REVISION QUESTIONS

1. Verify if the function $f(x) = x^3 - x^2 - 6x + 2$ satisfies the three hypotheses of Rolle's Theorem on the interval $[0,3]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.
   $[\boldsymbol{Answer: c = 1.786}]$

2. Given that $f(x) = (x^2 + 1)(2 - x)$. Find the points on the graph of f where the tangent line is horizontal. Show all calculations.
   $[\boldsymbol{Answer: \left(\frac{1}{3}, \frac{50}{27}\right) \ and \ (1, 2)}]$

3. Determine the absolute extreme values of $f(x) = (x + 1)^{4/3}$ on the interval $[-9,7]$ if they exist. Show all calculations.
   $[\boldsymbol{Answer: max = 16 \ and \ min = 0}]$

4. The average speed of a vehicle on the N1 between 6 am and 10 am on a typical weekday is approximated by the function:
   $$f(t) = 20t - 40\sqrt{t} + 50 \qquad (0 \le t \le 4)$$
   Where $f(t)$ is measured in kilometers per hour and t is measured in hours, with $t = 0$ corresponding to 6 am. Find the interval where f is increasing and the interval where f is decreasing and interpret the results.
   $[\boldsymbol{Answer: Average \ speed \ decreases \ from \ 6 \ am \ to \ 7 \ am \ and \ then \ picks \ up \ from \ 7 \ am \ to \ 10 \ am}]$

5. The percentage of south Africans homes with pre-paid electricity meters between 2018 and 2022 was approximated by:
   $$N(t) = 1.1375t^2 + 0.25t + 4.6 \qquad 0 \le t \le 4$$
   Where t is measured in years, with $t = 0$ corresponding to 2018.
   Show that the percentages of homes in South Africa with pre-paid electricity meters was increasing between 2018 and 2022. Explain your answer.
   $[\boldsymbol{Answer: Increasing \ between \ 2018 \ and \ 2022}]$

6. A company introduces a new product for which the number of units sold S is:
   $$S(t) = 200\left(5 - \frac{9}{2 + t}\right)$$
   Where t is the time in months.
   a. Calculate the average rate of change of $S(t)$ during the first year.
   b. During what month of the year does $S'(t)$ equal the average rate of change.

   $[\boldsymbol{a. Answer: S(t) = \frac{450}{7}}]$

   $[\boldsymbol{b. Answer: t = 3.29 \ months, \quad S'(t) = The \ average \ value \ in \ April}]$

7. A train is travelling from Pretoria to Zanzibar. The train uses $5x^2$ of fuel per hour when it travels at a speed of $x\ kilometers\ per\ hour$. There are additional expenses of operating the train that amount to $R2000$ per hour.
   Calculate the speed that minimizes the cost of a $500 - kilometer$ trip.

   [Hint: Express cost in terms of speed and time.
   The constraint equation is $distance = speed \times Time$]
   [*Answer*: *The speed is* 20 *kilometers per hour*]

8. Dunlop Tyres expects to sell 600 000 tyres in the upcoming year. Tyres sales tend to be roughly the same from month to month. Setting up each production run costs for the company to be R15 000. The costs of carrying inventory, based on the average number of tyres in storage, amounts to R5 per year for one tyre.
   a. Determine the costs incurred if there are 10 production runs during the year.
   b. Determine the production run size that minimizes the overall cost of producing the tyres.

   NB: Let x be the number of tyres produced in each production run, and let r be the number of runs in the year.
   [*a. Answer*: $C(10) = 300\ 000$]
   [*b. Answer*: *Each production run produces* $x = 60\ 000$ *tyres*]

9. The height of a tulip tree in meters after t months is:
$$f(t) = \frac{1}{0.05 + e^{-4t}}$$
   Calculate the growth of the tulip tree after 7 months.
   [*Answer*: *The rate of growth is about* 2 *meters per month*]