Mathematics 1A

ITMTA1-B44

Limits and Derivatives

With

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Lecture 5
Week 2

Continuation of…

2.2 The Limit of a Function

Infinite Limits; Vertical Asymptotes

Infinite Limits; Vertical Asymptotes

4 Intuitive Definition of an Infinite Limit

Let *f* be a function defined on both sides of ***a***, except possibly at ***a*** itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of *f*(*x*) can he made arbitrarily large (as large as we please) by taking *x* sufficiently close to *a*, but not equal to ***a***.

Infinite Limits; Vertical Asymptotes

Again, the symbol ∞ is not a number, but the expression

$$\lim_{x \to a} f(x) = \infty$$

is often read as

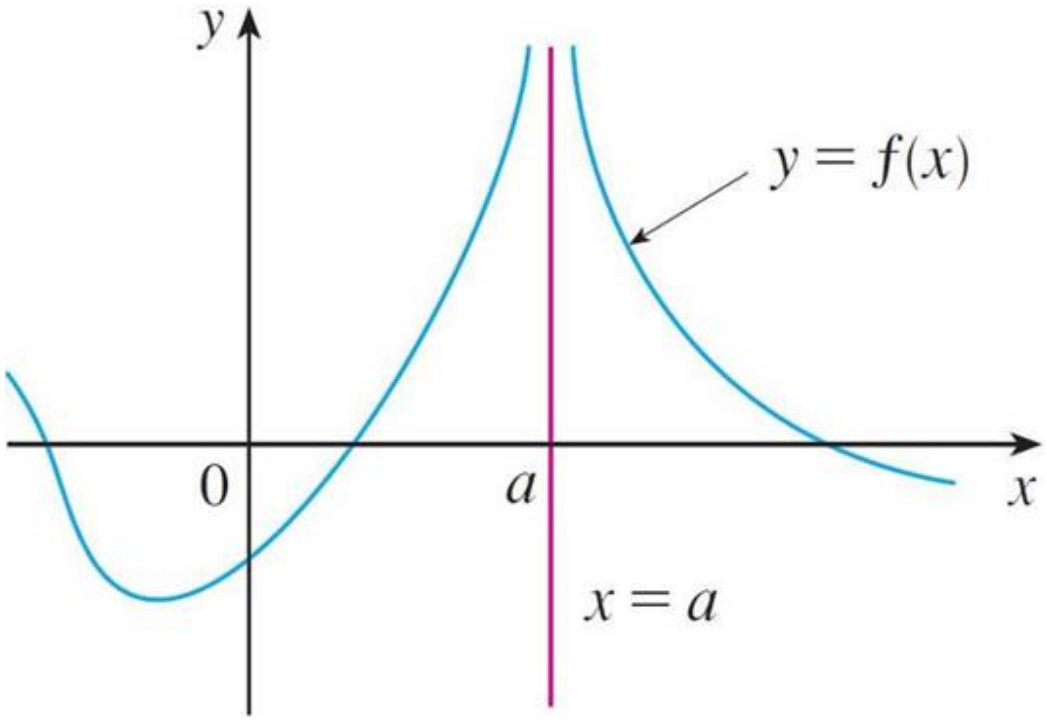
"the limit of $f(x)$, as x approaches a, is infinity"

or "$f(x)$ becomes infinite as x approaches a"

or "$f(x)$ increases without bound as x approaches a"

Infinite Limits; Vertical Asymptotes

This definition is illustrated graphically in Figure 10.

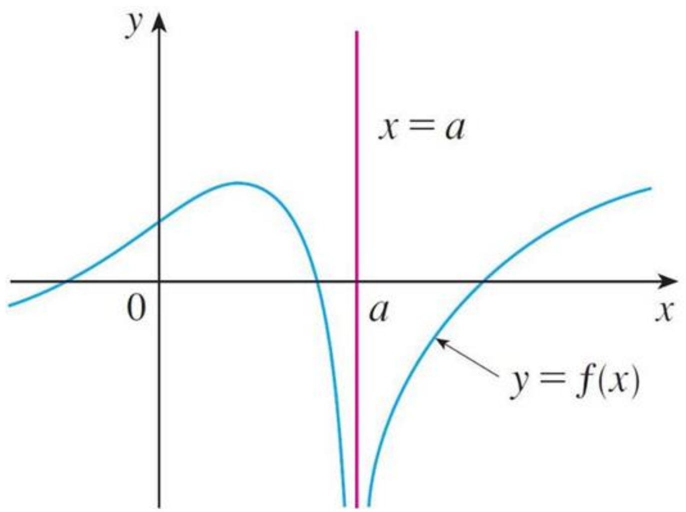


$$\lim_{x \to a} f\,(x) = \infty$$

Figure 10

Infinite Limits; Vertical Asymptotes

A similar sort of limit, for functions that become large negative as *x* gets close to *a*, is defined in Definition 5 and is illustrated in Figure 11.



$$\lim_{x \to a} f(x) = -\infty$$

Figure 11

Infinite Limits; Vertical Asymptotes

5 Definition

Let ***f*** be a function defined on both sides of ***a***, except possibly at ***a*** itself. Then

$$\lim_{x \to a} f\,(x) = -\infty$$

means that the values of *f*(*x*) can be made arbitrarily large negative by taking ***x*** sufficiently close to ***a***, but not equal to ***a***.

The symbol $\lim_{x \to a} f\,(x) = -\infty$ can be read as "the limit of *f*(*x*), as ***x*** approaches ***a***, is negative infinity" or "*f*(*x*) decreases without bound as *x* approaches *a*."

As an example we have

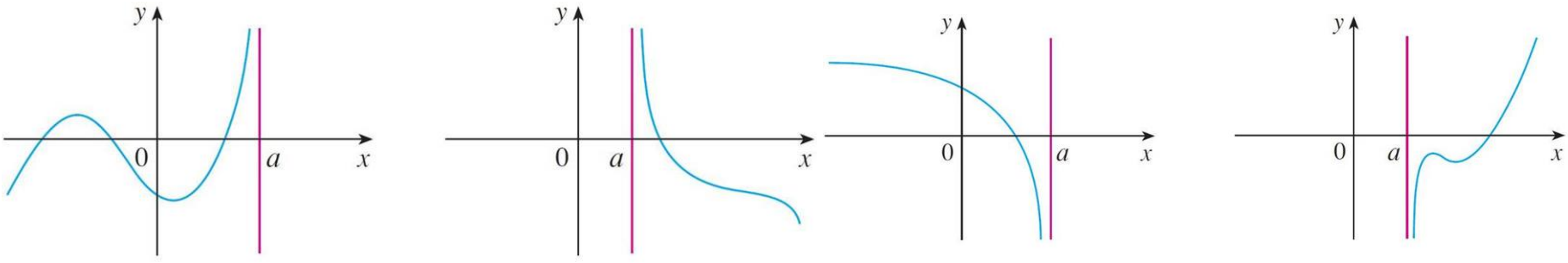
$$\lim_{x \to 0} \left(-\frac{1}{x^2}\right) = -\infty$$

Similar definitions can be given for the one-sided infinite limits

$$\lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

Illustrations of these four cases are given in Figure 12.



(a) $\lim_{x \to a^-} f(x) = \infty$ (b) $\lim_{x \to a^+} f(x) = \infty$ (c) $\lim_{x \to a^-} f(x) = -\infty$ (d) $\lim_{x \to a^+} f(x) = -\infty$

Figure 12

Infinite Limits; Vertical Asymptotes (8 of 8)

6 Definition

The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

Example 8

Find the vertical asymptotes of $f(x) = \tan x$.

Solution:

Because

$$\tan x = \frac{\sin x}{\cos x}$$

there are potential vertical asymptotes where $\cos x = 0$.

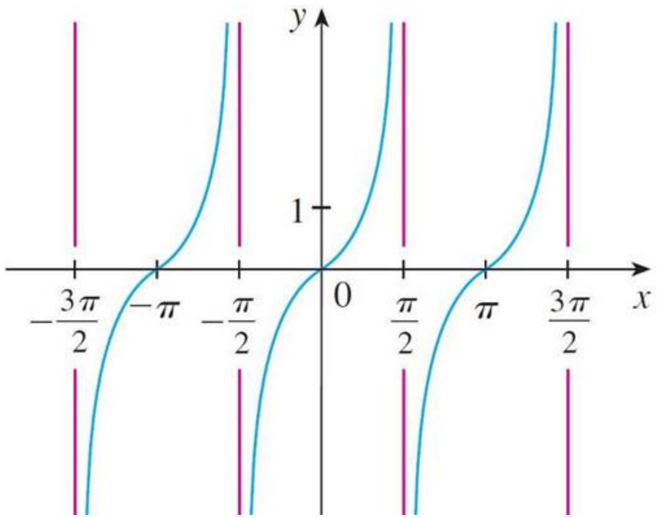
In fact, since $\cos x \to 0^+$ as $x \to (\pi/2)^-$ and $\cos x \to 0^-$ as $x \to (\pi/2)^+$, whereas $\sin x$ is positive (near 1) when x is near $\pi/2$, we have

$$\lim_{x\to(\pi/2)^-} \tan x = \infty \quad \text{and} \quad \lim_{x\to(\pi/2)^+} \tan x = -\infty$$

Example 8 – Solution

This shows that the line $x = \pi/2$ is a vertical asymptote. Similar reasoning shows that the lines $x = \pi/2 + n\pi,$ where *n* is an integer, are all vertical asymptotes of *f*(*x*) = tan *x*.

The graph in Figure 14 confirms this.



y = tan *x*

Figure 14

Exercise

Find the vertical asymptotes of the following:

1. $f(x) = \frac{x^2-25}{x^2+5x+6}$

$Answer: at\ x = -2\ and\ -3$

2. $f(x) = \frac{x-2}{x^2-7x+10}$

$Answer: at\ x = 5$

3. $f(x) = \frac{x-3}{x^2-9}$

$Answer: at\ x = -3$