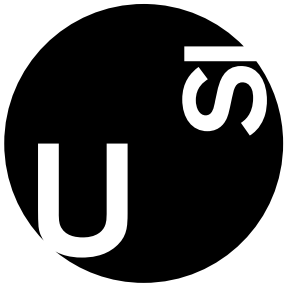

Università
della
Svizzera
italiana

Faculty
of
Informatics

Bachelor Thesis

May 30, 2023

Spectrally Accurate Resampling of High Quality Rotated Images

Dylan Reid Ramelli

Abstract

Advisor
Prof Rolf Krause
Co-Advisors
Dr Diego Rossinelli, Dr Patrick Zulian

Advisor's approval (Prof Rolf Krause): Date:

1 Introduction, Motivation

For many years the amount of digital imaging data has been increasing exponentially (citation) and as such the need for acquiring this large data and process it with care has become a central point of focus. The goal of this project is to process high quality signals, in this case images, by rotating them using techniques such as the Fourier Transform, Shift Theorem, FIR filtering and CUDA parallel processing. The accurate rotation of images is very useful in many applications such as Data Augmentation for example. For Data augmentation we want to increase the coverage of a certain dataset. In Convoluted Neural Networks we can augment the dataset by rotating the input images by some random rotation and then feeding them to the model to improve it without having to collect more data externally.

Another application of this technique can be seen in Data Visualization for multi-modal imaging. Images are taken at different times or in different ways, such as in a PET-CT scan and rotation can enable us to re-align the elements present in these images. In this case we can either acquire images at different times and then combine them together so that we can manipulate the images by rotation for example to align them correctly. Go into more detail.. ask Diego.

"Explain 2D to 3D."

FT: Direct solver instead of iterative, takes in a lot of data.

2 State of the art

For certain applications such as... it is required to be able to rotate images while maintaining the highest quality possible. To achieve satisfying results the general approach is to use bilinear and nearest neighbour interpolation.

Include images from Diego of blood vessels in the heart iI think

3 Methodology

To complete this project we made use of different tools such as, Python, Jupyter Notebook, C/C++ and CUDA.

In our project we will use a method of rotation which was described in the article titled "Convolution-based inteprolation for fast, high-quality rotation of images"[1] where the rotation matrix

$$R(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

is factorized as three matrices each of which represents a shearing in a cardinal direction.

$$R(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} = \begin{pmatrix} 1 & -tan\theta/2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ sin\theta & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -tan\theta/2 \\ 0 & 1 \end{pmatrix}$$

The first matrix shears the image in the x direction by $\Delta_x = -y \cdot tan(\theta/2)$, the second matrix shears the image in the y direction by $\Delta_y = x \cdot sin(\theta)$ and the last matrix shears again in the x direction by Δ_x.

Given this definition of a 2D rotation we can perform the operation as a sequence of three 1D translations on the 1D input array that represents our image.

3.1 Frequencies of a signal

$$freq_n = \lfloor \frac{n}{2} \rfloor + 1 \tag{1}$$

Visual representation with $n = 3$ of the discrete Fourier transform of a function $g(x)$

$$G_m = \sum_{k=0}^{N-1} g(k) e^{-i\frac{2\pi}{N}km}$$

To shift a signal by a fractional amount it is imperative to use the correct frequency when doing so.

Given that for n samples we have $\lfloor \frac{n}{2} \rfloor + 1$ distinct frequencies

Euler's Formula

$$e^{ix} = cos(x) + isin(x) \tag{2}$$

Useful property:

$$\cos(w) = \frac{1}{2}(e^{iw} + e^{-iw}) \tag{3}$$

Figure 1. Unit circle with N = 3,

3.2 Fourier Transform with Shift Theorem

Here we derive the shift theorem for a discrete signal starting from the normal Fourier Transform with g_k being our 1D input array, N the number of samples and m the frequency:

$$G_m = \sum_{k=0}^{N-1} g_k[k] e^{-i\frac{2\pi}{N}km}$$

Now instead of $g_k[k]$, we want $g_k[k-\delta]$ where δ is the amount we want to shift. So the above equation can be rewritten as:

$$Z_m = \sum_{k=0}^{N-1} g_k[k-\delta] e^{-i\frac{2\pi}{N}km}$$

$$Z_m = \sum_{r=0-\delta}^{N-1-\delta} g_k[r] e^{-i\frac{2\pi}{N}km}, r = k-\delta$$

Since $r = k-\delta$ then $k = r+\delta$, and as such:

$$Z_m = \sum_{r=-\delta}^{N-1-\delta} g_k[r] e^{-i\frac{2\pi}{N}(r+\delta)m}$$

We can then separate the exponential:

$$Z_m = \sum_{r=-\delta}^{N-1-\delta} g_k[r] e^{-i\frac{2\pi}{N}rm} e^{-i\frac{2\pi}{N}\delta m}$$

And factor it out of the sum:

$$Z_m = e^{-i\frac{2\pi}{N}\delta m} \sum_{r=-\delta}^{N-1-\delta} g_k[r] e^{-i\frac{2\pi}{N}rm}$$

The sum now has exactly the same range as before:

$$Z_m = e^{-i\frac{2\pi}{N}\delta m} \sum_{k=0}^{N-1} g_k[k] e^{-i\frac{2\pi}{N}km}$$

$$Z_m = e^{-i\frac{2\pi}{N}\delta m} G_m = H_m \cdot G_m \tag{4}$$

Here is a small example made in python:

The above equation 4 works well for any kind of shift that we want to perform on our signal but for any fractional amount we encounter some problems with the use of the correct frequency indexes. Here is a graphical example that was found to be useful: With an $N = 3$ number of samples of a signal we have exactly 2 distinct frequencies 1 but the total frequencies are actually 3 and they are $[0+i0, \frac{1}{2}+i\frac{\sqrt{3}}{2}, \frac{1}{2}-i\frac{\sqrt{3}}{2}]$. The last two frequencies are one the complex conjugate of the other.

"Explain why".

To solve this we need to take into consideration the negative frequencies and in particular if the number of samples is odd or even. Normally we multiply each sample by its corresponding phase, which is based on the frequency number m but since we need to take into consideration the negative frequencies we can define a function called wavenum that returns the correct frequency index to use in the phase shift:

$$wave\_n(m) = (m + \lfloor N/2 \rfloor) \bmod N - \lfloor N/2 \rfloor$$

$$H_m = e^{-i\frac{2\pi}{N}\delta \text{wave\_n}(m)}$$

$$Z_m = H_{\text{wave\_n}(m)} \cdot G_m \tag{5}$$

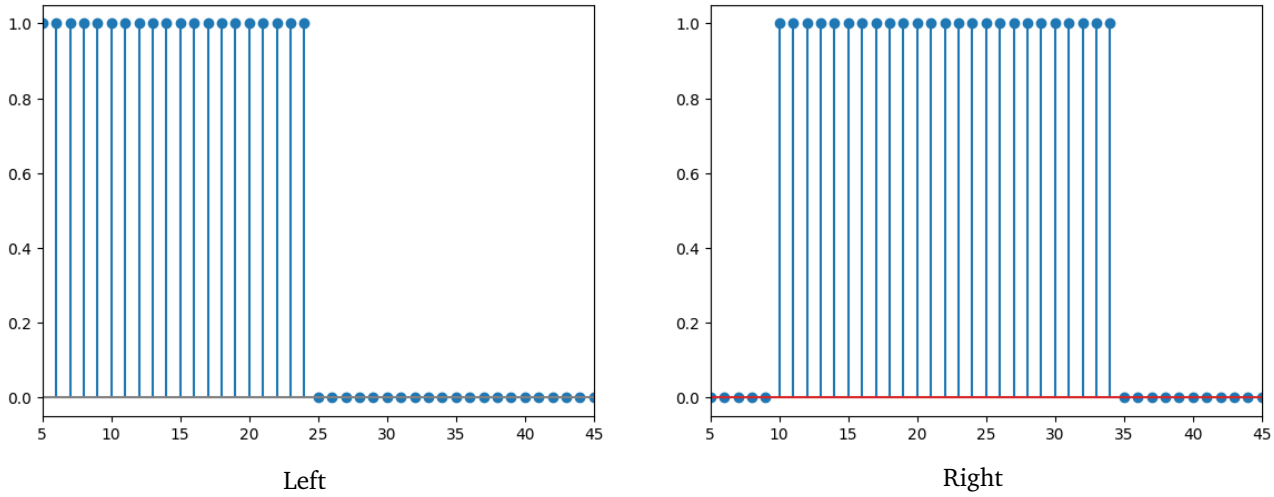

Figure 2. Original 1D function.

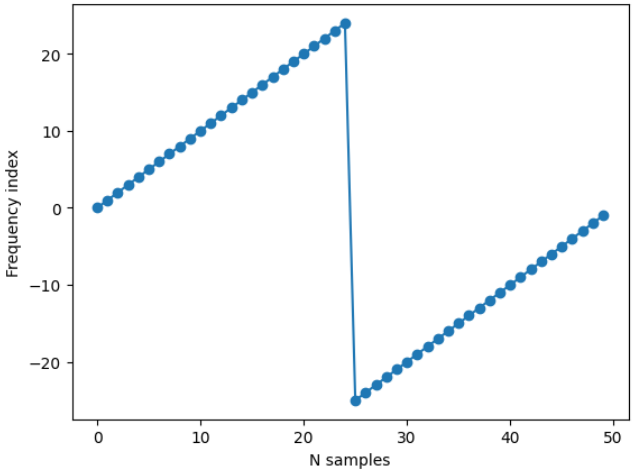

Figure 3. Wavenum function with $N = 50$

3.3 Lanczos Filter and FIR

To alleviate the problems that arise when shifting a signal by a fractional amount we can use a filter L that is convoluted with the shifted signal.

$$L_m = \frac{\sin(a)}{a},$$
$$a = \frac{2\pi wave\_n(m)}{N}$$

Apply lanczos filter to fractional shift to smooth the result.

$$Z_m = L_m \cdot H_m \cdot G_m$$

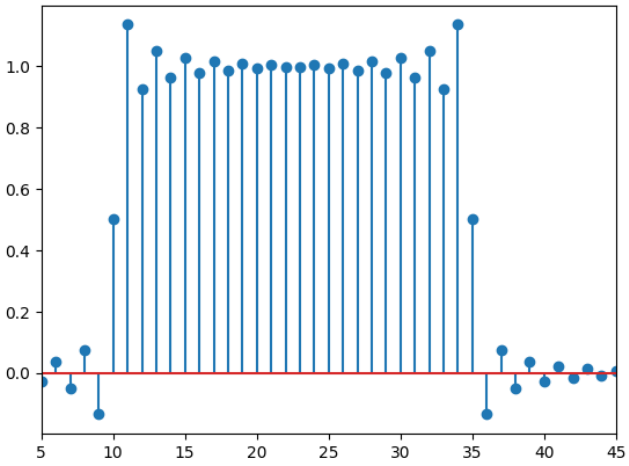

Figure 4. Shifted function by $\delta = 10.5$

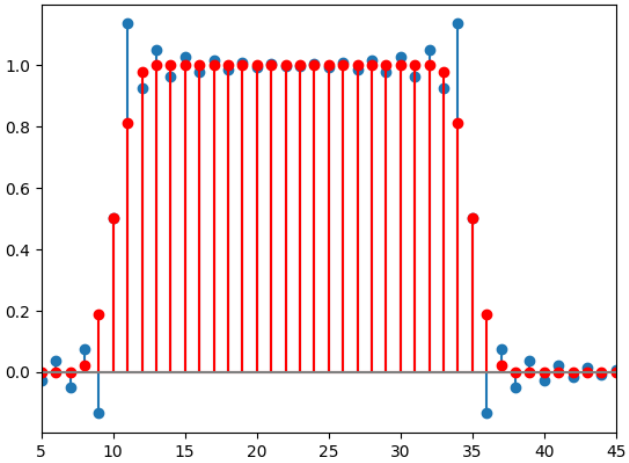

Figure 5. Shifted function by $\delta = 10.5$ with lanczos filter applied (red), no filter(blue).

4 Project Design

Since the premise of this project is based on the fact that a rotation can be performed with three 1D operations, the bottom-up approach was a no-brainer. We started with the fundamentals of signal processing by exploring the Fourier Transform and Shift Theorem to build our shift kernel. We used Jupyter Notebook extensively to test our method and quickly plot the results, while also documenting everything so that it would be easy to write this report. Since we used the bottom-up approach we were able to re-use parts of the code to test some other aspect of the shift. We started with simply shifting a 1D signal by an integer value which proved to be quite simple and then we tried to shift the same signal by a fractional amount. We then applied the same methodology to a 2D signal, which in our case was an image. We built the 2D kernel based on the 1D one that we created before.

Explain 1D,2D,3D transposition. Simplification of 2D rotation using 1D translations.
Bottom-up , start with 1d kernel then build 2d kernels starting from 1d.

5 Implementation

- Problems while implementing the code
- Indexing frequencies...
- practical problems...

6 Result

- Rotate image many times like in the paper
- look for artefacts
- compare with C code gathernoloss.c
- quality, quantity

Università della Svizzera italiana

Spectrally Accurate Resampling of High Quality Rotated Images

Student: Dylan Reid Ramelli Advisor: Prof. Rolf Krause Co-Advisors: Dr. Diego Rossinelli, Dr. Patrick Zulian

Motivation:

Image

Image Image Image

Week	Activity	Duration in Weeks	Status
W1	Finish all 2D image rotations in C and start rotations with CUDA.	1W	COMPLETED
W2	Complete 2D Rotations with CUDA and read/fully understand research article "Convolution-based Interpolation for fast, high quality Rotation of Images."	1W	DONE. CUDA rotations are not complete for gether no loss. Need to undserstand better when you can call kernels and where.
W3	Meet and ask questions about research article, keep learning paper. Finish gathernoloss in C, keep looking at CUDA examples.	1W	Started summarizing the article and also a reference book about digital signal processing, which is helping a lot in understanding the article. DONE
W4	Fully implement CUDA code for gathernoloss and fully understand article.	1W	Able to implement CUDA coda for gathernoloss still need this week for article. DONE
W5	Plot a sinusoid function around multiple circles to create a n image to serve as test for rotations. Understand how to translate an array by a fractional value	1W	DONE
W6	Progress Report , start writing report, start with abstract. Continue with translate signal.	1W	NOT DONE
W7	Start implementing interpolation using 3 pass algorithm in 2d test image	2-3W	IN PROGRESS
W8	Continue implementing interpolation using 3 pass algorithm in 2d test image and start looking into parallelizing the code with either MPI or CUDA	1W	CANCELLED
W9	Continue implementing interpolation using 3 pass algorithm in 2d test image	1W	IN PROGRESS
W10	Continue implementing interpolation using 3 pass algorithm in 2d test image ... and finish report	2-3W	IN PROGRESS
W11	Continue implementing interpolation using 3 pass algorithm in 2d test image ... and finish report	1W	IN PROGRESS
W12	Maybe add CUDA or MPI support and finish report	1W	IN PROGRESS

Table 1. Plan

7 Solution

8 Validation

9 Conclusion

- Summary
- limitations

References

[1] IEEE Philippe Thévenaz Michel Unser, Senior Member and Leonid Yaroslavsky. Convolution-based interpolation for fast, high-quality rotation of images.