COT 2000
Foundations of Computing

Summer 2024

Lecture 11 – part 1

Lab 6
Exam 2 – 06/21/24

Lecture 11 – part 2

Review

Review

- Logic review
- Logic exercises
- Quantified statements
- Arguments and fallacies with quantifiers

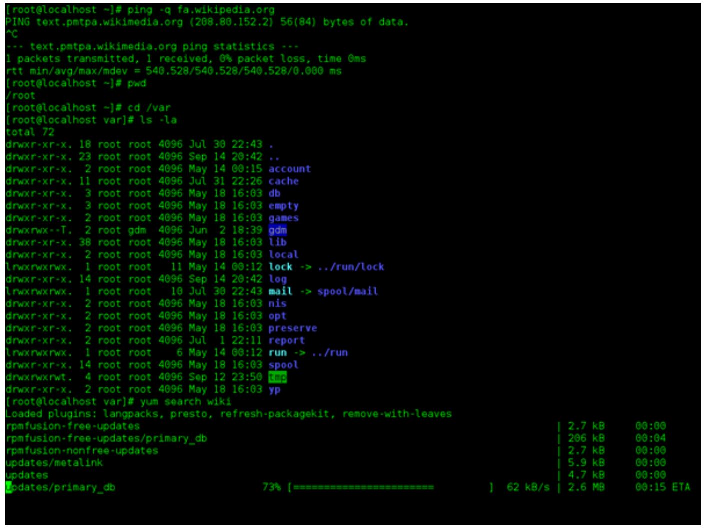
Lecture 11 – part 3

Bash/Terminal (Lab 6)

Bash/Terminal

- **Terminal:** An interface to interact with the computer system. Allows direct input and output from/to the user.
- **CLI (Command Line Interface).** Interface where users type commands to operate the computer.
  - Faster operations once commands are known.
  - Offers more powerful functionalities.
  - Challenge: Requires learning and memorizing commands.
- **Shell**
  - Software that interprets and executes CLI commands.
  - Acts as an interface between user and the operating system's kernel.
- **Bash**
  - A popular Unix shell. Known for scripting capabilities and user-friendliness.

https://en.wikipedia.org/wiki/Computer_terminal



https://en.wikipedia.org/wiki/Command-line_interface#Command-line_interpreter

Why is Bash Important for Computer Science?

- **Linux Everywhere:**
  - Powers cloud servers, serverless infrastructures, and VMs.
  - Runs on edge devices, IoT gadgets, and more.
  - Dominates modern tech landscape: understanding Linux = indispensable skill.
- **Ubiquity in Cloud:**
  - Default shell in major cloud ecosystems.
  - Automates cloud tasks, making deployments and configurations efficient.
- **CLI vs. GUI:**
  - CLI (like Bash) offers precision, speed, and scriptability.
  - Essential for many backend, cloud, and infrastructure tasks.
  - GUIs are limited, but CLIs empower deeper system interactions.
- **Future-Proofing Careers:**
  - As technology leans more towards automation and cloud, Bash proficiency becomes a valuable asset.
  - Opens doors to roles in DevOps, Cloud Engineering, IoT development, and more

SOME COMMANDS

ls - List files/directories.
Example: ls -l (Lists files in detailed view)

cd <directory> - Change to <directory>.
Example: cd Documents/ (Changes the directory to 'Documents')

pwd - Show current directory.
Example: pwd (Displays the path of the current directory)

touch <filename> - Create an empty file.
Example: touch newfile.txt (Creates a new file named newfile.txt)

cat <filename> - Display file content.
Example: cat myfile.txt (Displays the content of myfile.txt)

cp <source> <destination> - Copy file.
Example: cp file1.txt file2.txt (Copies file1.txt to file2.txt)

mv <source> <destination> - Move/rename file.
Example: mv oldname.txt newname.txt (Renames oldname.txt to newname.txt)

rm <filename> - Delete file.
Example: rm unwantedfile.txt (Deletes the file named unwantedfile.txt)

grep "pattern" <filename> - Search for a pattern in a file.
Example: grep "hello" myfile.txt (Searches for the word "hello" in myfile.txt)

find . -name "filename" - Search for a file in current and sub-directories.
Example: find . -name "notes.txt" (Searches for notes.txt in the current directory and all sub-directories)

man <command> - Display the manual for a command.
Example: man ls (Displays the manual for the 'ls' command)

<command> --help - Get a quick help for a command.
Example: ls --help (Provides a brief help documentation for the 'ls' command)

Lecture 11 – part 4

Sequences

Introduction to Sequences

An ordered list of numbers (or elements).

Example: Imagine that a person decides to count his ancestors.

2, 4, 8, 16, 32, 64, 128, . . .

Position in the row	1	2	3	4	5	6	7 . . .
Number of ancestors	2	4	8	16	32	64	128 . . .

$$n = 1, 2, 3, 4, \ldots$$

$$2^n = 2, 4, 8, 16, \ldots$$

$$A_k = 2^k$$

• Definition

A **sequence** is a function whose domain is either all the integers between two given integers or all the integers greater than or equal to a given integer.

$$a_m, a_{m+1}, \ldots, a_n$$

Finite Sequence

$$a_m, a_{m+1}, a_{m+2}, \ldots$$

Infinite Sequence

a_k is called a term.

k is the subscript or index.

Example:

Define sequences $a_1, a_2, a_3, \ldots$ and $b_2, b_3, b_4, \ldots$ by the following explicit formulas:

$$a_k = \frac{k}{k+1} \quad \text{for all integers } k \geq 1,$$

$$b_i = \frac{i-1}{i} \quad \text{for all integers } i \geq 2.$$

Compute the first five terms of both sequences.

Solution:

$$a_1 = \frac{1}{1+1} = \frac{1}{2} \qquad b_2 = \frac{2-1}{2} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3} \qquad b_3 = \frac{3-1}{3} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4} \qquad b_4 = \frac{4-1}{4} = \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} = \frac{4}{5} \qquad b_5 = \frac{5-1}{5} = \frac{4}{5}$$

$$a_5 = \frac{5}{5+1} = \frac{5}{6} \qquad b_6 = \frac{6-1}{6} = \frac{5}{6}$$

Example:

Compute the first six terms of the sequence $c_0, c_1, c_2, \ldots$ defined as follows:

$$c_j = (-1)^j \quad \text{for all integers } j \geq 0.$$

Solution:

$$\begin{aligned}
c_0 &= (-1)^0 = 1 \\
c_1 &= (-1)^1 = -1 \\
c_2 &= (-1)^2 = 1 \\
c_3 &= (-1)^3 = -1 \\
c_4 &= (-1)^4 = 1 \\
c_5 &= (-1)^5 = -1
\end{aligned}$$

Example:

Find an explicit formula for a sequence that has the following initial terms:

$$1, \quad -\frac{1}{4}, \quad \frac{1}{9}, \quad -\frac{1}{16}, \quad \frac{1}{25}, \quad -\frac{1}{36}, \ldots.$$

Solution:

$$\begin{array}{cccccc} \frac{1}{1^2}, & \frac{(-1)}{2^2}, & \frac{1}{3^2}, & \frac{(-1)}{4^2}, & \frac{1}{5^2}, & \frac{(-1)}{6^2}. \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{array}$$

$$a_k = \frac{\pm 1}{k^2}.$$

When k is odd, $k+1$ is even: $(-1)^{k+1} = +1$ and when k is even, $k+1$ is odd: $(-1)^{k+1} = -1$.

$$a_k = \frac{(-1)^{k+1}}{k^2} \quad \text{for all integers } k \geq 1.$$

Note that making the first term a_0 would have led to the alternative formula

$$a_k = \frac{(-1)^k}{(k+1)^2} \quad \text{for all integers } k \geq 0.$$

Summation Notation

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 126.$$

• Definition

If m and n are integers and $m \le n$, the symbol $\sum_{k=m}^{n} a_k$, read the **summation from *k* equals *m* to *n* of *a*-sub-*k***, is the sum of all the terms $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$. We say that $a_m + a_{m+1} + a_{m+2} + \ldots + a_n$ is the **expanded form** of the sum, and we write

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + a_{m+2} + \cdots + a_n.$$

We call k the **index** of the summation, m the **lower limit** of the summation, and n the **upper limit** of the summation.

Example:

Let $a_1 = -2$, $a_2 = -1$, $a_3 = 0$, $a_4 = 1$, and $a_5 = 2$. Compute the following:

a. $\sum_{k=1}^{5} a_k$ b. $\sum_{k=2}^{2} a_k$ c. $\sum_{k=1}^{2} a_{2k}$

Solution:

a. $\sum_{k=1}^{5} a_k = a_1 + a_2 + a_3 + a_4 + a_5 = (-2) + (-1) + 0 + 1 + 2 = 0$

b. $\sum_{k=2}^{2} a_k = a_2 = -1$

c. $\sum_{k=1}^{2} a_{2k} = a_{2\cdot 1} + a_{2\cdot 2} = a_2 + a_4 = -1 + 1 = 0$

Example:

Write the following summation in expanded form:

$$\sum_{i=0}^{n} \frac{(-1)^i}{i+1}.$$

Solution:

$$\begin{aligned}
\sum_{i=0}^{n} \frac{(-1)^i}{i+1} &= \frac{(-1)^0}{0+1} + \frac{(-1)^1}{1+1} + \frac{(-1)^2}{2+1} + \frac{(-1)^3}{3+1} + \cdots + \frac{(-1)^n}{n+1} \\
&= \frac{1}{1} + \frac{(-1)}{2} + \frac{1}{3} + \frac{(-1)}{4} + \cdots + \frac{(-1)^n}{n+1} \\
&= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^n}{n+1}
\end{aligned}$$

Example:

Express the following using summation notation:

$$\frac{1}{n} + \frac{2}{n+1} + \frac{3}{n+2} + \cdots + \frac{n+1}{2n}.$$

Solution:

$$\frac{1}{n} + \frac{2}{n+1} + \frac{3}{n+2} + \cdots + \frac{n+1}{2n} = \sum_{k=0}^{n} \frac{k+1}{n+k}.$$

Product Notation

$$\prod_{k=1}^{5} a_k = a_1 a_2 a_3 a_4 a_5.$$

• Definition

If m and n are integers and $m \le n$, the symbol $\prod_{k=m}^{n} a_k$, read the **product from k equals m to n of a-sub-k**, is the product of all the terms $a_m, a_{m+1}, a_{m+2}, \ldots, a_n$. We write

$$\prod_{k=m}^{n} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdots a_n.$$

Example:

Compute the following products:

a. $\prod_{k=1}^{5} k$

b. $\prod_{k=1}^{1} \frac{k}{k+1}$

Solution:

a. $\prod_{k=1}^{5} k = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

b. $\prod_{k=1}^{1} \frac{k}{k+1} = \frac{1}{1+1} = \frac{1}{2}$

Properties

If $a_m, a_{m+1}, a_{m+2}, \ldots$ and $b_m, b_{m+1}, b_{m+2}, \ldots$ are sequences of real numbers and c is any real number, then the following equations hold for any integer $n \geq m$:

1. $\displaystyle\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$

2. $\displaystyle c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k$ generalized distributive law

3. $\displaystyle\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \prod_{k=m}^{n} (a_k \cdot b_k).$

Example:

Let $a_k = k + 1$ and $b_k = k - 1$ for all integers k. Write each of the following expressions as a single summation or product:

a. $\displaystyle\sum_{k=m}^{n} a_k + 2\cdot\sum_{k=m}^{n} b_k$ b. $\displaystyle\left(\prod_{k=m}^{n} a_k\right)\cdot\left(\prod_{k=m}^{n} b_k\right)$

Solution:

a.
$$\begin{aligned}
\sum_{k=m}^{n} a_k + 2\cdot\sum_{k=m}^{n} b_k &= \sum_{k=m}^{n}(k+1) + 2\cdot\sum_{k=m}^{n}(k-1) && \text{by substitution}\\
&= \sum_{k=m}^{n}(k+1) + \sum_{k=m}^{n} 2\cdot(k-1) && \text{by Theorem 5.1.1 (2)}\\
&= \sum_{k=m}^{n}((k+1) + 2\cdot(k-1)) && \text{by Theorem 5.1.1 (1)}\\
&= \sum_{k=m}^{n}(3k-1) && \text{by algebraic simplification}
\end{aligned}$$

b.

$$\left(\prod_{k=m}^{n} a_k\right) \cdot \left(\prod_{k=m}^{n} b_k\right) = \left(\prod_{k=m}^{n} (k+1)\right) \cdot \left(\prod_{k=m}^{n} (k-1)\right)$$ by substitution

$$= \prod_{k=m}^{n} (k+1) \cdot (k-1)$$ by Theorem 5.1.1 (3)

$$= \prod_{k=m}^{n} (k^2 - 1)$$ by algebraic simplification

Transforming a Sum by a Change of Variable

Example: Transform the following summation by making the specified change of variable

$$\textit{summation: } \sum_{k=0}^{6} \frac{1}{k+1} \qquad \textit{change of variable: } j = k+1$$

Solution: First calculate the lower and upper limits of the new summation:

$$\text{When } k = 0, \quad j = k+1 = 0+1 = 1.$$
$$\text{When } k = 6, \quad j = k+1 = 6+1 = 7.$$

Since $j = k+1$, then $k = j-1$.

Hence $\dfrac{1}{k+1} = \dfrac{1}{(j-1)+1} = \dfrac{1}{j}$.

$$\sum_{k=0}^{6} \frac{1}{k+1} = \sum_{j=1}^{7} \frac{1}{j}.$$

Upper Limit change

Example: a. Transform the following summation by making the specified change of variable.

$$\textit{summation: } \sum_{k=1}^{n+1} \left(\frac{k}{n+k} \right) \qquad \textit{change of variable: } j = k - 1$$

b. Transform the summation obtained in part (a) by changing all j's to k's.

Solution:

a) $$\frac{k}{n+k} = \frac{j+1}{n+(j+1)}$$

$$\sum_{k=1}^{n+1} \frac{k}{n+k} = \sum_{j=0}^{n} \frac{j+1}{n+(j+1)}.$$

b) $$\sum_{j=0}^{n} \frac{j+1}{n+(j+1)} = \sum_{k=0}^{n} \frac{k+1}{n+(k+1)}$$

Factorial Notation

• Definition

For each positive integer n, the quantity ***n* factorial** denoted ***n*!,** is defined to be the product of all the integers from 1 to n:

$$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1.$$

Zero factorial, denoted $0!$, is defined to be 1:

$$0! = 1.$$

Recursive definition

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n \geq 1. \end{cases}$$

Example: Fist 10 factorials

$$\begin{aligned} 0! &= 1 & 1! &= 1 \\ 2! &= 2 \cdot 1 = 2 & 3! &= 3 \cdot 2 \cdot 1 = 6 \\ 4! &= 4 \cdot 3 \cdot 2 \cdot 1 = 24 & 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \\ 6! &= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 & 7! &= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040 \\ 8! &= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 & 9! &= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 40{,}320 & &= 362{,}880 \end{aligned}$$

Example: Simplify the following expressions:

a. $\frac{8!}{7!}$ b. $\frac{5!}{2!\cdot 3!}$ c. $\frac{1}{2!\cdot 4!}+\frac{1}{3!\cdot 3!}$ d. $\frac{(n+1)!}{n!}$ e. $\frac{n!}{(n-3)!}$

Solution:

a. $\frac{8!}{7!}=\frac{8\cdot \cancel{7!}}{\cancel{7!}}=8$

b. $\frac{5!}{2!\cdot 3!}=\frac{5\cdot 4\cdot \cancel{3!}}{2!\cdot \cancel{3!}}=\frac{5\cdot 4}{2\cdot 1}=10$

c. $\frac{1}{2!\cdot 4!}+\frac{1}{3!\cdot 3!}=\frac{1}{2!\cdot 4!}\cdot\frac{3}{3}+\frac{1}{3!\cdot 3!}\cdot\frac{4}{4}$ by multiplying each numerator and denominator by just what is necessary to obtain a common denominator

$=\frac{3}{3\cdot 2!\cdot 4!}+\frac{4}{3!\cdot 4\cdot 3!}$ by rearranging factors

$=\frac{3}{3!\cdot 4!}+\frac{4}{3!\cdot 4!}$ because $3\cdot 2!=3!$ and $4\cdot 3!=4!$

$=\frac{7}{3!\cdot 4!}$ by the rule for adding fractions with a common denominator

$=\frac{7}{144}$

d. $\frac{(n+1)!}{n!}=\frac{(n+1)\cdot \cancel{n!}}{\cancel{n!}}=n+1$

e. $\frac{n!}{(n-3)!}=\frac{n\cdot(n-1)\cdot(n-2)\cdot\cancel{(n-3)!}}{\cancel{(n-3)!}}$

$=n\cdot(n-1)\cdot(n-2)$

$=n^3-3n^2+2n$

(n choose r) Notation

- Binomial coefficient $\binom{n}{r}$

• Definition

Let n and r be integers with $0 \leq r \leq n$. The symbol

$$\binom{n}{r}$$

is read "***n* choose *r***" and represents the number of subsets of size r that can be chosen from a set with n elements.

• Formula for Computing $\binom{n}{r}$

For all integers n and r with $0 \leq r \leq n$,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

also called *combinations*

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example: Use the formula for computing $\binom{n}{r}$ to evaluate the following expressions:

a. $\binom{8}{5}$ b. $\binom{4}{0}$ c. $\binom{n+1}{n}$

Solution:

a.
$$\begin{aligned}\binom{8}{5} &= \frac{8!}{5!(8-5)!} \\ &= \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3\cdot 2\cdot 1}}{(\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1)\cdot(\cancel{\cdot 3\cdot 2\cdot 1})} \\ &= 56.\end{aligned}$$

b.
$$\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!0!} = \frac{\cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{(4\cdot 3\cdot 2\cdot 1)}(1)} = 1$$

c.
$$\binom{n+1}{n} = \frac{(n+1)!}{n!((n+1)-n)!} = \frac{(n+1)!}{n!1!} = \frac{(n+1)\cdot \cancel{n!}}{\cancel{n!}} = n+1$$

Lecture 11 – part 5

Sequences Exercises

Lecture 11 – part 6

Mathematical Induction

Mathematical Induction

Principle of Mathematical Induction

Let $P(n)$ be a property that is defined for integers n, and let a be a fixed integer. Suppose the following two statements are true:

1. $P(a)$ is true.
2. For all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true.

Then the statement

$$\text{for all integers } n \geq a,\ P(n)$$

is true.

Method of Proof by Mathematical Induction

Consider a statement of the form, "For all integers $n \geq a$, a property $P(n)$ is true." To prove such a statement, perform the following two steps:

Step 1 (basis step): Show that $P(a)$ is true.

Step 2 (inductive step): Show that for all integers $k \geq a$, if $P(k)$ is true then $P(k+1)$ is true. To perform this step,

> **suppose** that $P(k)$ is true, where k is any particular but arbitrarily chosen integer with $k \geq a$.
>
> *[This supposition is called the* **inductive hypothesis.***]*

Then

> **show** that $P(k+1)$ is true.

Example

Let $P(n)$ be the proposition that the sum of the first n odd numbers is n^2.

That is,

$$P(n) : 1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

The kth odd number is $2k - 1$, and the next odd number is $2k + 1$.

STEP 1: Observe that $P(1)$ is true: $1 = 1^2$.

STEP 2: Assuming $P(k)$ is true, we add $2k + 1$ to both sides, obtaining:

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = k^2 + (2k + 1).$$

This simplifies to:

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = (k + 1)^2.$$

By the principle of mathematical induction, $P(n)$ is true for all positive integers n.

$$P(1) : 1 = 1^2$$
$$P(2) : 1 + 3 = 2^2$$
$$P(3) : 1 + 3 + 5 = 3^2$$
$$P(4) : 1 + 3 + 5 + 7 = 4^2$$

Example

Any whole number of cents of at least 8¢. I can be obtained using 3¢ and 5¢ coins.

Number of Cents	How to Obtain It
8¢	3¢ + 5¢
9¢	3¢ + 3¢ + 3¢
10¢	5¢ + 5¢
11¢	3¢ + 3¢ + 5¢
12¢	3¢ + 3¢ + 3¢ + 3¢
13¢	3¢ + 5¢ + 5¢
14¢	3¢ + 3¢ + 3¢ + 5¢
15¢	5¢ + 5¢ + 5¢
16¢	3¢ + 3¢ + 5¢ + 5¢
17¢	3¢ + 3¢ + 3¢ + 3¢ + 5¢

For all integers $n \geq 8$, n¢ can be obtained using 3¢ and 5¢ coins.

Proof (by mathematical induction):

Let the property $P(n)$ be the sentence

n¢ can be obtained using 3¢ and 5¢ coins. ← $P(n)$

Show that P(8) is true:
$P(8)$ is true because 8¢ can be obtained using one 3¢ coin and one 5¢ coin.

Show that for all integers $k \geq 8$, if P(k) is true then P(k+1) is also true:

[Suppose that P(k) is true for a particular but arbitrarily chosen integer $k \geq 8$. That is:]
Suppose that k is any integer with $k \geq 8$ such that

k¢ can be obtained using 3¢ and 5¢ coins. ← $P(k)$
inductive hypothesis

[We must show that $P(k+1)$ is true. That is:] We must show that

$(k+1)$¢ can be obtained using 3¢ and 5¢ coins. ← $P(k+1)$

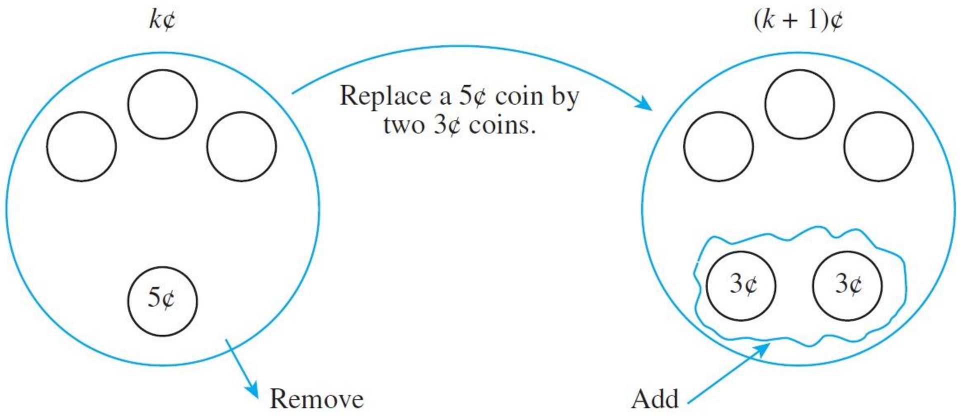
Case 1 (There is a 5¢ coin among those used to make up the k¢.): In this case replace the 5¢ coin by two 3¢ coins; the result will be $(k+1)$¢.

Case 2 (There is not a 5¢ coin among those used to make up the k¢.): In this case, because $k \geq 8$, at least three 3¢ coins must have been used. So remove three 3¢ coins and replace them by two 5¢ coins; the result will be $(k+1)$¢.

Thus in either case $(k+1)$¢ can be obtained using 3¢ and 5¢ coins *[as was to be shown]*.

[Since we have proved the basis step and the inductive step, we conclude that the proposition is true.]

Case 1



Case 2