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CSCI 3412

A math equations and numbers

AI-generated content may be incorrect.HW 2

For n = 1 (Best Case):

Inductive Hypothesis:

A math equations and numbers

AI-generated content may be incorrect.Inductive Step:

For n =7 (Best Case):

* 3(7) = 21
* 5040 > 21, base case holds

Inductive Hypothesis:

* Assume k! > 3k for some k 7

Inductive Step:

* Prove that
* Since ,
* Thus,

**Q1 - 3) (5 points) Finding a counterfeit token**

You are given *3n* identical-looking coins, where one of them is counterfeit and has a different weight (either lighter or heavier). Your task is to find the counterfeit coin using a balance scale, and you want to determine the minimum number of weighings required.

Prove by mathematical induction that the minimum number of weighings required to find the counterfeit coin among *3n* coins is *n* weighings

For n = 1 (Best Case):

* 31 = 3
* If we place one coin on the left side of the balance, one on the right, and leave one aside:
* If the balance tips, the counterfeit coin is in one of the two coins on the scale, and we can tell whether it is heavier or lighter.
* If the balance is equal, then the counterfeit coin is the one left out.
* This requires exactly 1 weighing, so the base case holds.

Inductive Hypothesis:

* Assume that for some *n = k*, the minimum number of weighings required to find the counterfeit coin among *3k* coins is exactly *k*.
* *3k* coins can be solved in *k* weighings

Inductive Step:

1. Given *3k+1* coins, we can divide them into three equal groups of *3k* coins each.
2. Then, we place one group on the left side of the balance and another group on the right side.
3. The balance can show us **three possible outcomes**:
   * If the balance tilts, the counterfeit is in one of the two groups being weighed, and we know whether it is heavier or lighter.
   * If the balance stays even, the counterfeit is in the third group.
4. After the first weighing, we are left with exactly *3k* coins, and we already assumed (by **inductive hypothesis**) that finding the counterfeit among *3k* coins requires *k* weighings.
5. Since we already did **one weighing**, we need *k* more weighings to complete the search.
6. Thus, the total number of weighings required is: (k + 1)