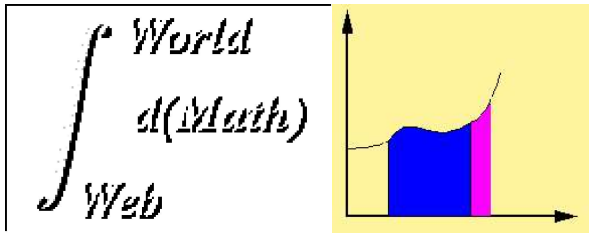

Formal Definition of Limits

For a function $f(x)$,

$$\lim_{x \to a} f(x) = L$$

if and only if for each positive number

$$\epsilon$$

(the lowercase Greek ``epsilon''), there exists a positive number

$$\delta$$

(the lowercase Greek letter "delta") with the property that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta.$$

That may be quite a bit to swallow all at once. You might want to shake those epsilons and deltas from your head and take a step back. The limit is concerned with what $f(x)$ looks like *around* the point $x = a$. The formal statement says that the limit L is the number such that if you take numbers arbitrarily close to a (or, values of x within delta of a) that the result of f applied to those numbers must be arbitrarily close to L (or, within epsilon of L).

One of the important things is that nowhere is the formal definition mention *anything* about the actual value of $f(x)$ at $x = a$. The value of $f(a)$ does not affect the limit, and may not even be defined.

Calculus page | World Web Math Categories Page

---

watko@mit.edu
Last modified August 19, 1998