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Mathematics 1A

ITMTA1-B44

Derivatives

With

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Lecture 2
Week 5

3 Differentiation Rules

3.4 The Chain Rule

Derivatives of General Exponential Functions

Derivatives of General Exponential Functions

We can use the Chain Rule to differentiate an exponential function with any base $b > 0$.

Given b^x, to differentiate this:

$$\begin{aligned}\frac{d}{dx}(b^x) &= \frac{d}{dx}\left(e^{\ln b^x}\right)\\ &= \frac{d}{dx}\left(e^{\ln b^x}\right)\\ &= \left[e^{\ln b^x}\right]\cdot[\ln e]\cdot\left[\frac{d}{dx}(\ln b^x)\right]\\ &= \left[e^{\ln b^x}\right]\cdot\left[\frac{d}{dx}(x\ln b)\right]\\ &= \boldsymbol{b^x \ln b}\end{aligned}$$

Example 10

Find the derivative of each of the functions.

(a) $g(x) = 2^x$ (b) $h(x) = 5^{x^2}$

Solution:

(a) We use Formula 5 with $b = 2$:

$$g'(x) = \frac{d}{dx}(2^x) = 2^x \ln 2$$

This is consistent with the estimate

$$\frac{d}{dx}(2^x) \approx (0.693)2^x$$

because ln 2 ≈ 0.693147.

Example 10 – Solution

(b) The outer function is an exponential function and the inner function is the squaring function, so we use Formula 5 and the Chain Rule to get

$$h'(x) = \frac{d}{dx}(5^{x^2})$$

$$h'(x) = 5^{x^2} \ln 5 \cdot \frac{d}{dx}(x^2)$$

$$h'(x) = 5^{x^2} \ln 5 \cdot 2x$$

$$h'(x) = 2x \cdot 5^{x^2} \ln 5$$

Derivative of ln$x$

If $f(x) = ln\, x$

Then

$$f'(x) = \frac{d}{dx}(\ln x) = \frac{1}{x}, for\ x \geq 0$$

If $f(x) = ln\, |x|$

Then

$$f'(x) = \frac{d}{dx}(\ln |x|) = \frac{1}{x}, for\ x \neq 0$$

Examples

Find the first derivative of the following functions:

1. $f(x) = 3\ln(\cos 2^x)$

2. $f(x) = x^2 \ln 3x^4$

Exercises

Find the first derivative of the following functions:

1. $f(x) = \sin 3^2$
2. $f(x) = \ln x^2$
3. $f(x) = \ln(\cos 4^{-x})$
4. $f(x) = 5x \ln 4x^3$

$\boldsymbol{f'(x) = \frac{2}{x}}$

$f'(x) = 4^{-x} \ln 4(\tan 4^{-x})$

$\boldsymbol{f'(x) = 5(3 + ln\ 4x^3)}$

Homework

Find the first derivative of the following functions:

1. $f(x) = -e^{-2x}\ln(\sin 3^x)$

$\boldsymbol{f'(x) = -e^{-2x}[3^x ln3\ (cot\ 3^x) - 2\ ln\ (sin\ 3^x)]}$

2. $f(x) = \log_e \frac{\sqrt{e^{-2x}\sin 7x^3}}{\ln x}$

Chapter 4, p265