



BSc COMPUTER SCIENCE

CM1015 Computational Mathematics

Midterm coursework

INSTRUCTIONS TO STUDENTS:

This coursework assignment consists of 5 questions. You should answer **ALL** the questions.

There are 100 marks available in this coursework assignment. The marks for each question are indicated at the end of the part in [.] brackets. **Full marks will be awarded for complete answers to a total of 5 questions. Make sure to show your work clearly in each of the questions.**

The aim of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics. You must submit your answers as a single pdf document. Ideally you should submit work that is properly formatted using the maths mode of your word processor. However, you may submit photos of your work, combined into a single pdf file. If submitting photos of your work, please ensure that **all answers are written clearly.**

Question 1

Show your working. Full marks will only be awarded if workings are shown.

(a) Using number bases, find the value of k for the following:

$$\text{if } 4_6 \times (k3)_4 = 2_5 \times (k4)_7.$$

[4 marks]

(b) If $123_x = 22_8$, what is the value of the base x .

[3 marks]

(c) Explain your answer for the following questions:

i- State the decimal value of the underlined digit of a number in base eight. $(2\underline{3}76)_8$

ii- State the decimal value of the underlined digit of a number in base two. $(\underline{1}1011)_2$

[2 marks]

(d) Here are three binary numbers:

10100110 , 1110111, and 1110001

i- Add up all three numbers.

ii- Add up the two larger numbers.

iii- Subtract the smallest number from the second smaller numbers.

[6 marks]

Question 2

Show your working. Full marks will only be awarded if workings are shown.

- (a) Find all the terms in between $a_1 = -8$ and $a_7 = 10$ of an arithmetic sequence.

[6 marks]

- (b) Find the general term of an arithmetic sequence where $a_3 = -1$ and $a_{10} = 48$.

[6 marks]

- (c) A geometric series is $a + ar + ar^2 + \dots$

- i- Prove that the sum of the first n terms of the series is given by:

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

- ii- Find:

$$\sum_{k=1}^{10} 100(2^k)$$

- iii- Find the sum to infinity of the geometric series:

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \dots$$

[9 marks]

- (d) Which common quadratic equation can be used to derive the golden ratio directly?

[3 marks]

- (e) Ali has bought 9 books. The first one cost 1 dollar, he paid 2 dollars for the second book, the third cost 4 dollars, and the fourth, 8 dollars, and so on. How much did Ali pay for the 9 books?

[6 marks]

Question 3

Show your working. Full marks will only be awarded if workings are shown.

(a) Find the value of x if $14 \equiv 172 \pmod{x}$.

[3 marks]

(b) When 67 is divided by a number, say x , a remainder of 3 is obtained. If $x < 50$, then what is the largest number that x can be?

[4 marks]

(c) What is the shortest (quickest) way to calculate the remainder when:
 $3 \times 52 \times 97 \times 22 \times 438$ is divided by 7.

[4 marks]

(d) Using your knowledge about modular arithmetic, find the last two digits of 2^{20} . For example, $2^7 = 128$, then the last two digits are 28.

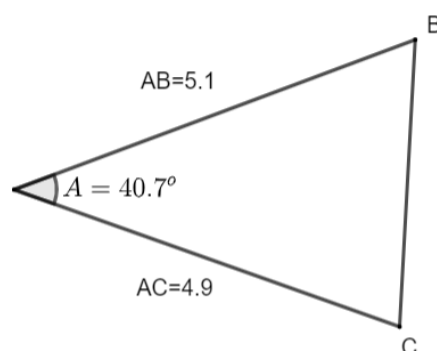
[4 marks]

Question 4

Show your working. Full marks will only be awarded if workings are shown.

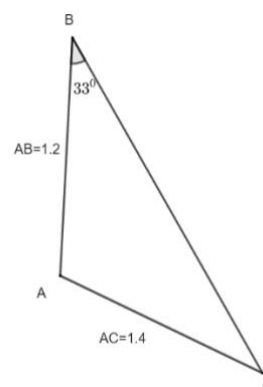
- (a) Find the length of the side BC. Please note that the triangle is not drawn to scale.

[5 marks]



- (b) ABC is a triangle as shown below. Please note that the triangle is not drawn to scale.
Find the angle C in degrees.

[5 marks]



- (c) Solve the following trigonometric equation for $0 \leq \theta \leq 360^\circ$

$$2 + 4\cos^2\theta = 7\cos\theta\sin\theta$$

[5 marks]

- (d) Simplify $(1 + \tan^2 A) + (1 + \frac{1}{\tan^2 A})$

[5 marks]

Question 5

Show your working. Full marks will only be awarded if workings are shown.

(a) Find the answer for each of the following questions:

i- If $f(x) = \frac{x-3}{4}$ find $f^{-1}(x)$

ii- Find $f^{-1}(x)$ if $f(x) = \sqrt{x} + 6$

[4 marks]

(b) Test the injection, surjection, and bijection of the function:

$$g: N \rightarrow N, g(n) = n + 1 \text{ where } N \text{ is the positive integer numbers.}$$

If the function is bijective then find the inverse. Justify your answer.

[4 marks]

(c) Show whether the function $f: R^* \rightarrow R \setminus \{1\}$ defined by $f(x) = \frac{x+1}{x}$ is a one-to-one or not. If it is one-to-one, determine its inverse.

[5 marks]

(d) A bicycle pedalling along a straight road at 20 km/h accelerates at 2 m/s^2 for 4 s . Find the final speed.

[3 marks]

(e) Two balls are thrown upwards from the same spot. 1.15 seconds apart. The first ball had an initial velocity of 15 m/s and the second was 12 m/s . At what height do they collide?

[4 marks]

[END OF COURSEWORK ASSIGNMENT]