

# Tutorial 7

Complete the following practice examples in class. Feel free to work in pairs in class:

1. **Inequalities:** Solve the given inequalities:

- $2 + 3x < 5$
- $16x + 64 \leq 16$
- $x^2 - 1 < 0$
- $x^3 - 2x^2 + x \geq 0$
- $\frac{1}{x} < x$
- $\frac{x^2-9}{x+1} > 0$
- $\frac{1}{x-1} + \frac{4}{x-6} > 0$
- $\frac{2x-6}{x^2-6x+5} < 0$
- $|x| < 4$
- $|3x + 1| > 5$
- $|x - 2| < \frac{1}{2}$
- $0 < \left|x - \frac{1}{2}\right| < 2$

2. **Distance and Midpoint:** Find the distance and the midpoint of the line between the following sets of coordinates:

- (2,4) and (6,8)
- (2,-3) and (7,-3)
- (3,-1) and (-1,5)
- (a,3) and (6,a)

3. **Straight line equation:** Determine the equation of the straight line passing through the following points:

- (2,4) and (6,8)
- (2,-3) and (7,-3)
- (3,-1) and (-1,5)
- (a,3) and (6,a)

4. **Domain and Range:** Find the domain and range of the following functions:

- $f(x) = |x|$
- $f(x) = 2x - 3$
- $f(x) = \frac{1}{x^2}$
- $f(x) = \sqrt{1-x}$
- $f(x) = \sqrt{7-x} - 1$
- $f(x) = \frac{1}{\sqrt{4-x^2}}$

5. **Limits:** Determine if the following limits exist or not and evaluate if they do exist:

- $\lim_{x \rightarrow 0} (2x - 1)$
- $\lim_{x \rightarrow 2} (x^2 - 2x + 4)$
- $\lim_{x \rightarrow 3} (|x| - 2)$
- $\lim_{x \rightarrow 1} \left(\frac{3}{x+1}\right)$
- $\lim_{x \rightarrow -1} \left(\frac{-2}{x+1}\right)$
- $\lim_{x \rightarrow 3} \left(\frac{2x-6}{x-3}\right)$

6. **Continuity:** Determine whether the following functions are continuous at the indicated point or not:

- $f(x) = x^3 - 5x + 1; x = 2$
- $f(x) = \sqrt{(x-1)^2 + 5}; x = 1$
- $f(x) = \sqrt{x^2 + 9}; x = 3$
- $f(x) = |4 - x^2|; x = 2$
- $f(x) = \begin{cases} x^2 + 4, & x < 2 \\ x^3, & x \geq 2 \end{cases}; x = 2$
- $f(x) = \begin{cases} x^2 + 5, & x < 2 \\ x^3, & x \geq 2 \end{cases}; x = 2$

7. **Derivatives:** Find the derivative of the following functions from first principles, i.e. find:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- a.  $f(x) = 4$
  - b.  $f(x) = 2 - 3x$
  - c.  $f(x) = 5x - x^2$
  - d.  $f(x) = x^4$
  - e.  $f(x) = \sqrt{x-1}$
  - f.  $f(x) = \frac{1}{x^2}$
8. Find the derivative ( $f'(x)$ ) of the given function:
- a.  $f(x) = 1 - x$
  - b.  $f(x) = 11x^5 - 6x^3 + 8$
  - c.  $f(x) = \frac{3}{x^2}$
  - d.  $f(x) = ax^2 + bx + c$   $a, b, c$  constants
  - e.  $f(x) = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - \frac{x}{1}$
  - f.  $f(x) = (x^2 - 1)(x - 3)$
  - g.  $f(x) = \frac{x^3}{1-x}$
  - h.  $f(x) = \frac{x^2-1}{2x+3}$
  - i.  $f(x) = (1 - 2x)^{-1}$
  - j.  $f(x) = (x^5 - x^{10})^{20}$
  - k.  $f(x) = \sqrt{x^2 + 1}$
  - l.  $f(x) = [(2x + 1)^2 + (x + 1)^2]^3$
  - m.  $f(x) = x^2\sqrt{x^2 + 1}$
9. Find an equation for the tangent line to the graph of  $f$  at the point  $(a, f(a))$ :
- a.  $f(x) = \frac{x}{x+2}$ ,  $a = -4$
  - b.  $f(x) = (x^3 - 2x + 1)(4x - 5)$ ,  $a = 2$
10. Use either the first or second derivative tests to find the local extreme values of the following functions:

- a.  $f(x) = 2x^3 - 3x^2 - 12x + 5$
  - b.  $f(x) = x^3$
  - c.  $f(x) = x^4 - 2x^3$
  - d.  $f(x) = \frac{1}{x-1}$
  - e.  $f(x) = 2x^{5/3} + 5x^{2/3}$
11. Evaluate the following integrals:
- a.  $\int_0^1 (2x - 6x^4 + 5) dx$
  - b.  $\int_{-1}^1 (x - 1)(x + 2) dx$
  - c.  $\int_0^1 (2x - 3) dx$
  - d.  $\int_{-1}^0 5x^4 dx$
  - e.  $\int_1^4 2\sqrt{x} dx$
  - f.  $\int_1^5 2\sqrt{x-1} dx$
  - g.  $\int_0^1 (x^{3/2} - x^{1/2}) dx$
  - h.  $\int \frac{1}{x^4} dx$
  - i.  $\int \left( \frac{x^3+1}{x^5} \right) dx$
  - j.  $\int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$
  - k.  $\int (x - a)(x - b) dx$
12. Find the area between the graph of  $f$  and the  $x$ -axis:
- a.  $f(x) = 2 + x^3$ ,  $x \in [0, 1]$
  - b.  $f(x) = x^2(3 + x)$ ,  $x \in [0, 8]$
  - c.  $f(x) = (2x^2 + 1)^2$ ,  $x \in [0, 1]$
  - d.  $f(x) = x^2 - 4$ ,  $x \in [1, 2]$
13. Determine the derivative of the following functions:
- a.  $f(x) = \ln 4x$
  - b.  $f(x) = \ln(x^3 + 1)$
  - c.  $f(x) = \ln \sqrt{1 + x^2}$
  - d.  $f(x) = x^2 \ln x$

- e.  $f(x) = \ln[(x + 1)^3]$
- f.  $f(x) = (\ln x)^3$
- g.  $f(x) = e^{-2x}$
- h.  $f(x) = e^{x^2-1}$
- i.  $f(x) = e^x \ln x$
- j.  $f(x) = e^{-x} x^{-1}$
- k.  $f(x) = (e^{x^2} + 1)^2$
- l.  $f(x) = e^{4 \ln x}$

14. Evaluate the following integrals:

- a.  $\int \frac{1}{x+1} dx$
- b.  $\int \frac{1}{3-x} dx$
- c.  $\int \frac{x+1}{x^2} dx$

15. Find the approximate value of  $\int_0^3 \sqrt{4 + x^3} dx$  for  $n = 6$  using:

- a. The left-endpoint estimate
- b. The right-endpoint estimate
- c. The midpoint estimate
- d. The trapezoidal estimate
- e. The parabolic estimate (Simpson's rule)

16. Use a 4-th order Taylor polynomial to estimate the following functions:

- a.  $\sqrt{e}$
- b.  $\sin 1$
- c.  $\ln 1.2$
- d.  $e^{0.8}$

17. Find the Taylor/Maclaurin series expansion of the following function:

- a.  $f(x) = e^{2x}$