

# Interview 1 Answers

The following answers are **not** model answers, they are just one possible answer.

These answers do not make use of register machines from previous questions, however answers that do are equally valid.

PS: Register machines are analogous to Turing machines: a mathematical model of computation. If you are interested in reading more, you can read the Cambridge University lecture notes on register machines [here].

Below we use the notation  $R_n := m$  to represent setting the register  $R_n$  to the value  $m$ , and the notation  $R_n := R_m$  to represent setting the register  $R_n$  to the value stored in register  $R_m$ .

$(R_n := R_m), (R_m := 0)$  represents setting register  $R_n$  to the value of register  $R_m$ , and subsequently setting the register  $R_m$  to the value 0.

## Introduction

**Can you informally reason about why this machine represents the function  $f(x) = x$  for all  $x$ ?**

This machine works by incrementing  $R_0$  each time we decrement  $R_1$ . The machine only ends once  $R_1$  reaches 0. To reach this point, we must decrement from  $R_1$   $x$  number of times, and so  $R_0 = x$  at the end of the program.

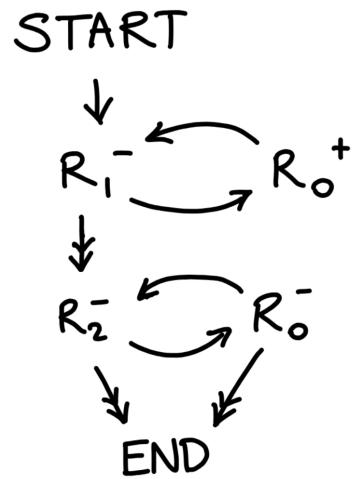
**What function does the following two-argument register machine represent?**

$$f(x, y) = x + y$$

We first perform  $R_0 := R_0 + x$ , followed by  $R_0 := R_0 + y$ . Since  $R_0$  starts at 0, the end result must be  $R_0 = x + y$ .

## Question 1

The idea here is to set  $R_0 := R_1$ , and keep subtracting  $R_2$  from  $R_0$  until either  $R_2$  or  $R_0$  reaches 0.



## Question 2

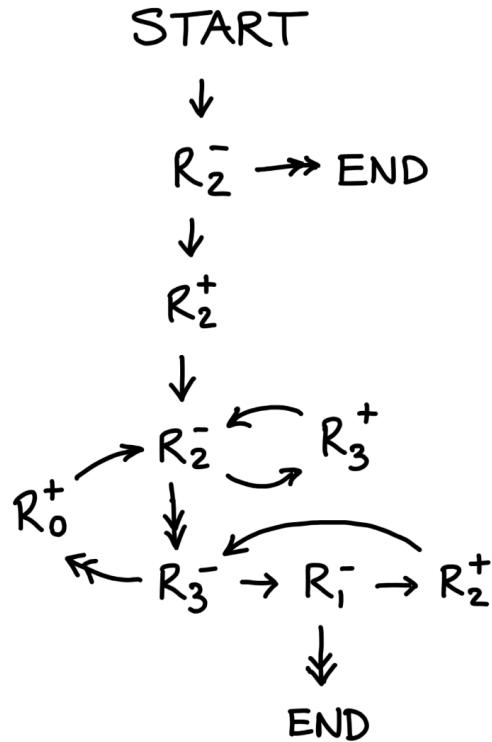
The idea here is to keep subtracting  $R_2$  from  $R_1$  until  $R_1$  reaches zero. We will keep track of the number of complete subtractions in the result register  $R_0$ .

If  $R_2$  is 0 we want to immediately exit with  $R_0 = 0$ .

Else we want to count the number of times we can subtract  $R_2$  from  $R_1$ . We do this by introducing a dummy register  $R_3$ , and setting  $(R_3 := R_2), (R_2 := 0)$  so that now  $R_3$  contains  $y$ , and  $R_2$  contains 0.

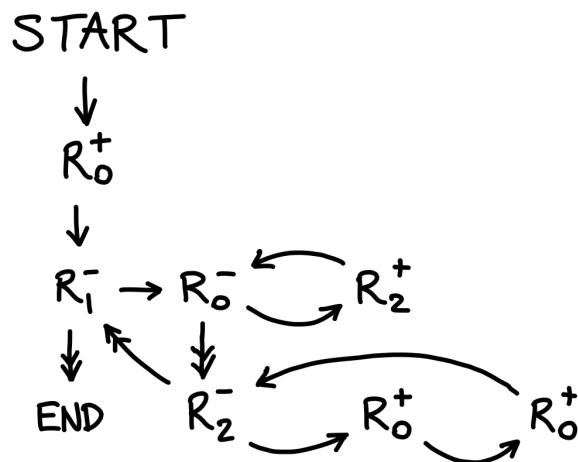
We then subtract  $R_3$  from  $R_1$  (using similar ideas to Question 1), and for each decrement of  $R_1$  we increment  $R_2$ . Hence, when  $R_3$  reaches 0,  $R_2$  now contains  $y$ . By swapping  $R_2$  and  $R_3$ , we reset the program state so that once again  $R_3$  contains  $y$ , and  $R_2$  contains 0.

We can then continue subtracting  $y$  from  $R_1$ ; each time we complete a full subtraction we increment  $R_0$  by one. Once  $R_1$  reaches 0, we exit the program.



### Question 3

This machine works by setting  $R_0 := 1$  and then doubling this register  $R_1$  number of times. The doubling works by setting  $(R_2 := R_0)$ ,  $(R_0 := 0)$ , and then adding  $R_2$  back to  $R_0$  twice. Hence, at the end of the loop,  $(R_0 := R_2 + R_2)$ ,  $(R_2 := 0)$ .



## Question 4

The idea is to extend our answer to question 2, by counting the number of times we can divide  $R_1$  by 2.

We first set  $R_2 := 2$ , to act as our logarithm base. To do this, we clear the value of  $R_2$  with continuous decrements until it reaches 0, and then increment  $R_2$  twice (clearing  $R_2$  only becomes relevant once we loop back around to these nodes).

We then divide  $R_1$  by  $R_2$  and keep the result in  $R_4$ . If  $R_4$  is 0 then no more divisions of  $R_1$  can be made and so we immediately exit. Else, we set  $(R_1 := R_4), (R_4 := 0), (R_3 := 0)$  to set  $R_1$  to its halved value, and reset  $R_4$  and  $R_3$  back to 0.

We then increment  $R_0$  by 1 as  $R_1$  has been halved, and repeat this loop.

