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Mathematics 1A

ITMTA1-B44

Limits and Derivatives 2

With

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Lecture 9
Week 3

2 Limits and Derivatives

Infinite Limits at Infinity

Infinite Limits at Infinity (1 of 1)

The notation

$$\lim_{x \to \infty} f(x) = \infty$$

is used to indicate that the values of $f(x)$ become large as x becomes large. Similar meanings are attached to the following symbols:

$$\lim_{x \to -\infty} f(x) = \infty \quad \lim_{x \to \infty} f(x) = -\infty \quad \lim_{x \to -\infty} f(x) = -\infty$$

Example 9

Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

Solution:

When *x* becomes large, x^3 also becomes large. For instance,

$$10^3 = 1000 \qquad 100^3 = 1{,}000{,}000 \qquad 1000^3 = 1{,}000{,}000{,}000$$

In fact, we can make x^3 as big as we like by requiring *x* to be large enough.

Therefore we can write

$$\lim_{x\to\infty} x^3 = \infty$$

Example 9 – Solution

Similarly, when x is large negative, so is x^3. Thus

$$\lim_{x \to -\infty} x^3 = -\infty$$

These limit statements can also be seen from the graph of $y = x^3$ in Figure 11.

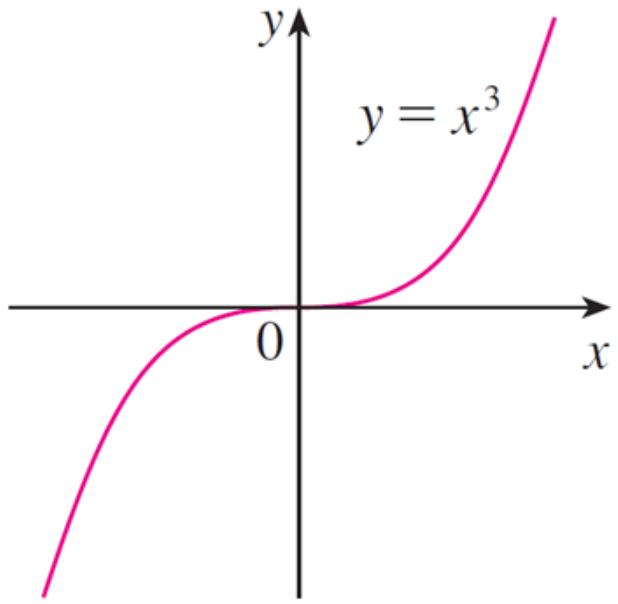

Figure 11

$$\lim_{x \to \infty} x^3 = \infty, \lim_{x \to -\infty} x^3 = -\infty$$

Example 6 – Solution

EXAMPLE 10 Find $\lim_{x \to \infty} (x^2 - x)$.

SOLUTION It would be **wrong** to write

$$\lim_{x \to \infty} (x^2 - x) = \lim_{x \to \infty} x^2 - \lim_{x \to \infty} x = \infty - \infty$$

The Limit Laws can't be applied to infinite limits because ∞ is not a number ($\infty - \infty$ can't be defined). However, we *can* write

$$\lim_{x \to \infty} (x^2 - x) = \lim_{x \to \infty} x(x - 1) = \infty$$

because both x and $x - 1$ become arbitrarily large and so their product does too.

Exercises

15–38 Find the limit or show that it does not exist.

15. $\lim_{x \to \infty} \dfrac{3x - 2}{2x + 1}$

16. $\lim_{x \to \infty} \dfrac{1 - x^2}{x^3 - x + 1}$

17. $\lim_{x \to -\infty} \dfrac{x - 2}{x^2 + 1}$

18. $\lim_{x \to -\infty} \dfrac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$

19. $\lim_{t \to \infty} \dfrac{\sqrt{t} + t^2}{2t - t^2}$

20. $\lim_{t \to \infty} \dfrac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5}$

21. $\lim_{x \to \infty} \dfrac{(2x^2 + 1)^2}{(x - 1)^2(x^2 + x)}$

22. $\lim_{x \to \infty} \dfrac{x^2}{\sqrt{x^4 + 1}}$

23. $\lim_{x \to \infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

24. $\lim_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

25. $\lim_{x \to \infty} \left(\sqrt{9x^2 + x} - 3x\right)$

26. $\lim_{x \to -\infty} \left(x + \sqrt{x^2 + 2x}\right)$

Exercises

27. $\lim_{x \to \infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 + bx} \right)$

28. $\lim_{x \to \infty} \sqrt{x^2 + 1}$

29. $\lim_{x \to \infty} \dfrac{x^4 - 3x^2 + x}{x^3 - x + 2}$

30. $\lim_{x \to \infty} (e^{-x} + 2 \cos 3x)$

31. $\lim_{x \to -\infty} (x^4 + x^5)$

32. $\lim_{x \to -\infty} \dfrac{1 + x^6}{x^4 + 1}$

33. $\lim_{x \to \infty} \arctan(e^x)$

34. $\lim_{x \to \infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

35. $\lim_{x \to \infty} \dfrac{1 - e^x}{1 + 2e^x}$

36. $\lim_{x \to \infty} \dfrac{\sin^2 x}{x^2 + 1}$

37. $\lim_{x \to \infty} (e^{-2x} \cos x)$

38. $\lim_{x \to 0^+} \tan^{-1}(\ln x)$