Discrete Mathematics

Tutorial sheet

Introduction to Proofs

Question 1.
Prove that the sum of any two even integers is even. In an other way show that:

$\forall n, m \in \mathbb{Z},$ if n and m are even numbers then $n+m$ is also an even number.

Question 2.
Use direct proof to show that: $\forall n, m \in \mathbb{Z}$, if n is an even number and m is an odd number then $3n + 2m$ is also an even number.

Question 3.
Prove that the sum of any two odd integers is even. In an other way show that:

$\forall n, m \in \mathbb{Z},$ if n and m are odd numbers then $n + m$ is an even number.

Question 4.

Show that for any odd number integer n, n^2 is also odd. in another way show that:

$$\forall n \in \mathbb{Z}, \text{ if } n \text{ is odd then } n^2 \text{ is also odd.}$$

Question 5.
Show that: $\forall x \in \mathbb{R}\ \forall m \in \mathbb{Z}, \lfloor x + m \rfloor = \lfloor x \rfloor + m.$

Question 6.

Use proof by contraposition show that for any integer n, if n^2 is even then n is even

Question 7.

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Use proof by contraposition show that for any integer n, if $5 \nmid n^2$ then $5 \nmid n$

Question 8.

Use proof by contradiction to show that for any integer n, if n^2 is even then n is even

Question 9.

Use proof by contradiction to show that for any integer n, $3n + 2$ is not divisible by 3.

Question 10.

Use proof by contradiction to show that for any integer n, $7n + 4$ is not divisible by 7.

Question 11.
Write the following series in $\sum$ notation:

1. $1 + 3 + 5 \cdots (2n - 1)$
2. $1 + 2 + 4 + 8 + 16 + \cdots + 1024$

Question 12.
Given the following formulae

$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Evaluate the following

1. $\sum\limits_{k=1}^{10} (4k - 2)$
2. $\sum\limits_{k=41}^{100} k$
3. $3 + 6 + 9 + 12 + \cdots + 300$

Question 13.
Given the following arithmetic sequence:

$$a_n : 2, 5, 8, 11, 14, \cdots$$

1. Find the common difference d
2. Calculate the next term;
3. Write down the n^{th} term in terms of n.
4. Let $S_n = \sum\limits_{k=1}^{n} a_n$ be the sum of the first n^{th} terms of this sequence. Write down S_n in terms of n and a_1.
5. Workout the value of S_{100}

Question 14.

Let the sequence u_n be defined by the recurrence relation

$$u_{n+1} = u_n + 2n, \quad \text{for } n = 1, 2, 3, ... \text{ and let } u_1 = 1.$$

Use mathematical induction to show that the nth term, where $n \geq 0$, is given by

$$u_n = n^2 - n + 1.$$

Question 15.

Screencast 4
Let S_n be a series defined as follows:

$$S_n = 1^2 + 2^2 + 3^3 + \cdots + n^2 = \sum_{k=1}^{n} k^2$$

Use mathematical induction to prove that each positive integer n, $S_n = \frac{n(n+1)(2n+1)}{6}$.

Question 16.
Let $S_n = \sum\limits_{i=1}^{i=n}(2i-1) = n^2$ for all $n \in \mathbb{Z}^+$.

1. Find S_1 and S_2.
2. Prove by induction that $S_n = n^2$ for all $n \in \mathbb{Z}^+$.

Question 17.

Use mathematical induction to show that for all integer $n \geq 3$, $2n + 1 < 2^n$

Question 18.
Given the following sequence defined by

$$u_{n+2} = 4u_{n+1} - 3u_n$$

and initial terms $u_1 = 4$ and $u_2 = 10$.

1. Calculate u_3
2. Use strong mathematical induction to prove that

$$u_n = 3^n + 1, \quad \forall\ n \geq 1.$$

Question 19.

Use strong mathematical induction to prove that if n is an integer greater than 1, then it is either a prime or can be written as the product of primes.