Discrete Mathematics

Tutorial sheet | Propositional Logic

Question 1.
Which of the following statements are propositions:

1. $2 + 2 = 4$
2. $2 + 2 = 5$
3. $x^2 + 2 = 11$
4. $x + y > 0$
5. This coffee is strong

Question 2.
Let s and i be the following propositions:
s:"stocks are increasing"
i:"interest rates are steady"
Write each of the following sentences symbolically:

1. Stocks are increasing but interest rates are steady
2. Neither are stocks increasing nor are interest rates steady

Question 3.
Let h, s and r be the following three propositions:
h:"It is hot"
s:"It is sunny"
r:"It is raining"
Write each of the following sentences symbolically:

1. It is not hot but it is sunny
2. It is neither hot nor sunny
3. It is either hot and sunny or it is raining
4. It is sunny or it is raining but not both

Question 4.
Let l denote one of the letters in the word "$software$". The following propositions relate to l

$$p: \text{“}l \text{ is a vowel”}; \quad q: \text{“}l \text{ comes after the letter } k \text{ in the alphabet”}.$$

Use the *listing method* to specify the truth sets corresponding to each of the following statements:

$$\neg q; \qquad p \wedge \neg q; \qquad \neg p \vee q.$$

Question 5.
Let p and q be two propositions. Construct a truth table to show the truth value of each of the following logical statements:

$$p \vee q, \qquad \neg p \vee \neg q, \qquad p \wedge q, \qquad \neg(p \wedge q)$$

What can we say about the following two statements: $\neg p \vee \neg q$ and $\neg(p \wedge q)$?
Question 6.
Let h, s and r be the following three propositions:
h:"It is hot"
s:"It is sunny"
r:"It is raining"
Write each of the following sentences symbolically:

1. It is sunny or it is raining but not both
2. It is hot only if it is sunny
3. It is hot only if it is sunny and not raining.

Question 7.
Let p, q be propositions. Construct a truth table to show the truth value of each of the following statements:

$$p \to q, \qquad \neg p \vee q, \qquad \neg q \to \neg p.$$

What can we say the above three logical statements?
Question 8.
Let p and q be the following propositions concerning a positive integer n.

$$p: \text{“}n \text{ is divisible by 5”}; \quad q: \text{“}n \text{ is even”}.$$

1. Express in words the following statements.

$$(i)\ p \vee \neg q; \quad (ii)\ \ p \wedge\ q.$$

2. List the elements of the truth sets corresponding to each of the statements in (1).

3. Express each of the following conditional statements symbolically.

(i) *if* n *is odd then* n *is divisible by* 5.
(ii) n *is even or* n *is divisible by* 5 *but not both.*

Question 9.
Let p and q be two propositions. Show that $p \vee \neg(p \wedge q)$ is a tautology.
Question 10.

Copy and complete the following table by giving the truth value of each of the statements $p,\ q,\ p \to q,\ q \to p$ and $p \leftrightarrow q$.

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
0	0			
0	1			
1	0			
1	1			

Question 11.
Write the inverse, the converse and the contrapositive of the following statement.

If it is November 5th then we have fireworks.

Question 12.
Let p denote the following statement about integers n:

If n is divisible by 15, then it is divisible by 3 or divisible by 5.

Write the inverse, the converse and the contrapositive of p.
Question 13.
Let p and q be two propositions. Show, by constructing the truth table or otherwise, that the following statements are equivalent:

$$p \to q \text{ and } \neg(\neg(p \wedge q) \wedge p)$$

Question 14.
Let p and q be two propositions, show that $(p \wedge \neg q) \vee (p \wedge q) = p$.
Question 15.
Let p, q and r be three propositions, show that $p \to (q \to r)$ and $(p \wedge q) \to r$ are two equivalent statements.

End of questions