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Mathematics 1A

ITMTA1-B44

Derivatives

With

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Lecture 2
Week 4

3 Differentiation Rules

3.1 Derivatives of Polynomials and Exponential Functions

Derivatives of Polynomials and Exponential Functions (1 of 1)

In this section we learn how to differentiate the following:

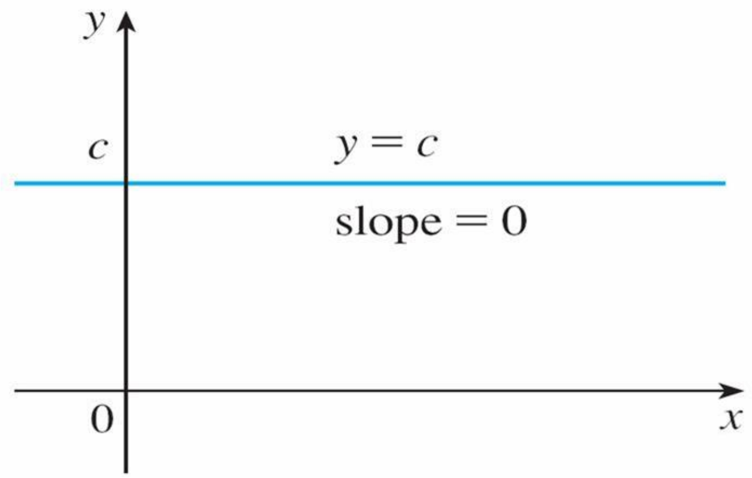
1. Constant functions
2. Power functions
3. Polynomials, and
4. Exponential functions.

Constant Functions

Constant Functions

Let's start with the simplest of all functions, the constant function $f(x) = c$.

The graph of this function is the horizontal line $y = c$, which has slope 0, so we must have $f'(x) = 0$. (See Figure 1.)



The graph of $f(x) = c$ is the line $y = c$, so $f'(x) = 0$.

Figure 1

Constant Functions

In Leibniz notation, we write this rule as follows.

Derivative of a Constant Function

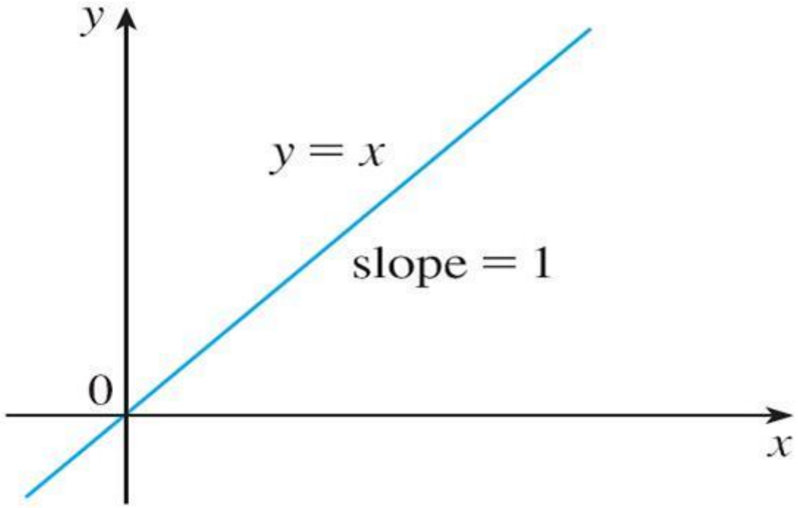
$$\frac{d}{dx}(c) = 0$$

Power Functions

Power Functions

We next look at the functions $f(x) = x^n$, where *n* is a positive integer.

If $n = 1$, the graph of $f(x) = x$ is the line $y = x$, which has slope 1. (See Figure 2.)



The graph of $f(x) = x$ is the line $y = x$, so $f'(x) = 1$.

Figure 2

Power Functions

So

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^2) = 2x \quad \frac{d}{dx}(x^3) = 3x^2$$

Power Functions

For n = 4 we find the derivative of $f(x) = x^4$ as follows:

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} \\
&= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h} \\
&= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \\
&= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) \\
&= 4x^3
\end{aligned}
$$

Power Functions

Thus

$$\frac{d}{dx}(x^4) = 4x^3$$

The Power Rule If *n* is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Exercises

(a) If $f(x) = x^6$,

then $f'(x) = 6x^5$.

(b) If $y = x^{1000}$

then $y' = 1000x^{999}$.

(c) If $y = t^4$,

then $\dfrac{dy}{dt} = 4t^3$.

(d) $\dfrac{d}{dr}(r^3) =?$

$\dfrac{d}{dr}(r^3) = 3r^2$

(e) $\dfrac{d}{dr}(10^3) =?$

$\dfrac{d}{dr}(10^3) = 0$

New Derivatives from Old

New Derivatives from Old

When new functions are formed from old functions by addition, subtraction, or multiplication by a constant, their derivatives can be calculated in terms of derivatives of the old functions.

In particular, the following formula says that *the derivative of a constant times a function is the constant times the derivative of the function.*

The Constant Multiple Rule If *c* is a constant and *f* is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Example 4

(a) $\frac{d}{dx}(3x^4)$

$$\begin{aligned} \frac{d}{dx}(3x^4) &= 3\frac{d}{dx}(x^4) \\ &= 3(4x^3) \\ &= 12x^3 \end{aligned}$$

(b) $\frac{d}{dx}(-x)$

$$\begin{aligned} \frac{d}{dx}(-x) &= \frac{d}{dx}[(-1)x] \\ &= (-1)\frac{d}{dx}(x) \\ &= -1(1) \\ &= -1 \end{aligned}$$

New Derivatives from Old

The next rule tells us that *the derivative of a sum (or difference) of functions is the sum (or difference) of the derivatives.*

The Sum and Difference Rules If *f* and *g* are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

The Sum Rule can be extended to the sum of any number of functions. For instance, using this theorem twice, we get

$$(f + g + h)' = [(f + g) + h]' = (f + g)' + h' = f' + g' + h'$$

New Derivatives from Old

The Constant Multiple Rule, the Sum Rule, and the Difference Rule can be combined with the Power Rule to differentiate any polynomial.

Exponential Functions

Exponential Functions

Let's try to compute the derivative of the exponential function $f(x) = b^x$ using the definition of a derivative:

$$f'(x) \quad = b^x \log_e b$$

OR

$$f'(x) \quad = b^x \ln b$$

Exponential Functions

If we put $b = e$ and, therefore,

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Also note that if $y = e^{nx}$

$$\frac{dy}{dx} = e^{nx} . \frac{d}{dx}(nx)$$

Example 8

If $f(x) = e^x - x$, find f' and f''. Compare the graphs of f and f'.

Solution:

Using the Difference Rule, we have

$$\begin{aligned} f'(x) &= \frac{d}{dx}(e^x - x) \\ &= \frac{d}{dx}(e^x) - \frac{d}{dx}(x) \\ &= e^x - 1 \end{aligned}$$

Example 8 – Solution

We defined the second derivative as the derivative of f', so

$$
\begin{aligned}
f''(x) &= \frac{d}{dx}(e^x - 1) \\
&= \frac{d}{dx}(e^x) - \frac{d}{dx}(1) \\
&= e^x
\end{aligned}
$$

Derivative of Trigonometric functions

Derivative of Trigonometric functions

Given a trigonometric function, we can find the derivative, f', using certain rules of differentiation:

Given $y = \sin x$,

$$\frac{dy}{dx} = \cos x$$

$y = cos\, x$,

$$\frac{dy}{dx} = -\sin x$$

Given $y = tan\, x$,

$$\frac{dy}{dx} = sec^2\, x$$

Examples

Find the first and second derivatives of the following functions, using any technique of your choice:

1. $f(x) = e^{\pi}$
2. $f(x) = x^{e}$
3. $f(x) = 3^{x}$
4. $f(x) = \frac{1}{x}$
5. $f(x) = \sqrt{x}$
6. $f(x) = e^{-2x^{5}}$
7. $f(x) = 5e^{2x}$

Exercises

Find the first derivatives of the following functions, using any technique of your choice:

1. $f(x) = 5x^2$ $\quad \mathbf{10x}$
2. $f(x) = \frac{2}{\sqrt{x}}$ $\quad -\frac{\mathbf{1}}{\sqrt{(x)^3}}$
3. $f(x) = \frac{1}{2}x^2$ $\quad \mathbf{x}$
4. $f(x) = x^{-2}$ $\quad -\frac{\mathbf{2}}{x^3}$
5. $f(x) = 3x^5$ $\quad \mathbf{15x^4}$
6. $f(x) = \frac{1}{2\sqrt{x}}$ $\quad -\frac{\mathbf{1}}{4\sqrt{x^3}}$
7. $f(x) = 2e^{-3x}$ $\quad -6e^{-3x}$
8. $f(x) = e^2$ $\quad \mathbf{0}$
9. $f(x) = 5^x$ $\quad 5^x \ln 5$
10. $f(x) = \frac{1}{\pi e^x}$ $\quad -\frac{1}{\pi e^x}$
11. $f(x) = \sqrt{3x}$ $\quad \frac{\sqrt{3}}{2\sqrt{x}}$
12. $f(x) = e^{-x^3}$ $\quad -3x^2e^{-x^3}$

Home Work

Find the first and second derivatives of the following functions, using any technique of your choice:

1. $f(x) = x^{2e} + 5$
2. $f(x) = 3x^2 - 7$
3. $f(x) = x^2 + 2x$
4. $f(x) = x^3 - x$
5. $f(x) = \frac{2}{x} + \sqrt{x}$
6. $f(x) = \pi^2 + 2x$
7. $f(x) = e^x$
8. $f(x) = e^{2x} + \pi^2 + 2x$