Advanced Calculus and Algebra Questions

1. Inequalities (Advanced)

1. Solve the inequality: $\frac{x^2-5x+6}{x^2-x-6} \leq 0$

2. Find the solution set of $\left|\frac{2x-3}{x+1}\right| < 2$

3. Determine the values of x such that $\sqrt{x^2 - 4x + 5} < 3$

2. Distance and Midpoint (With Variables)

1. Find the distance between the points $(2a, 3b)$ and $(a+1, b-4)$

2. Determine the midpoint of the segment joining (x, x^2) and $(2x, 3x^2)$

3. Straight Line Equation (Implicit and Parametric)

1. Find the equation of the line passing through $(3, 7)$ and perpendicular to the line $2x - 3y + 6 = 0$

2. Express parametrically the line through $(1, 2)$ with slope 5

4. Domain and Range (Non-trivial Functions)

1. Find the domain and range of $f(x) = \frac{1}{\sqrt{4-x^2}}$

2. Determine the domain and range of $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

5. Limits (Piecewise and Rational Functions)

1. Evaluate $\lim_{x \to 2} \frac{x^2-4}{x-2}$

2. Determine $\lim_{x \to 0^-} \frac{|x|}{x}$

3. Evaluate $\lim_{x \to 0} \frac{\sin(3x)}{x}$

6. Continuity (Discontinuous at a Point)

1. Discuss the continuity of the function:

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

2. Determine if $f(x) = \frac{1}{x-1}$ is continuous at $x = 1$

7. Derivatives from First Principles (Complex Functions)

1. Using the definition, find $f'(x)$ for $f(x) = \sqrt{x}$

8. Derivatives (Chain Rule, Product Rule, Quotient Rule)

1. $f(x) = \frac{e^{x^2}}{\ln x}$

2. $f(x) = (\sin x + x^2)^3$

3. $f(x) = x^3 \ln(x^2 + 1)$

9. Tangents (Non-linear Functions)

1. Find the tangent line to $f(x) = \ln(x^2 + 1)$ at $x = 1$

10. Extrema (With Rational or Logarithmic Functions)

1. Use the first derivative test to find local extrema of $f(x) = \frac{x^2 - 1}{x}$

11. Integrals (Non-standard Bounds or Substitutions)

1. $\int_1^4 \frac{\ln x}{x^2} \, dx$

2. $\int_0^1 x\sqrt{1 - x^2} \, dx$

3. $\int \frac{1}{x\sqrt{\ln x}} \, dx$

12. Area Under Curves

1. Find the area between the curve $y = x^2$ and the line $y = 2x$

13. Taylor Series

1. Find the 4-th order Maclaurin series of $\cos(x)$

2. Estimate $\ln(1.5)$ using 3rd-order Taylor polynomial

14. Mixed Advanced Problems

1. Evaluate the limit: $\lim_{x \to 0} \frac{e^x - \cos x}{x}$

2. Determine the critical points of $f(x) = x^4 - 4x^3 + 6x^2$ and classify them.

3. Solve the inequality: $\ln(x^2 + 1) > x$

4. Find the inverse function of $f(x) = \frac{3x+2}{x-1}$ and state its domain.

5. Show that the function $f(x) = x^3 + x$ is one-to-one and find its inverse locally around $x = 0$.

15. Challenging Calculus and Series

1. Find the derivative of $f(x) = x^x$ for $x > 0$

2. Evaluate the improper integral $\int_1^\infty \frac{1}{x(\ln x)^2} dx$

3. Determine whether the series $\sum_{n=1}^\infty \frac{(-1)^n}{n + \sin n}$ converges.

4. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Is $f'(x)$ continuous at $x = 0$?

5. Use the definition of derivative to show that $\frac{d}{dx}(e^x) = e^x$