COT 2000

Foundations of Computing

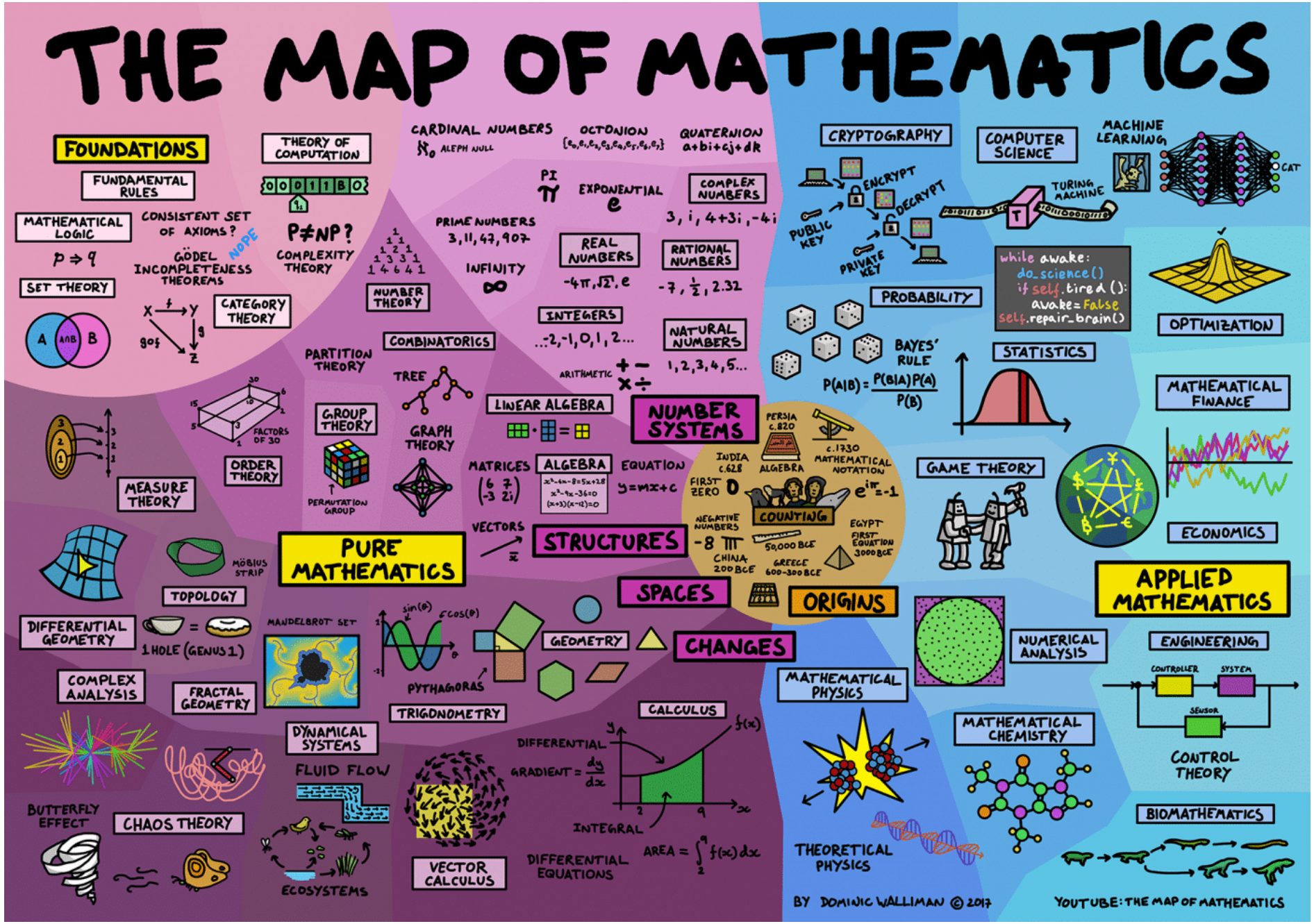
Spring 2024

Lecture 1 – part 1

Introduction

Introduction

- Introduction to the **mathematical topics** that are useful in computer science.
- Foundational knowledge in **discrete mathematics** and its application in computing.



Domain of Science is produced by physicist Dominic Walliman

https://store.dftba.com/products/map-of-mathematics-poster

Why discrete mathematics ?

- Core of many **algorithms and data structures**.
- Basis for **computer hardware** design (digital logic).
- Essential for **formal reasoning & proof** in computer science.
- Guides **efficiency and optimization** in computing tasks.

Lecture 1 – part 1

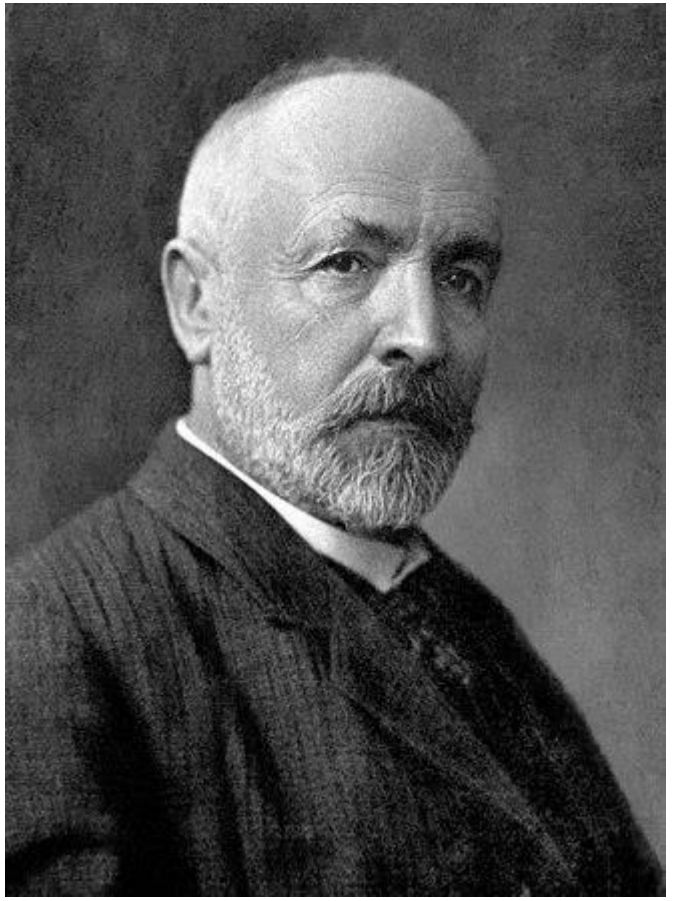
Syllabus

Lecture 1 – part 2

Language of Sets

Set theory

- While rudimentary concepts of sets and groupings can be traced back to ancient civilizations, a systematic and formal approach to set theory was not developed **until the 19th century**.
- The word set as a formal mathematical term was introduced in 1879 by **Georg Cantor** (1845–1918).
- **Set theory** is commonly employed as a foundational system for the **whole of mathematics !**.



https://en.wikipedia.org/wiki/Georg_Cantor

Set notation

- A set is a collection of objects (elements)
- For instance, if C is the set of all countries that are currently in the United Nations, then the United States is **an element of** C.
- If I is the set of all integers from 1 to 100, then the number 57 is **an element of** I .
- Symbol for an element that belongs to a set.
- Set-roster notation.
- Set-builder notation.

The symbol $\in$

Let's define a set A that consists of the first five positive integers:

$$A = \{1, 2, 3, 4, 5\}$$

Using the set-roster notation, the set A is represented by explicitly listing its elements within curly braces.

Belonging:

The symbol $\in$ denotes "belongs to" or "is an element of". For instance, considering the number 3 and set A:

$$3 \in A$$

This reads as "3 belongs to A" or "3 is an element of A".

Not Belonging:

The symbol $\notin$ denotes "does not belong to" or "is not an element of". Let's consider the number 7 and set A:

$$7 \notin A$$

This reads as "7 does not belong to A" or "7 is not an element of A".

Using the Set-Roster Notation

1. Let $A = \{1, 2, 3\}$, $B = \{3, 1, 2\}$, and $C = \{1, 1, 2, 3, 3, 3\}$. What are the elements of A, B, and C? How are A, B, and C related?
2. Is $\{0\} = 0$?
3. How many elements are in the set $\{1, \{1\}\}$?
4. For each nonnegative integer n, let $U_n = \{n, -n\}$. Find U_1, U_2, and U_0.

Solution

1. A, B, and C have exactly the same three elements: 1, 2, and 3. Therefore, A, B, and C are simply different ways to represent the same set.
2. $\{0\} \neq 0$ because $\{0\}$ is a set with one element, namely 0, whereas 0 is just the symbol that represents the number zero.
3. The set $\{1, \{1\}\}$ has two elements: 1 and the set whose only element is 1.
4. $U_1 = \{1, -1\}$, $U_2 = \{2, -2\}$, $U_0 = \{0, -0\} = \{0, 0\} = \{0\}$.

Set-builder Notation

Let S denote a set and let $P(x)$ be a property that elements of S may or may not satisfy. We may define a new set to be the set of all elements x in S such that $P(x)$ is true. We denote this set as follows:

$$\{x \in S \mid P(x)\}$$

Using the Set-Builder Notation

Given that $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{Z}$ the set of all integers, and $\mathbb{Z}^+$ the set of all positive integers, describe each of the following sets.

1. $\{x \in \mathbb{R} \mid -2 < x < 5\}$
2. $\{x \in \mathbb{Z} \mid -2 < x < 5\}$
3. $\{x \in \mathbb{Z}^+ \mid -2 < x < 5\}$

Solution

1. $\{x \in \mathbb{R} \mid -2 < x < 5\}$ is the open interval of real numbers (strictly) between -2 and 5.
2. $\{x \in \mathbb{Z} \mid -2 < x < 5\}$ is the set of all integers (strictly) between -2 and 5. It is equal to the set $\{-1, 0, 1, 2, 3, 4\}$.
3. Since all the integers in $\mathbb{Z}^+$ are positive, $\{x \in \mathbb{Z}^+ \mid -2 < x < 5\} = \{1, 2, 3, 4\}$.

Subsets

A basic relation between sets is that of subset.

Definition:
If A and B are sets, then A is called a subset of B, written $A \subseteq B$, if every element of A is also an element of B.

Symbolically:

$$A \subseteq B$$

means if $x \in A$ then $x \in B$.

If A is not a subset of B, we write $A \nsubseteq B$. This means there's at least one element in A which is not in B.

The phrases "A is contained in B" and "B contains A" are alternative ways of saying that A is a subset of B.

Definition:
Let A and B be sets. A is a **proper subset** of B if every element of A is in B, but there exists an element in B which is not in A.

More exercises

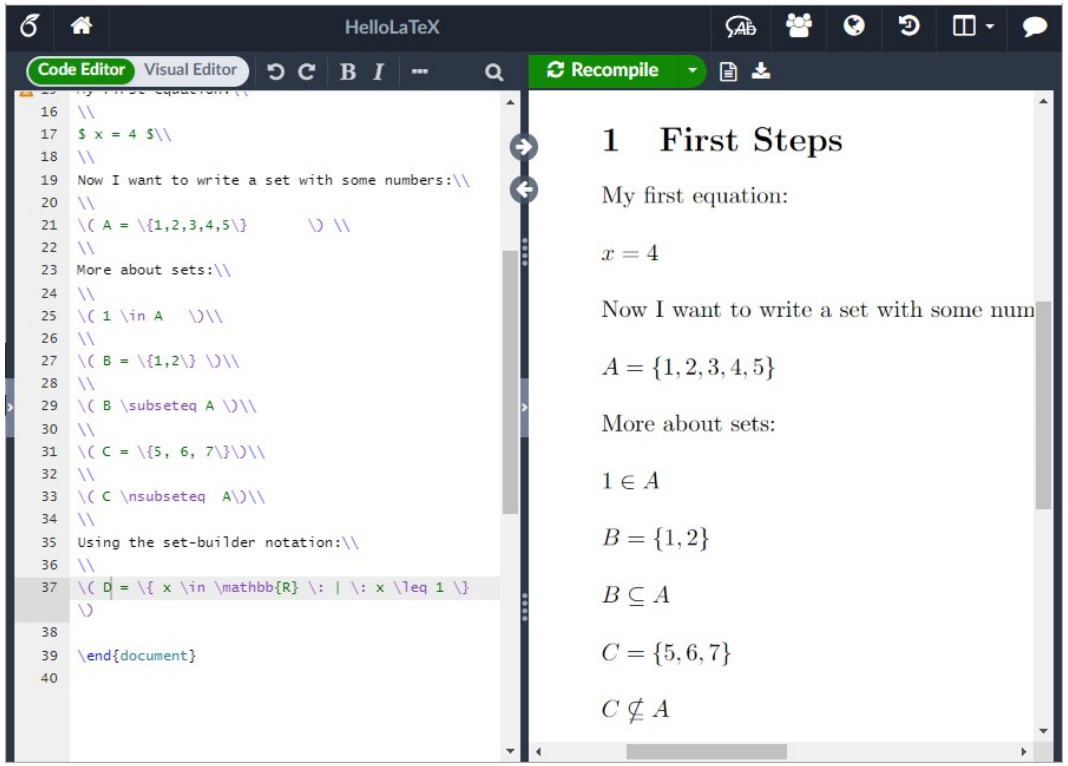
Lecture 1 – part 3

LaTex – Lab 1

LaTeX

- **LaTeX** is a high-quality typesetting system.
- **LaTeX** was created by Leslie Lamport in the 1980s.
- Designed for the production of technical and scientific documentation.
- Different paradigm than “**WYSIWYG**”.
- Authors can focus on content.
- A TeX engine renders the style.

https://www.latex-project.org/



text PDF

LaTeX code

```
1205          \end{tabular}
1206          \label{unitvectorderivatives}
1207      \end{center}
1208  \end{table}
1209
1210  \subsection{Partial derivatives with respect to
      $\theta$:}\label{thetaderivatives}
1211  We proceed by individually applying the operation
      $\displaystyle\frac{\partial ()}{\partial \theta}$ to each component of
      equation \eqref{E:VPartialDerivativesTheta}. This detailed process unfolds
      through the following subsections (a)-(h), each illustrating a separate
      derivative calculation:
1212
1213  % WITH RESPECT TO THETA LEFT
1214  % First component
1215  \subsubsection{(a) The $\displaystyle\frac{\partial}{\partial
      \theta}\left[\left(\frac{-t_r}{r}\right)\mathbf{e}_r\right]$ component:}
1216
1217  \begin{align}
1218      \frac{\partial}{\partial
      \theta}\left[\left(\frac{-t_r}{r}\right)\mathbf{e}_r\right]
1219      &=  \frac{\partial}{\partial
      \theta}\left(\frac{-t_r}{r}\right)\mathbf{e}_r +
      \left(\frac{-t_r}{r}\right)\frac{\partial\mathbf{e}_r}{\partial \theta}
1220      \notag\\
1221      &=  (-t_r)\frac{\partial}{\partial
      \theta}\left(\frac{1}{r}\right)\mathbf{e}_r
      + \left(\frac{1}{r}\right)\frac{\partial(-\mathbf{t} \cdot
      \mathbf{e}_r)}{\partial \theta}\mathbf{e}_r
1223      + \left(\frac{-t_r}{r}\right)\cos(\phi)\mathbf{e}_\theta
1224      \notag\\
1225      &=  (-t_r)\left(\frac{-1}{r^2}\frac{\partial r}{\partial
```

Line: 1210 Column: 47 INSERT

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Process started: pdflatex.exe -synctex=1 -interaction=nonstopmode "JuanYepes-PhD-Dissertation-2023-v3".tex

Process exited normally

PDF output

6.2.1 Partial derivatives with respect to θ:

We proceed by individually applying the operation $\frac{\partial()}{\partial\theta}$ to each component of equation (6.8). This detailed process unfolds through the following subsections (a)-(h), each illustrating a separate derivative calculation:

(a) The $\frac{\partial}{\partial\theta}\left[\left(\frac{-t_r}{r}\right)\mathbf{e}_r\right]$ *component:*

$$
\begin{aligned}
\frac{\partial}{\partial\theta}\left[\left(\frac{-t_r}{r}\right)\mathbf{e}_r\right] &= \frac{\partial}{\partial\theta}\left(\frac{-t_r}{r}\right)\mathbf{e}_r + \left(\frac{-t_r}{r}\right)\frac{\partial\mathbf{e}_r}{\partial\theta} \\
&= (-t_r)\frac{\partial}{\partial\theta}\left(\frac{1}{r}\right)\mathbf{e}_r + \left(\frac{1}{r}\right)\frac{\partial(-\mathbf{t}\cdot\mathbf{e}_r)}{\partial\theta}\mathbf{e}_r + \left(\frac{-t_r}{r}\right)\cos(\phi)\mathbf{e}_\theta \\
&= (-t_r)\left(\frac{-1}{r^2}\frac{\partial r}{\partial\theta}\right)\mathbf{e}_r + \left(\frac{1}{r}\right)\left(\frac{\partial(-\mathbf{t})}{\partial\theta}\cdot\mathbf{e}_r\right)\mathbf{e}_r \\
&+ \left(\frac{1}{r}\right)\left(-\mathbf{t}\cdot\frac{\partial\mathbf{e}_r}{\partial\theta}\right)\mathbf{e}_r + \left(\frac{-t_r}{r}\right)\cos(\phi)\mathbf{e}_\theta \\
&= \frac{t_r}{r}\left(\frac{1}{r}\frac{\partial r}{\partial\theta}\right)\mathbf{e}_r + \left(\frac{1}{r}\right)\left(\frac{\partial(-\mathbf{t})}{\partial\theta}^{0}\cdot\mathbf{e}_r\right)\mathbf{e}_r + cos(\phi)\left(\frac{-\mathbf{t}\cdot\mathbf{e}\theta}{r}\right)\mathbf{e}_r \\
&+ \left(\frac{-t_r}{r}\right)\cos(\phi)\mathbf{e}_\theta \\
&= \left[L\left(\frac{1}{r}\frac{\partial r}{\partial\theta}\right) + \cos(\phi)\left(\frac{-t_\theta}{r}\right)\right]\mathbf{e}_r - L\cos(\phi)\mathbf{e}_\theta \qquad (6.10)
\end{aligned}
$$

Why **LaTeX** is relevant for the course ?

LaTeX

Several reasons:

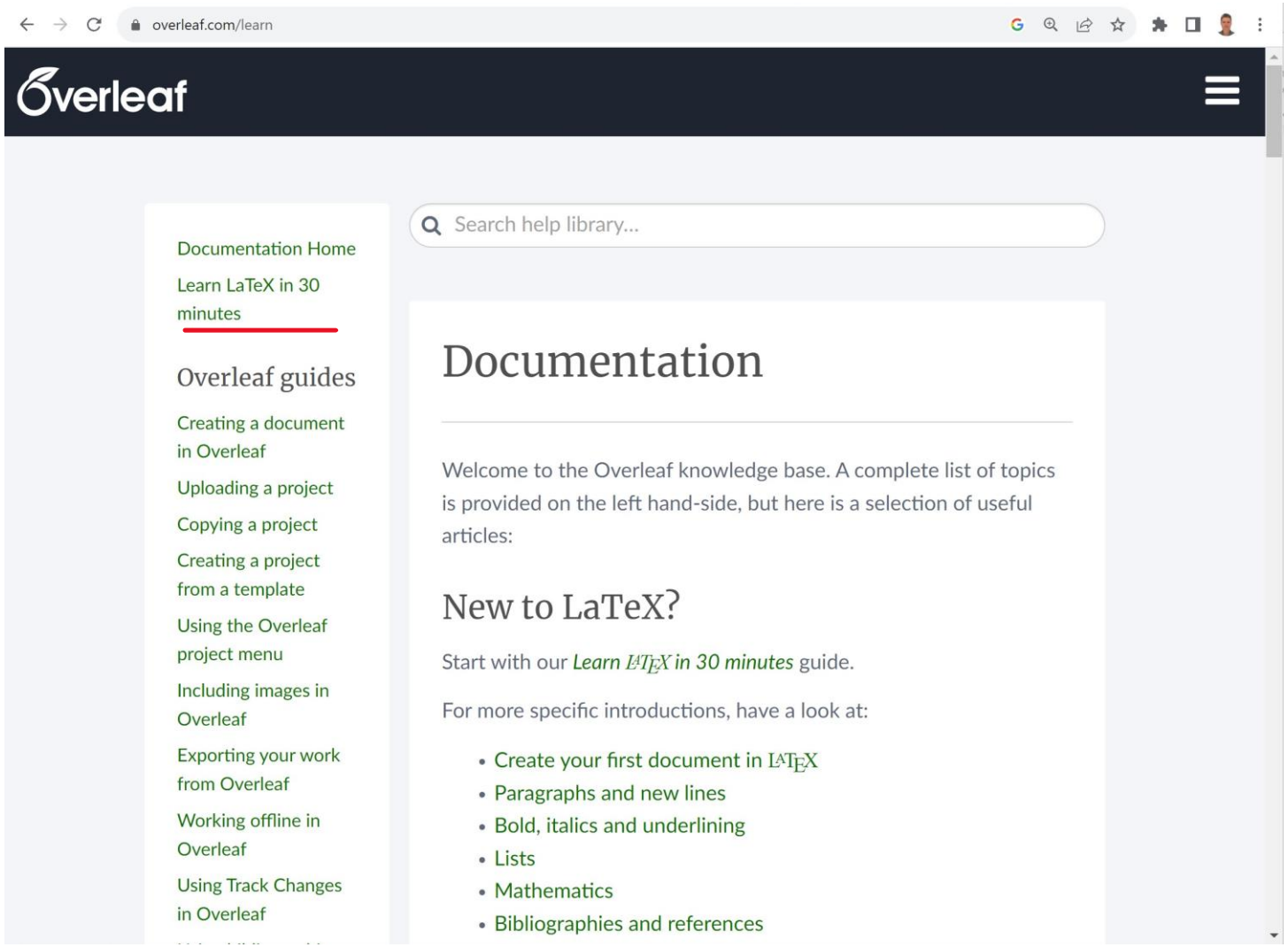
- **Mathematical Notation** - one of the best tools available.
- **Clarity and Precision** - LaTeX ensures that the clarity of thought.
- **Homework and Assignments** – Look more professional.
- **Preparation for Further Courses and Research** – Graduate courses.
- **Standard in Academia** – Papers in Computer Science, publications.
- **Skills Beyond the Classroom** – Valuable skill, thesis, technical documentation.

How I get started with LaTeX ?

- You can use an online tool - https://www.overleaf.com/
- You can download a tool - https://www.texstudio.org/

Let’s play with Overleaf Online Tool

I want to learn more…



https://www.overleaf.com/learn