Discrete Mathematics

Tutorial sheet

Predicate Logic

Question 1.
Let $P(x)$ be the predicate "$x^2 > x$" with the domain the set $\mathbb{R}$ of all real numbers. Write $P(2)$, $P(\frac{1}{2})$, and $P(-\frac{1}{2})$ and indicate which of these statements are true and which are false.

Question 2.
Let $P(x)$ be the predicate "$x^2 > x$" with the domain the set $\mathbb{R}$ of all real numbers. What are the values $P(2) \wedge P(\frac{1}{2})$, and $P(2) \vee P(\frac{1}{2})$?

Question 3.

1. Let $D = \{1, 2, 3, 4\}$, and consider the following statement:

   $$\forall x \in D, x^2 \geq x.$$

   Write one way to read this statement, and show that is it true.

2. Show that the following statement is false.

   $$\forall x \in \mathbb{R}, x^2 \geq x$$

Question 4.

1. Consider the following statement:

   $$\exists n \in \mathbb{Z}^+ \text{ such that } n^2 = n$$

   Write one way to read this statement, and show that is it true.

2. Let $E = \{5, 6, 7, 8\}$, and consider the following statement:

   $$\exists n \in E, n^2 = n.$$

   Show that this statement is false.

Question 5.
Rewrite each of the following statements formally, Use quantifiers and variables.

1. All triangles have three sides.

2. No dogs have wings.

3. Some programs are structured.

Question 6.
Rewrite the following statements in form of $\forall$________ if _______ then _______

1. If a real number is an integer, then it is a rational number

2. All bytes have eight bits

3. No fire trucks are green

Question 7.
A **prime number** is an integer greater than 1 whose only positive integer factors are itself and 1. Consider the following predicate **Prime**(n): "n is is prime " and **Even**(n): "n is even". Use the notation **Prime**(n) and **Even**(n) to rewrite the following statement:

"There is an integer that is both prime and even "

Question 8.
Determine the truth value each of the following where $P(x, y) : y < x^2$, where x and y are real numbers:

1. $(\forall x)(\forall y)P(x, y)$

2. $(\exists x)(\exists y)P(x, y)$

3. $(\forall y)(\exists x)P(x, y)$

4. $(\exists x)(\forall y)P(x, y)$

Question 9.
Let $P(x)$ denote the statement x is taking discrete mathematics course. The domain of discourse is the set of all students. Write each of the following statements in words.

$$\forall x P(x), \quad \forall x \neg P(x), \quad \neg(\forall x P(x)), \quad \exists x P(x), \quad \exists x \neg P(x), \quad \neg(\exists x P(x)).$$

Question 10.
Let $P(x)$ denote the statement 'x is a professional athlete', and let $Q(x)$ denote the statement 'x plays football'. The domain of discourse is the set of all people. Write each of the following in words.

1. $\forall x(P(x) \to Q(x))$
2. $\exists x(Q(x) \to P(x))$
3. $\forall x(P(x) \land Q(x))$

Question 11.
Let $P(x)$ denote the statement 'x is a professional athlete', and let $Q(x)$ denote the statement 'x plays football'. The domain of dis- course is the set of all people. Write the negation of each proposition symbolically and in words.

1. $\forall x(P(x) \to Q(x))$
2. $\exists x(Q(x) \to P(x))$
3. $\forall x(P(x) \land Q(x))$

Question 12.
Let P and Q denote the following propositional functions:

- $P(x)$: "x is greater than 2"
- $Q(x)$: "x^2 is greater than 4 "

where, the universe of discourse for both $P(x)$ and $Q(x)$ is the set of real number, $\mathbb{R}$.

1. Use quantifiers and logical operators to write the following statement formally

   " if a real number is greater 2, then its square is greater than 4."

2. Write a formal and informal contrapositive, converse and inverse of the statement above in (1).

Question 13.

1. Rewrite each of the following statements in English as simply as possible without using the symbols $\forall$ or $\exists$ or variables.

   (a) $\forall$ color c, $\exists$ an animal a such that a is colored c.

   (b) $\exists$ a book b such that $\forall$ person p, p has read b.

   (c) $\forall$ odd integer n, $\exists$ an integer k such that $n = 2k + 1$.

   (d) . $\forall$ $x \in \mathbb{R}$, $\exists$ a real number y such that $x + y = 0$.

2. Write a negation for each of the statements above.

Question 14.
Rewrite the statement "No good cars are cheap " in the form "$\forall x$,if $P(x)$ then $\neg Q(x)$". Indicate whether each of the following arguments is valid or invalid, and justify your answers.

1. No good care are cheap
   A Ferrari is a good car
   $\therefore$ A Ferrari is not cheap

2. No good cars are cheap
   A BMW is not cheap
   $\therefore$ A BMW is no a good car

Question 15.
Let x be any student and C(x), B(x) and P(x) be the following statements:
$C(x)$: "x is in this class".
$B(x)$: "x has read the book".
$P(x)$: "x has passed the first exam".
Rewrite the following symbolically and state whether it a valid argument.
A student in this class has not read the book
Everyone in this class passed the first exam
$\therefore$ Someone who passed the first exam has not read the book

End of questions