

# Homework 5

**Due Date: 07/05/2024**

- Please submit your answers as a single consolidated PDF file, and upload the file to Canvas.
- You may submit multiple times, but only the last submission made before the due date will be considered for grading.
- Make sure you submit the right file to Canvas. Wrong file submissions will not be graded.
- Note: Inquiries about homework must be sent to the TAs or instructor within 3 days after grades are published.
- **10 Bonus points** if you create your document in LaTeX and submit the compiled result in .pdf.

## Functions

1. Fill in each blank with the word *most* or *least*.
  - a. A function  $F$  is one-to-one if, and only if, each element in the co-domain of  $F$  is the image of at \_\_\_\_\_ one element in the domain of  $F$ .
  - b. A function  $F$  is onto if, and only if, each element in the co-domain of  $F$  is the image of at \_\_\_\_\_ one element in the domain of  $F$ .
2. When asked to state the definition of one-to-one, a student replies, "A function  $f$  is one-to-one if, and only if, every element of  $X$  is sent by  $f$  to exactly one element of  $Y$ ." Give a counterexample to show that the student's reply is incorrect.
3. Let  $X = \{1, 5, 9\}$  and  $Y = \{3, 4, 7\}$ .
  - (a) Define  $f : X \rightarrow Y$  by specifying that

$$f(1) = 4,$$

$$f(5) = 7,$$

$$f(9) = 4.$$

Is  $f$  one-to-one? Is  $f$  onto? Explain your answers.

(b) Define  $g : X \rightarrow Y$  by specifying that

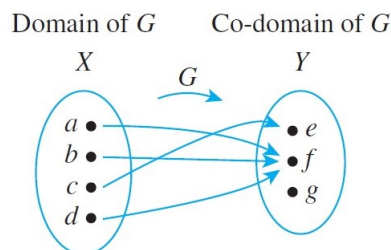
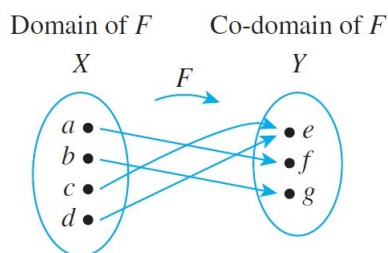
$$g(1) = 7,$$

$$g(5) = 3,$$

$$g(9) = 4.$$

Is  $g$  one-to-one? Is  $g$  onto? Explain your answers.

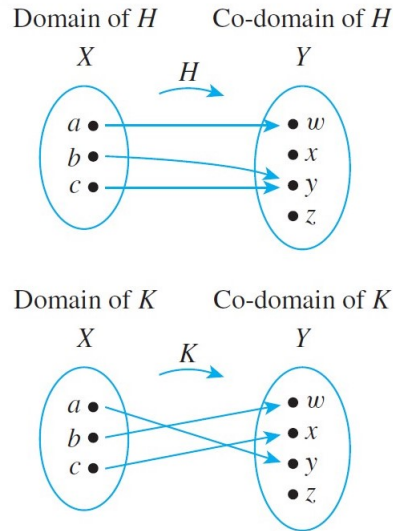
4. Let  $X = \{a, b, c, d\}$  and  $Y = \{e, f, g\}$ . Define functions  $F$  and  $G$  by the arrow diagrams below.



(a) Is  $F$  one-to-one? Why or why not? Is it onto? Why or why not?

(b) Is  $G$  one-to-one? Why or why not? Is it onto? Why or why not?

5. Let  $X = \{a, b, c\}$  and  $Y = \{w, x, y, z\}$ . Define functions  $H$  and  $K$  by the arrow diagrams below.



- (a) Is  $H$  one-to-one? Why or why not? Is it onto? Why or why not?
- (b) Is  $K$  one-to-one? Why or why not? Is it onto? Why or why not?
6. Let  $X = \{1, 2, 3\}$ ,  $Y = \{1, 2, 3, 4\}$ , and  $Z = \{1, 2\}$ .
- (a) Define a function  $f : X \rightarrow Y$  that is one-to-one but not onto.
- (b) Define a function  $g : X \rightarrow Z$  that is onto but not one-to-one.
- (c) Define a function  $h : X \rightarrow X$  that is neither one-to-one nor onto.
- (d) Define a function  $k : X \rightarrow X$  that is one-to-one and onto but is not the identity function on  $X$ .
7. (a) Define  $g : \mathbf{Z} \rightarrow \mathbf{Z}$  by the rule  $g(n) = 4n - 5$ , for all integers  $n$ .
- i. Is  $g$  one-to-one? Prove or give a counterexample.
- ii. Is  $g$  onto? Prove or give a counterexample.
- (b) Define  $G : \mathbf{R} \rightarrow \mathbf{R}$  by the rule  $G(x) = 4x - 5$  for all real numbers  $x$ . Is  $G$  onto? Prove or give a counterexample.
8. (a) Define  $H : \mathbf{R} \rightarrow \mathbf{R}$  by the rule  $H(x) = x^2$ , for all real numbers  $x$ .
- i. Is  $H$  one-to-one? Prove or give a counterexample.
- ii. Is  $H$  onto? Prove or give a counterexample.
- (b) Define  $K : \mathbf{R}^{\text{nonneg}} \rightarrow \mathbf{R}^{\text{nonneg}}$  by the rule  $K(x) = x^2$ , for all non-negative real numbers  $x$ . Is  $K$  onto? Prove or give a counterexample.

**In each of the following a function  $f$  is defined on a set of real numbers. Determine whether or not  $f$  is one-to-one and justify your answer.**

9.  $f(x) = \frac{x+1}{x}$ , for all real numbers  $x \neq 0$
10.  $f(x) = \frac{x}{x^2+1}$ , for all real numbers  $x$