Initial Online Assessment Paper

Faculty Name:	Information Technology
Module Name:	Mathematics 1A Block 2
Module Code:	ITMTA1-B22
Start Date:	13/07/2021
Submission:	3 Hours
Total Marks:	80
Examiner:	Jennifer Mafemba
External Moderator:	Vingin Leung
Copy Editor:	Kevin Levy
Resources Required:	A non-programmable calculator
Section A: Multiple Choice Question(s)	10 Marks
Section A: Application-Based Question(s)	30 Marks
Section B: Scenario-Based Question(s)	40 Marks

This assessment contributes 50% towards the final mark.

Instructions to Student

1. Ensure that you are writing the correct assessment.
2. Read each question carefully.
3. No late submissions allowed.

Section A

Multiple Choice Questions   **10 Marks**

There is only one right answer. If more than one option is listed, only the first option appearing in the answer book will be marked.

Question 1

1.1 Express the given function, $y = \frac{9}{x^2} + 4$ as a composite of functions f and g such that $y = f(g(x))$.

   a. $f(x) = \frac{9}{x^2}$, $g(x) = 4$

   b. $f(x) = \frac{1}{x}$, $g(x) = \frac{9}{x} + 4$

   c. $f(x) = x + 4$, $g(x) = \frac{9}{x^2}$

   d. $f(x) = x$, $g(x) = \frac{9}{x} + 4$

1.2 Determine whether the graph of the equation, $f(x) = 9x^4 - 5x - 5$ has symmetry about the x-axis, the y-axis, and the origin.

   a.   Origin

   b.   x - axis

   c.   x – axis, origin

   d.   No symmetry

1.3 The distance D, in meters that an object has fallen after t seconds is given by $D(t) = 16t^2$.

   i.   Evaluate $D(3)$ and $D(4)$.

   ii.   Calculate the slope of the secant line through $D(3)$ and $D(4)$ on the graph of D and interpret the answer in terms of the average rate of change of D from 3 to 4.

   a.   (i) 144, 256

         (ii) 112, the objects average speed from 3 to 4 is 112 meters/second.

   b.   (i) 48, 64

         (ii) 112, the objects average speed from 3 to 4 is 112 meters/second.

   c.   (i) 144 256

         (ii) 16, the objects average speed from 3 to 4 is 16 meters/second.

   d.   (i) 48, 64

         (ii) 16, the objects average speed from 3 to 4 is 16 meters/second.

1.4   Find the limit of the function, $\lim\limits_{x \to 2}(x^3 + 5x^2 - 7x + 1)$.

   a.   15

   b.   29

   c.   0

   d.   It does not exist.

1.5   Find the domain and the range of the function, $f(x) = -3 + \sqrt{x}$.

   a.   Domain: $(-\infty, 0]$, Range: $(-\infty, -3]$

   b.   Domain: $[0, \infty)$, Range: $(-\infty, \infty)$

   c.   Domain: $(-\infty, \infty)$, Range: $[-3, \infty)$

   d.   Domain: $[0, \infty)$, Range: $[-3, \infty)$

1.6   If $f(x) = 4x + 9$ and $g(x) = 3x - 1$, find $f\big(g(x)\big)$.

   a. $12x + 13$

   b. $12x + 8$

   c. $12x + 5$

   d. $12x + 26$

1.7   If $f(x) = -7x - 6$ and $g(x) = -8x^2 - 4x + 3$, find $g\big(f(3)\big)$.

   a.   393

   b. -177

   c. -1857

   d. -279

1.8   Find $-f(x)$ when $f(x) = 2x^2 + 5x + 2$.

   a. $2x^2 - 5x + 2$

   b. $-2x^2 - 5x - 2$

   c. $-2x^2 + 5x + 2$

   d. $-2x^2 - 5x + 2$

1.9 Write the equation that results in the desired transformation:

The graph of $y = x^3$, vertically compressed by a factor of 0.8.

    a. $y = 0.8\sqrt[3]{x}$

    b. $y = (x - 0.8)^3$

    c. $y = (x + 0.8)^3$

    d. $y = 0.8x^3$

1.10 Find an equation for the tangent to the curve, $y = x - x^2$ at the given point, $(-1, -2)$.

    a. $y = -3x + 1$

    b. $y = -x + 1$

    c. $y = -x - 1$

    d. $y = 3x + 1$

(10 × 1 Mark)

End of Section A

Section B
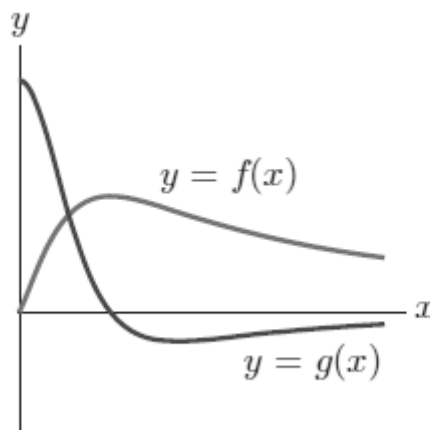
Application-Based Question(s)   **30 Marks**

Question 2

2.1   Suppose that $f(t) = 3t + 2 - 12t$.

    a.   What is the average rate of change of $f(t)$ over the interval 2 to 3?   (4 Marks)

    b.   What is the (instantaneous) rate of change of $f(t)$ when $t = 2$?   (2 Marks)

2.2   Determine which function is the derivative of the other. Explain your answer.

(4 Marks)

[Sub Total 10 Marks]

Question 3

Calculate the derivatives of the following functions using the appropriate rules of derivatives.

3.1 $y = (x^2 + 1)^2 + 3(x^2 - 1)^2$ (5 Marks)

3.2 $y = 2\sqrt[4]{x^2 + 1}$ (5 Marks)

[Sub Total 10 Marks]

Question 4

Calculate the derivatives of the following functions using either the product, quotient or chain rule. Simplify your answer in each case.

4.1 $s = \frac{\sqrt{t}}{1+\sqrt{t}}$ (5 Marks)

4.2 $g(x) = \left(\frac{3x^2-2}{2x+3}\right)^{-2}$ (5 Marks)

[Sub Total 10 Marks]

End of Section B

Section C

Scenario-Based Question(s)   **40 Marks**

Question 5

Find the critical numbers of the function, $g(t) = t\sqrt{4-t}$ where $t < 3$.   (10 Marks)

Question 6

6.1   Given the function, $f(x) = \frac{x^2-24}{x-5}$, find the intervals on which the function is increasing or

decreasing.   (8 Marks)

6.2   Find the local maximum and minimum values of $f(x) = \frac{x^2-24}{x-5}$.   (2 Marks)

[Sub Total 10 Marks]

Question 7

7.1   Determine whether the Mean Value Theorem applies to the following function, $f(x) = \frac{x}{(x+2)}$

on the interval [-1, 2].   (1 Mark)

7.2   If so, determine the point(s) that are guaranteed to exist by the Mean Value Theorem.

(5 Marks)

7.3   Draw a sketch of the function and the line that passes through $(a, f(a))$ and $(b, f(b))$. Mark

the points P at which the slope of the function equals the slope of the secant line. Sketch

the tangent line at P.   (4 Marks)

[Sub Total 10 Marks]

Question 8

A model used for the yield Y of an agricultural crop as a function of the nitrogen level, N in the

soil (measured in appropriate units) is

$$Y = \frac{k\,N}{1 + N^2}$$

Where k is a positive constant. What nitrogen level gives the best yield?   (10 Marks)

End of Section C