Midterm CM1020 coursework assignment - dp261@london.ac.uk

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Question 1

(a)

I. 10 is in all three so X ∩ Y ∩ Z = { 10 }
(X \ Z) is the { 4, 6, 8 }
X ∪ (X \ Z) = { 2, 4, 6, 8, 10 }

II. X = { 2, 4, 6, 8, 10 }
The values of all X must be in the other two sets, however this is False. So X cannot be a subset

III. P (X union (Y and Z)) , therefore:

P(X ∪ (Y ∩ Z)) = { 2, 4, 6, 8, 10, 12 } = 2^6 = 64

(b)

Powerset (A) subset {∅, {∅}, {{∅}}} , All empty sets , we test this

{ ∅ , {∅}} will be an empty set of the set and the general empty set = A could be a subset of these

{{ Ø }} is a nested subset, however this set would be an empty set and a single set like { Ø , {Ø}}

Therefore both would be equal, the powerset of these two would then True

(c)

A ⊆ B (if and only if) A ∩ C ⊆ B ∩ C for all sets of C

I will try disapprove this statement:

A ⊄ B, however A ∩ C subset B ∩ C for all C

A = {1}, B = {∅}, ∴ A ⊄ C, hence true

A ∩ C = {1} but B ∩ C = ∅

And {1} cannot be a subset of {∅}, so fail and the statement is True

(d)

If A ⊆ B and C ⊆ B`, then A ∩ C = ∅        B` = complement of B

Universal Set U.

So if A is a subset of B , and C is a subset of B`

A ∩ B ⊆ C ∩ `B , A and C = empty set

Therefore: A is in B, C are not in B , they can not share elements

Question 2

(a)

I. $f(x) = \frac{1}{ln(x-1)}$: $if\ x = 1,\ then\ \frac{1}{ln(1-1)}$ so then the equation will be undefined. However the real number can be negative, zero or positive. **Is Not a function**

II. $f(x) = x^3 - 2x + 5$: This is an exponential function, As a set of integers can be positive, zero or negative. **Is a function**

III. $f(x) = \sqrt{x - 3}$: This is a square root function, the number can be positive,negative or zero. This also does not pass the function test. **So it Not a function**

b)

$(\sqrt{x + 3})^2 + b = \sqrt{(x^2 + b) + 3}$

Left side:

$x + 3 + b$

Right side:

$\sqrt{x^2 + b + 3}$

$x + 3 + b = \sqrt{x^2 + b + 3}$

$(x + 3 + b)^2 = x^2 + b + 3$

$x^2 + b^2 + xb^2 + 6x + 6b + 9 = x^2 + b + 3$

$b^2 + xb^2 + 6x + 6b + 6 = b$

$6x + 6 = b - b^2 - 6b - xb^2$

If I divide by x, I will not get a final value. ∴ There is no real value for b

c)

$log_4(x^2) - log_4(3x - 2) = 0$

$log_4(\frac{x^2}{3x-2})$ = 0

We Will tackle the brackets: $x^2(3x - 2)^{-1} = x^2 - 3x + 2$

∴ $(x - 1)(x - 2) = 0$

$x = 1 \; or \; x = 2$

d)

I. $f'(x) = e^x + 1 > 0$ ∴ x is increasing, **SO yes is one-to-one**

II. $f(0) = 1$ The range is cut at 0, so x cannot equal 0, SO 0 is exclusive to the domain. **No, not onto** e)

Let $g(f(x_1) = g(f(x_2))$, then we can say f is injective. Because $f(x_1) = f(x_2)$

Therefore $g \circ f$ is one-to-one

Question 3

a)

I.

p	q	r	$p \oplus q$	$p \oplus q \rightarrow r$	$p \vee q$	$p \wedge r$	$(p \vee q) \rightarrow (p \wedge r)$
t	t	t	f	t	t	t	t
t	t	f	f	t	t	f	f
t	f	t	t	t	t	t	t
t	f	f	t	f	t	f	f
f	t	t	t	t	t	f	f
f	t	f	t	f	t	f	f
f	f	t	f	t	f	f	t
f	f	f	f	t	f	f	t

II. $(p \oplus q)$ r is false when $p \oplus q = T\, and\, r = F$
So that means row 4 and 6. **NO tautology**

$(p \lor q) \rightarrow (p \land r)\ is\ False\ too$

b)

Right side:
$\neg p = T,\ \neg p \land q = T\ and\ T = T$
$r = F,\ hence\ F \rightarrow T = T$
True

Left side
$p \lor \neg r = F\ or\ T = T$
And $q \rightarrow s = T$
True

∴ True

c)

I. $r \rightarrow (p \land q)$
II. $r \rightarrow (p \oplus q)$
III. $r \leftrightarrow (p \land q)$

d)

Contrapositive

$\neg Q(x) \rightarrow \neg P(x)$

$if\ x \in [1,\ 2],\ then\ x^2 - 3x + 2 \leq 0$

$If\ 1 \leq x \leq 2,\ then\ x^2 - 3x + 2 \leq 0$

Converse

$Q(x) \rightarrow P(x)$

$if\ x > 2\ or\ x < 1,\ then\ x^2 - 3x + 2 > 0$

Inverse

$\neg P(x) \rightarrow \neg Q(x)$

$If\ x^2 - 3x + 2 \leq 0,\ then\ x \in [1, 2]$

$If\ x^2 - 3x + 2 \leq 0,\ then\ 1 \leq x \leq 2$

e)

Left side

$(p \wedge q) = T$

And $r \rightarrow s = F \rightarrow F = T$

LHS = T

Right side
$p \vee r = T, (p \vee r) \rightarrow r = T \rightarrow F = F$
And $q \vee r = T, (q \vee r) \rightarrow s = T \rightarrow F = F$
RHS = F

∴ LHS $\neq$ RHS -> NOt a Tautology

Question 4

a)

I. $D(x)$: Doctor
$M(x)$: Wears Mask

$\therefore \forall (D(x) \rightarrow M(x))$

II. $W(x)$: is watered
$H(x)$: grows healthy

$\therefore \forall (W(x) \rightarrow H(x)$

III. $Domain$: $\mathbb{R}$ numbers
$L(x, y)$: $x > y$

$\therefore \exists x\, \forall y\, (x \neq y \rightarrow L(x, y))$

IV. $W(x)$: wings
$F(x)$: fly

$\therefore \forall x (W(x) \rightarrow F(x))$

b)

I. Every Integer, find a nonzero → xy < 1
Hence, try: for all x, y and xy for values [0, 1)

$\therefore$ **True**

II. Every Non-zero, every Integer xy > 1
Hence try:x, y, and xy for values (-1, 1) but values cannot equal 0

$\therefore$ **False**

III. So xy = 2, we could $\frac{2}{x}$ as x cannot equal 0. Therefore exists

∴ True

c)

d)

We are given the equation

1. $s \rightarrow (p \vee q)$ (assuming F)
2. $\neg p \rightarrow r$ (assuming T)
3. $\neg q \rightarrow r$ (assuming T)
4. $\neg s$

   $\therefore r$

This argument is **not valid , there's just not enough guarantee**

Question 5

a)

I.

$\overline{(p \cdot \overline{q} \cdot r)} \cdot \overline{(\overline{r} \cdot s)}$ (de morgan's law)

First term

$\overline{p \cdot \overline{q} \cdot r} = \overline{p} + q + \overline{r}$

Second term

$\overline{(\overline{r} \cdot s)} = r + \overline{s}$

Simplify

$(\overline{p} + q + \overline{r}) \cdot (r + \overline{s})$

II. First

$\overline{(x + \bar{y})} = x \cdot \bar{y}$

Second

$\overline{(\bar{x} + y)} = \bar{x} \cdot y$

Third

$\overline{(\bar{y} + z)} = y \cdot \bar{z}$

Simplified

$x \cdot \bar{y} \cdot \bar{x} \cdot y \cdot y \cdot \bar{z}$

$x \cdot \bar{x} = 0$ and $y \cdot \bar{y} = 0$

∴ ii is equal to zero

b)

I. $Q = \overline{(A \cdot \overline{B}) + (\bar{C} \cdot D)}$

II. De morgan's law

$\overline{(A \cdot \bar{B} + \bar{C} \cdot D}$ = $\overline{A \cdot \bar{B}} \cdot \overline{\bar{C} \cdot D}$

Q = $(\bar{A} + B) \cdot (C + \bar{D})$

c)

To do Duality we will need to modify the existing equation to work

LHS:

$a \cdot b + c \cdot d \Rightarrow (a + b) \cdot (c + d)$

RHS:

$(a + c)(a + d)(b + c)(b + d) \Rightarrow (a \cdot c) + (a \cdot d) + (b \cdot c) + (b \cdot d)$

Therefore

$(a + b) \cdot (c + d) \; = \; (a \cdot c) + (a \cdot d) + (b \cdot c) + (b \cdot d)$

d)

I.

A	B	C	D	$f = (A, B, C, D)$
0	1	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1

II. Karnaugh map

	CD				
		00	01	11	10
AB					
00		0	0	0	0
01		0	1	0	0
11		1	1	0	0
10		0	1	1	1

III.

$$F = A\overline{C} + A\overline{B}C + B\overline{C}D$$

End of assignment