specifying that

$f(1) = 4$,

$f(5) = 7$,

$f(9) = 4$.

Is f one-to-one? Is f onto? Explain your answers.

- One-To-One: No, because $f(1) = 4$ and $f(9) = 4$
- Onto: No, because 3 in Y is not an image of any element in X

(b) Define $g : X \to Y$ by specifying that

$g(1) = 7$,

$g(5) = 30$,

$g(9) = 4$.

Is g one-to-one? Is g onto? Explain your answers.

- One-To-One: Yes, because all images are distinct.
- Onto: Yes, because every element in Y is an image of some element in X

Question 4

Let $X = \{a, b, c, d\}$ and $Y = \{e, f, g\}$. Define functions F and G by the arrow diagrams below.

(a) Is F one-to-one? Why or why not? Is it onto? Why or why not?

- No its not One-To-One, because c and d map to e.
- But it is Onto, because all elements of Y are covered by an element of X

(b) Is G one-to-one? Why or why not? Is it onto? Why or why not?

- Neither One-To-One or Onto, because f is covered by a,b,and d. And g is not covered by any element.

Question 5

Let $X = \{a, b, c\}$ and $Y = \{w, x, y, z\}$. Define functions H and K by the arrow diagrams below.

(a) Is H one-to-one? Why or why not? Is it onto? Why or why not?

- H is neither, y is covered by b and c, and x and z are not covered.

(b) Is K one-to-one? Why or why not? Is it onto? Why or why not?

- K is One-To-One, because every element of X maps to only one element of Y, but it is not onto because z is not covered by any element of X.

Question 6

Let X = {1, 2, 3}, Y = {1, 2, 3, 4}, and Z = {1, 2}.

(a) Define a function Let f : X → Y that is one-to-one but not onto.

- $f(1)=1,

f(2)=2,

f(3)=3$

(b) Define a function g : X → Z that is onto but not one-to-one.

- $g(1)=1,

g(2)=2,

g(3)=2$

(c) Define a function h : X → X that is neither one-to-one nor onto.

- $h(1)=2,

h(2)=2,

h(3)=2$

(d) Define a function k : X → X that is one-to-one and onto but is not the identity function on X.

- $k(1)=2,

k(2)=3,

k(3)=1$

Question 7

(a) Define $g : Z \to Z$ by the rule $g(n) = 4n - 5$, for all integers n.

i. Is g one-to-one? Prove or give a counterexample.

- Yes. Proof: $g(n_1) = g(n_2)$, then $4n_1 - 5 = 4n_2 - 5$. So, $n_1 = n_2$

ii. Is g onto? Prove or give a counterexample.

- Yes, Proof: For any $m \in \mathbb{Z}$, $n = \frac{m+5}{4}$ is an integer, so $g(n) = m$

(b) Define $G : R \to R$ by the rule $G(x) = 4x - 5$ for all real numbers x.

Is G onto? Prove or give a counterexample.

- Yes, Proof: For any $y \in \mathbb{R}, x = \frac{y+5}{4}$ is a real number, so $G(x) = y$

Question 8

(a) Define $H : R \to R$ by the rule $H(x) = x2$, for all real numbers x.

i. Is H one-to-one? Prove or give a counterexample.

- No, Counter: $H(1) = 1$ and $H(-1) = 1$

ii. Is H onto? Prove or give a counterexample.

- No, Counter: There is no $x \in \mathbb{R}$ that $x^2 = -1$

(b) Define $K : R^{nonneg} \to R^{nonneg}$ by the rule $K(x) = x2$, for all non- negative real numbers x. Is K onto? Prove or give a counterexample.

- Yes. Proof: For any $y \geq 0, x = \sqrt{y}$ is a non-neg real number, so $K(x) = y$

In each of the following a function f is defined on a set of real numbers. Determine whether or not f is one-to-one and justify your answer.

Question 9

$f(x) = \frac{x+1}{x}$, for all real numbers $x \neq 0$

- Yes, Proof: if $\frac{x_1+1}{x_1} = \frac{x_2+1}{x_2}$, then $x_1 = x_2$

Question 10

$f(x) = \frac{x}{x^2+1}$, for all real numbers x

- No, Counter: $f(1) = \frac{1}{2}$ and $f(-1) = \frac{-1}{2}$