COT2000 - Foundations of Computing
Exam 4

Instructions

Instruction page – please read very carefully.

Date: Friday, August 2, 2024
Time Window: 9:00 am - 9:00 pm

Format:

- **Location:** This test is administered remotely; there's no need to come to the classroom. The test will be accessible for download on Canvas during the designated time window.
- This is an open-book test. You may use: textbooks, lecture notes, personal notes, formulae pages, handouts, other supplementary materials prepared in advance. These materials can be either paper or electronic format.
- **Individual Work:** This test is meant to be completed independently. Collaboration is strictly prohibited. Do not discuss or share any details about the test or its solutions with anyone.

Submission:

- Download the test and print it to answer. If you cannot print it, write your answers clearly on separate sheets of paper. You can also use electronic form as long as you submit in PDF.
- **5 Bonus points if you submit a pdf compiled document using LaTeX.**
- Clearly show and explain your work for each question, where necessary.
- After completion, scan your test and submit it online via Canvas. Set aside at least 10 minutes for this process.
- Use a scanning app to convert your test into a single PDF. Ensure your submission is in the form of a single PDF file.
- Clearly write your name and Z number on your test.
- **File Naming Convention:** `[Your Name]_[Z Number].pdf`
- While Canvas does allow multiple submissions, only the last one will be considered for grading.
- **Do Not share any information about the test or its solutions with others.**
- Exclude the instruction page from your submission.
- Do not include your formula sheets in your test submission.

Please review these instructions thoroughly to ensure a smooth testing experience. Best of luck!

Note: No late submissions will be accepted after the time window concludes and after the solution key is published.

COT2000 - Foundations of Computing
Exam 4

Your Name:

Your zNumber:

Questions

1. **(20 points)**: Consider a graph G consisting of vertices $V = \{v_1, v_2, v_3, v_4, v_5\}$ and edges $E = \{e_1 = (v_1, v_2), e_2 = (v_2, v_3), e_3 = (v_3, v_1), e_4 = (v_2, v_4), e_5 = (v_4, v_5), e_6 = (v_4, v_4)\}$.
   (a) Assume G is directed. Are there any circuits in G? If so, identify them.
   (b) Draw G and label each vertex and edge accordingly.
   (c) Determine the in-degree and out-degree of each vertex in G.
   (d) Create an induced sub-graph F by removing v_2
   (e) Has F different connected components ?

2. **(20 points)** Define a multigraph and explain how it differs from a simple graph. Provide an example of a multigraph that includes at least **two loops** and multiple directed edges between any two vertices. In your example, describe a possible real-world scenario that this multi-graph could represent.

3. **(20 points)**: Given the following list of edges in a directed graph:
   $E = \{(A, B), (B, C), (C, D), (D, A), (C, E), (E, C)\}$,
   (a) Draw the directed graph.
   (b) Identify any circuits within the graph. If any, classify them as simple or not.
   (c) Does this graph have an Euler path or circuit? Justify your answer. **Definition (Euler Path):** An Euler path in a graph is a path that uses every edge of the graph exactly once. If such a path exists, the graph is said to be *traversable*. Note that an Euler path does not need to start and end at the same vertex. In contrast, an ***Euler Circuit*** is an Euler path which starts and ends on the same vertex, using every edge exactly once.

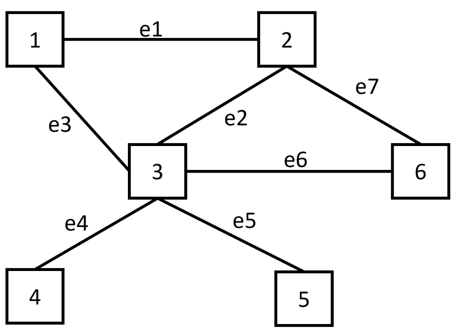
4. **(20 points)**: Analyze the following graphs:
   - G_1 with vertices $\{a, b, c, d\}$ and edges $\{(a, b), (b, c), (c, d), (d, a), (a, c)\}$,
   - G_2 with vertices $\{w, x, y, z\}$ and edges $\{(w, x), (x, y), (y, z), (z, w), (w, y), (x, z)\}$.

   (a) Draw both G_1 and G_2.
   (b) Assess whether G_1 and G_2 are isomorphic by:
      i. Defining a function $f : V(G_1) \to V(G_2)$ to map vertices from G_1 to G_2.
      ii. Defining a function $h : E(G_1) \to E(G_2)$ to map edges from G_1 to G_2, ensuring each edge in G_1 corresponds to a unique edge in G_2 based on the vertex mapping f.
      iii. Demonstrating that f and h are one-to-one correspondence verifying that for every edge $e = (u, v)$ in $E(G_1)$, there is an edge $h(e) = (f(u), f(v))$ in $E(G_2)$.

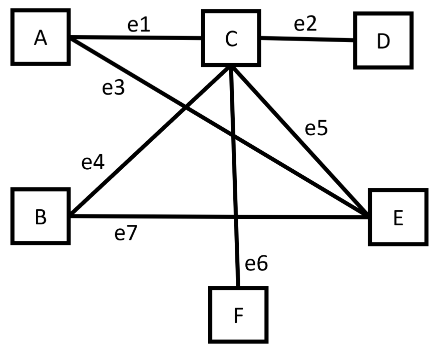
Utilize the definitions of graph isomorphism, function f, and function h as discussed in class to substantiate your arguments.

5. **(20 points)** Consider the following graphs G1 and G2.

G1



G2



   (a) Assess whether G_1 and G_2 are isomorphic. Explain.
   (b) Represent G1 using adjacency matrix.
   (c) Represent G2 using an adjacency matrix.
   (d) What is the degree of each vertex on G1 and G2.

6. **(BONUS 10 points)** Consider the un-directed sub-graph G_1 represented by the following 3x3 adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now, consider another un-directed graph G_2 represented by this 4x4 adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   (a) How many 3x3 subgraphs can be obtained from the 4x4 matrix of G_2 that are isomorphic to G_1? Describe the process of determining this.
   (b) Draw G1 and G2.