The Implications of Zero-Growth

Analysed with an Agent-Based Model

Abstract

The ever-approaching limits of the Earth's biosphere and the potentially catastrophic consequences caused by climate change have begun to call into question the endless growth of the economy. There is increasing interest in the prospects of zero economic growth from the degrowth and post-growth literature. In particular, can a zero-growth policy in a capitalist system with interest-bearing debt be economically stable? There have been several answers to this question using macroeconomic models; some find a zero-growth trajectory is stable, while other models show an economic breakdown. However, the capitalist system in a period of growth is not guaranteed to be stable. Hence, a more appropriate methodology is to compare the relative stability between a growth and zero-growth scenario on the same model. Such a question has not yet been answered at the microeconomic level; what are the consequences of zero-growth on market share instability and concentration, bankruptcy rates, distributional effects, and the impact on credit network risk? To answer such questions, we develop a macroeconomic agent-based model which incorporates Minskyan financial dynamics. The growth and zero-growth scenarios are accomplished by changing an average productivity growth parameter for the firms in the model. The results of the model showed that the real GDP growth rates were more stable in the zero-growth scenario, fewer economic crises, lower unemployment, a higher wage share of output for workers, and capital firm and bank market shares were relatively more stable. Some of the consequences of zero-growth were a higher rate of inflation than in the growth scenario, an increase in market concentration for both firms and banks, and a higher level of financial risk in the credit network.

1 Introduction

The increase of wealth is not boundless; that at the end of what they term the progressive state lies the stationary state.
John Stewart Mill (1884, p. 514)

As John Stewart Mill remarked, there is a culmination to economic growth, a point of transition to a stationary state. Albeit seemingly distant in sight, it is the ineluctable destination of our current trajectory. Contrary to the growth-oriented stance that pervades current macroeconomic thought. Growth rates across the developed world have already slowed to a “new normal” of 1% (Malmaeus and Alfredsson, 2017) and the increase in wealth from material and energy use becomes increasingly bound by the Earth’s biophysical limits. It is therefore imperative to discern the implications of a post-growth stationary state on economic welfare, for which current mainstream economic models and theory are unable to reveal.

The prosperity predicted by Keynes (1930) has not yet been reached, instead, mired by inequality (Piketty, 2014, p. 327) the alluring stationary state of affluence and abundance remains a distant ideal of our economic potential. Growth is still the remedy prescribed by the 2030 Agenda for Sustainable Development, adopted in 2015 by all UN member states, which explicitly states in the 8^{th} Sustainable Development Goal[1] (SDG 8) an aim to “promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all”. Achieving this goal while simultaneously tackling climate change relies on the concept of ‘decoupling’, to promote economic growth while reducing the use of natural resources and emissions. There is a growing body of evidence that decoupling policies, such as SDG 8, will not enable sufficiently rapid reductions in emissions (Haberl et al., 2020) and there are calls to look beyond decoupling to quickly reduce our impact on the environment (Elder and Olsen, 2019).

Moreover, the concept of decoupling is deeply rooted in the Neoclassical circular flow vision of the economy, which is inconsistent when viewed from the perspective of the ecology; in which, the economy is an open subsystem of a larger finite and closed total system, the Earth’s biosphere (Daly, 1996, p. 6). As outlined in Nicholas Georegescu-Roegen’s seminal work, the fundamentals of an economic process is to transform low entropy inputs into high entry outputs (Georgescu-Roegen, 1972, p. 276-283). Hence, it is impossible for the global economy to continuously grow without limit because low entropy inputs from the environment, such as natural resources, become depleted and high entropy outputs, such as material waste and emissions, are accumulated. This continuous growth dynamic is evidently unsustainable on a finite planet. Parrique et al. (2019) conclude that there is no empirical evidence to support decoupling and that it remains unlikely to occur in the near future. Furthermore, Ward et al. (2016) demonstrate using an IPAT[2] model that economic growth cannot be decoupled from growth in material and energy use. Wiedmann et al. (2020) show that resource use and pollutant emissions grow

[1] For Sustainable Development Goal 8, see here: `https://sdgs.un.org/goals/goal8`
[2] The IPAT model is a simple formulation of environmental impact (I) as a function of population (P), affluence (A), and technology (T): $I = PAT$.

more rapidly than the efficiency of technology, which relates to the Jevons' paradox, where gains in efficiency antithetically cause a rise in production and consumption (Alcott, 2005). Slameršak et al. (2024) further exhibit how low growth rates make it more feasible to reach mitigation goals consistent with a 1.5°C-2°C average temperate rise above pre-industrial levels. Therefore, it is unlikely that decoupling alone will be enough to mitigate climate change, resource depletion, and environmental breakdown. Zero-growth will be a necessary outcome due to these factors affecting the economy given no alternative, and governments will have to stop targeting real GDP growth.

Even the proponent of growth Walt W. Rostow questioned, "Where history offers us only fragments: what to do when the increase in real income itself loses its charm?" (Rostow, 1960, p. 16); for indeed this charm is beginning to wane. For decades, warnings of the potential consequences of our persistent growth have been ineffective in mitigating humanities' strain on the planet. Meadows et al. (1972) posited the potential catastrophic trajectory of continuous economic growth, where the standard run now seems to reflect our current course as outlined by Herrington (2021). Furthermore, the planetary boundaries framework, developed in Rockström et al. (2009) and Steffen et al. (2015), classifies nine boundaries for which human overshoot could lead to potential destabilising effects on the ecosystem. An update by Richardson et al. (2023) showed that six of the nine boundaries are transgressed, in which the authors state that the "Earth is now well outside of the safe operating space for humanity". This strain on the whole Earth system must be eased, a transition of our economic activity away from growth to a post-growth stationary state would alleviate this pressure within a realistic time frame.

There is a further debate on the growth imperative, whether interest-bearing debt necessitates a positive growth rate for economic stability and whether prolonged zero-growth is stable in a capitalist system. Barrett (2018) showed on a dynamic Minskyan macroeconomic model that given a positive interest rate on debt, there exists, for both growth and zero-growth policies, stable and unstable scenarios. In this paper, we endeavour to expand the domain of analysis in Barrett (2018) to also include the microeconomic level. The modelling framework that best suits this task is that of agent-based models (ABMs) developed in complex systems modelling. ABMs are a type of computational model in which there are numerous 'agents' interacting with each other, such as firms, households, or even particles or cells. This computational framework allows the modeller to create a system from the micro level upwards, leading to emergent properties[3], which allows the modeller to also analyse the macro dynamics of the system. Moreover, ABMs do not depend on historical data when informing changes in policy or regime. Thus, due to the lack of historical data for economies sustaining a zero-growth stationary state, the ABM framework is advantageous for discerning the implications of zero-growth at both the micro and macroeconomic levels. While the literature on zero-growth economics and ABMs has burgeoned in recent years, ABMs have not yet been used to explore the impacts of zero-growth economics.

Barrett (2018) performed a highly novel analysis of the stability of zero-growth on the macroeconomy by tweaking a productivity growth input parameter, which was set at either 2% or zero. The model, based on work by Keen (1995), incorporated elements of Minsky's

[3]Emergent properties is the phenomenon in which an entity has attributes that its individual parts do not have on their own.

Financial Instability Hypothesis (FIH). The FIH highlights the instability of the capitalist market mechanism and the role of financial attributes that are essential to capitalism in creating economic crises, especially regarding high levels of interest-bearing debt (Minsky, 1986, p. 173). This paper showed that zero-growth scenarios are generally no less stable than growth scenarios. Other models focused on analysing the impacts of zero-growth on an economy with interest-bearing debt and whether this creates an inherent 'growth imperative', which find a relatively stable zero-growth path include Berg et al. (2015), Rosenbaum (2015), Jackson and Victor (2015), Cahen-Fourot and Lavoie (2016), Jackson and Victor (2020), and D'Alessandro et al. (2020). Binswanger (2009) found that there does exist a growth imperative and that a zero-growth scenario becomes unstable. All these models have in common that they model the macroeconomic variables and cannot explain the impact that zero-growth will have on individual agent outcomes, such as bankruptcy rates, market instability, market concentration, and financial network risk, questions this paper intends to answer.

Moving now to the literature focused on the development of macroeconomic ABMs, the complex adaptive trivial systems (CATS) framework is prevalent in the ABM literature (Delli Gatti et al., 2003, 2005; Russo et al., 2007; Gaffeo et al., 2008; Delli Gatti et al., 2010), which focuses on industrial coordination and credit dynamics. Russo et al. (2007) and Gaffeo et al. (2008) expanded the framework to a more complete view of the economy. Furthermore, Assenza et al. (2015) improved the credit and capital dynamics of the CATS framework. Additionally, there was a zero-growth mechanism in the Assenza et al. (2015) model, showing that an ABM can have a stable zero-growth path in the long-run. However, the analysis was not focused on zero-growth and this decision was only made as a simplifying assumption. There are also the Keynes meets Schumpeter (K-S) variety of ABMs (Dosi et al., 2006, 2009, 2010, 2013), these models cultivate a capital sector more complex than those found in the CATS models, where technical development can either occur from innovation or imitation due to the R&D of capital firms. The labour productivity of consumption firms is then embodied in the vintages of their capital machinery. This is the driving force of economic growth in the K-S models, following a Schumpetarian approach of 'creative destruction' to firm growth behaviour. Furthermore, another relevant contribution to macroeconomic ABMs is the model presented in Caiani et al. (2016), which develops a stock-flow consistent (SFC) approach to ABMs and creates a stylised macroeconomic version of the model to initialise the model simulation. The model has constant labour productivity, which again results in a zero-growth model. However, the authors are not concerned with this analysis and focus on other features of the model. Nevertheless, this does highlight the viability of zero-growth in an ABM with positive interest rates on debt. Botte et al. (2021) establish a synthesis between ecological macroeconomics and macroeconomic ABMs. This paper presents a unique analysis of the implications of a transition to a net zero carbon economy on a macroeconomic ABM. The model (TRansit) is extensive in its construction, consisting of a consumption goods sector, an energy sector, a capital goods sector, a government sector, households/consumer sector, and a banking sector. Similarly to the K-S models, the main driver of growth in TRansit comes from the capital goods sector, which engages in R&D to improve capital firms' own productivity, energy efficiency, and labour productivity of a given vintage of machine, hence, growth is endogenous to the model. For a further in-depth review of the ABM literature see Axtell and Farmer (2022).

The structure of the paper is as follows. Section 2 provides an overview of the model details and assumptions regarding the agent's behaviour. Section 3 presents the results from the model, demonstrating the model reproduces stylised facts seen in real economic data and then focuses on a stability analysis comparing growth and zero-growth scenarios. Section 4 is a discussion regarding the results. Finally, section 5 contains a summary and concluding remarks.

2 The Model

2.1 The Agents

The model is populated with four different types of heterogeneous agents: (i) there are $i = \{1, ..., \mathbf{N}_C\}$ consumption sector firms (C-firms) producing a homogeneous consumption good; (ii) a smaller number of $j = \{1, ..., \mathbf{N}_K\}$ capital sector firms (K-firms) producing a homogeneous capital good (the symbol $\iota = \{i, j\}$ is used throughout to refer to both C-firms and K-firms); (iii) $h = \{1, ..., \mathbf{N}_H\}$ households that work and consume; and, (iv) $b = \{1, ..., \mathbf{N}_B\}$ banks which extends loans to firms and hold deposits. Each agent makes decisions in discrete time $t \in \mathcal{T} := \{0, \Delta t, 2\Delta t..., N\Delta t = T\}$, where T is the number of years of the simulation, N is the number of steps, and Δt is the time period interval, which is assumed to correspond to a quarter of a year.

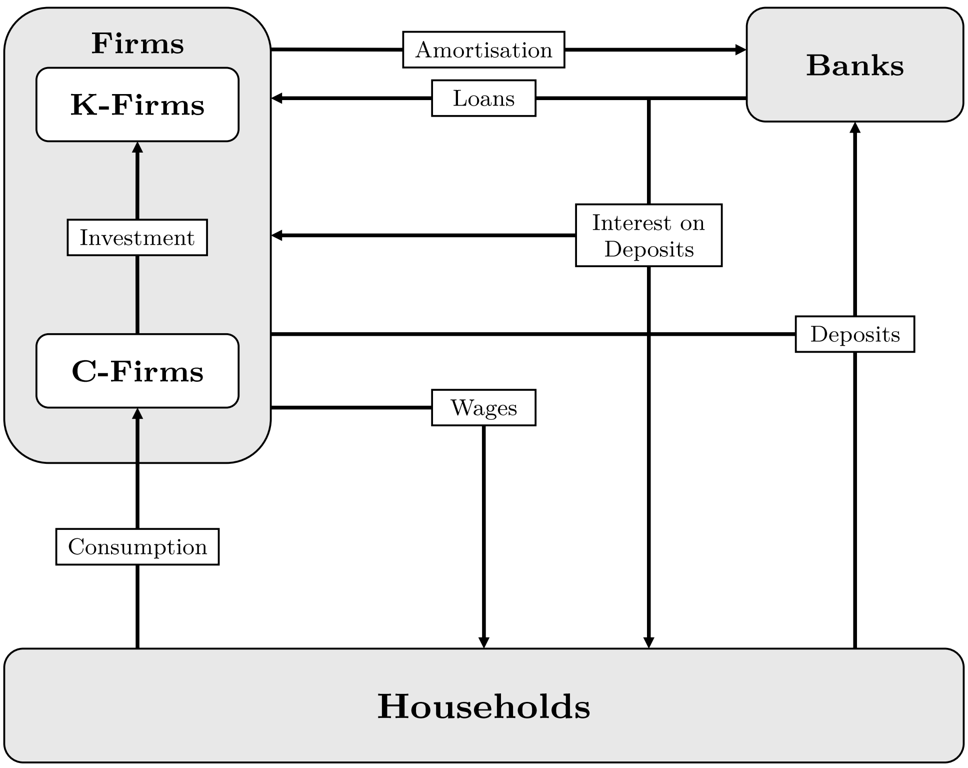


Figure 1: Model ontology, the flow diagram of payments between the different agents in the model. Arrows point from paying agents to receiving agents.

Figure 1 gives a visual representation of the model and how the agents interact with one another through financial flows. In summary, households work at either C-firms or K-firms and are paid a wage for their work each period, they put any savings at banks as deposits and receive interest from the bank. Households also consume consumption goods from C-firms using their wage from employment and/or their deposits at the bank. C-firms invest in new capital from K-firms using their internal finance and loans from banks. Additionally, both C-firms and K-firms also seek external finance from banks if their wage bill exceeds internal finance capabilities. Firms use an amortisation schedule to pay down both the interest and principal of the loan. Furthermore, banks pay interest to firms and households for any deposits they hold at banks.

2.2 Sequence of Events

The sequence of events that occur during each time period is given below:

1. At the beginning of each time step t, new entrants enter their respective markets to replace firms that went bankrupt in the previous period.
2. A decentralised market for labour opens where C-firms and K-firms post vacancies and update their wage rate to attract new employees, households then send out applications to firms that have open vacancies and firms hire employees from their application pool.
3. C-firms and K-firms engage in production, pay their employees for their work, and update the price of their goods.
4. A decentralised market for consumption goods opens and households visit and buy goods from C-firms for consumption.
5. A decentralised market for capital goods opens and C-firms visit and buy goods from K-firms to use as investment in their capital stock for production in the next period.
6. A decentralised credit market opens and C-firms and K-firms demand loans from banks, which are supplied depending on the bank's risk tolerance.
7. C-firms and K-firms update their accounts and exit the market if they have run out of deposits to cover their payments, households employed by bankrupt firms become unemployed, and banks absorb any outstanding loans to bankrupt firms into their equity.
8. Banks become bankrupt if they have negative equity and are bailed out by their depositors, both firms and households.

2.3 Households

2.3.1 Income & Expenditure

The hth household receives both a wage from their employer, firm ι, and interest from their bank on deposits as income:

$$Y_{h,t} = \mathbb{1}_{h,t} w_{\iota,t} + r^M M_{h,t}, \tag{1}$$

where $w_{\iota,t}$ denotes the wage rate of firm ι defined in their labour contract, r^M is bank b's interest rate on deposits (uniform across all banks), $M_{h,t}$ are household h's deposits at bank b, and $\mathbb{1}_{h,t}$ is a dummy variable determining the employment status of household h:

$$\mathbb{1}_{h,t} = \begin{cases} 1 & \text{if household } h \text{ is employed} \\ 0 & \text{if household } h \text{ is unemployed.} \end{cases} \tag{2}$$

Household h's desired consumption expenditure, $E^d_{h,t}$, is assumed, for simplicity, to be given by a simple Keynesian budget constraint in which household h desires to spend all of their income and a linear proportion of their deposits:

$$E^d_{h,t} = Y_{h,t} + cM_{h,t}, \tag{3}$$

where $c \in (0, 1)$ is the marginal propensity to consume out of deposits.

2.3.2 Accounting

Household h keeps any involuntary savings at bank b where household h earns an interest on their deposits. Hence, household h updates their deposits by:

$$M_{h,t+1} = M_{h,t} + Y_{h,t} - E_{h,t}, \tag{4}$$

where $E_{h,t}$ is household h's actual expenditure on C-goods, defined in the market for consumption goods, see Section 2.8.

2.4 Consumption Firms

2.4.1 Production

The output of the ith C-firm, $Y_{i,t}$, is assumed to be given by a Leontief production function with constant returns to scale for both labour, $N_{i,t}$, and capital, $K_{i,t}$:

$$Y_{i,t} = \min\left\{a_{i,t}N_{i,t}, \frac{K_{i,t}}{\nu}\right\}, \tag{5}$$

where $a_{i,t}$ is firm i's labour productivity and ν^{-1} is a constant capital-output ratio. Labour productivity, which is the essential variable of growth in the model, determines the efficiency of firm i's production process. It is assumed that labour productivity evolves according to the stochastic differential equation (SDE) for geometric Brownian motion (GBM):

$$\mathrm{d}a_{i,t} = a_{i,t}(g\mathrm{d}t + \sigma_a \mathrm{d}W_{i,t}), \tag{6}$$

where g is the constant average growth rate of labour productivity, σ_a is the standard deviation of labour productivity, and $W_{i,t}$ is a Brownian motion. This equation for labour productivity was chosen because it captures exponential growth with random fluctuations from the deterministic growth path set by the g parameter. This will allow the growth rate of the model to be set exogenously. Furthermore, the expected value of the SDE in eq. 6 is given by $\mathbb{E}[a_{i,t}] = a_{i,0}\exp\{gt\}$, which corresponds to the deterministic model used in Barrett (2018). The SDE for labour productivity can be simulated on a discrete time grid using the exact solution for GBM:

$$a_{i,t} = a_{i,t-1}\exp\left\{g - \frac{1}{2}\sigma_a^2 + \sigma_a\varepsilon_{i,t}\right\}, \tag{7}$$

where $\varepsilon_{i,t} \sim \mathcal{N}(0,1)$ is a random variable drawn from a standard normal distribution. Hence, when $g > 0$, labour productivity grows exponentially. Furthermore, due to the functional form in Eq. (7), for a given time period t, firm labour productivity will be distributed according to a log-normal distribution, such that $\ln(a_{i,t}) \sim \mathcal{N}\left((g - \sigma_a^2/2)t, \sigma_a^2 t\right)$.

2.4.2 Desired Output & Inventories

Firm i's desired output, $Y_{i,t+1}^d$, is assumed to be equal to their expected demand, $Z_{i,t+1}^e$, in the next period:

$$Y_{i,t+1}^d = Z_{i,t+1}^e, \tag{8}$$

where expected demand is adaptively updated according to firm i's actual demand, $Z_{i,t}$:

$$Z_{i,t+1}^e = Z_{i,t}^e + \gamma_Z(Z_{i,t} - Z_{i,t}^e), \tag{9}$$

where $\gamma_Z \in (0,1)$ determines the speed of adjustment to actual demand. From Eq. (9) it is evident after repeated substitution that the expected demand is equal to the sum of all past demand with geometrically decaying weights.

It is assumed that C-goods are perishable and do not last longer than one period, hence, any leftover inventories are disposed of by firm i at no extra cost. Thus, firm i's level of involuntary inventories are given by:

$$V_{i,t} = Y_{i,t} - Q_{i,t}, \tag{10}$$

where $Q_{i,t}$ is the actual quantity of sold C-goods, determined on the consumption good market, see Section 2.8.

2.4.3 Prices

Firm i sets their price according to both internal and external factors. In particular, firm i will stochastically increase (decrease) their price if involuntary inventories are zero (positive), as this signals there is excess demand (supply) for firm i's goods. Furthermore, firm i will also adjust their price towards the average price level of other C-firms to stay competitive. Hence, the pricing mechanism of firm i is given by:

$$P_{i,t} = \begin{cases} P_{i,t-1}(1 + \sigma_P|\varepsilon_{i,t}|) + \gamma_P(\bar{P}_{t-1}^C - P_{i,t-1}) & \text{if } V_{i,t-1} = 0 \\ P_{i,t-1}(1 - \sigma_P|\varepsilon_{i,t}|) + \gamma_P(\bar{P}_{t-1}^C - P_{i,t-1}) & \text{if } V_{i,t-1} > 0 \end{cases} \tag{11}$$

where σ_P is the standard deviation of firm i's price growth, $\varepsilon_{i,t} \sim \mathcal{N}(0,1)$, therefore $|\varepsilon_{i,t}|$ is distributed according to a folded normal distribution, $\gamma_P \in (0,1)$ is a speed of adjustment parameter, and $\bar{P}^C_{t-1}$ is the weighted average of consumption goods prices:

$$\bar{P}^C_t = \frac{\sum_{k=1}^{\mathbf{N}_C} P_{k,t} Y_{k,t}}{\sum_{k=1}^{\mathbf{N}_C} Y_{k,t}}. \tag{12}$$

2.4.4 Investment & Capital

Firm i finances new investment using both internal and external finance. Firm i determines external investment financing using their desired debt-to-output ratio for investment finance, given by:

$$d^d_{i,t+1} = d_0 + d_1 \alpha_{i,t} + d_2 \pi_{i,t} \tag{13}$$

where $d_0, d_1, d_2 > 0$ are parameters, $\alpha_{i,t} = \ln(a_{i,t}) - \ln(a_{i,t-1})$ is the log difference of firm i's labour productivity, and $\pi_{i,t} = \Pi_{i,t}/(P_{i,t}Y_{i,t})$ is firm i's profit share. This is the essential equation determining the level of debt in the model, which has been taken from Barrett (2018). Firm i's desired debt for investment is therefore given by:

$$D^d_{i,t+1} = d^d_{i,t+1} P_{i,t} Y_{i,t}. \tag{14}$$

Hence, firm i's desired loan for investment financing is given by:

$$IL^d_{i,t+1} = \max\left\{D^d_{i,t+1} - D_{i,t}, 0\right\}, \tag{15}$$

where $D_{i,t}$ is firm i's current debt. Firm i's desired investment expenditure is then given by:

$$IE^d_{i,t+1} = \max\left\{IL^d_{i,t+1} + \Pi_{i,t} + M_{i,t} - W_{i,t}, 0\right\}, \tag{16}$$

where $M_{i,t}$ is firm i's deposits at bank b and $W_{i,t}$ is firm i's current wage bill, where firm i is assumed to keep enough internal finance to cover their future wage bill, which firm i assumes will remain constant in the next period.

After firm i has purchased their desired amount of K-goods on the capital market, firm i updates their capital stock by:

$$K_{i,t+1} = K_{i,t}(1-\delta) + I_{i,t+1}, \tag{17}$$

where δ is the depreciation rate of K-goods, and $I_{i,t+1}$ is firm i's new investment orders. Firm i also updates their total capital expenditure, which is similarly given by:

$$KE_{i,t+1} = KE_{i,t}(1-\delta) + IE_{i,t+1}, \tag{18}$$

where $IE_{i,t+1}$ is firm i's actual investment expenditure. $I_{i,t+1}$ and $IE_{i,t+1}$ are both determined on the capital good market, see Section 2.9.

2.4.5 Labour & Wages

Firm i's desired labour in the next period is determined using eq. 5 and their desired capital utilisation:

$$N^d_{i,t+1} = \upsilon^d_{i,t+1} \frac{K_{i,t+1}}{\nu a^e_{i,t+1}}, \tag{19}$$

where $\upsilon^d_{i,t+1}$ is firm i's desired capital utilisation, similar in form to that specified in Caiani et al. (2016):

$$\upsilon^d_{i,t+1} = \min\left\{\frac{\nu Y^d_{i,t+1}}{K_{i,t+1}}, 1\right\}, \tag{20}$$

where, $Y^d_{i,t+1}$ is the desired output of firm i, defined in eq. 8. The expected labour productivity in the next period, $a^e_{i,t+1}$, is derived from eq. 7 as:

$$a^e_{i,t+1} = a_{i,t} \exp\{g\}. \tag{21}$$

Firm i then decides how many employees they wish to hire or fire in the next period using:

$$\eta_{i,t+1} = N^d_{i,t+1} - N_{i,t}. \tag{22}$$

The wage that firm i offers their employees in the next period depends on firm i's employment decisions and the average wage in the previous period. When firm i desires to hire $\eta_{i,t+1} \geq 0$ (or fire $\eta_{i,t+1} < 0$) labour then firm i stochastically increases (decreases) their nominal wage. Furthermore, firm i will also adjust their wage towards the average wage to stay competitive. Hence, firm i's wage is updated as:

$$w_{i,t} = \begin{cases} w_{i,t-1}(1 + \sigma_w|\varepsilon_{i,t}|) + \gamma_w(\bar{w}_{t-1} - w_{i,t-1}) & \text{if } \eta_{i,t} \geq 0 \\ w_{i,t-1}(1 - \sigma_w|\varepsilon_{i,t}|) + \gamma_w(\bar{w}_{t-1} - w_{i,t-1}) & \text{if } \eta_{i,t} < 0, \end{cases} \tag{23}$$

where σ_w is the standard deviation of firm i's wage growth, $\varepsilon_{i,t} \sim \mathcal{N}(0,1)$, hence $|\varepsilon_{i,t}|$ is distributed according to a folded normal distribution, $\gamma_w \in (0,1)$ is a speed of adjustment parameter, and $\bar{w}_{t-1}$ is the average market wage in the previous period.

2.4.6 Debt

Due to imperfect market conditions, the cost of external finance is greater for firm i than using internal funds, hence, in line with the pecking order theory of finance (Myers, 1984) and the methodology of other ABMs (Gaffeo et al., 2008; Assenza et al., 2015; Caiani et al., 2016), firm i will seek external financing when their internal flow of funds (profits and deposits) have been exhausted. However, it is rare that a firm will ever completely exhausts their internal finances, therefore, firm i keeps a buffer of internal funds for precautionary reasons. This buffer is assumed to be proportional to their wage bill. Thus, firm i's actual desired loan is given by:

$$L^d_{i,t+1} = \max\left\{IE_{i,t+1} + W_{i,t} - \Pi_{i,t} - M_{i,t}, 0\right\}. \tag{24}$$

where $IE_{i,t+1}$ is firms i's actual investment expenditure, $W_{i,t} = w_{i,t}N_{i,t}$ is firm i's wage bill, $\Pi_{i,t}$ is firm i's profits, and $M_{i,t}$ is firm i's deposits at bank b.

Firm i takes out a single loan each period from a given bank b, see Section 2.10, denoted $L_{i\to b,t}$, with an interest rate $r^L_{b,t}$. Firm i then calculates the amortisation cost to bank b on this loan using the following formula:

$$A_{i\to b,t} = L_{i\to b,t}\frac{r^L_{b,t}(1+r^L_{b,t})^n}{(1+r^L_{b,t})^n - 1}, \tag{25}$$

where n is the number of repayment periods. In each period, firm i repays a proportion of the principal to bank b, $\rho L_{i\to b,t}$, where $\rho = 1/n$, which remains constant until the loan has been repaid in full.

Firm i pays interest to bank b on the loan $L_{i\to b,t}$ as the difference between the amortisation cost and principal payment, $A_{i\to b,t} - \rho L_{i\to b,t}$, until the loan has been repaid in full. Hence, the total interest payments of firm i in period t is given by the sum of the difference between the amortisation cost and principal payments to each bank that firm i still has an outstanding loan with in time t:

$$IP_{i,t} = \sum_{b\in\Omega_B}\sum_{\tau=0}^{t} A_{i\to b,\tau} - \rho L_{i\to b,\tau}, \tag{26}$$

where Ω_B is the set of banks that firm i still has an outstanding loan with. Additionally, firm i's total debt in period t is given by the sum of all firm i's outstanding loans:

$$D_{i,t} = \sum_{b\in\Omega_B}\sum_{\tau=0}^{t} L_{i\to b,\tau}. \tag{27}$$

2.4.7 Accounting

Firm i's profits are given by their revenue and interest on deposits minus the wage bill and total interest payments:

$$\Pi_{i,t} = P_{i,t}Q_{i,t} + r^M M_{i,t} - W_{i,t} - IP_{i,t}, \tag{28}$$

where $P_{i,t}$ denotes firm i's price, $Q_{i,t}$ is the quantity of C-goods sold, r^M is the interest rate on deposits, $M_{i,t}$ are deposits at the bank, and $IP_{i,t}$ are total interest payments on all outstanding loans.

The balance sheet identity of firm i implies that their assets must be equal to their liabilities and equity:

$$KE_{i,t} + M_{i,t} = D_{i,t} + E_{i,t}, \tag{29}$$

where $KE_{i,t}$ is firm i's total capital expenditure, $D_{i,t}$ is firm i's debt, and $E_{i,t}$ is firm i's equity. The equity of firm i is updated based on their profits:

$$E_{i,t+1} = E_{i,t} + \Pi_{i,t}. \tag{30}$$

Hence, given the above, the deposits of firm i are derived as:

$$M_{i,t+1} = M_{i,t} + \Pi_{i,t} + L_{i,t+1} - \rho D_{i,t} - IE_{i,t+1}, \tag{31}$$

where $L_{i,t+1}$ is the actual loan firm i takes out from bank b, $\rho D_{i,t}$ is the cost of all principal repayments, and $IE_{i,t}$ is firm i's actual investment expenditure.

2.5 Capital Firms

2.5.1 Production

The jth K-firm is assumed only to use labour, $N_{j,t}$, as a factor of production, with constant returns to scale. Hence, firm j's production function is given by:

$$Y_{j,t} = a_{j,t} N_{j,t}, \tag{32}$$

where $a_{j,t}$ is firm j's labour productivity. As with C-firms, the jth K-firm updates their own production efficiency, which evolves according to a GBM:

$$a_{j,t} = a_{j,t-1} \exp\left\{ g - \frac{1}{2}\sigma_a^2 + \sigma_a \varepsilon_{j,t} \right\}, \tag{33}$$

where g is the average growth rate of labour productivity, σ_a is the standard deviation of labour productivity, and $\varepsilon_{j,t} \sim \mathcal{N}(0,1)$ is a standard normal random variable.

2.5.2 Desired Output & Inventories

Capital goods are assumed to be durable, hence, firm j can keep inventories from one period to the next. Therefore, firm j's desired output, $Y^d_{j,t+1}$, is determined by their expected demand, $Z^e_{j,t+1}$, and their level of inventories, $V_{j,t}$:

$$Y^d_{j,t+1} = Z^e_{j,t+1}(1+\xi) - V_{j,t}(1-\delta), \tag{34}$$

where $\xi \in (0,1)$ is firm j's desired excess capacity to meet future variations in demand that were not forecasted, $V_{j,t}(1-\delta)$ are firm j's inventories from the previous period that have depreciated by δ, and $Z^e_{j,t+1}$ is firm j's expected demand, adaptively updated according to their actual demand:

$$Z^e_{j,t+1} = Z^e_{j,t} + \gamma_Z (Z_{j,t} - Z^e_{j,t}). \tag{35}$$

Due to the assumption that K-goods are durable and depreciate at a rate δ each period, firm j carries it's inventories over from the previous period:

$$V_{j,t} = V_{j,t-1}(1-\delta) + Y_{j,t} - Q_{j,t}, \tag{36}$$

where $Q_{j,t}$ is firm j's actual quantity of sold K-goods, determined on the capital good market, see Section 2.9. Additionally, because firm j's desired excess capacity is positive, this implies that firm j has desired inventories equal to their desired excess capacity:

$$V^d_{j,t+1} = \xi Y_{j,t}. \tag{37}$$

2.5.3 Prices

Similarly to C-firms, the jth K-firm sets their price according to both internal and external factors. In particular, firm j will adjust their price according to their inventories and the average capital price index, given by:

$$P_{j,t} = \begin{cases} P_{j,t-1}(1+\sigma_P|\varepsilon_{j,t}|) + \gamma_P(\bar{P}^K_{t-1} - P_{j,t-1}) & \text{if } V_{j,t-1} \leq V^d_{j,t} \\ P_{j,t-1}(1-\sigma_P|\varepsilon_{j,t}|) + \gamma_P(\bar{P}^K_{t-1} - P_{j,t-1}) & \text{if } V_{j,t-1} > V^d_{j,t} \end{cases} \tag{38}$$

where $\varepsilon_{j,t} \sim \mathcal{N}(0,1)$, therefore $|\varepsilon_{j,t}|$ is distributed according to a folded normal distribution, $\gamma_P \in (0,1)$ is an adjustment parameter, and $\bar{P}^K_t$ the weighted average of capital good prices.

2.5.4 Labour & Wages

Firm j uses their desired output and expected productivity in the next period to determine their desired amount of labour, derived from eq. 32 as:

$$N^d_{j,t+1} = \frac{Y^d_{j,t+1}}{a^e_{j,t+1}}, \tag{39}$$

where $Y^d_{j,t+1}$ is firm j's desired output, defined in eq. 34. Similarly to C-firms, firm j's expected productivity is derived from the functional form for labour productivity (eq. 33) as:

$$a^e_{j,t+1} = a_{j,t}\exp\{g\}. \tag{40}$$

Firm j then decides how many employees they wish to hire or fire in the next period:

$$\eta_{j,t+1} = N^d_{j,t+1} - N_{j,t}. \tag{41}$$

As with C-firms, K-firm j updates their wage rate according to their labour demand and the average wage, given by:

$$w_{j,t} = \begin{cases} w_{j,t-1}(1+\sigma_w|\varepsilon_{j,t}|) + \gamma_w(\bar{w}_{t-1} - w_{j,t-1}) & \text{if } \eta_{j,t} \geq 0 \\ w_{j,t-1}(1-\sigma_w|\varepsilon_{j,t}|) + \gamma_w(\bar{w}_{t-1} - w_{j,t-1}) & \text{if } \eta_{j,t} < 0, \end{cases} \tag{42}$$

where σ_w is the standard deviation of firm i's wage growth, $\varepsilon_{j,t} \sim \mathcal{N}(0,1)$, hence $|\varepsilon_{j,t}|$ is distributed according to a folded normal distribution, $\gamma_w \in (0,1)$ is a speed of adjustment parameter, and $\bar{w}_{t-1}$ is the average market wage in the previous period.

2.5.5 Debt

K-firms also experience higher costs for external finance, hence, they adhere to the pecking order theory of finance in which they initially use internal finance and then seek external funding on imperfect capital markets. As with C-firms, K-firms will never completely exhaust their internal finances, therefore, firm j keeps a buffer of internal funds for precautionary reasons. This buffer is assumed to be proportional to their wage bill. Thus, firm j's desired new loan is given by:

$$L^d_{j,t+1} = \max\{W_{j,t} - \Pi_{j,t} - M_{j,t}, 0\}. \tag{43}$$

where $W_{j,t} = w_{j,t}N_{j,t}$ is firm j's wage bill, $\Pi_{j,t}$ is firm j's profits, and $M_{j,t}$ is firm j's deposits. K-firms use the same method as C-firms when calculating the amortisation cost of a new loan (eq. 25), total interest payments (eq. 26), and total debt (eq. 27).

2.5.6 Accounting

Firm j's profits are given by their revenue and interest on deposits minus the wage bill and total interest payments:

$$\Pi_{j,t} = P_{j,t}Q_{j,t} + r^M M_{j,t} - W_{j,t} - IP_{j,t}, \tag{44}$$

where $P_{j,t}$ denotes firm j's price, $Q_{j,t}$ is the quantity of C-goods sold, r^M is the interest rate on deposits, $M_{j,t}$ are deposits at the bank, and $IP_{j,t}$ are total interest payments on all outstanding loans.

The balance sheet identity of firm j implies that their assets must be equal to their liabilities and equity. However, capital firms do not have any physical assets only monetary assets, hence firm j's balance sheet is reduced to:

$$M_{j,t} = D_{j,t} + E_{j,t}, \tag{45}$$

where $D_{j,t}$ is firm j's debt and $E_{j,t}$ is firm j's equity. The equity of firm j is updated based on their profits:

$$E_{j,t+1} = E_{j,t} + \Pi_{j,t}. \tag{46}$$

Hence, given the above, the deposits of firm j are derived as:

$$M_{j,t+1} = M_{j,t} + \Pi_{j,t} + L_{j,t+1} - \rho D_{i,t}, \tag{47}$$

where $L_{j,t+1}$ is the actual loan firm j takes out from bank b and $\rho D_{i,t}$ is the cost of all principal repayments.

2.6 Banks

2.6.1 Accounting

The balance sheet identity of bank b is given by:

$$R_{b,t} + L_{b,t} = A_{b,t} + M_{b,t} + E_{b,t}, \tag{48}$$

where $R_{b,t}$ are the reserves of bank b, assumed to be deposited at an unmodelled central bank, $L_{b,t}$ is bank b's total stock of loans extended to firms, $A_{b,t}$ are central bank advances, $M_{b,t}$ is the total amount of household and firm deposits held by bank b, and $E_{b,t}$ is bank b's equity.

Bank b's reserves are simply derived from the above accounting identity:

$$R_{b,t} = A_{b,t} + M_{b,t} + E_{b,t} - L_{b,t}. \tag{49}$$

For simplicity, it is assumed that the central banks offers an interest free advance to banks when the have negative reserves, therefore, banks b's advances from the central bank are given by:

$$A_{b,t+1} = \max\{-R_{b,t}, 0\}. \tag{50}$$

Bank b's loans are given by the sum of all outstanding loans extended to both C-firms and K-firms:

$$L_{b,t} = \sum_{\iota \in \Omega_{F_L}} \sum_{\tau=0}^{t} L_{\iota \to b,\tau}, \tag{51}$$

where Ω_{F_L} is the set of all C-firms and K-firms with outstanding loans to bank b. Bank b's deposits are given by the sum of all deposits held by C-firms, K-firms and households at bank b:

$$M_{b,t} = \sum_{\iota \in \Omega_{F_M}} M_{\iota,t} + \sum_{h \in \Omega_{H_M}} M_{h,t}, \tag{52}$$

where Ω_{F_M} is the set of all C-firms and K-firms with deposits at bank b, and Ω_{H_M} is the set of all households with deposits at bank b. The equity of bank b is updated by their profits, $\Pi_{b,t}$, and bank b is assumed to absorb all the losses from bad loans $B_{b,t}$, which is the sum of all outstanding loans extended to insolvent firms. Moreover, when bank b's equity becomes negative they are assumed to be bailed out by their depositors, they are bailed out by the amount of their desired capital ratio times their assets, see Section 2.11 for details. Hence, bank b's equity is given by:

$$E_{b,t+1} = \begin{cases} E_{b,t} + \Pi_{b,t} - B_{b,t} & \text{if } E_{b,t} > 0 \\ CR^d_{b,t+1}(L_{b,t} + R_{b,t}) & \text{if } E_{b,t} \leq 0, \end{cases} \tag{53}$$

where $CR^d_{b,t+1}$ is bank b's desired capital ratio, defined in eq. 55. The profits of bank b are given by the difference between between the sum of interest received from loans and interest paid on deposits:

$$\Pi_{b,t} = \sum_{\iota \in \Omega_{F_L}} \sum_{\tau=0}^{t} (A_{\iota \to b,\tau} - \rho L_{\iota \to b,\tau}) - r^M M_{b,t}, \tag{54}$$

where Ω_{F_L} is the set of all C-firms and K-firms that have outstanding loans at bank b, $A_{\iota \to b,t} - \rho L_{\iota \to b,t}$ is the amount of interest paid by firm ι to bank b in period t, where $A_{\iota \to b,t}$ is firm ι's amortisation cost paid to bank b, $\rho L_{\iota \to b,t}$ is firm i's principal repayment to bank b, and r^M is bank b's interest rate on deposits, which is assumed to be constant and uniform across all banks.

2.6.2 Loans

Bank b is assumed to lend to firm ι based on bank b's risk tolerance, which is measured by bank b's desired capital ratio. It is assumed that if bank b's ratio of expected bad loans to total loans increases, then bank b becomes more risk averse, and vice versa. Therefore, bank b's desired capital ratio is given by a linear function of their expected bad loans ratio:

$$CR^d_{b,t+1} = \kappa_1 + \kappa_2 \beta^e_{b,t+1}, \tag{55}$$

where κ_1, $\kappa_2 > 0$ are the intercept and slope, respectively, of the desired capital ratio, and $\beta^e_{b,t+1} = L_{b,t}/B^e_{b,t+1}$ is bank b's expected bad loans ratio, where $B^e_{b,t+1}$ is bank b's total expected bank loans, determined as the sum of outstanding loans to firms multiplied by the each firm's probability of default:

$$B^e_{b,t+1} = \sum_{\iota \in \Omega_{F_L}} p_t(\lambda^e_{\iota,t+1}) \sum_{\tau=0}^{t} L_{\iota \to b,\tau}. \tag{56}$$

Similar to Assenza et al. (2015), firm ι's probability of default is estimated each period by bank b using a logistic regression model of firm ι's expected leverage ratio, $\lambda^e_{\iota,t+1}$:

$$p_t(\lambda^e_{\iota,t+1}) = \frac{1}{1 + \exp\{-(\hat{\theta}_0 + \hat{\theta}_1 \lambda^e_{\iota,t+1})\}}, \tag{57}$$

where $\hat{\theta}_0$ and $\hat{\theta}_1$ are estimated using a time series of firm defaults (0 no default, 1 default) and expected leverage ratios, where firm ι's expected leverage ratio is defined as:

$$\lambda^e_{\iota,t+1} = \frac{D^e_{i,t+1}}{M_{\iota,t} + \Pi_{\iota,t} + D^e_{\iota,t+1}}, \tag{58}$$

where firm ι's expected debt level is $D^e_{\iota,t+1} = D_{\iota,t}(1-\rho) + L^d_{\iota,t+1}$. Moreover, Bank b separates firms into C-firms and K-firms and estimates the bankruptcy probability of each firm type separately.

Bank b supplies firm ι's desired loan in full if bank b's desired capital ratio is lower than their actual capital ratio, banks b does not supply the loan if their desired capital ratio is higher than their actual capital ratio, which ensures bank b does not take on too much risk:

$$L^s_{\iota \to b,t+1} = \begin{cases} L^d_{\iota,l+1} & \text{if } CR^d_{b,t+1} < CR_{b,t} \\ 0 & \text{if } CR^d_{b,t+1} \geq CR_{b,t}. \end{cases} \tag{59}$$

where bank b's actual capital ratio is given by:

$$CR_{b,t} = \frac{E_{b,t}}{L_{b,t}}. \tag{60}$$

2.6.3 Interest Rate

Bank b also uses their desired capital ratio to determine the interest rate they set on loans. Bank b will stochastically increase (decrease) their loan interest rate to make themselves less (more) desirable to firms looking to take out a loan, which will increase (decrease) their capital ratio. Furthermore, bank b also adjusts their loan interest rate towards an assumed fixed natural rate of interest:

$$r^L_{b,t+1} = \begin{cases} r^L_{b,t}(1+\sigma_r|\varepsilon_{b,t}|) + \gamma_r(r^N - r^L_{b,t}) & \text{if } CR^d_{b,t+1} \geq CR_{b,t} \\ r^L_{b,t}(1-\sigma_r|\varepsilon_{b,t}|) + \gamma_r(r^N - r^L_{b,t}) & \text{if } CR^d_{b,t+1} < CR_{b,t}, \end{cases} \tag{61}$$

where σ_r is the standard deviation of bank b's loan interest rate growth, $\varepsilon_{b,t} \sim \mathcal{N}(0,1)$, hence $|\varepsilon_{b,t}|$ is distributed according to a folded normal distribution, $\gamma_r \in (0,1)$ is a speed of adjustment parameter, and r^N is a fixed natural rate of interest which is uniform across all banks.

2.7 The Market for Labour

The labour market is characterised by a search and matching mechanism similar to the one presented in Gaffeo et al. (2008). Initially, firm ι determines their employment decision for the next period, to either hire or fire employees depending on their employment variable $\eta_{\iota,t}$, eq. 22. If $\eta_{\iota,t} < 0$, then firm ι wants to fire $|\eta_{\iota,t}|$ employees, and when $\eta_{i,t} > 0$, firm ι will post vacancies equal to $\eta_{i,t}$. Firm ι offers potential employees a single-period labour contract with a guaranteed wage of $w_{\iota,t}$ for the current period. It is assumed that the labour contract is periodically renewed and updated with the current wage offered by the firm until the firm decides to fire the employee.

The labour market then opens and firms post their vacancies with the corresponding wage rate. If household h is unemployed, they will randomly visit n_F firms each period, household h does not incur any travel costs when they visit the initial n_F firms, however, after this number is reached, travel costs become prohibitively high. The probability that household h visits firm ι is equal to firm ι's labour market share, $ms^N_{\iota,t} = N_{\iota,t}/(\sum_{c=1}^{\mathbf{N}_C} N_{c,t} + \sum_{k=1}^{\mathbf{N}_K} N_{k,t})$. Household h then sorts the firms they visited by descending wage rate and will send an application to all firms on their list which have open vacancies, $\eta_{\iota,t} > 0$.

Firms then hire or fire employees based on the value of $\eta_{\iota,t}$. When $\eta_{\iota,t} < 0$, it is assumed that firm ι randomly chooses $\min\{|\eta_{\iota,t}|, N_{\iota,t} - 1\}$ workers to fire from their set of employees, hence firm ι always keeps at least 1 employee. When $\eta_{\iota,t} > 0$, it is assumed that firm ι randomly hires $\min\{\eta_{\iota,t}, ||\Omega_A||\}$ new workers from their set of applications, denoted Ω_A, where $||\Omega_A||$ is the number of households in firm ι's set of applications. Thus, the number of households firm ι can hire is constrained by the number of applications they receive. Furthermore, the set of firm ι's applications is reset each period.

2.8 The Market for Consumption Goods

Households randomly visit n_C C-firms each period, that which travel costs permit. The probability that household h will visit C-firm i is equal to C-firm i's consumption market share, $ms_{i,t} = Y_{i,t}/\sum_{k=1}^{\mathbf{N}_C} Y_{k,t}$. Household h then sorts the C-firms they visited by ascending price and demands to consume C-goods worth $C^d_{h,t} = E^d_{h,t}/P_{i,t}$ from the first C-firm on their list. If C-firm i's inventories run out before household h exceeds their desired expenditure, household h purchases the remaining C-goods from C-firm i and moves to the next C-firm on their list.

Households continue to demand C-goods from their list of C-firms until they have reached their desired expenditure or exhausted their list of C-firms. Hence, household h's actual expenditure is given by:

$$E_{h,t} = \min\left\{E^d_{h,t}, \sum_{i\in\Omega_C} P_{i,t}V_{i,t}\right\}, \tag{62}$$

where Ω_C is the set of C-firms visited by household h. Furthermore, the actual demand for C-goods faced by C-firm i is given by:

$$Z_{i,t} = \sum_{h\in\Omega_H} \frac{E^d_{h,t}}{P_{i,t}}, \tag{63}$$

where Ω_H is the set of all households that demanded C-goods from C-firm i. C-firm i then calculates their actual quantity of sold C-goods as:

$$Q_{i,t} = \min\left\{Z_{i,t}, Y_{i,t}\right\}. \tag{64}$$

2.9 The Market for Capital Goods

If the ith C-firm desired to invest in new capital good, $I^d_{i,t+1} > 0$, then C-firm i will randomly visit n_K K-firms each period, that which travel costs permit. The probability that C-firm i will visit K-firm j is equal to K-firm j's capital market share, $ms_{j,t} = Y_{j,t}/\sum_{k=1}^{\mathbf{N}_K} Y_{k,t}$. C-firm i then sorts the K-firms they visited by ascending price and demands to consume K-goods worth $I^d_{i,t}$ from the first K-firm on their list. If K-firm j's inventories run out before C-firm i exceeds their desired expenditure, then C-firm i purchases the remaining K-goods from K-firm j and moves to the next K-firm on their list. C-firms continue to demand K-goods from their list of K-firms until they have reached their desired investment or exhausted their list of K-firms. Hence, C-firm i's actual investment is given by:

$$I_{i,t+1} = \min\left\{I^d_{i,t+1}, \sum_{j\in\Omega_K} V_{j,t}\right\}, \tag{65}$$

where Ω_K is the set of K-firms visited by C-firm i. Similarly, C-firms i's investment expenditure is given by:

$$IE_{i,t+1} = \min\left\{IE^d_{i,t+1}, \sum_{j\in\Omega_K} P_{j,t}V_{j,t}\right\}. \tag{66}$$

Furthermore, the actual demand for K-goods faced by K-firm j is given by:

$$Z_{j,t} = \sum_{i\in\Omega_C} I^d_{i,t+1}, \tag{67}$$

where Ω_C is the set of all C-firms that demanded K-goods from K-firm j. K-firm j then calculates their actual quantity of sold K-goods as:

$$Q_{j,t} = \min\left\{Z_{j,t}, Y_{j,t}\right\}. \tag{68}$$

2.10 The Market for Credit

C-firms and K-firms randomly visit a single bank b each period. The probability that firm ι visits bank b is equal to bank b's credit market share, $ms_{b,t} = L_{b,t}/\sum_{k=1}^{\mathbf{N}_B} L_{k,t}$. Firm ι then demands loans worth $L^d_{\iota,t+1}$ from the selected bank b. Bank b supplies the loan depending on their risk tolerance. Therefore, the actual loan received by firm ι is equal to the loan supplied from bank b:

$$L_{\iota\to b,t+1} = L^s_{\iota\to b,t+1}, \tag{69}$$

where $L^s_{\iota\to b,t+1}$ is defined in eq. 59.

Furthermore, due to the rigorous accounting framework that is used, when firm ι takes out a loan from bank b, bank b simultaneously credits firm ι's deposit account, while also increasing banks b's stock of loans. Hence, this action increases total bank assets and liabilities at the same time. Hence, in aggregate, the bank's balance sheets have increased by the size of the loan, creating new money. Moreover, when firm ι repays a proportion of a loan, this destroys money, because banks record this transaction as both a reduction in deposits and loans, which simultaneously decreases both assets and liabilities by the size of the loan repayment.

2.11 Entry & Exit Dynamics

The number of all agents apart from firms is fixed throughout the simulation. The bankruptcy condition of firm ι is assumed to be when their deposits are less than or equal to zero, $M_{\iota,t} \leq 0$. There is a one-to-one replacement of firms, thus, if firm ι exits the market a new firm will enter. New entrants are a random copy of incumbent firms with no debt, a single employee (chosen from the pool of unemployed households), and average market values for their price $P_{\iota,t} = \bar{P}^F_t$ ($F = \{C, K\}$ for either C-market or K-market respectively) and wage $w_{\iota,t} = \bar{w}_t$.

Furthermore, banks become bankrupt when their equity is less than or equal to zero, $E_{b,t} \leq 0$, however, unlike firms, banks are bailed out by their depositors (firms and households) and the new equity of the bankrupt bank in the next period is $E_{b,t+1} = CR^d_{b,t+1}(L_{b,t} + R_{b,t})$, which is the banks desired amount of equity to their assets. Each depositor's deposits are reduced proportionally by the size of their deposit account at the bankrupt bank.

2.12 Stock-Flow Consistency

In this section, it is shown that the model is stock-flow consistent (SFC) at the aggregate level, which ensures that all financial transactions are accounted for, such that each payment from one agent goes to another agent in the model. Therefore, every financial stock is recorded as a liability for one agent and an asset for another agent.

Table 1 provides a financial balance sheet representation of the model for each agent class. Where values in the table represent the aggregate stock variables for each type of agent[4]. It is evident from Table 1 that all the columns and rows dealing with financial assets must sum to zero, where tangible capital (K) is the only asset that appears once on the balance and thus

[4]Each variable with a subscript refers to that agent's share of the total, e.g. M_H are household deposits, M_C are C-firm deposits, and M_K are K-firm deposits. The same variable without a subscript refers to the total of that variable, e.g. total deposits are the sum of all agents' deposits: $M = M_H + M_C + M_K$.

does not sum to zero. Hence, the sum of all agents' equities must be equal to tangible capital, in line with the reasoning set out by Godley and Lavoie (2007).

	Households	**C-Firms**	**K-Firms**	**Banks**	**Central Bank**	$\sum$
Capital		K				K
Deposits	M_H	M_C	M_K	$-M$		0
Debt		$-D_C$	$-D_K$	D		0
Reserves				R	$-R$	0
Advances				$-A$	A	0
Equity	$-E_H$	$-E_C$	$-E_K$	$-E_B$	$-E_{CB}$	$-K$
$\sum$	0	0	0	0	0	0

Table 1: Macro financial balance sheet matrix.

Table 2 provides a transaction flow matrix, which shows the aggregate flows of financial transactions between the different agents of the model. Each column and row of the transaction flow matrix must sum to zero for the model to be SFC. The upper part of the transaction flow matrix in Table 2 reproduces the national income statistics presented in Table 1, and the lower part of Table 2 represents the inter-sectoral flow of funds in the model.

	Households	**C-Firms**		**K-Firms**		**Banks**		**CB**	$\sum$
		Current	Capital	Current	Capital	Current	Capital		
Wages	W	$-W_C$		$-W_K$					0
Consumption	$-C$	C							0
Investment			$-I$	I					0
Loan repayments			$-\rho D_C$		$-\rho D_K$		ρD		0
Loan Interest		$-IP_C$		$-IP_K$		IP			0
Deposit Interest	$i^M M_H$	$i^M M_C$		$i^M M_K$		$-i^M M$			0
Profits		$-\Pi_C$	Π_C	$-\Pi_K$	Π_K	$-\Pi_B$	Π_B		0
Inventories				ΔV	$-\Delta V$				0
Change in Deposits	$-\Delta M_H$		$-\Delta M_C$		$-\Delta M_K$		ΔM		0
Change in Debt			L_C		L_K		$-L$		0
Change in Reserves							$-\Delta R$	ΔR	0
Change in Advances							ΔA	$-\Delta A$	0
Loan Defaults			B_C		B_K		$-B$		0
$\sum$	0	0	0	0	0	0	0	0	0

Table 2: Macro transaction flow matrix.

3 Simulation Results

In this section, the output of the results from the model described in section 2 have been analysed. Firstly, we show that the model can reproduce the stylised facts present in empirical data for the baseline scenario of 2% mean productivity growth ($g = 0.02$). The stability between growth and zero-growth scenarios for differing firm debt dynamics is then analysed at both the macro and micro levels. Each scenario was run for a batch of 50 Monte Carlo simulations ($s \in \mathcal{S} = \{1, 2, ..., 50 = S\}$), to avoid atypical behaviour that may be present due to the implementation of random variables. To avoid differences in the scenario caused by the generation of random numbers, the same random seed was used in each simulation for the different scenarios. Therefore, the only difference between scenarios was the change in parameter values because each random number was the same across scenarios. Furthermore, each simulation was run for 400 periods, hence, given the assumption that each time step represents a quarter of a year, this equates to a total simulation length of 100 years. Additionally, the simulation was run for an initial 200 periods (50 years), which were discarded to remove any transient behaviour from the analysis.

3.1 Stylised Facts

We first show the time-series behaviour of a typical run of the simulation ($s = 5$, randomly chosen) as seen in Fig. 2. It is evident in Sub-Fig. 2a that investment (dotted) is more volatile than consumption (dashed), which is in line with empirical U.S. data[5]. Furthermore, Sub-Fig. 2b shows that before most of the recessions throughout the simulation, the debt to GDP ratio (solid) peaks and then drops during the recession, this demonstrates that the model produces credit cycles similar to that in Barrett (2018). Thus, there is an emergent Minskyan dynamic, where corporate debt builds up during a boom and reduces during a bust. The Minskyan cycle in corporate debt to GDP has recently been empirically validated by Stockhammer and Gouzoulis (2023) for the U.S. from 1889 to 2014. Sub-Fig. 2f further supports the corporate debt to GDP cycles present in the model, where for 8 of the 12 recessions in the simulation the credit rate, defined as the log difference of debt, was negative. Therefore, when the model was in recession, corporate debt usually decreased. The unemployment rate dynamics in Sub-Fig. 2e has a peak during a recession and a low during a boom, which is again reflected in empirical U.S. data[6].

Further stylised facts emerging from the single simulation ($s = 5$) are the linear relationship between key economic variables. Fig. 3 shows these key relationships, where there is a weak but statistically significant negative relationship between price-inflation and unemployment, a price-related Phillips curve in Sub-Fig. 3a. Furthermore, Sub-Fig. 3b presents a wage-related (standard) Phillips curve, which again shows a weak negative linear relationship between the rate of change of the nominal average wage and the unemployment rate. Sub-Fig. 3c shows the credit-related Phillips curve, which is the relationship between credit, the annual percentage change of debt, and the unemployment rate. There is a significant negative relationship, therefore increasing credit will reduce unemployment (Keen, 2014). Finally, Sub-Fig. 3d shows the Okun curve, which shows a strong negative relationship between real GDP growth and the change in the unemployment rate.

From the above, it has been shown that a randomly chosen simulation ($s = 5$) replicates the stylised facts seen in empirical economic data. We also tested these results for other simulations which showed similar findings. However, to be more thorough in the analysis of the stylised facts the model produces, we show that the average values across all simulations replicate the statis-

[5] The standard deviation of the annual log difference for simulated investment is 8.9% and 9.9% for U.S. investment, whereas for simulated consumption it is only 2.6% and 1.9% for U.S. consumption. Data taken from the Federal Reserve Economic Data (FRED) database at: `https://fred.stlouisfed.org/`. PCECC96 for consumption and GPDIC1 for investment

[6] See the unemployment rate for the U.S. (UNRATE) from the FRED database.

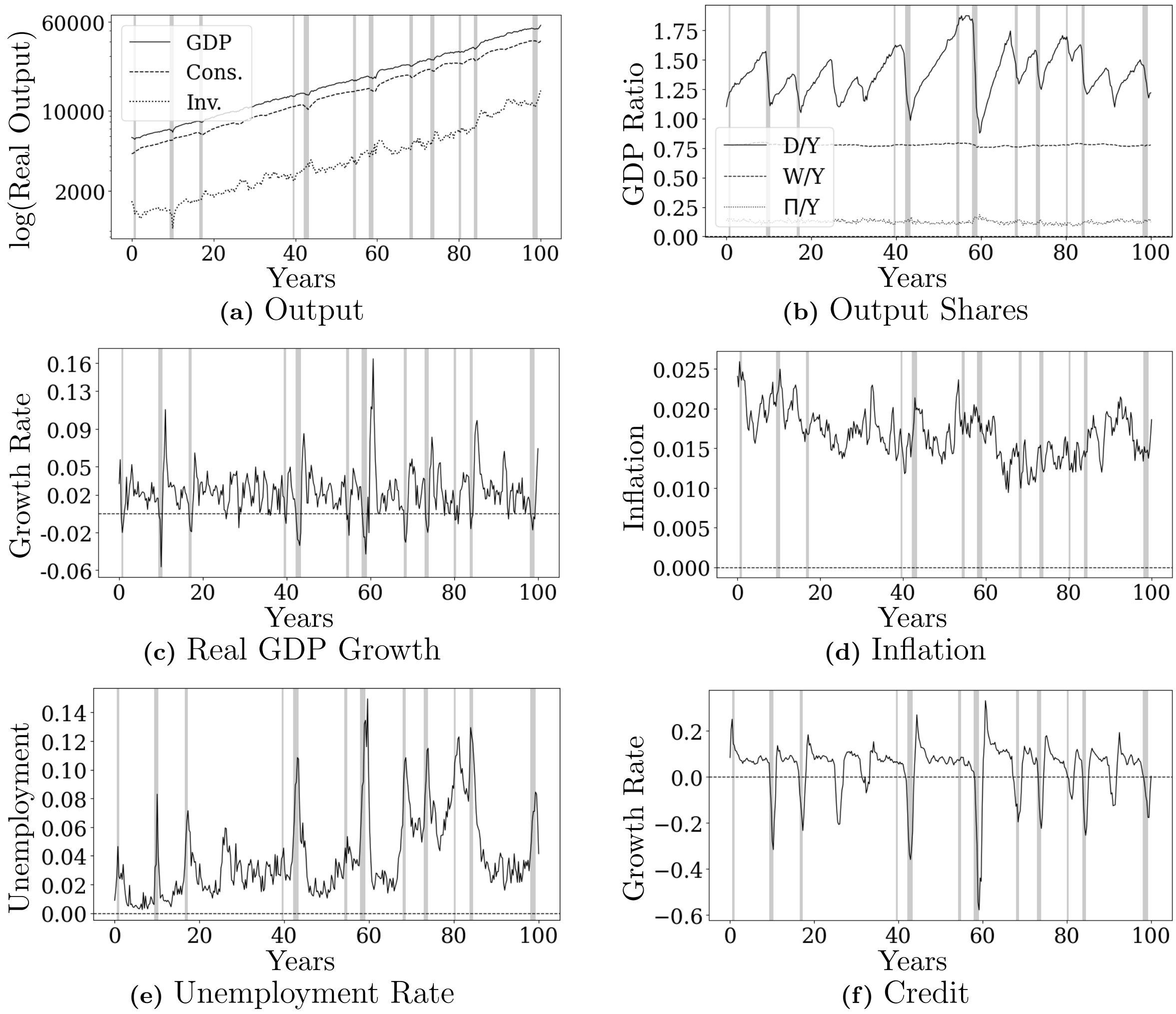


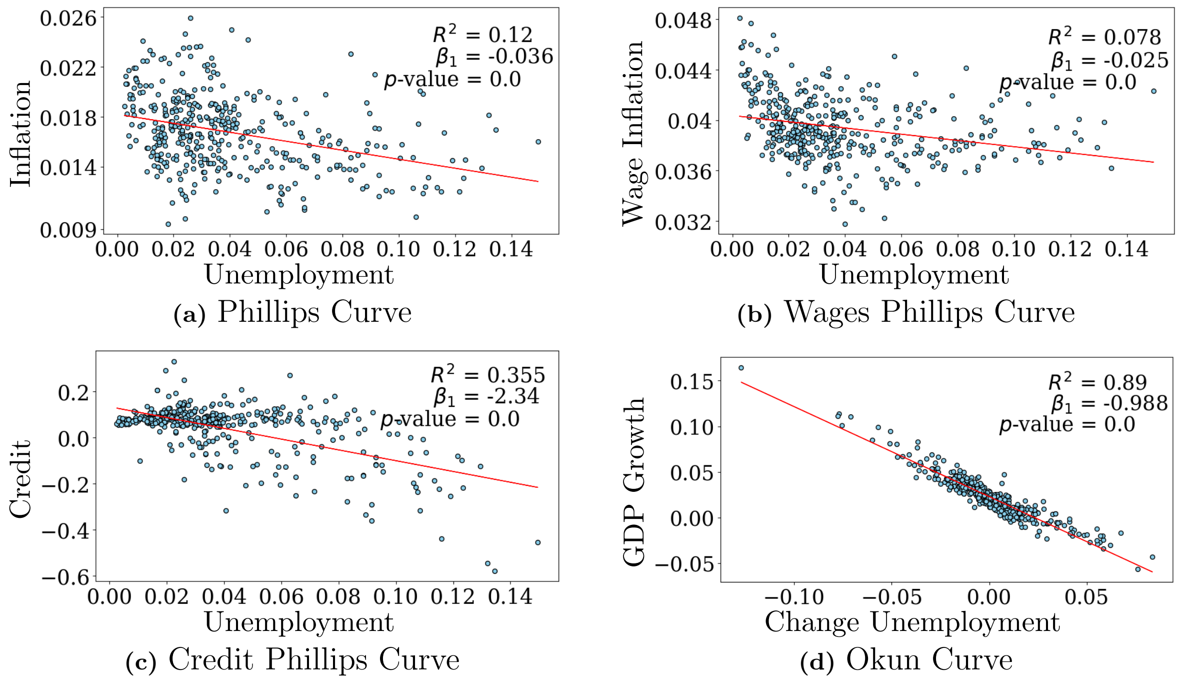
Figure 2: Stylised facts of time-series for a typical run ($s = 5$). The shaded area highlights an economic recession (two or more quarters of negative real GDP growth). Panel (a) shows real GDP (solid), consumption (dashed), and investment (dotted). Panel (b) shows the debt to GDP ratio (solid), wage share of GDP (dashed), and profit share of GDP (dotted). Panel (c) shows the growth rate of real GDP. Panel (d) shows the CPI inflation rate. Panel (e) shows the unemployment rate. Finally, Panel (f) shows the credit rate, defined as the annual percentage change of debt.

tical properties found in empirical data. The analysis will now focus on a comparison between the properties of the simulated time-series, for the baseline scenario of 2% mean productivity growth ($g = 0.02$), and empirical time-series of U.S. data[7]. Both the simulated and empirical time-series were passed through a Hodrick-Prescott (HP) filter[8] so as to remove the trend from the data so that the cyclical component of each time-series could be analysed. It can be noted from Figure 4 that overall the simulated time-series display similar levels of autocorrelation, which is the correlation in time t with time $t - lag$, for the cyclical components of real GDP, productivity, consumption, investment, the unemployment rate, and corporate debt.

Figure 5 presents a further comparison between the statistical attributes of the simulated

[7]Empirical U.S. time-series taken from the FRED database at: `https://fred.stlouisfed.org/`. GDPC1 for real GDP, PCECC96 for consumption, GPDIC1 for investment, CE16OV for employment, UNRATE for the unemployment rate, and TBSDODNS for corporate debt. All time-series had a quarterly frequency from Q1 1950 to Q4 2019 so as to avoid anomalous data caused by the global pandemic.

[8]We use a value of $\lambda = 1600$ for the HP filter.



(a) Phillips Curve

(b) Wages Phillips Curve

(c) Credit Phillips Curve

(d) Okun Curve

Figure 3: Stylised facts of the relationship between key economic variable for a typical baseline run ($s = 5$). Panel (a) shows the price-related Phillips curve. Panel (b) shows the wage-related Phillips curve. Panel (c) shows the credit-related Phillips curve. Finally, panel (d) shows the Okun curve.

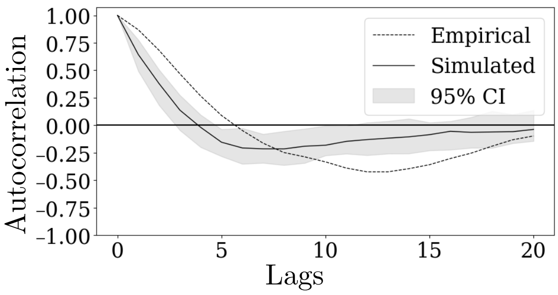
time-series with the empirical time-series, displaying plots of the cross-correlation for the cyclical components of the time-series from the HP filter. The cross-correlation is the correlation of two series as a function of the displacement of one time-series in $t \pm lag$ with the other time-series in time t. From Figure 5, it is shown that the cross-correlations for the cyclical components of the simulated time-series replicate those seen in the empirical time-series. In particular, the cross correlation of GDP vs GDP, GDP vs unemployment, GDP vs debt, and debt vs unemployment, have a very close match to the empirical cross-correlations. However, although the simulated cross-correlations for GDP vs consumption and GDP vs investment are relatively similar to the empirical data, they do not replicate the statistical properties as accurately as the other cross-correlations. Where the cross-correlation of GDP vs consumption is more positively skewed in the simulated time-series than in the empirical time series, and the cross-correlation of GDP vs investment is more negatively skewed in the simulated time-series than in the empirical time-series.

To further test the statistical properties of the simulated macroeconomic time-series, we look at the distribution of real GDP growth rates. The empirical data suggests that real GDP growth rates are not normally distributed and have fat tails. Fagiolo et al. (2008) show that output growth rates for OECD countries are well approximated by the exponential power (EP) or Subbiton family of distributions[9]. Sub-Fig. 6a shows the probability density distribution

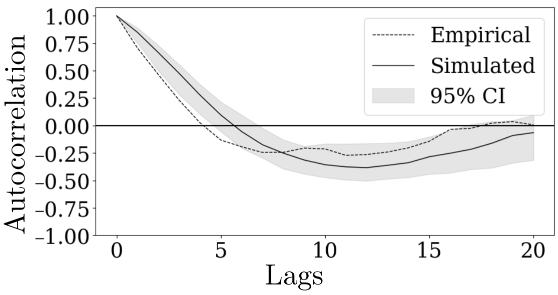
[9]The exponential power or Subbotin distribution has a density function given by:

$$f(x;\beta,\alpha,\mu) = \frac{\beta}{2\alpha\Gamma(1/\beta)} \exp\left\{-\left|\frac{x-\mu}{\alpha}\right|^{\beta}\right\}, \tag{70}$$

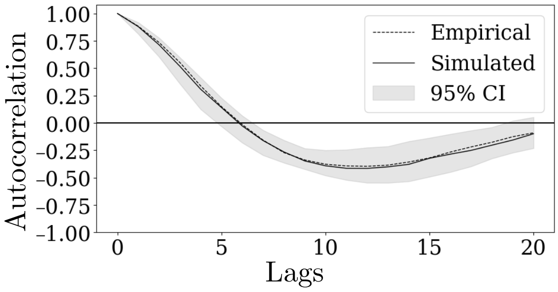
where $\Gamma(\cdot)$ is the gamma function, β is a shape parameter, α is a scale parameter, and μ is a location parameter. For $\beta = 1$ the distribution reduces to a Laplace distribution, and for $\beta = 2$ the distribution reduces to a Gaussian



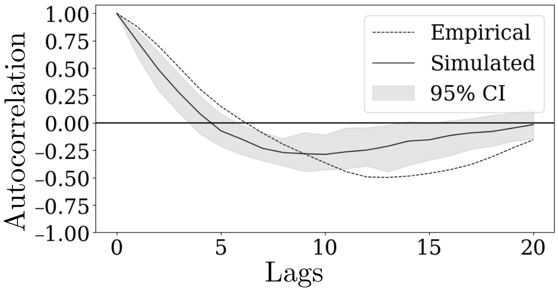
(a) Real GDP



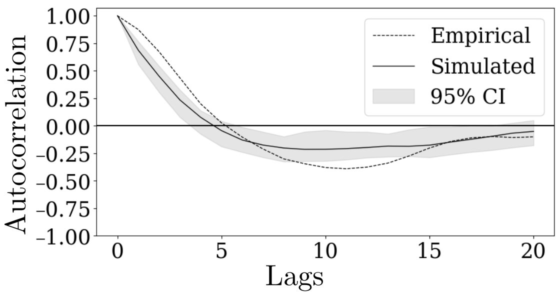
(b) Productivity



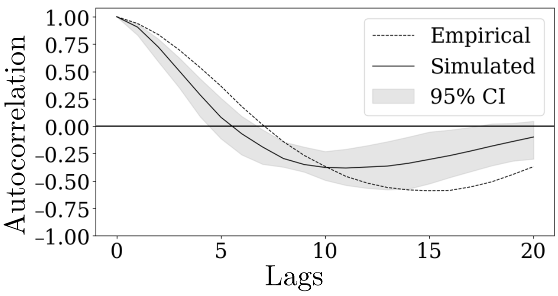
(c) Consumption



(d) Investment



(e) Unemployment Rate



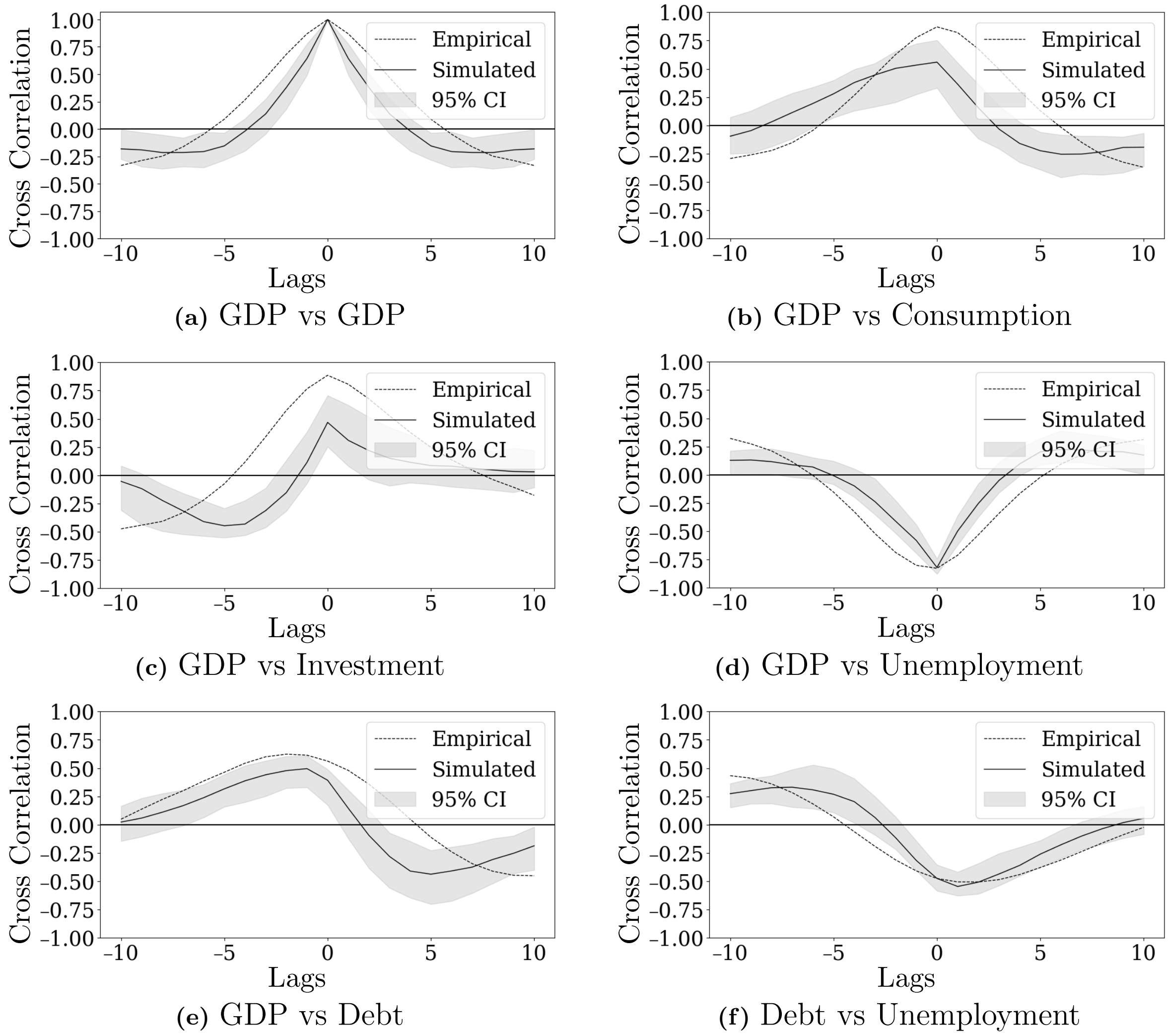
(f) Corporate Debt

Figure 4: Autocorrelation of simulated and empirical time-series. The dashed line is the empirical autocorrelation, the solid line is the median autocorrelation across all simulations, and the shaded area is the 95% confidence interval (CI) across all simulations.

of simulated real GDP growth rates, where they show super-Laplace fat tailed behaviour because the estimated shape parameter $\hat{\beta} = 0.9197$ is less then 1. Additionally, the estimated shape parameter $\hat{\beta}$ is within the parameter range found by Fagiolo et al. (2008) for the growth rate distributions of OECD countries, ranging from $\hat{\beta} = 1.6452$ for Canada to $\hat{\beta} = 0.8063$ for Denmark. Moreover, Table 7 in Appendix C shows the test results for the Kolmogorov-Smirnov, Shapiro-Wilk, and Anderson-Darling tests of normality. The null hypothesis, that the distribution is normal, could be rejected for each test at the 1% significance level, providing additional confirmation that the distribution of simulated real GDP growth rates is not normally distributed.

From the above analysis, it has been shown that the macroeconomic behaviour of the model replicates the stylised facts found in empirical data. A strength of the agent-based approach to modelling is that the microeconomic behaviour of the agents can also be examined. In line with empirical evidence on firm growth rates, the model reproduces the tent-shaped distribution with fat tails as seen in Sub-Fig. 6b. Again, C-firm growth rates have been fitted with an EP

(normal) distribution.



(a) GDP vs GDP

(b) GDP vs Consumption

(c) GDP vs Investment

(d) GDP vs Unemployment

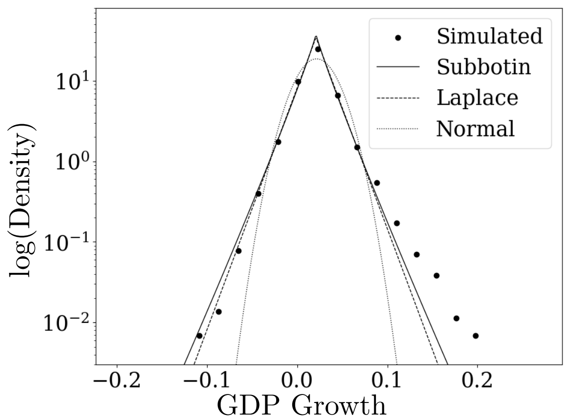
(e) GDP vs Debt

(f) Debt vs Unemployment

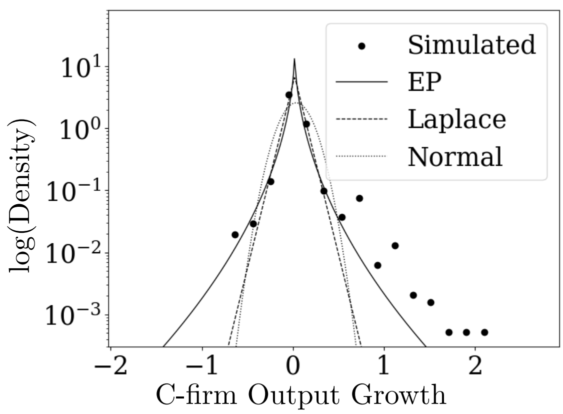
Figure 5: Cross correlation of simulated and empirical time-series. Where the dashed line is the empirical cross-correlation, the solid line is the median cross-correlation across all simulation, and the shaded area is the 95% confidence interval (CI) across all simulations.

distribution with an estimated shape parameter $\hat{\beta} = 0.5023$. This is lower than that seen in empirical data which is usually closer to a Laplace distribution with $\beta \sim 1$ (Bottazzi and Secchi, 2006). This is largely due to the fact that the simulated data has a large proportion of small firms, whereas empirical datasets do not often have a large sample of small firms. When we remove firms with less than 10 employees, we find that the estimated shape parameter is closer to that found in the empirical data, with $\hat{\beta} = 1.0441$.

Furthermore, in Fig. 7 the complementary cumulative distribution functions (CCDFs) for C-firm and K-firm output and leverage are shown, as well as distributions of bank loans and leverage. The model reproduces the well-known fact that the distribution of firm size has fat tails and is positively skewed (Axtell, 2001; de Wit, 2005). Furthermore, the model also produces heavy-tailed distributions for C-firm and K-firm leverage, as well as bank loans and leverage. The tail of the distribution for firm and bank leverage ratios is particularly well approximated by the power-law fit as seen in Fig. 7, suggesting that highly leveraged firms and banks are not unusual in the model. Furthermore, when a firm becomes highly leveraged in the model this increases the bank's estimated probability of default of that firm, thus, reducing the bank's



(a) Distribution of GDP Growth



(b) Distribution of C-firm Output Growth

Figure 6: Panel (a) shows the distribution of real GDP growth over all Monte Carlo simulations. Solid line is the exponential power (Subbotin) fit ($\hat{\beta} = 0.9197$, $\hat{\alpha} = 0.0127$, $\hat{\mu} = 0.0209$), dashed line is the Laplace distribution fit ($\hat{\beta} = 1$, $\hat{\alpha} = 0.0145$, $\hat{\mu} = 0.0209$), dotted line is the normal distribution fit ($\hat{\beta} = 2$, $\hat{\alpha} = 0.0213$, $\hat{\mu} = 0.0216$). Panel (b) shows the distribution of C-firm output growth rates over all Monte Carlo simulations taken at a midpoint snapshot ($s \in \{1, 2, ...50\}$ and $t = 300$). Solid line is the exponential power (Subbotin) fit ($\hat{\beta} = 0.5023$, $\hat{\alpha} = 0.0118$, $\hat{\mu} = 0.0185$), dashed line is the Laplace distribution fit ($\hat{\beta} = 1$, $\hat{\alpha} = 0.0728$, $\hat{\mu} = 0.0190$), dotted line is the normal distribution fit ($\hat{\beta} = 2$, $\hat{\alpha} = 0.1549$, $\hat{\mu} = 0.0347$).

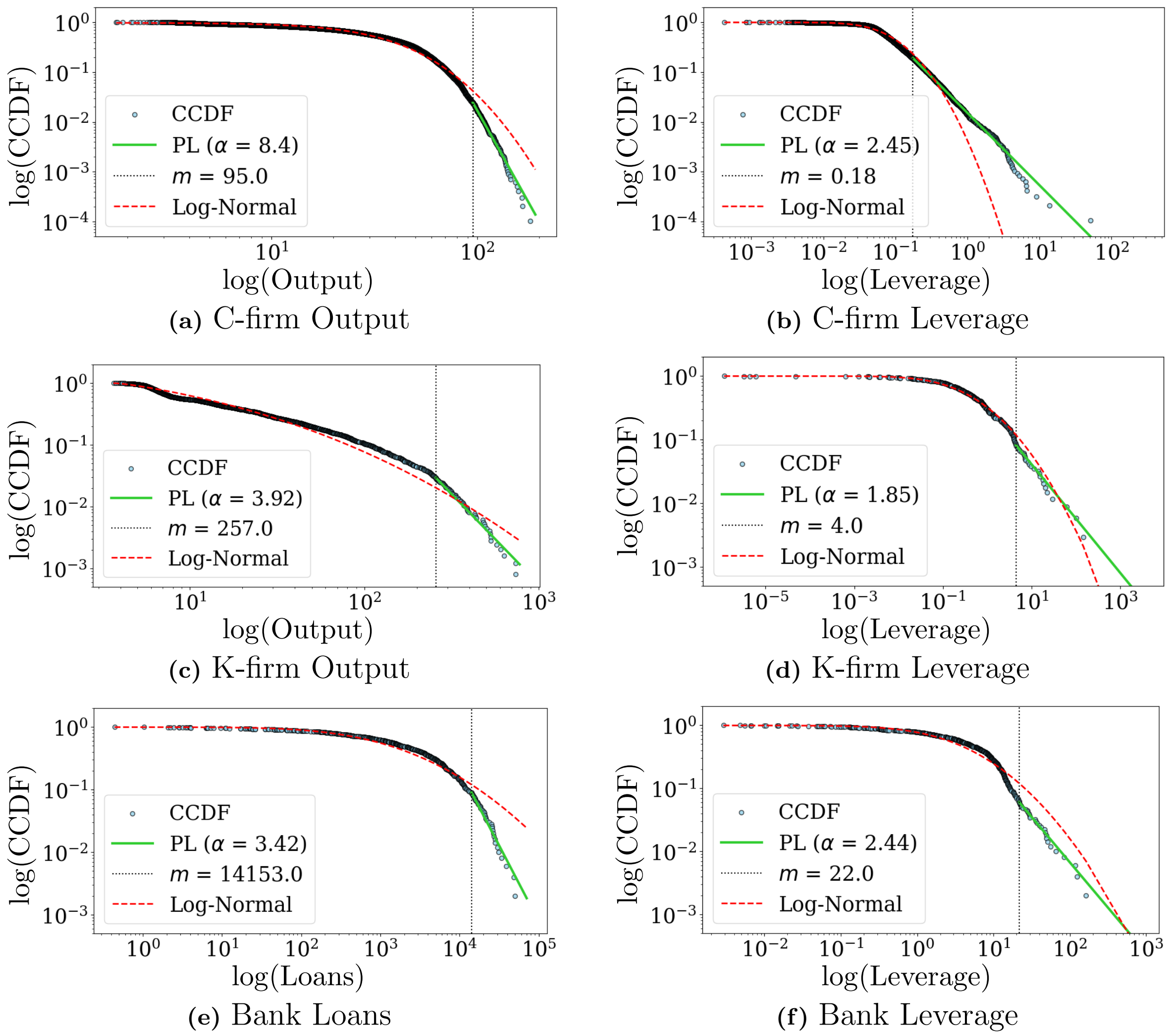
willingness to lend to that firm. Hence, a highly leveraged firm may become bankrupt if banks are not willing to lend to the firm in order for the firm to cover its costs. Therefore, leverage ratios play a key role in the dynamics of the model, and from this distributional shape, it is shown that high leverage ratios are not uncommon and are a feature of the model.

3.2 Stability Analysis

The baseline model has been shown to account for a broad class of empirical regularities. Encouraged by its performance we now explore a larger set of simulation possibilities. In particular, we are interested in analysing how firm debt dynamics impact the relative stability between a growth and zero-growth scenario, for which we performed four separate batches of simulations. First, we ran the growth scenario with an average labour productivity growth rate of 2% ($g = 0.02$) for two different simulations, one where C-firms desired investment debt ratio (eq. 13) is low, which we refer to as Growth S1, and another where C-firms desired investment debt ratio is high, which we refer to as Growth S2. Then, we ran the zero-growth scenario, with an average labour productivity growth rate of 0% ($g = 0$), for the same two different batch simulations, the first where C-firms desired investment debt ratio is low, which we refer to as Zero-Growth S1, and another where C-firms desired investment debt ratio is high, which we refer to as Zero-Growth S2. Table 3 shows the parameters used for each of the different batch simulations.

The results for the primary macroeconomic variables of the model for each of the batch simulation scenarios are presented in Table 4, which displays mean values grouped by simulation for the average and standard deviation of each variable over time. Growth rates, i.e real GDP growth, inflation, wage inflation, and the rate of credit, were calculated using the log-difference[10]

[10]The growth rate g_t of a given time-series variable x_t is well approximated by the log-difference between the



(a) C-firm Output **(b)** C-firm Leverage **(c)** K-firm Output **(d)** K-firm Leverage **(e)** Bank Loans **(f)** Bank Leverage

Figure 7: Complementary cumulative distribution function (CCDF) of key agent characteristics over all Monte Carlo simulations taken at a midpoint snapshot ($s \in \{1, 2, ...50\}$ and $t = 300$). Power-law fit (green line), power-law cut-off (black dotted), and log-normal fit (red dashed).

Parameters	**Growth S1**	**Growth S2**	**Zero-Growth S1**	**Zero-Growth S2**
g	0.02	0.02	0	0
d_0	0.5	0.5	0.5	0.5
d_1	3	5	3	5
d_2	2	3	2	3

Table 3: Parameter values used for each of the different batch simulations.

from one period to the next. The statistic for the Gini Coefficient, which measures household wealth inequality, is calculated using the method in Dixon et al. (1987), where, given an ordered list of household wealth the Gini Coefficient in time t can be defined as:

$$G_t = \frac{1}{\mathbf{N}_H}\left(\mathbf{N}_H + 1 - 2\left(\frac{\sum_{h=1}^{\mathbf{N}_H}(\mathbf{N}_H + 1 - h)M_{h,t}}{\sum_{h=1}^{\mathbf{N}_H} M_{h,t}}\right)\right), \tag{71}$$

current and previous time step: $g_t = (x_t - x_{t-1})/x_{t-1} \approx \log(x_t) - \log(x_{t-1})$.

where $\mathbf{N}_H$ is the number of households and $M_{h,t}$ is the wealth of household h at time t. The standard measure of an economic crisis being two quarters of negative real GDP growth was unsuitable in our analysis, this is because it is biased towards the growth scenario (less crises), as the random fluctuations in real GDP growth around the average value for g meant that the zero-growth scenarios would have more crises even with a lower standard deviation of real GDP growth as the mean was closer to 0. Therefore, we adopted the framework in Dosi et al. (2013) where they define an economic crises as a period in which real GDP growth is below -3%.

Variable	**Growth S1**		**Growth S2**		**Zero-Growth S1**		**Zero-Growth S2**	
	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
Real GDP Growth	0.022	0.021	0.020	0.026	0.004	0.013	0.004	0.015
	(0.0001)	(0.0004)	(0.0001)	(0.0004)	(0.0001)	(0.0002)	(0.0002)	(0.0004)
Unemployment	0.034	0.025	0.028	0.028	0.008	0.008	0.008	0.009
	(0.0009)	(0.0007)	(0.0006)	(0.0006)	(0.0001)	(0.0002)	(0.0001)	(0.0003)
Inflation	0.017	0.003	0.019	0.005	0.029	0.008	0.029	0.008
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0002)	(0.0003)	(0.0002)
Wage Inflation	0.039	0.003	0.039	0.004	0.033	0.008	0.033	0.009
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0002)	(0.0003)	(0.0002)
Credit Rate	0.040	0.112	0.039	0.122	0.030	0.161	0.031	0.168
	(0.0003)	(0.0017)	(0.0004)	(0.0020)	(0.0005)	(0.0033)	(0.0005)	(0.0038)
Debt Ratio	1.344	0.208	1.530	0.254	1.052	0.269	1.081	0.255
	(0.0066)	(0.0054)	(0.0086)	(0.0057)	(0.0139)	(0.0148)	(0.0108)	(0.0076)
Profit Share	0.126	0.018	0.131	0.023	0.090	0.035	0.093	0.037
	(0.0009)	(0.0004)	(0.0011)	(0.0006)	(0.0021)	(0.0015)	(0.0023)	(0.0016)
Wage Share	0.779	0.011	0.776	0.014	0.860	0.037	0.858	0.039
	(0.0008)	(0.0003)	(0.0010)	(0.0005)	(0.0026)	(0.0013)	(0.0026)	(0.0014)
Gini	0.798	0.005	0.799	0.007	0.796	0.014	0.795	0.015
	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0010)	(0.0010)	(0.0009)	(0.0010)
Crises	0.050	0.213	0.124	0.341	0.036	0.178	0.044	0.203
	(0.0035)	(0.0088)	(0.0062)	(0.0085)	(0.0030)	(0.0088)	(0.0033)	(0.0075)

Table 4: Mean values grouped by simulation for averages (Avg.) and standard deviations (Std. Dev.) of macroeconomic variables over time. Standard errors are in parenthesis below.

Table 4 allows us to compare the relative differences between each batch simulation scenario. It can be first noted that the average values for real GDP growth are close to their given parameter values. This suggests that eq. 7 for labour productivity was successful in controlling the aggregate growth rate of the simulation. The standard deviation of real GDP growth is lower in the Growth S1 scenario than in the Growth S2 scenario, which suggests that when firms desire more debt this produces instability in the model. Additionally, the standard deviation is again higher in the Zero-Growth S2 than S1 scenario, which again suggests that higher desired debt is a destabilising factor at the macroeconomic level. Furthermore, the standard deviation of real GDP growth is lower in both the Zero-Growth S1 and S2 scenarios when compared to both Growth S1 and S2 scenarios. This indicates that the zero-growth scenario has less volatility at the macroeconomic level and is therefore more stable. The unemployment rate in the Growth S1 and S2 scenarios is much higher than in both the Zero-Growth scenarios. Furthermore, the standard deviation of the unemployment rate is also lower in both the Zero-Growth scenarios. Hence, the zero-growth scenarios have better employment outcomes for households. The rate of inflation in the Growth S1 and S2 scenarios are relatively similar and are both lower than in the Zero-Growth S1 and S2 scenarios, with marginally lower standard deviations. Thus, zero-growth tends to produce more inflationary pressure but the debt dynamics have little impact on the rate of inflation. However, average wage inflation is sightly higher in both growth scenarios than in both the zero-growth scenarios, with slightly lower standard deviations, suggesting there is little impact on the rate of average wage inflation between a growth and zero-growth scenario. The credit rate is higher in both the Growth S1 and S2 scenarios than in the Zero-Growth S1 and S2 scenarios, suggesting that firms take on more debt in a growth regime. However, the standard deviation of credit is higher in the zero-growth scenarios, therefore, although the increase in debt may be lower, there is more volatility. This is further evidenced with the debt ratio[11], where the Growth S1 and S2 scenarios have significantly higher debt ratios than the Zero-Growth S1 and S2 scenarios but lower standard deviations. This indicates that the growth scenarios have a larger but more stable financial sector relative to GDP than the zero-growth scenarios. The profit share[12] in the growth scenarios is higher than in both the zero-growth scenarios, hence, more nominal GDP is going to firms than workers in the growth scenario. This is again shown in the wage share[13], which is lower in both the growth scenarios than the zero-growth scenarios, which means that households have a larger proportion of GDP and are therefore better off. The Gini Coefficient is relatively similar across all scenarios, suggesting little impact in household inequality between a growth and zero-growth regime. Crises are more likely to occur in the S2 high debt scenario for both growth and zero growth, suggesting that firm debt dynamics play an essential role in creating economic crises. Importantly, crises occur less often under zero-growth, with both the low and high-debt zero-growth scenarios leading to fewer crises than the low-debt growth scenario.

Fig. 8 shows the simulation average with 95% confidence intervals for the debt ratio, employment rate, wage share, profit share, and real GDP growth for each scenario over time. From Fig. 8, it is evident that the debt ratio starts at a similar level for each scenario, however, in both growth scenarios the debt ratio either maintains the initial level or slightly increases. In the zero-growth scenarios, the debt ratio reduces significantly in the initial 20 to 30 years of the simulation and then finds a new equilibrium level. Moreover, it was shown in Table 4 that the wage share was higher and the profit share was lower in the zero-growth scenarios, which is also true in Fig. 8. Although, the wage share is decreasing and the profit share is increasing in both zero-growth scenarios, suggesting that this result may not hold indefinitely.

To further investigate the effects of debt dynamics and productivity growth on real GDP growth rates, we show the probability density distribution for each scenario in Fig. 9. Here, in both the growth and zero-growth scenarios, increasing desired debt ratios widens the distribution of real GDP growth rates, with more probability mass located in the tails. Hence, a higher desired debt ratio will increase the volatility of real GDP growth rates for both growth and zero-growth scenarios. This is particularly evident in the Zero-Growth S2 scenario, which not only displays super-Laplace tails but also super-Subbotin tails. Therefore, a zero-growth regime with high debt dynamics will increase the likelihood of a deep recession as well as a roaring boom.

We now focus the analysis on the microeconomic level, where we use the Hymer-Pashigian

[11]The debt ratio is defined as nominal corporate debt to nominal GDP.
[12]The profit share is defined as nominal profits to nominal GDP.
[13]The wage share is defined as nominal wage to nominal GDP.

(a) GDP Shares - Growth S1

(b) GDP Shares - Zero-Growth S1

(c) GDP Shares - Growth S2

(d) GDP Shares - Zero-Growth S2

Figure 8: Time-series of macroeconomic variables with 95% confidence intervals for each simulation scenario. Debt ratio (red), employment rate (yellow), wage share (green), profit share (blue), and real GDP growth (black).

Instability Index (HPI) to measure the instability of market shares for firms and banks (Hymer and Pashigian, 1962), given by:

$$HPI_t = \sum_{a=1}^{\mathbf{N}_A} |ms_{a,t} - ms_{a,t-1}|, \tag{72}$$

where $\mathbf{N}_A$ is the total number of agents in market $A \in \{C, K, B\}$ ($\mathbf{N}_C$ for the number of C-firms, $\mathbf{N}_K$ for the number of K-firms, and $\mathbf{N}_B$ for the number of banks) and $ms_{a,t}$ is the market share of agent $a \in \{i, j, b\}$ (C-firms $a = i$, K-firms $a = j$, or banks $a = b$). $HPI_t \in (0, 1)$ measures the relative market share instability in time t. For HPI_t close to 0, this indicates a relatively stable market, and for HPI_t close to 1, this indicates a relatively unstable market. Furthermore, to measure relative market concentration, we use the Herfindahl-Hirschman Index (HHI), defined as[14] (Hirschman, 1945):

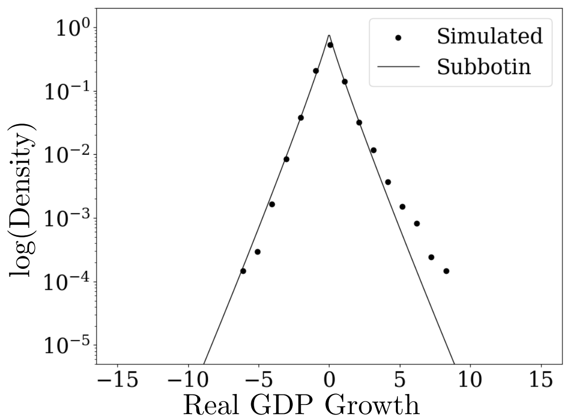
$$HHI_t = \sum_{a=1}^{\mathbf{N}_A} ms_{a,t}^2, \tag{73}$$

where $\mathbf{N}_A$ is the total number of agents in market $A \in \{C, K, B\}$ and $ms_{a,t}$ is the market share of agent $a \in \{i, j, b\}$. Furthermore, because $HHI_t \in [1/\mathbf{N}_A, 1]$, we normalise HHI_t such that $HHI_t^* \in [0, 1]$, given by:

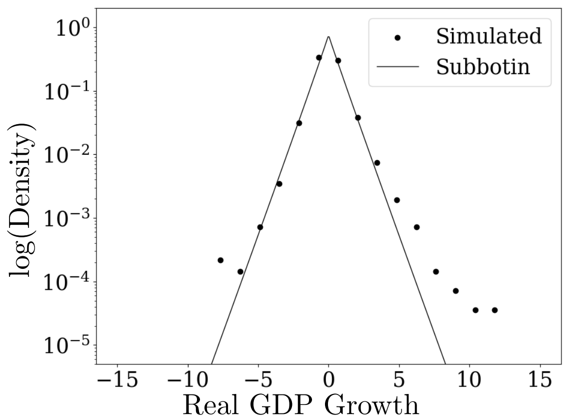
$$HHI_t^* = \frac{HHI_t - \frac{1}{\mathbf{N}_A}}{1 - \frac{1}{\mathbf{N}_A}}. \tag{74}$$

Thus, for HHI_t^* close to 0, this indicates a highly competitive market, and for HHI_t^* close to

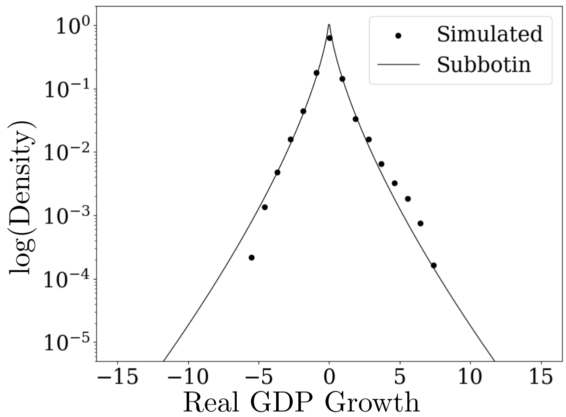
[14] Orris C. Herfindahl also developed the statistic independently in his unpublished doctoral thesis "Concentration in the U.S. Steel Industry" (Columbia University, 1950).



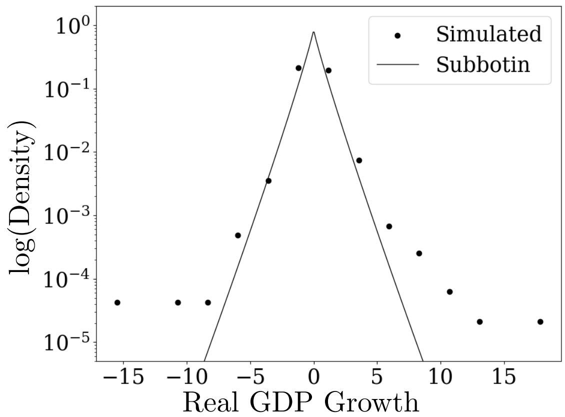
(a) Distribution of Real GDP Growth Rates - Growth S1



(b) Distribution of Real GDP Growth Rates - Zero-Growth S1



(c) Distribution of Real GDP Growth Rates - Growth S2



(d) Distribution of Real GDP Growth Rates - Zero-Growth S2

Figure 9: Distribution of standardised real GDP growth over all Monte Carlo simulations for each scenario. Solid line is the exponential power (Subbotin) fit. $\hat{\beta} = 0.9205$ and $\hat{\alpha} = 0.5981$ for Growth S1; $\hat{\beta} = 0.6887$ and $\hat{\alpha} = 0.3010$ for Growth S2; $\hat{\beta} = 0.9784$ and $\hat{\alpha} = 0.6603$ for Zero-Growth S1; and, $\hat{\beta} = 0.9151$ and $\hat{\alpha} = 0.5686$ for Zero-Growth S2. Due to standardisation, all estimated $\hat{\mu} \approx 0$.

1, this indicates a highly concentrated market.

We also measure the annual bankruptcy rate for firms and banks, which we define as:

$$BR_t = \frac{1}{\mathbf{N}_A} \sum_{\tau=t-n+1}^{t} B_\tau^A, \tag{75}$$

where $\mathbf{N}_A$ is the total number of agents $A \in \{C, K, B\}$ in the market, n is the number of time periods in a year, $n = 1/\Delta t$, and B_t^A is the total number of bankrupt of agents $A \in \{C, K, B\}$ in a given period.

Additionally, to measure the relative risk in the financial sector we use the measure of DebtRank proposed by Battiston et al. (2012). More specifically, because the model produces a bipartite credit network between banks and firms, we use the methodology from Aoyama et al. (2013), which apply the DebtRank measure to a bipartite credit network between Japanese banks and firms. DebtRank measures the amount of financial risk present in a credit network in a given time period. The higher (lower) the value of DebtRank, the more (less) financial risk is present in the credit network, see Appendix D for a detailed construction of the DebtRank measure for the credit network in the model.

Variable	Growth S1		Growth S2		Zero-Growth S1		Zero-Growth S2	
	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.
C-Firm HPI	0.093	0.063	0.105	0.102	0.096	0.139	0.100	0.161
	(0.0009)	(0.0013)	(0.0013)	(0.0027)	(0.0014)	(0.0048)	(0.0015)	(0.0066)
K-Firm HPI	0.274	0.133	0.222	0.152	0.072	0.123	0.071	0.123
	(0.0043)	(0.0024)	(0.0041)	(0.0025)	(0.0019)	(0.0061)	(0.0018)	(0.0056)
Bank HPI	0.162	0.076	0.153	0.085	0.135	0.059	0.134	0.070
	(0.0018)	(0.0016)	(0.0027)	(0.0026)	(0.0021)	(0.0015)	(0.0025)	(0.0023)
C-Firm HHI*	0.003	0.001	0.005	0.001	0.088	0.057	0.095	0.063
	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0036)	(0.0032)	(0.0037)	(0.0034)
K-Firm HHI*	0.062	0.021	0.079	0.026	0.131	0.053	0.121	0.043
	(0.0033)	(0.0018)	(0.0042)	(0.0023)	(0.0128)	(0.0053)	(0.0114)	(0.0038)
Bank HHI*	0.218	0.152	0.299	0.175	0.309	0.186	0.358	0.207
	(0.0093)	(0.0076)	(0.0148)	(0.0094)	(0.0096)	(0.0090)	(0.0134)	(0.0083)
C-Firm BR	0.044	0.076	0.057	0.114	0.047	0.095	0.051	0.109
	(0.0009)	(0.0015)	(0.0015)	(0.0028)	(0.0008)	(0.0021)	(0.0010)	(0.0029)
K-Firm BR	0.023	0.049	0.020	0.051	0.034	0.082	0.032	0.079
	(0.0005)	(0.0013)	(0.0006)	(0.0018)	(0.0008)	(0.0020)	(0.0008)	(0.0019)
Bank BR	0.101	0.193	0.085	0.183	0.098	0.194	0.091	0.185
	(0.0019)	(0.0023)	(0.0021)	(0.0030)	(0.0015)	(0.0027)	(0.0020)	(0.0033)
DebtRank	1.709	0.354	1.916	0.440	2.097	0.573	2.082	0.613
	(0.0281)	(0.0236)	(0.0487)	(0.0275)	(0.0563)	(0.0432)	(0.0615)	(0.0503)

Table 5: Mean values grouped by simulation for averages (Avg.) and standard deviations (Std. Dev.) of microeconomic variables over time. Standard errors are in parentheses below. Abbreviations: HPI is the Hymer-Pashigian Instability Index, HHI is the Herfindahl-Hirschman Index, and BR is the Bankruptcy Rate.

Table 5 shows the mean values grouped by simulation for the averages and standard deviations for each microeconomic measure over time. It is evident from the table that the consumption market instability (C-firm HPI) is similar between all growth and zero-growth scenarios. However, the standard deviation in the zero-growth scenarios is higher, suggesting that the instability of C-firm market shares is more volatile in the zero-growth scenarios. The capital market instability (K-firm HPI) is significantly higher in both growth scenarios than in either zero-growth scenarios and displays higher standard deviations. Therefore, the market shares of K-firms are more unstable in a growth regime. Additionally, bank HPI is also higher in both growth scenarios, with higher standard deviations, which indicates that the growth scenarios produce more unstable financial markets. Alternatively, the concentration of markets, as measured by HHI*, shows that the consumption (C-firm HHI*), capital (K-firm HHI*), and credit (bank HHI*) markets are all more concentrated in the zero-growth scenarios than the growth scenarios. This implies that markets are less competitive in a zero-growth regime. The bankruptcy rates (BR) of C-firms are slightly higher in the S2 compared to the S1 scenarios for both the growth and zero-growth scenarios, suggesting that higher desired debt ratios lead to more bankruptcies, regardless of the growth regime. However, K-firm bankruptcy rates are higher in the zero-growth scenarios than in the growth scenarios. Bank bankruptcy rates are higher in the growth and zero-growth S1 scenarios than in the S2 for either regime, which implies that higher desired debt ratios are good for banks. Finally, the DebtRank of the growth S1 scenario is lower than the growth S2 scenario, similarly for zero-growth S1 and zero-growth S2,

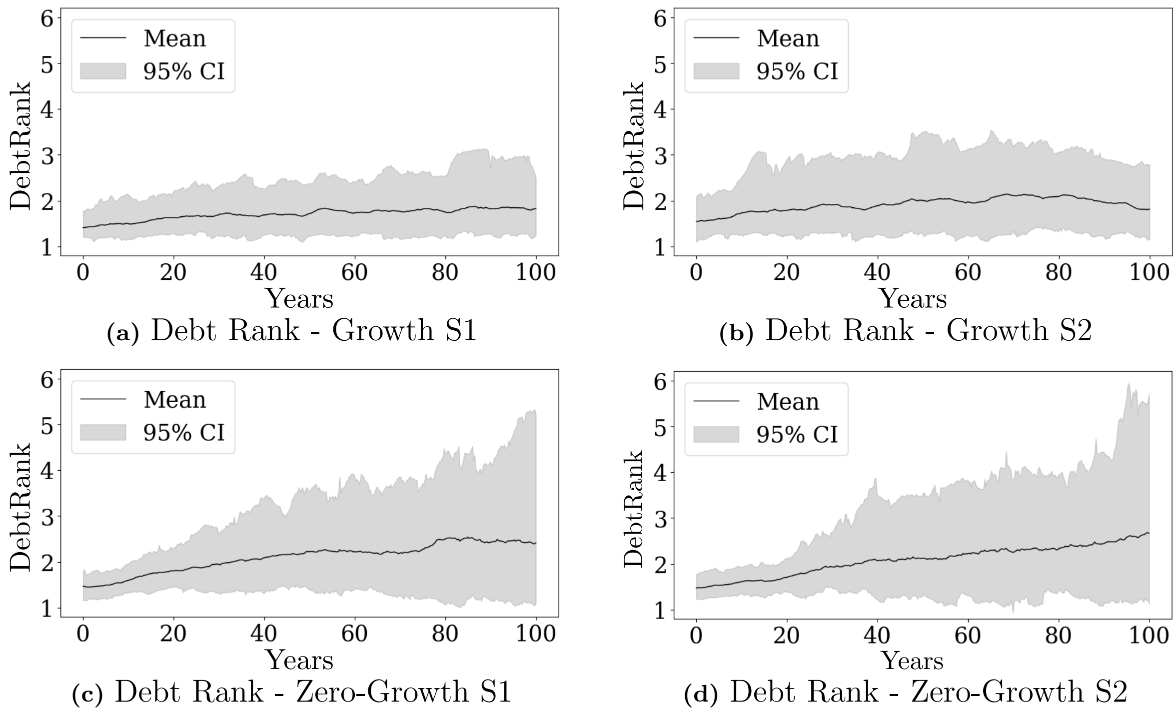
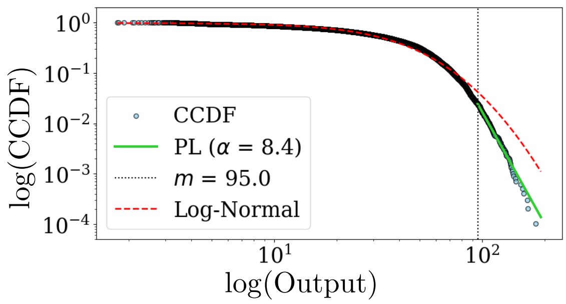


Figure 10: DebtRank for each scenario over time. The black line is the mean of all simulations for each scenario and the shaded area is the 95% confidence interval.

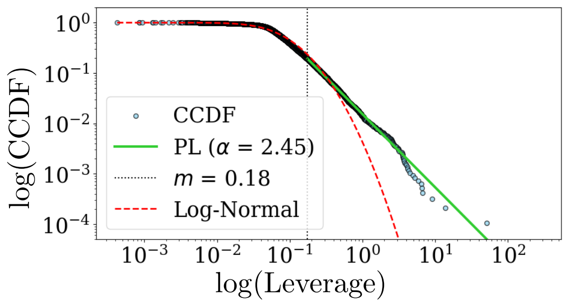
which infers that higher desired debt ratios lead to more financial risk in the credit network. Additionally, the DebtRank of both growth scenarios is much lower than the zero-growth scenarios, hence, a zero-growth regime increases the amount of financial risk in the credit network.

Fig. 10 shows the mean DebtRank over time for each scenario and the 95% confidence interval. The results from 10 confirm those found in Table 5, where the zero-growth scenarios have a higher DebtRank than the growth scenarios. Furthermore, Fig. 10 gives insight into the time evolution of financial risk for each scenario. Both the growth and zero-growth scenarios initially start with similar levels of DebtRank. However, over time the the average DebtRank in the zero-growth scenarios significantly increases when compared to the growth scenarios. Hence, the financial risk present in the network becomes greater throughout the simulation in the zero-growth scenarios. Whereas, the financial risk in the growth scenarios is relatively constant throughout the simulation. Furthermore, the 95% confidence interval of the zero-growth scenarios is much wider towards the later time periods, suggesting greater divergence in the risk profile of the financial network between simulations within each scenario. Whereas, the 95% confidence interval of the growth scenarios is relatively more constant over time, which suggests that the financial risk of the network more similar between simulations.

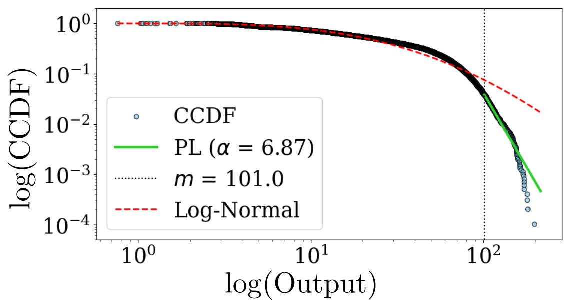
Fig. 11 shows the complementary cumulative distribution functions (CCDFs) for C-firm size (output) and leverage in each scenario. The CCDF for C-firm size is similar between the Growth S1 and Growth S2 scenarios and between the Zero-Growth S1 and Zero-Growth S2 scenarios. Therefore, higher desired debt ratios do not significantly impact the distribution of C-firm size. However, when comparing the Growth and Zero-Growth scenarios, it is clear that the tails of the distribution for C-firm size are larger in the zero-growth scenarios. This is in line with the evidence from C-firm HHI* in Table 5 because a larger tail in the distribution



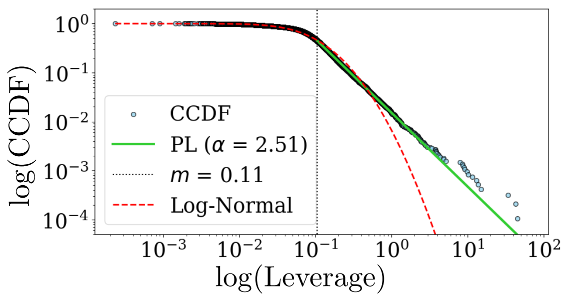
(a) C-firm Output - Growth S1



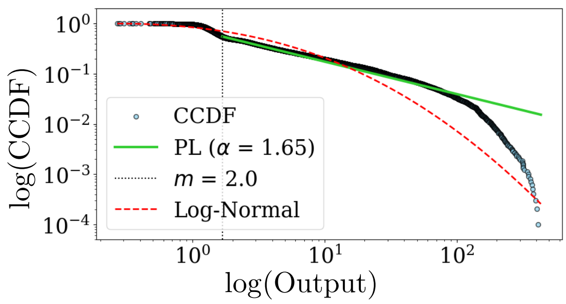
(b) C-firm Leverage - Growth S1



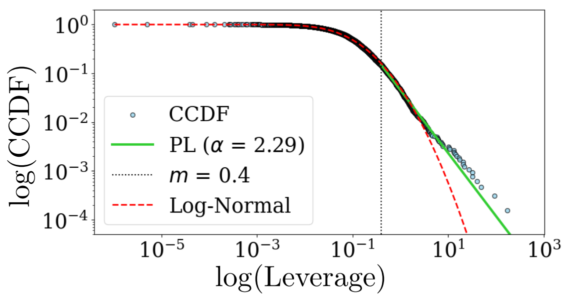
(c) C-firm Output - Growth S2



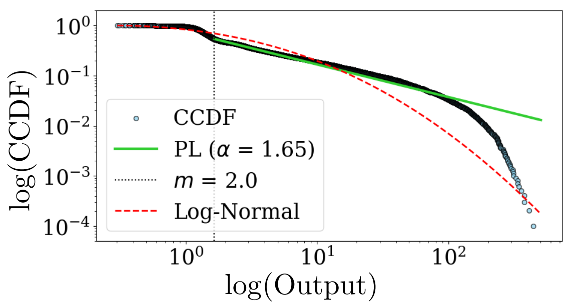
(d) C-firm Leverage - Growth S2



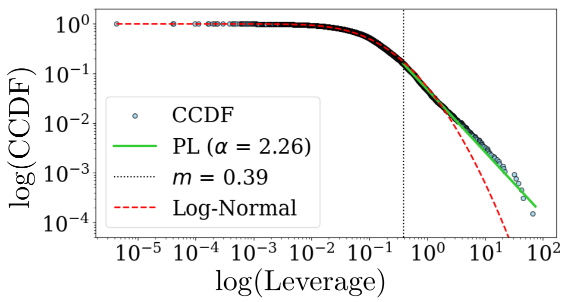
(e) C-firm Output - Zero-Growth S1



(f) C-firm Leverage - Zero-Growth S1



(g) C-firm Output - Zero-Growth S2



(h) C-firm Leverage - Zero-Growth S2

Figure 11: Complementary cumulative distribution functions (CCDF) for C-firm output and leverage, shown in log-log scale over all Monte Carlo simulations, taken at a midpoint snapshot ($s \in \{1, 2, ...50\}$ and $t = 300$). The red dashed line shows the log-normal fit, the thick green line shows the power-law fit, and the black dotted line shows the minimum power-law cut-off.

of C-firm size indicates a greater probability of large C-firms and a more concentrated market. Additionally, the power-law minimum is significantly lower in the Zero-Growth scenarios, which fits a power-law distribution well until the end of the distribution for C-firm size, where there is some exponential cut-off. C-firm leverage, measured as debt divided by equity ($\lambda_{i,t} = D_{i,t}/E_{i,t}$, different to the expected leverage, $\lambda^e_{i,t+1}$, defined in eq. 58), also displays power-law behaviour. Particularly in the Growth scenarios, which have a lower power-law cut-off than the Zero-

Growth scenarios. The Zero-Growth S1 scenario has the most leveraged C-firms. Therefore, zero-growth could increase how leveraged C-firms become. There is little difference between the low and high debt scenarios, which suggests that a higher desired debt ratio does not actually increase leverage in the C-firm market.

4 Discussion

The aim of this paper was to extend the analysis in Barrett (2018) to the microeconomic level, incorporating Minskyan debt dynamics into a macroeconomic ABM. The model was then run for four scenarios; two growth scenarios with low and high debt dynamics and two zero-growth scenarios with low and high debt dynamics. These results were then compared across scenarios to determine the stability of zero-growth and whether it offers a path viable for the economy, whether from a more deliberate policy-focused approach or due to a natural slowdown in economic growth. The scenario analysis found that zero-growth scenarios were no less stable than growth scenarios. Where, at the aggregate level, there were fewer economic crises present in the zero-growth scenario. However, there was more financial risk in the credit network.

The macroeconomic results from the model are similar to those found in Barrett (2018), such as the lower debt share of output, higher wage share, and lower profit share in the zero-growth scenarios than in the growth scenarios. Hence, similarly to Barrett (2018), this contradicts Piketty (2014) analysis that there is a trade-off between more sustainable low-growth rates and inequality between workers and capitalists. Furthermore, the unemployment rate was also lower in the zero-growth scenarios, and it was found that within household inequality measured by the Gini Index, was relatively similar across all scenarios. Another similar finding to Barrett (2018) was that increasing the firm's desired debt ratio for investment increased the instability of the model. However, rather than identifying model instability as a complete breakdown of the model's dynamics, we define an economic crisis as a period in which real GDP growth is less that -3%. With this, it was found that zero-growth scenarios had fewer economic crises than the growth scenarios.

A key departure from Barrett (2018) was the inclusion of endogenously determined prices set by firms. At the macro level, this led to a higher inflation rate in the zero-growth scenarios than in the growth scenarios. To explain such an emergent property, we must deviate to where the debate surrounding whether interest-bearing debt creates a growth imperative in a capitalist system has its roots, in the work of Frederick Soddy's Wealth, Virtual Wealth and Debt (1926). Soddy argues, that the growth imperative may be twofold, 1) there is an imperative to grow the physical goods within an economy to pay off the interest on debt, or 2) there is an imperative for the price of goods in an economy to grow to pay off the interest on debt. Therefore, it is expected that the zero-growth scenarios will have a higher inflation rate because, without the ability to produce more, firms will have to increase their prices instead to meet the interest requirements set by the banks in the model. This is evident in the results, where the zero-growth scenarios have a significantly higher inflation rate than the growth scenarios. Furthermore, there is another explanation for the higher inflation rate in the zero-growth scenarios, given the model produces a relatively constant wage share, which is evidenced in Fig. 8, this implies that the growth rate of the wage share should be approximately zero. Thus, given the wage share,

$$\omega = \frac{W}{PY}, \tag{76}$$

where ω is the wage share, W is the macro wage bill, P is the price level index, and Y is real GDP. Then, taking logs and differentiating to get the growth rates for each variable and setting to zero (constant wage share), yields:

$$\frac{1}{\omega}\frac{d\omega}{dt} = \frac{1}{W}\frac{dW}{dt} - \frac{1}{P}\frac{dP}{dt} - \frac{1}{Y}\frac{dY}{dt} = 0. \tag{77}$$

Then, solving for the growth rate of the price level index (inflation) gives:

$$g_P = g_W - g_Y, \tag{78}$$

where $g_P = \frac{1}{P}\frac{dP}{dt}$, $g_W = \frac{1}{W}\frac{dW}{dt}$, and $g_Y = \frac{1}{Y}\frac{dY}{dt}$. Therefore, given similar nominal wage growth rates between the growth and zero-growth scenarios, 0.039 and 0.033 respectively for the baseline scenario in Table 4. The inflation rate is therefore expected to be higher in the zero-growth scenario than in the growth scenario because the growth rate of real GDP is lower. Furthermore, by checking Table 4, the relationship in eq. 78 holds in each scenario.

Moreover, the effects of zero-growth were also analysed at the microeconomic level, for which Barrett (2018) analysis was unable to explain. Zero-growth scenarios increased market concentration for all agents compared to growth scenarios. There was less market instability for K-firms and banks in the zero-growth scenario. Bankruptcy rates were higher for C-firms in both growth and zero-growth S2 scenarios than in the S1 scenarios, thus, higher desired debt ratios lead to more bankruptcies. However, bank bankruptcies were lower in all S2 scenarios compared to S1 scenarios, suggesting that higher C-firm desired debt ratios are better for banks. Additionally, financial risk is higher in the credit network for both the zero-growth scenarios when compared to the growth scenario, when measured using DebtRank. However, at the aggregate level, there are fewer economic crises, hence, the zero-growth scenarios are more robust to financial risk.

The model presented in this paper demonstrates the necessity of analysing the effects of zero-growth at both the macro and microeconomic levels. The current literature on post-growth and degrowth economics mainly focuses on the macroeconomic analysis of zero-growth, which may overlook effects at the microeconomic level.

5 Concluding Remarks

This model is unique insofar as its comparison of a growth and zero-growth scenario done for a first time on a multi-agent macroeconomic ABM. This paper has shown that a stable zero productivity growth scenario, with interest bearing debt, was possible on the model. Moreover, it has been shown that the model, given a 2% labour productivity growth rate, captures the cyclical dynamics present in real world data. Furthermore, the results found that real GDP growth rates were more stable in the zero-growth scenario, there were less economic crises, unemployment was lower, there was a higher wages share which is better for workers, and K-firm and bank market shares were relatively more stable than in the growth scenarios. However, some of the consequences of zero-growth include a higher rate of inflation, for which we supported with theoretical reasoning, an increase in market concentration for both firms and banks, and a greater level of financial risk in the bank-firm credit network as measured by the total DebtRank. This paper demonstrates that the end of growth is not instability inducing on the economy, and could provide a new path for humanity to abate the strain of anthropogenic perturbations on the earths biophysical limits.

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A Parameters

Symbol	Description	Value
$\mathbf{N}_H$	Number of households	2000
$\mathbf{N}_C$	Number of C-firms	200
$\mathbf{N}_K$	Number of K-firms	50
$\mathbf{N}_B$	Number of banks	10
n_C	Number of C-firms visited by households	2
n_K	Number of K-firms visited by C-firms	2
n_F	Number of job applications sent to firms by households	4
n_B	Number of banks visited by firms	2
g	Average growth rate of labour productivity	0.02 or 0
σ_α	Standard deviation of productivity growth	0.03
σ_P	Standard deviation of prices	0.03
σ_w	Standard deviation of wages	0.03
σ_r	Standard deviation of interest rates	0.03
γ_Z	Demand speed of adjustment	0.1
γ_P	Demand speed of adjustment	0.1
γ_w	Demand speed of adjustment	0.1
γ_r	Demand speed of adjustment	0.1
c	Household marginal propensity to consume wealth	0.1
ν	C-firm capital accelerator	3
d_0	C-firm desired debt ratio intercept parameter	0.5
d_1	C-firm desired debt ratio productivity growth parameter	5 or 3
d_2	C-firm desired debt ratio profit share parameter	3 or 2
δ	Depreciation rate of capital	0.07
ξ	K-firm desired excess capacity	0.1
n	Bank loan repayment periods	40
κ_1	Bank desired capital ratio intercept	0.06
κ_2	Bank desired capital ratio slope	1
r^M	Bank interest rate on deposits	0.001
r^N	Bank natural interest rate on loans	0.02

Table 6: Model parameters.

Parameters were chosen based on estimated values from real data, such as the average growth rate of labour productivity[15], capital acceleration, and depreciation of capital[16]. Other parameter values were chosen such that the model did not display degenerate dynamics. In particular, hyperinflation, collapse in GDP towards zero, sustained unemployment rate over 50%, and sustained bankruptcy rates over 50%. Furthermore, because a quarterly time step that

[15]Values were taken from the OECD, as in Barrett (2018), see data from: `https://data.oecd.org/lprdty/labour-productivity-and-utilisation.htm#indicator-chart`. For which 2% was a typical value for our g parameter, during the economically stable period from 1981-2006

[16]Values for capital acceleration and depreciation were taken from Jackson and Victor (2015).

was chosen, values that represent real world estimates of annual growth rates were multiplied by $\Delta t = 1/4$ each period.

B Initial Values

Initial values for each agent in the simulation.

C-firm initial values:

It is assumed that each C-firm in the simulation is initialised with the same initial values for each variable. Labour productivity and prices are assumed to be normalised to 1 at the start of the simulation. Furthermore, all households are assumed to be employed at the start of the simulation, where each household is randomly allocated to a C-firm or K-firm. Hence, given the assumed starting values for labour productivity, prices, and labour, all other initial values can be derived. To get the starting values for the debt ratio and profit share, we assume that C-firms start in equilibrium. Moreover, all desired variables are initialised with their real counterparts, e.g. desired labour will be initialised to the C-firms actual labour.

$$
\begin{aligned}
a_{i,0} &= 1, \\
P_{i,0} &= 1, \\
N_{i,0} &= \frac{\mathbf{N}_H}{\mathbf{N}_C + \mathbf{N}_K}, \\
Y_{i,0} &= N_{i,0}, \\
Z_{i,0} &= Y_{i,0}, \\
K_{i,0} &= \nu Y_{i,0}, \\
d_{i,0} &= \frac{d_0 + g d_1 + \nu(g+\delta) d_2}{1 + g d_2}, \\
\pi_{i,0} &= \nu(g+\delta) - g d_{i,0}, \\
\omega_{i,0} &= 1 - \pi - r^L_{b,0} d_{i,0}, \\
D_{i,0} &= d_{i,0} Y_{i,0}, \\
\Pi_{i,0} &= \pi_{i,0} Y_{i,0}, \\
w_{i,0} &= \omega_{i,0}, \\
W_{i,0} &= w_{i,0} N_{i,0}, \\
M_{i,0} &= \Pi_{i,0} + D_{i,0}, \\
E_{i,0} &= K_{i,0} + M_{i,0} - D_{i,0},
\end{aligned}
\tag{79}
$$

for all $i \in \{1, \ldots, \mathbf{N}_C\}$.

K-firm initial values:

Similarly to C-firms, it is assumed that each K-firm in the simulation is initialised with the same initial values for each variable. Labour productivity and prices are assumed to be normalised to 1 at the start of the simulation. Households are again randomly allocated to K-firms. Hence, given the assumed starting values for labour productivity, prices, and labour, all other initial values for K-firms can be derived. Again, all desired variables are initialised with their real counterparts, e.g. desired labour will be initialised to the K-firms actual labour.

$$
\begin{aligned}
a_{j,0} &= 1, \\
P_{j,0} &= 1, \\
N_{j,0} &= \frac{\mathbf{N}_H}{\mathbf{N}_C + \mathbf{N}_K}, \\
Y_{j,0} &= N_{j,0}, \\
Z_{j,0} &= Y_{j,0}, \\
K_{j,0} &= \nu Y_{j,0}, \\
d_{j,0} &= 0, \\
w_{j,0} &= w_{i,0}, \\
W_{j,0} &= w_{j,0} N_{j,0}, \\
\Pi_{j,0} &= Y_{j,0} - W_{j,0}, \\
M_{j,0} &= \Pi_{j,0}, \\
E_{j,0} &= M_{j,0},
\end{aligned}
\tag{80}
$$

for all $j \in \{1, \ldots, \mathbf{N}_K\}$.

Household initial values:

Households are randomly allocated to C-firms or K-firms as employees, hence, the initial income of households is equal to the firms wage rate (uniform across all firms in the initial period). The initial deposits of households is then easily derived as being equal to their initial income.

$$
\begin{aligned}
Y_{h,0} &= w_{\iota,0}, \\
M_{h,0} &= Y_{h,0},
\end{aligned}
\tag{81}
$$

for all $h \in \{1, \ldots, \mathbf{N}_H\}$.

Bank initial values:

Is is assumed that the annual loan interest rate of banks has an initial annual value of 2% (equal to 0.5% for a quarterly time step). C-firms and K-firms are then randomly allocated to banks. The bank calculates their initial loans as the sum of C-firm debt for each C-firm that was allocated to the bank (K-firms can be ignored as they are initialised with zero debt). Banks then derive their initial equity as their desired capital ratio multiplied by their amount of loans. The desired capital ratio reduces to simply the intercept value, κ_1 because the expected bad loans ratio will be equal to zero as no firms have become bankrupt yet for the bank to estimate their amount of bad loans.

$$
\begin{aligned}
r^L_{b,0} &= r^N, \\
L_{b,0} &= \sum_{i \in \Omega_{F_L}} D_{i,0}, \\
E_{b,0} &= \kappa_1 L_{b,0},
\end{aligned}
\tag{82}
$$

for all $b \in \{1, \ldots, \mathbf{N}_B\}$.

C Statistical Tests

Variable	KS Test		SW Test		AD Test	
	stat.	p-value	stat.	p-value	stat.	p-value
Real GDP Growth	0.478	0.000	0.924	0.000	333.01	0.000
C-firm Output Growth	0.405	0.000	0.637	0.000	1057.67	0.000
C-firm Output	0.996	0.000	0.949	0.000	98.23	0.000
C-firm Debt	0.999	0.000	0.925	0.000	123.61	0.000
C-firm Leverage	0.500	0.000	0.012	0.000	3323.73	0.000
K-firm Output	0.996	0.000	0.499	0.000	411.60	0.000
K-firm Debt	0.977	0.000	0.283	0.000	152.99	0.000
K-firm Leverage	0.500	0.000	0.037	0.000	126.34	0.000
Bank Loans	0.988	0.000	0.638	0.000	49.62	0.000
Bank Interest Rate	0.505	0.000	0.867	0.000	9.25	0.000
Bank Leverage	0.659	0.000	0.149	0.000	117.79	0.000

Table 7: Statistical tests of normality for baseline scenario (Growth S1). Column 2 shows the test statistic (stat.) and p-value for the Kolmogorov–Smirnov (KS) test, column 3 the Shapiro-Wilk (SW) test, and column 4 the Anderson-Darling (AD) test.

D DebtRank

To construct a measure of DebtRank on a bipartite graph, we follow the methodology in Aoyama et al. (2013). An edge connecting between bank b and firm ι ($\iota = i$ for C-firms and $\iota = j$ for K-firms) in period t is associated with the total lending (credit) of bank b to firm ι, defined as:

$$C_{b\iota} = \sum_{\tau=0}^{t} L_{\iota\to b,\tau}. \tag{83}$$

Let the matrix $\mathbf{C}$ represent the credit network between banks and firms in time t, where the elements are given by $(\mathbf{C})_{b\iota} = C_{b\iota}$, hence for $\mathbf{N}_B$ banks and $\mathbf{N}_F$ firms in the credit market, the matrix $\mathbf{C}$ is of size $(\mathbf{N}_B \times \mathbf{N}_F)$. Similarly to Aoyama et al. (2013), we define two network propagation matrices, the bank propagation matrix $\mathbf{W}_B$ and the firm propagation matrix $\mathbf{W}_F$, with elements:

$$(\mathbf{W}_B)_{b\iota} = \frac{C_{b\iota}}{C_b}, \tag{84}$$

$$(\mathbf{W}_F)_{\iota b} = \frac{C_{b\iota}}{C_\iota}. \tag{85}$$

C_b is bank b's loans (total loans to firms), defined as the row sum of matrix $\mathbf{C}$:

$$C_b = \sum_{\iota=1}^{\mathbf{N}_F} C_{b\iota}. \tag{86}$$

C_ι is firm ι's debt (total loans from banks), defined as the column sum of matrix $\mathbf{C}$:

$$C_\iota = \sum_{b=1}^{\mathbf{N}_B} C_{b\iota}. \tag{87}$$

Hence, the bank propagation matrix $\mathbf{W}_B$ is of size $(\mathbf{N}_B \times \mathbf{N}_F)$ and is row stochastic such that $\sum_{\iota=1}^{\mathbf{N}_F} (\mathbf{W}_B)_{b\iota} = 1$. Similarly, the firm propagation matrix $\mathbf{W}_F$ is of size $(\mathbf{N}_F \times \mathbf{N}_B)$ and is also row stochastic, such that $\sum_{b=1}^{\mathbf{N}_B} (\mathbf{W}_B)_{\iota b} = 1$.

The DebtRank is calculated at each period t of the simulation, where it is initialised at time period $\tau = t$ and runs until the algorithm stops at some time period $\tau = t + T$. To calculate the DebtRank of the network for time period t of the simulation, each node is associated with two variables, the distress of the node $h_k \in [0,1]$ ($k = \iota$ for firms and $k = b$ for banks) and the state of the node $s_k \in \{U, D, I\}$, with three discrete states, undistressed (U), distressed (D), and inactive (I). At the initial time period $\tau = t$, the set of distressed nodes is given by Ω_D. Hence, the initial conditions are given by: $h_k(t) = \psi \; \forall k \in \Omega_D$, $h_k(t) = 0 \; \forall k \notin \Omega_D$, $s_k(t) = D \; \forall k \in \Omega_D$, and $s_k(t) = U \; \forall k \notin \Omega_D$. For simplicity, we set $\psi = 1$, such that the initial set of distressed nodes are maximally distressed. The distress and state variables are then updates as:

$$\begin{aligned} h_k(\tau) &= \min\left\{1, h_k(\tau-1) + \sum_{\ell | s_\ell(\tau-1) = D} W_{\ell k} h_\ell(\tau-1)\right\}, \\ s_k(\tau) &= \begin{cases} D & \text{if } h_k(\tau) > 0 \;\&\; s_k(\tau-1) \neq I \\ I & \text{if } s_k(\tau-1) = D \\ s_k(\tau-1) & \text{else.} \end{cases} \end{aligned} \tag{88}$$

Where all h_k variables are updated in parallel, followed by all s_k variables in parallel. Furthermore, after a finite number of steps T, the dynamics stop as each nodes state is either $s_k(t+T) = U$ or $s_k(t+T) = I$. As in Aoyama et al. (2013), these equations are applied to both the bank and firm propagation matrix, where the bank DebtRank is calculated as a weighted sum of the distress, with weights equal to the banks asset size:

$$DR_t^B = \sum_{b=1, \notin \Omega_D}^{\mathbf{N}_B} h_b(t+T) \frac{L_{b,t} + R_{b,t}}{\sum_{b'=1, \notin \Omega_D}^{\mathbf{N}_B} L_{b',t} + R_{b',t}}, \tag{89}$$

where bank b's assets are given by $L_{b,t} + R_{b,t}$ from eq. 48, where $L_{b,t}$ are bank b's loans and $R_{b,t}$ are bank b's reserves in time t.

Similarly, the firm DebtRank is calculated as a weighted sum of the distress, with weights equal to the firms asset size, which is broken into the sum of C-firms DebtRank and K-firms DebtRank:

$$DR_t^F = \sum_{i=1, \notin \Omega_D}^{\mathbf{N}_C} h_i(t+T) \frac{M_{i,t} + KE_{i,t}}{\sum_{i'=1, \notin \Omega_D}^{\mathbf{N}_C} M_{i',t} + KE_{i',t}} + \sum_{j=1, \notin \Omega_D}^{\mathbf{N}_K} h_j(t+T) \frac{M_{j,t}}{\sum_{j'=1, \notin \Omega_D}^{\mathbf{N}_K} M_{j',t}}, \tag{90}$$

where C-firm i's assets are given by $M_{i,t} + KE_{i,t}$ from eq. 29, where $M_{i,t}$ are C-firm i's deposits and $KE_{i,t}$ is C-firm i's capital expenditure in time t, and K-firm j's assets are only given by their deposits, $M_{j,t}$ from eq. 45.

We are then interested in the total DebtRank, the sum of the bank DebtRank and firm DebtRank, produced by the credit network:

$$DR_t = DR_t^B + DR_t^F, \tag{91}$$

which gives us a numerical value for the systemic financial risk of the credit network. The higher (lower) the value of DR_t the more (less) financial risk is present in the credit network.