UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Monday 4 March 2024 at 12:00 midday Greenwich Mean Time
Close date: Tuesday 5 March 2024 by 12:00 midday Greenwich Mean Time
Time allowed: 4 hours to submit

INSTRUCTIONS TO CANDIDATES:
Part A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs). You should attempt to answer **ALL** the questions in **Part A.** The maximum mark for Part A is **40**.

Candidates must answer **TWO** out of the **THREE** questions in **Part B**. The maximum mark for Part B is **60**.

Part A and Part B will be completed online together on the Inspera exam platform. You may choose to access either part first upon entering the test area but must complete both parts within **4 hours** of doing so.

A handheld non-programmable calculator may be used when answering questions on this paper, but it must not be able to display graphics, text, or algebraic equations. Please hold your calculator to the camera at the start of the examination to clearly show the make and type.

You may use **ONE** A4 page of previously prepared notes in this examination. Please hold up your notes to the camera at the start of the examination.

File upload is permitted in this examination.

Do not write your name anywhere in your answers.

PART A

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) in Part A.

PART B

Candidates should answer any **TWO** questions from Part B.

Question 1

(a) List the elements of the following sets:

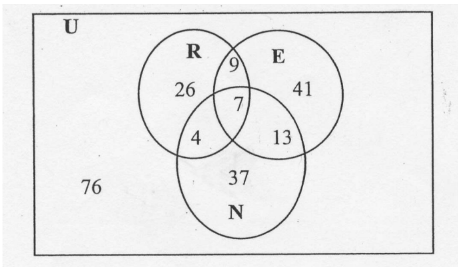
i. $\{x | x \in \mathbb{Z} \wedge (x^2 = 6)\}$

ii. $\{x | x \in \mathbb{Z} \wedge (x^2 = 9)\}$

iii. $\{x | x \in \mathbb{N} \wedge (x \mod 2 = 1) \wedge (x < 10)\}$

[3 marks]

(b) Let E, R and N be three subsets of the universal set U shown in the diagram below. The number given on each region represents the number of elements in that region.



Use the above diagram to find the cardinality for each of the following sets:

$$N,\ \overline{E}, (R \cap E) \cup N, E - R, E \cap R \cap E, \text{ and } U$$

[6 marks]

(c) Prove that if A and B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. [6 marks]

(d) Prove whether the following set identities are correct or not:

i. $(A \cap B) \cup (A \cap \overline{B}) = A$

ii. $(A - B) - C \subseteq A - C$

iii. $(A - C) \cap (C - B) = \emptyset$

[6 marks]

(e) Let p and q be two propositions. Is $(p \to q) \to [(p \to q) \to q]$ a tautology? Explain your answer. [5 marks]

(f) The domain of discourse consists of all positive intergers, $\mathbb{Z}^+$. What are the truth values for each of the following:

i. $\forall x \exists y (x < y)$

ii. $\forall x \exists y (x + y = 0)$

iii. $\exists x \forall y (x < y)$

iv. $\exists x \exists y (x * y < 0)$

[4 marks]

Question 2

(a) Re-write the following statements without any negations on quantifiers:

i. $\neg \exists x P(x)$

ii. $\neg \exists x \neg \exists y P(x, y)$

iii. $\neg \exists x \forall y P(x, y)$

[3 marks]

(b) Given the following truth table for a four-input logic circuit:

A	B	C	D	Out
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

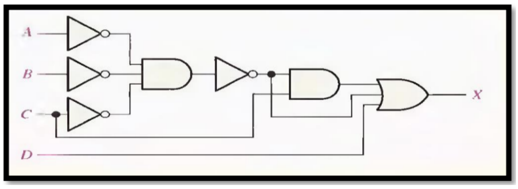
i. Write the sum-of-products expression for this truth table. [2 marks]

ii. Use the laws of Boolean algebra to reduce this expression to its simplest form. [2 marks]

iii. Translate the truth table into its corresponding Karnauph map. [2 marks]

iv. Minimise the sum-of-production expression in (i) using karnauph map. [2 marks]

(c) Given the following logical circuit with three inputs A, B and C:



i. Use the boolean algebra notation and write down the boolean expression of the output x of this circuit. [4 marks]

ii. Simplify the logical expression in (i). Explain your answer. [5 marks]

(d) The function f is given by:

$f(x) = \ln(4x - 2)$ where $x \in \mathbb{R}$ and $x > \frac{1}{2}$

i. Find an expression for $f^{-1}(x)$ in its simplest form. [3 marks]

ii. State the range of $f^{-1}(x)$ [2 marks]

iii. Solve the equation $f(x) = 1$ [2 marks]

(e) How many binary sequences of length 10 start with a 1 and end with a 0?

[3 marks]

Question 3

(a) Either draw a graph with the following specified properties, or explain why no such graph exists:

i. A graph with four vertices having the degrees of its vertices 1, 2, 3, and 4.

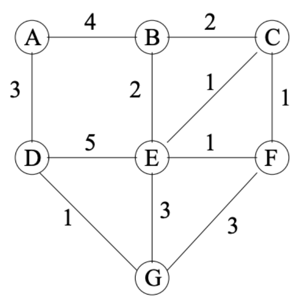
ii. A simple graph with five vertices with degrees 2, 3, 3, 3, and 5.

iii. A simple graph in which each vertex has degree 3 and which has exactly six edges.

iv. A graph with four vertices having the degrees of its vertices 1, 1, 2, and 6.

[8 marks]

(b) Given the following weighted undirected graph:



i. Execute Prim's algorithm on the graph above starting at vertex A. If there are any ties, the vertex with the lower letter comes first. List the edges in the order in which they are added to the tree. [3 marks]

ii. Execute Kruskal's algorithm on the graph above. Assume that equal weight edges are ordered lexicographically by the labels of their vertices assuming that the lower labeled vertex always comes first when specifying an edge, e.g. (C, E) is before (C, F) which in turn is before (D, G). List the edges in the order in which they are added to the developing forest.

[3 marks]

(c) Given S is the set of integers $\{2, 3, 4, 5, 6, 7, 8\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,

$$\forall x, y \in S, xRy \iff x - y \mod 2 = 0$$

i. Draw the digraph of $\mathcal{R}$. [2 marks]

ii. Show that $\mathcal{R}$ is an equivalence relation. [6 marks]

iii. Find the equivalence classes for $\mathcal{R}$. [2 marks]

(d) Suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ are equivalence relations on a set A. Define the relation $\mathcal{R}$ on A by $x\mathcal{R}y$ if ($x\mathcal{R}_1y$ and $x\mathcal{R}_2y$). Give the first two steps of the proof that $x\mathcal{R}_1y$ is an equivalence relation by showing that $\mathcal{R}$ is reflexive and symmetric. [6 marks]

END OF PAPER

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