UNIVERSITY OF LONDON

BSc COMPUTER SCIENCE

CM1020 Discrete Mathematics

Midterm coursework assignment

INSTRUCTIONS TO CANDIDATES:

This coursework assignment consists of **FIVE** questions. You should answer **ALL** the questions.

There are 100 marks available in this coursework assignment. The marks for each question are indicated at the end of the part in [.] brackets. **Full marks will be awarded for complete answers to a total of FIVE questions. Make sure to show your work clearly in each of the questions**.

The aim of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics.

Submission requirements

Please submit **one** PDF document for this coursework assignment.

IMPORTANT: you must submit work that is properly formatted using the maths mode of your word processor. Any handwritten submissions will be subject to penalty of being capped at 40 marks.

Question 1

Show your work. Full marks will only be awarded if the workings are shown.

(a) Let $X = \{2, 4, 6, 8, 10\}$, $Y = \{4, 6, 8, 10, 12\}$, and $Z = \{2, 10, 12, 14, 16\}$.

i. Find $X \cap Y \cap Z$ and $X \cup (Y \setminus Z)$.

[2 marks]

ii. Is $X \subseteq X \cap Y \cap Z$? Justify your answer.

[2 marks]

iii. Find the number of elements in the power set $\mathcal{P}(X \cup (Y \cap Z))$.

[2 marks]

(b) Determine all sets A which satisfy

$$\mathcal{P}(A) \subseteq \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}.$$

[4 marks]

(c) Let A, B, and C be three sets. Show whether the following statement is correct or not. Explain your answer.

$$A \subseteq B \quad \text{if and only if} \quad A \cap C \subseteq B \cap C \quad \text{for all sets } C.$$

[6 marks]

(d) Let A, B, and C be subsets of a universal set U. Show whether the following statement is correct or not. Explain your answer.

$$\text{If } A \subseteq B \text{ and } C \subseteq \overline{B}, \text{ then } A \cap C = \emptyset.$$

[4 marks]

Question 2

Show your work. Full marks will only be awarded if the workings are shown.

(a) Show whether each of the following expressions is a function or not. If f is not a function explain why.

i. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{\ln(x-1)}$

ii. $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^3 - 2x + 5$

iii. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x-3}$

[3 marks]

(b) Given $f(x) = x^2 + b$ and $g(x) = \sqrt{x+3}$. Find all possible values of b for which $(f \circ g)(x) = (g \circ f)x$. Explain your answer.

[4 marks]

(c) Solve the following logarithmic equation: $2\log_4(x) - \log_4(3x-2) = 0$. Show your work.

[2 marks]

(d) Let $f : \mathbb{R}^* \to \mathbb{R}$ with $f(x) = e^x + x$, where $\mathbb{R}^*$ is the set of all real numbers different from 0.

i. Determine whether or not f is a one to one function.

ii. Determine whether or not f is an onto function.

[6 marks]

(e) Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if f and g are both one-to-one, then $g \circ f$ is also one-to-one.

[5 marks]

Question 3

Show your work. Full marks will only be awarded if the workings are shown.

(a) Let p, q and r be three propositions.

i. Construct a truth table for each of the following compound propositions

$$(p \oplus q) \to r, \qquad \text{and} \qquad (p \vee q) \to (p \wedge r)$$

ii. Determine whether either of the above compound propositions is a tautology. Explain your answer.

[8 marks]

(b) Let p, q, r and s be four propositions. Assuming that p and r are false and that q and s are true, find the truth value of the following compound proposition:

$$((p \vee \neg r) \wedge (q \to s)) \leftrightarrow ((\neg p \wedge q) \vee (r \to s))$$

[2 marks]

(c) Let p, q and r be three propositions where:

p means "The employee completes the safety training."

q means "The employee passes the skills assessment."

r means "The employee qualifies for the fieldwork assignment."

Express the following compound propositions symbolically using p, q, r, and appropriate logical symbols.

i. *'To qualify for the fieldwork assignment, it is necessary for the employee to both complete the safety training and pass the skills assessment.'*

ii. *'The employee qualifies for the fieldwork assignment only if they either complete the safety training or pass the skills assessment, but not both.'*

iii. *'The employee will qualify for the fieldwork assignment if, and only if, they complete the safety training and pass the skills assessment.'*

[3 marks]

(d) Give the contrapositive, the converse and the inverse of the following statement:

$$\forall x \in \mathbb{R}, \text{if } x^2 - 3x + 2 > 0 \text{ then } x > 2 \text{ or } x < 1$$

[3 marks]

(e) A tautology is a proposition that is always true. Let p, q and r be three propositions. Using the laws of propositions only, prove or disprove that $((p \wedge q) \vee (r \to s)) \Leftrightarrow ((p \vee r) \to s) \wedge ((q \vee r) \to s)$ is a tautology.

[4 marks]

Question 4

(a) Translate the following statements into predicate logic using appropriate quantifiers and predicates:

i. "Every doctor in the hospital wears a mask."

ii. "If a plant is watered regularly, then it grows healthy."

iii. "There exists a number that is larger than any other number.

iv. "All animals that have wings can fly."

[8 marks]

(b) Indicate which of the following statements are true and which are false. Justify your answer.

i. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}^*$ such that $xy < 1$

ii. $\forall x \in \mathbb{R}^*, \forall y \in \mathbb{Z}$ such that $xy > 1$

iii. $\forall x \in \mathbb{R}^*, \exists y \in \mathbb{R}$ such that $xy = 2$

[6 marks]

(c) Negate the following quantified statement:

$$\forall x \, \exists y \, (M(x) \wedge N(y)) \vee \forall z \, (K(z) \to L(z))$$

[2 marks]

(d) Let p, q, r and s be three propositions. Say whether or not the following argument is a valid argument. Explain your answer.

$s \to (p \vee q)$
$\neg p \to r$
$\neg q \to r$
$\neg s$

$\therefore r$

[4 marks]

Question 5

Show your work. Full marks will only be awarded if the workings are shown.

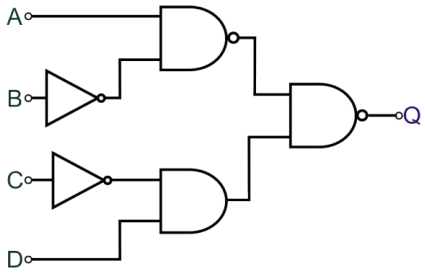
(a) Use DeMorgan's laws to simplify the following expressions:

i. $\overline{(\overline{p \cdot \overline{q} \cdot r}) + \overline{(\overline{r} \cdot s)}}$

ii. $\overline{(\overline{x} + y)} \cdot \overline{(x + \overline{y})} \cdot \overline{(\overline{y} + \overline{z})}$

[6 marks]

(b) Given the following logic circuit:



i. Find a Boolean expression for the output Q for this logic circuit.

ii. Use the laws of Boolean algebra to simplify the output of the above circuit.

[5 marks]

(c) Use the duality principle to find out the dual of the following equation:

$$a.b + c.\overline{d} = (a + c).(a + \overline{d}).(b + c).(b + \overline{d})$$

[2 marks]

(d) For the following Boolean expression, give:

$$F(A, B, C, D) = A \cdot \overline{B} \cdot C + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot C \cdot \overline{D}$$

i. the truth table

ii. the Karnaugh map

iii. the minimal sum of products expression

[7 marks]

[END OF COURSEWORK ASSIGNMENT]

.