Homework 4

Due Date: 06/28/2024

- Please submit your answers as a single consolidated PDF file, and upload the file to Canvas.
- You may submit multiple times, but only the last submission made before the due date will be considered for grading.
- Make sure you submit the right file to Canvas. Wrong file submissions will not be graded.
- Note: Inquiries about homework must be sent to the TAs or instructor within 3 days after grades are published.
- **10 Bonus points** if you create your document in LaTeX and submit the compiled result in .pdf.

Questions

1. Write the first four terms of the sequences defined by the formulas:
   (a) $a_k = \dfrac{2k}{5+k}$, for all integers $k \geq 1$.
   (b) $b_j = \dfrac{4-j}{4+j}$, for all integers $j \geq 1$.
2. $e_m = 2 + \left(\dfrac{1}{3}\right)^m$, for all integers $m \geq 0$.
   (a) Write the first four terms of the sequence.
   (b) Show that the sequence converges to 2 as m approaches infinity.
3. Find explicit formulas for sequences of the form $a_1, a_2, a_3, \ldots$ with the initial terms given in the following exercise:
   (a) $1, -1, 1, -1, 1, -1$
   (b) $0, -1, 2, -3, 4, -5$
   (c) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

4. Compute the summations and products:

   (a) $\sum_{k=1}^{6}(k+2)$

   (b) $\prod_{k=3}^{5} k^2$

5. Prove the following proposition using mathematical induction:

$$P(n) : 1 + 2 + 4 + 8 + \cdots + 2^n = 2^{n+1} - 1$$

   (a) Prove the base case $P(0)$.

   (b) Assume $P(k)$ is true and prove $P(k+1)$.