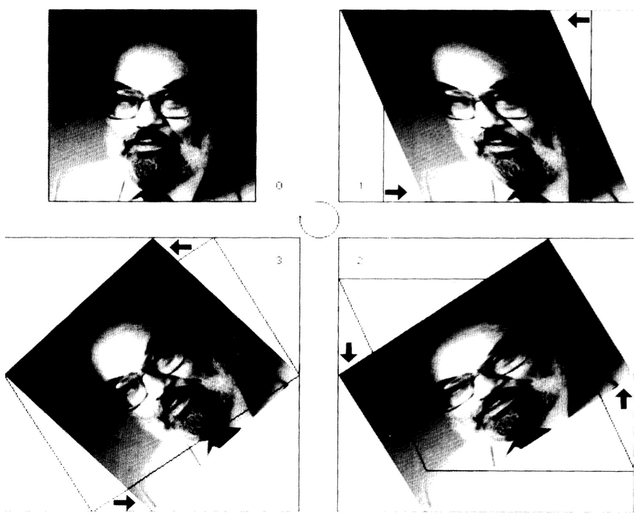
Rotation of Multidimensional Signals with Spectral Schemes

Student: Dylan Reid Ramelli Advisor: Prof Rolf Krause Co-Advisors: Dr Diego Rossinelli, Dr Patrick Zulian

Large-scale in-silico investigation of homeostasis in the subarachnoid space of the human optic nerve [Rossinelli et al. 2023, in-preparation]. Homeostasis is directly related with the interaction between the cerebrospinal fluid flow (not shown here) and the meningeal surface. Blue denotes poor levels of homeostasis , whereas red denotes sufficient material exchange between fluid and structure.

Light transport calculation

- Light attenuation by massive ray casting.
- Efficiency through volumetric rotations.
- 3D rotations as series of 2D rotations.



[Convolution-Based Interpolation for Fast, High-Quality Rotation of Images, Unser et. al, 1995]

Design Strategies

$$R(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

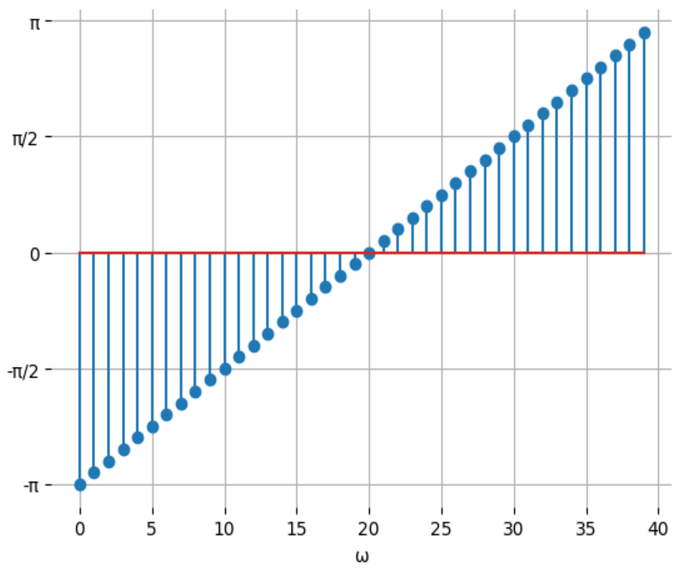
$$= \begin{pmatrix} 1 & -tan\theta/2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ sin\theta & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -tan\theta/2 \\ 0 & 1 \end{pmatrix}$$

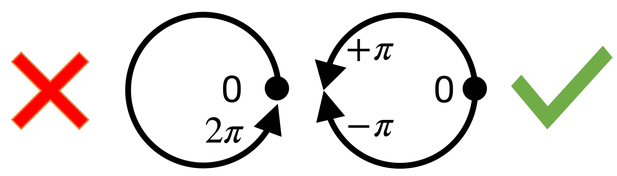
- 2D Rotations as 1D translations (!)
- Favor local computational schemes.
- Perform direct convolution:

$$y(n) = \sum_{k=0}^{Q-1} h(k)x(n-k) = h(k) * x(n)$$

- Compact filter support compared to original signal.
- Ideal execution on superscalar architectures.
- Exploiting spatiotemporal locality (data cache).
- Dense Linear Algebra (data-level parallelism).
- Scalable (thread-level parallelism).

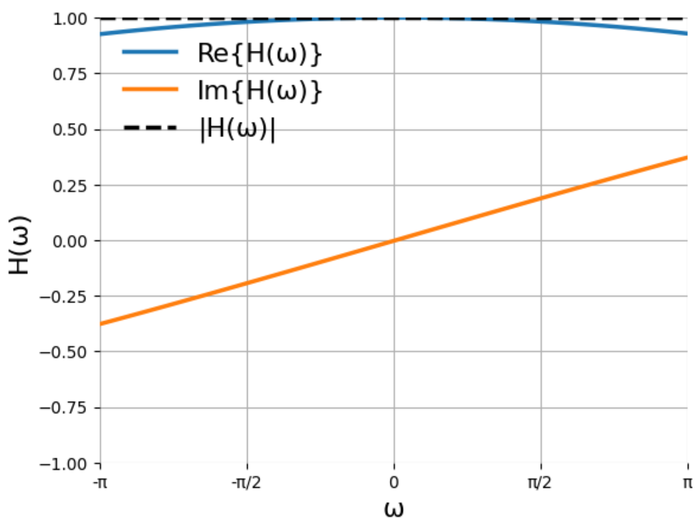
Interpretation of frequencies





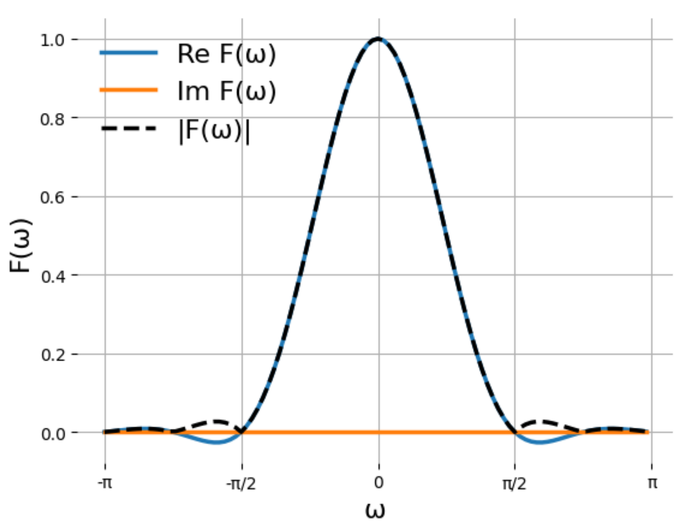
Own Contributions

$H(\omega) = e^{-i\omega\delta}$, δ known at runtime.



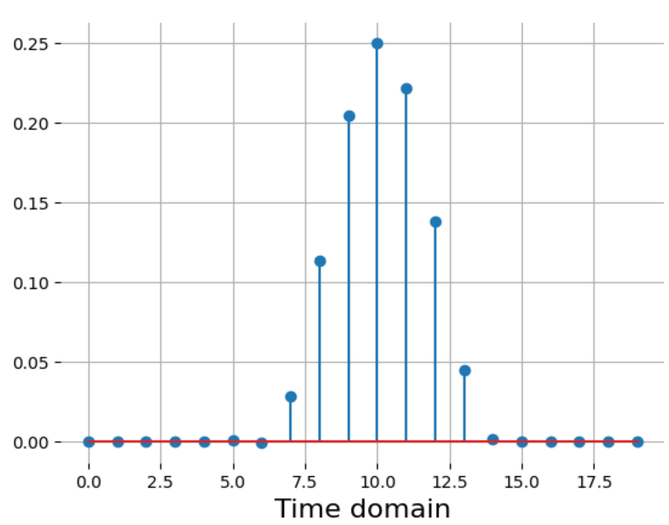
Interpolant, translation by $\delta = 0.1234$

$$F(\omega) = \int_{-M}^{M} \cos(\frac{\pi}{M}t)e^{-i\omega t}dt$$



Smoothing (Low-Pass filter) for $M = 2$

$$DFT^{-1}(\{H(\omega_i)F(\omega_i)\}_i)$$



Combined (Cascaded) Impulse Response