COT2000 - Foundations of Computing
Exam 1

Instructions

Instruction page – please read very carefully.

Date: Friday, May 31, 2024
Time Window: 9:00 am - 9:00 pm

Format:

- **Location:** This test is administered remotely; there's no need to come to the classroom. The test will be accessible for download on Canvas during the designated time window.
- This is an open-book test. You may use: textbooks, lecture notes, personal notes, formulae pages, handouts, other supplementary materials prepared in advance. These materials can be either paper or electronic format.
- **Individual Work:** This test is meant to be completed independently. Collaboration is strictly prohibited. Do not discuss or share any details about the test or its solutions with anyone.

Submission:

- Download the test and print it to answer. If you cannot print it, write your answers clearly on separate sheets of paper. You can also use electronic form as long as you submit in PDF.
- **5 Bonus points if you submit a pdf compiled document using LaTeX.**
- Clearly show and explain your work for each question, where necessary.
- After completion, scan your test and submit it online via Canvas. Set aside at least 10 minutes for this process.
- Use a scanning app to convert your test into a single PDF. Ensure your submission is in the form of a single PDF file.
- Clearly write your name and Z number on your test.
- **File Naming Convention:** [Your Name]_[Z Number].pdf
- While Canvas does allow multiple submissions, only the last one will be considered for grading.
- **Do Not share any information about the test or its solutions with others.**
- Exclude the instruction page from your submission.
- Do not include your formula sheets in your test submission.

Please review these instructions thoroughly to ensure a smooth testing experience. Best of luck!

Note: Inquiries about test results, homework and extra credit grades must be sent to the instructors within 3 days after grades are published.

COT2000 - Foundations of Computing
Exam 1 - Solution Key

Questions

1. **(15 points) Given the sets:**

   $A = \{2, 3, \{4\}\}$

   $B = \{3, 2, \{4\}\}$

   $C = \{\{4\}, +\sqrt{9}, 2\}$

   $D = \{3, 2, 4\}$

   $E = \{\text{``apple''}, \text{``banana''}, \{\text{``cherry''}\}\}$

   $F = \{x \in \mathbb{Z} \mid 1 < x < 5\}$

   (a) Specify which of the given sets are equal to each other.

   (b) For each set, determine the number of elements it contains.

   (c) Determine if the following statements are true or false:

       i. $\{4\} \subseteq A$

       ii. $\{4\} \subseteq D$

       iii. $\{\text{``cherry''}\} \subseteq E$

       iv. $\{\{4\}\} \subseteq C$

       v. $\{2, 3\} \subseteq B$

2. **(15 points) Use the set-roster notation to write the following sets, and indicate the number of elements:** ($\mathbb{Z}$ means the integers)

   (a) $V = \{t \in \mathbb{Z} \mid t > -3 \text{ and } t < 7\}$.

   (b) $V = \{t \in \mathbb{Z} \mid t < -3 \text{ or } t > 7\}$.

   (c) Let $A = \{p, q, r\}$ and $B = \{x, y\}$, Find $B \times A$

3. **(15 points) Let $A = \{m, n, o, p\}$ and $B = \{g, h\}$.**

   Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

   (a) $A \times B$

   (b) $B \times A$

   (c) $A \times A$

   (d) $B \times B$

4. **(20 points) Answer each of the following:**

   Let $A = \{10, 11, 12\}$ and $B = \{2, 3, 4\}$. Define a relation R from A to B as follows:

   $$\text{For all } (x, y) \in A \times B, (x, y) \in R \text{ if and only if } \frac{x}{y+1} \text{ is an integer.}$$

   (a) Write $A \times B$ in set-roster notation

   (b) Determine the validity of the following:

       i. Is $12\ R\ 3$?

       ii. Is $(11, 4) \in R$?

   (c) Write R as a set of ordered pairs.

(d) Identify the domain and co-domain of R.

(e) Is R a function, explain.

5. **(15 points) Write each statement in symbolic form.**

Let $p = $ "I enjoy programming", $q = $ "I will graduate", and $r = $ "I will complete my project." Express each of the following propositions in symbolic form:

(a)　i. I enjoy programming and I will graduate.

ii. I will complete my project or I will not graduate.

iii. It is not true that I both enjoy programming and will complete my project.

iv. I will not complete my project and I will not graduate.

(b) For each of the following propositions, identify simple propositions, express the compound proposition in symbolic form, and determine whether it is true or false:

i. The moon is made of cheese or one plus one equals two.

ii. If 1,000,000 is a multiple of 5, then 1,000,000 is even.

iii. 7 is a prime number and 9 is not divisible by 3.

iv. $4 \in \mathbb{Z}$ and $4 \in \mathbb{Q}$.

v. $\dfrac{3}{4} \in \mathbb{Z}$ and $\dfrac{3}{4} \in \mathbb{Q}$. (Note: Q is the rational numbers and Z is the integer numbers.)

vi. The product of two odd integers is odd and the product of two even integers is even.

6. **(20 points) Write the truth table for the following statement forms:**

(a) $p \vee (\neg q \wedge r)$

(b) $(p \vee q) \wedge (\neg p \vee (q \wedge \neg r))$

(c) **Determine whether the statement forms are logically equivalent. Explain.**

$p \wedge (p \vee q)$ equivalent to the statement p

7. **(Bonus 5 points)** Expand the compound statement $(p \oplus q) \wedge r$ using the definition of Exclusive OR? Explain.