

# The DYNamic Agent-based MINskyan (DYNAMIN) Model Description

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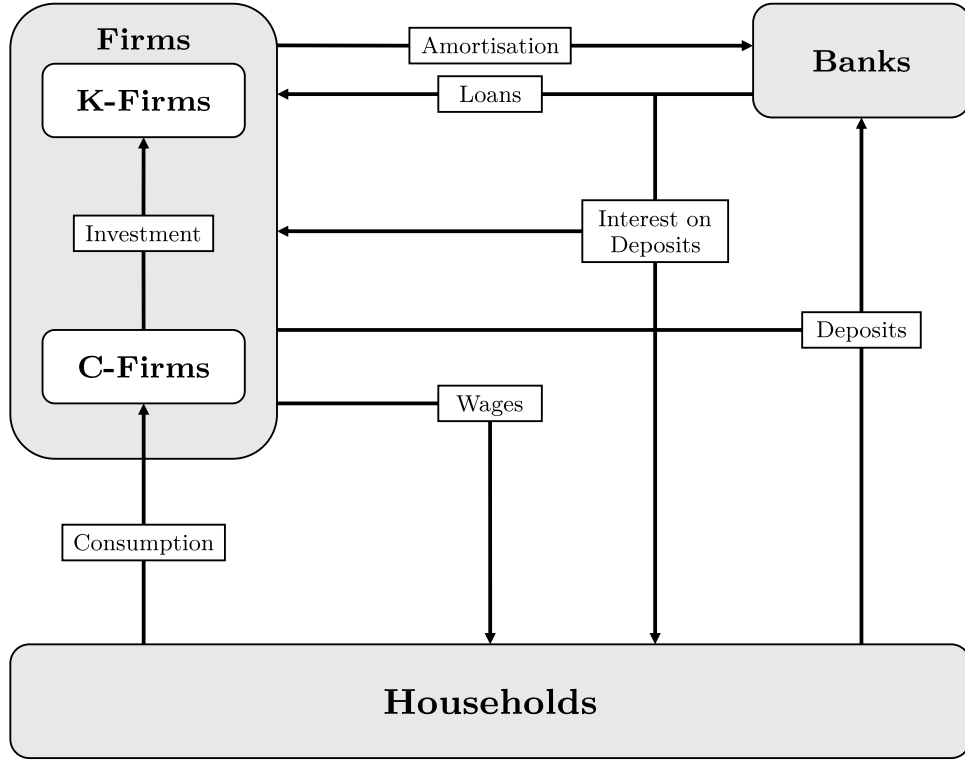
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## 1 Model Overview

The DYNamic Agent-based MINskyan (DYNAMIN) model contains four sectors: (i) a household sector comprising  $\mathbf{N}_H$  households; (ii) a consumption firm sector with  $\mathbf{N}_C$  consumption firms (C-firms) selling a homogenous consumption good (C-good); (iii) a capital firm sector with  $\mathbf{N}_K$  capital firms (K-firms) selling a homogenous capital good (K-good); and (iv) a banking sector with  $\mathbf{N}_B$  banks. As we are interested in the instability of the capitalist system, we chose to omit the government sector, which should act as a damping force against the cyclical dynamics of a purely capitalist economy, given the right choice of fiscal policy. With these sectors in place, we endeavoured to keep the model as simple as possible for a focused analysis of financial stability under growth and zero-growth scenarios. The model is thus less detailed than some of the larger models in the literature (Dosi et al., 2017; Lamperti et al., 2018; Reissl et al., 2025; Botte et al., 2021), yet it is capable of producing a rich set of stylised facts from real-world economies. The focus is fully on economic dynamics, and thus incorporating the environment into the model is left to future research. The model is stock-flow consistent (SFC) (Godley and Lavoie, 2007): each financial asset is related to an equivalent financial liability; and each payment flow goes explicitly from one agent in the model to another. Here we give an overview of the most important features and assumptions of the model.

Figure 1 gives a visual representation of the model and how the agents interact with one another through financial flows. In summary, households work at either C-firms or K-firms and are paid a wage for their work each period; they put any savings at banks as deposits, and receive interest from the bank. Households also consume consumption goods from C-firms using their wage from employment and/or their deposits at the bank. C-firms invest in new capital from K-firms using their internal finance and loans from banks. Additionally, both C-firms and K-firms also seek external finance from banks if their wage bill exceeds internal finance



**Figure 1:** Model ontology, showing the flow of payments between each category of agent in the model. Arrows point from paying agents to receiving agents.

capabilities. Firms use an amortisation schedule to pay down both the interest and principal of the loan. Furthermore, banks pay interest to firms and households for any deposits they hold at banks. Additionally, each period is assumed to correspond to a quarter of a year, where the sequence of events that occur during each time period is given below.

### 1.1 Sequence of events

1. At the beginning of each time step  $t$ , new entrants enter their respective markets to replace firms that went bankrupt in the previous period.
2. A decentralised market for labour opens where C-firms and K-firms post vacancies and update their wage rate to attract new employees. Households then send out applications to firms that have open vacancies, and firms hire employees from their application pool.
3. C-firms and K-firms engage in production, pay their employees for their work, and update the price of their goods.
4. A decentralised market for consumption goods opens, and households visit and buy goods from C-firms for consumption.
5. A decentralised market for capital goods opens, and C-firms visit and buy goods from K-firms to use as investment in their capital stock for production in the next period.

6. A decentralised credit market opens, and C-firms and K-firms demand loans from banks, which are supplied depending on the bank's risk tolerance.
7. C-firms and K-firms update their accounts and exit the market if they have run out of deposits to cover their payments. Households employed by bankrupt firms become unemployed, and banks absorb any outstanding loans to bankrupt firms with their equity.
8. Banks become bankrupt if they have negative equity, and are bailed out by their depositors, both firms and households.

## 1.2 Stock-flow consistency (SFC)

It is demonstrated here that the model is SFC at the aggregate level, ensuring that all financial transactions are accounted for, such that each payment from one agent is directed to another agent in the model. Therefore, every financial stock is recorded as a liability for one agent and an asset for another agent.

	Households	C-Firms	K-Firms	Banks	Central Bank	$\Sigma$
Capital		$K$				$K$
Deposits	$M_H$	$M_C$	$M_K$	$-M$		0
Debt		$-D_C$	$-D_K$	$D$		0
Reserves				$R$	$-R$	0
Advances				$-A$	$A$	0
Equity	$-E_H$	$-E_C$	$-E_K$	$-E_B$	$-E_{CB}$	$-K$
$\Sigma$	0	0	0	0	0	0

**Table 1:** Macro financial balance sheet matrix.

Table 1 provides a financial balance sheet representation of the model for each agent class. Where values in the table represent the aggregate stock variables for each type of agent<sup>1</sup>. It is evident from Table 1 that all the columns and rows dealing with financial assets must sum to zero, where tangible capital ( $K$ ) is the only asset that appears once on the balance sheet and thus does not sum to zero. Hence, the sum of all agents' equities must be equal to tangible capital, in line with the reasoning set out by Godley and Lavoie (2007).

Table 2 presents a transaction flow matrix showing the aggregate flows of financial transactions among the agents in the model. Each column and row of the transaction flow matrix must sum to zero for the model to be SFC. The upper part of the transaction flow matrix in Table 2 reproduces the national income statistics presented in Table 1. The lower part of Table 2 represents the inter-sectoral flow of funds in the model. The variables in the transaction flow matrix are defined as follows:  $W$  is the wage bill,  $C$  is household consumption,  $I$  is C-firm investment,  $\rho D$  is debt repayment,  $IP$  is loan interest payments,  $i^M M$  is deposit interest payments,  $\Pi$  is profits,  $\Delta M$  is the change in deposits,  $L$  is new loans,  $\Delta R$  is the change in reserves,  $\Delta A$  is the change in central bank (CB) advances, and  $B$  is firm bad debt (total non-performing loans).

<sup>1</sup>Each variable with a subscript refers to that agent's share of the total, e.g.  $M_H$  are household deposits,  $M_C$  are C-firm deposits, and  $M_K$  are K-firm deposits. The same variable without a subscript refers to the total of that variable, e.g. total deposits are the sum of all agents' deposits:  $M = M_H + M_C + M_K$ .

	Households	C-Firms		K-Firms		Banks		CB	$\Sigma$
		Current	Capital	Current	Capital	Current	Capital		
Wages	$W$	$-W_C$		$-W_K$					0
Consumption	$-C$	$C$							0
Investment			$-I$	$I$					0
Loan repayments			$-\rho D_C$		$-\rho D_K$		$\rho D$		0
Loan Interest		$-IP_C$		$-IP_K$		$IP$			0
Deposit Interest	$i^M M_H$	$i^M M_C$		$i^M M_K$		$-i^M M$			0
Profits		$-\Pi_C$	$\Pi_C$	$-\Pi_K$	$\Pi_K$	$-\Pi_B$	$\Pi_B$		0
Inventories				$\Delta V$	$-\Delta V$				0
Change in Deposits	$-\Delta M_H$		$-\Delta M_C$		$-\Delta M_K$		$\Delta M$		0
Change in Debt			$L_C$		$L_K$		$-L$		0
Change in Reserves							$-\Delta R$	$\Delta R$	0
Change in Advances							$\Delta A$	$-\Delta A$	0
Loan Defaults			$B_C$		$B_K$		$-B$		0
$\Sigma$	0	0	0	0	0	0	0	0	0

**Table 2:** Macro transaction flow matrix.

## 2 Model Description

In the following appendix, we present a full model description including the equations and algorithms used to simulate the model. If the reader is interested in the code used to simulate the model, see <https://github.com/DylanTerryDoyle/Py-DYNAMIN>.

### 2.1 Households

#### 2.1.1 Income & Expenditure

The  $h$ th household receives both a wage from their employer, firm  $\iota$ , and interest from their bank on deposits as income:

$$Y_h(t) = \mathbb{1}_h(t)w_\iota(t) + Div_h(t) + r^M M_h(t), \quad (1)$$

where  $w_\iota(t)$  denotes the wage rate of firm  $\iota$  defined in their labour contract,  $r^M$  is bank  $b$ 's interest rate on deposits (uniform across all banks),  $M_h(t)$  are household  $h$ 's deposits at bank  $b$ ,  $Div_h(t) = Div(t)(M_h(t)/M(t))$  are dividends distributed to households (total dividends,  $Div(t)$  are distributed proportional to household  $h$ 's share of total deposits  $M(t)$ ), and  $\mathbb{1}_h(t)$  is a dummy variable determining the employment status of household  $h$ :

$$\mathbb{1}_h(t) = \begin{cases} 1 & \text{if household } h \text{ is employed} \\ 0 & \text{if household } h \text{ is unemployed.} \end{cases} \quad (2)$$

Household  $h$ 's desired consumption expenditure,  $E_h^d(t)$ , is assumed, for simplicity, to be given by a simple Keynesian budget constraint in which household  $h$  desires to spend all of their income and a linear proportion of their deposits:

$$E_h^d(t) = c_Y Y_h(t) + c_M M_h(t), \quad (3)$$

where  $c_Y \in (0, 1)$  is the marginal propensity to consume out of income and  $c_M \in (0, 1)$  is the marginal propensity to consume out of deposits.

### 2.1.2 Accounting

Household  $h$  keeps any involuntary savings at bank  $b$  where household  $h$  earns interest on their deposits. Hence, household  $h$  updates its deposits by:

$$M_h(t+1) = M_h(t) + Y_h(t) - E_h(t), \quad (4)$$

where  $E_h(t)$  is household  $h$ 's actual expenditure on C-goods, defined in the market for consumption goods, see Section 2.7.

## 2.2 Consumption Firms

### 2.2.1 Production

The output of the  $i$ th C-firm,  $Y_i(t)$ , is assumed to be given by a Leontief production function with constant returns to scale for both labour,  $N_i(t)$ , and capital,  $K_i(t)$ :

$$Y_i(t) = \min \left\{ a_i(t)N_i(t), \frac{K_i(t)}{\nu} \right\}, \quad (5)$$

where  $a_i(t)$  is firm  $i$ 's labour productivity and  $\nu^{-1}$  is a constant capital-output ratio. Labour productivity, which is the essential variable of growth in the model, determines the efficiency of firm  $i$ 's production process. It is assumed that labour productivity evolves according to the stochastic differential equation (SDE) for geometric Brownian motion (GBM):

$$da_i(t) = a_i(t)(gdt + \sigma_a dW_i(t)), \quad (6)$$

where  $g$  is the constant average growth rate of labour productivity,  $\sigma_a$  is the standard deviation of labour productivity, and  $W_i(t)$  is a Brownian motion. This equation for labour productivity was chosen because it captures exponential growth with random fluctuations from the deterministic growth path set by the  $g$  parameter. This will allow the growth rate of the model to be set exogenously. Furthermore, the expected value of the SDE in Eq. 6 is given by  $\mathbb{E}[a_i(t)] = a_i(0) \exp\{gt\}$ , which corresponds to the deterministic model used in Barrett (2018). The SDE for labour productivity can be simulated on a discrete time grid using the exact solution for GBM:

$$a_i(t) = a_i(t-1) \exp \left\{ g - \frac{1}{2}\sigma_a^2 + \sigma_a \varepsilon_i(t) \right\}, \quad (7)$$

where  $\varepsilon_i(t) \sim \mathcal{N}(0, 1)$  is a random variable drawn from a standard normal distribution. Hence, when  $g > 0$ , labour productivity grows exponentially. Furthermore, due to the functional form in Eq. (7), for a given time period  $t$ , firm labour productivity will be distributed according to a log-normal distribution, such that  $\ln(a_i(t)) \sim \mathcal{N}(\ln(a_i(0)) + (g - \sigma_a^2/2)t, \sigma_a^2 t)$ .

### 2.2.2 Desired Output & Inventories

Firm  $i$ 's desired output,  $Y_i^d(t+1)$ , is assumed to be equal to their expected demand,  $Z_i^e(t+1)$ , in the next period:

$$Y_i^d(t+1) = Z_i^e(t+1), \quad (8)$$

where expected demand is adaptively updated according to firm  $i$ 's actual demand:

$$Z_i^e(t+1) = Z_i^e(t) + \gamma_Z(Z_i(t) - Z_i^e(t)), \quad (9)$$

where  $\gamma_Z \in (0, 1)$  determines the speed of adjustment to actual demand, and  $Z_i(t)$  is firm  $i$ 's actual demand, which is determined on the market for consumption goods, see Appendix 2.7. From Eq. (9), it is evident after repeated substitution that the expected demand is equal to the sum of all past demands with geometrically decaying weights.

It is assumed that C-goods are perishable and do not last longer than one period. Therefore, at the beginning of period  $t$ , C-firms set their inventories equal to output  $V_i(t) = Y_i(t)$ . Then, households consume from firm  $i$ 's inventories until they are depleted or firm  $i$  is left with involuntary inventories. Additionally, any leftover inventories are disposed of by firm  $i$  at no extra cost. Thus, firm  $i$ 's level of inventories after household consumption is given by:

$$V_i(t) = Y_i(t) - Q_i(t), \quad (10)$$

where  $Q_i(t)$  is the actual quantity of sold C-goods, determined on the consumption good market, see Section 2.7.

### 2.2.3 Prices

Firm  $i$  sets their price according to both internal and external factors. In particular, firm  $i$  will stochastically increase (decrease) their price if involuntary inventories are zero (positive), as this signals there is excess demand (supply) for firm  $i$ 's goods. Furthermore, firm  $i$  will also adjust their price towards the average price level of other C-firms to stay competitive. Hence, this pricing mechanism is similar to the CATS framework (Russo et al., 2007; Gaffeo et al., 2008; Assenza et al., 2015), with the addition of prices tending to adjust towards the average price level as in Kalecki (1954). Hence, the pricing mechanism of firm  $i$  is given by:

$$P_i(t) = \begin{cases} P_i(t-1)(1 + \sigma_P|\varepsilon_i(t)|) + \gamma_P(\bar{P}^C(t-1) - P_i(t-1)) & \text{if } V_i(t-1) = 0 \\ P_i(t-1)(1 - \sigma_P|\varepsilon_i(t)|) + \gamma_P(\bar{P}^C(t-1) - P_i(t-1)) & \text{if } V_i(t-1) > 0 \end{cases} \quad (11)$$

where  $\sigma_P$  is the standard deviation of firm  $i$ 's price growth,  $\varepsilon_i(t) \sim \mathcal{N}(0, 1)$ , therefore  $|\varepsilon_i(t)|$  is distributed according to a folded normal distribution,  $\gamma_P \in (0, 1)$  is a speed of adjustment parameter,  $V_i(t-1)$  are firm  $i$ 's inventories in the previous period, a proxy for excess demand ( $V_i(t-1) = 0$ ) or supply ( $V_i(t-1) > 0$ ), and  $\bar{P}^C(t-1)$  is the weighted average of consumption goods prices:

$$\bar{P}^C(t) = \frac{\sum_{k=1}^{N_C} P_k(t) Y_k(t)}{\sum_{k=1}^{N_C} Y_k(t)}. \quad (12)$$

### 2.2.4 Investment & capital

Firm  $i$  finances new investment using both internal and external finance. Firm  $i$  determines external investment financing using their desired debt-to-output ratio for investment finance, given by:

$$d_i^d(t+1) = d_0 + d_1\alpha_i(t) + d_2\pi_i(t) \quad (13)$$

where  $d_0, d_1, d_2 > 0$  are parameters,  $\alpha_i(t) = \ln(a_i(t)) - \ln(a_i(t-1))$  is the log difference of firm  $i$ 's labour productivity, and  $\pi_i(t) = \Pi_i(t)/(P_i(t)Y_i(t))$  is firm  $i$ 's profit share. This is the essential equation determining the level of debt in the model, which has been taken from Barrett (2018).  $d_0$  controls the level of the desired investment debt ratio, given no labour productivity growth or profit share, and  $d_1$  and  $d_2$  determine the strength with which the desired debt ratio is influenced by firm  $i$ 's productivity growth and profit share, respectively. Therefore, when firm  $i$  is successful, and so has higher  $\alpha_i(t)$  and  $\pi_i(t)$ , this will lead to a higher desired investment debt ratio. Additionally, firm  $i$ 's nominal desired debt level for investment can be derived from Eq. 13 as:

$$D_i^d(t+1) = d_i^d(t+1)P_i(t)Y_i(t). \quad (14)$$

Hence, firm  $i$ 's desired loan for investment financing is given by:

$$IL_i^d(t+1) = \max \{D_i^d(t+1) - D_i(t), 0\}, \quad (15)$$

where  $D_i(t)$  is firm  $i$ 's current debt. Firm  $i$ 's desired investment expenditure is then given by:

$$IE_i^d(t+1) = \max \{IL_i^d(t+1) + \Pi_i(t) - Div_i(t) + M_i(t) - W_i(t), 0\}, \quad (16)$$

where  $M_i(t)$  is firm  $i$ 's deposits at bank  $b$  and  $W_i(t)$  is firm  $i$ 's current wage bill. Firms are assumed to keep enough internal finance to cover their future wage bill, and they assume the wage bill will remain the same in the next period.

After firm  $i$  has purchased their desired amount of K-goods on the capital market, firm  $i$  updates their capital stock by:

$$K_i(t+1) = K_i(t)(1 - \delta) + I_i(t+1), \quad (17)$$

where  $\delta$  is the depreciation rate of capital, and  $I_i(t+1)$  is firm  $i$ 's new investment orders. Firm  $i$  also updates their total capital expenditure, which is similarly given by:

$$KE_i(t+1) = KE_i(t)(1 - \delta) + IE_i(t+1), \quad (18)$$

where  $IE_i(t+1)$  is firm  $i$ 's actual investment expenditure.  $I_i(t+1)$  and  $IE_i(t+1)$  are both determined on the capital good market, see Section 2.8.

### 2.2.5 Labour & Wages

Firm  $i$ 's desired labour in the next period is determined using Eq. 5 and their desired capital utilisation:

$$N_i^d(t+1) = v_i^d(t+1) \frac{K_i(t+1)}{\nu a_i^e(t+1)}, \quad (19)$$

where  $v_i^d(t+1)$  is firm  $i$ 's desired capital utilisation, similar in form to that specified in Caiani et al. (2016):

$$v_i^d(t+1) = \min \left\{ \frac{\nu Y_i^d(t+1)}{K_i(t+1)}, 1 \right\}, \quad (20)$$

where,  $Y_i^d(t+1)$  is the desired output of firm  $i$ , defined in Eq. 8. The expected labour productivity in the next period,  $a_i^e(t+1)$ , is derived from Eq. 7 as the expectation of a discrete-time GBM:

$$a_i^e(t+1) = a_i(t) \exp\{g\}. \quad (21)$$

Firm  $i$  then decides how many employees they wish to hire or fire in the next period using:

$$\eta_i(t+1) = N_i^d(t+1) - N_i(t). \quad (22)$$

The wage that firm  $i$  offers their employees depends on firm  $i$ 's employment decisions and the average wage in the previous period. When firm  $i$  desires to hire  $\eta_i(t) \geq 0$  (or fire  $\eta_i(t) < 0$ ) labour then firm  $i$  stochastically increases (decreases) their nominal wage. Furthermore, firm  $i$  will also adjust their wage towards the average wage to stay competitive. Hence, firm  $i$ 's wage is updated as:

$$w_i(t) = \begin{cases} w_i(t-1)(1 + \sigma_w |\varepsilon_i(t)|) + \gamma_w (\bar{w}(t-1) - w_i(t-1)) & \text{if } \eta_i(t) \geq 0 \\ w_i(t-1)(1 - \sigma_w |\varepsilon_i(t)|) + \gamma_w (\bar{w}(t-1) - w_i(t-1)) & \text{if } \eta_i(t) < 0, \end{cases} \quad (23)$$

where  $\sigma_w$  is the standard deviation of firm  $i$ 's wage growth,  $\varepsilon_i(t) \sim \mathcal{N}(0, 1)$ , so  $|\varepsilon_i(t)|$  is distributed according to a folded normal distribution,  $\gamma_w \in (0, 1)$  is a speed of adjustment parameter, and  $\bar{w}(t-1)$  is the average wage in the previous period. The actual labour hired by C-firm  $i$ ,  $N_i(t)$ , is determined on the labour market, see Appendix 2.6.

### 2.2.6 Debt

Due to imperfect market conditions, the cost of external finance is greater for firm  $i$  than using internal funds, hence, in line with the pecking order theory of finance (Myers, 1984) and the methodology of other ABMs (Gaffeo et al., 2008; Assenza et al., 2015; Caiani et al., 2016), firm  $i$  will seek external financing when their internal flow of funds (profits and deposits) have been exhausted. However, it is rare that a firm will ever completely exhausts their internal finances, therefore, firm  $i$  keeps a buffer of internal funds for precautionary reasons. This buffer is assumed to be proportional to their wage bill. Thus, firm  $i$ 's actual desired loan is given by:

$$L_i^d(t+1) = \max \{ IE_i(t+1) + W_i(t) + Div_i(t) - \Pi_i(t) - M_i(t), 0 \}. \quad (24)$$



where  $IE_i(t+1)$  is firms  $i$ 's actual investment expenditure,  $W_i(t) = w_i(t)N_i(t)$  is firm  $i$ 's wage bill,  $\Pi_i(t)$  is firm  $i$ 's profits, and  $M_i(t)$  is firm  $i$ 's deposits at bank  $b$ .

Firm  $i$  is assumed to request a single loan  $\ell$  each period from a given bank  $b$ , see Appendix 2.9 for the credit market, denoted  $L_{ib,\ell}(t)$ , with an interest rate  $r_b^L(t)$ . Firm  $i$  then calculates the amortisation cost to bank  $b$  on this loan using the following formula:

$$A_{ib,\ell}(t) = L_{ib,\ell}(t) \frac{r_b^L(t)(1 + r_b^L(t))^n}{(1 + r_b^L(t))^n - 1}, \quad (25)$$

where  $n$  is the number of repayment periods. In each period, firm  $i$  repays a proportion of the principal of loan  $\ell$  to bank  $b$ ,  $\rho L_{ib,\ell}(t)$ , where  $\rho = 1/n$  is the repayment rate and  $L_{ib,\ell}(t)$  is the value of the loan that was taken out in period  $t$ . Hence, loan repayments remain constant until the loan has been repaid in full. Assuming that a loan was taken out in period  $\tau$ , the current outstanding value of the loan  $\ell$  in period  $t$  is:

$$L_{ib,\ell}(t) = \max\{L_{ib,\ell}(t-1) - \rho L_{ib,\ell}(\tau), 0\}, \quad (26)$$

where  $\tau$  is the period in which the loan was taken out from bank  $b$ , and  $L_{ib,\ell}(\tau)$  is the initial value of the loan  $\ell$ , the loan principal. Therefore, assuming no defaults, it will take the firm  $i$   $n$  periods to repay the loan in full, hence,  $t - n \leq \tau \leq t$ .

Firm  $i$  pays interest to bank  $b$  on the loan  $\ell$ , as the difference between the initially calculated amortisation cost,  $A_{ib,\ell}(\tau)$  and principal payment,  $\rho L_{ib,\ell}(\tau)$ , that was taken out in period  $\tau$ :

$$IP_{ib,\ell}(t) = A_{ib,\ell}(\tau) - \rho L_{ib,\ell}(\tau), \quad (27)$$

until the loan has been repaid in full,  $L_{ib,\ell}(t) = 0$  from Eq. 26. Hence, the total interest payments of firm  $i$  in period  $t$  are given by the sum of interest payments to each bank  $b$  that firm  $i$  still has an outstanding loan with in time  $t$ :

$$IP_i(t) = \sum_{b \in \mathcal{B}_i(t)} \sum_{\ell \in \mathcal{L}_{ib}(t)} IP_{ib,\ell}(t), \quad (28)$$

where  $\mathcal{B}_i(t)$  is the set of banks that firm  $i$  still has an outstanding loan with in period  $t$  and  $\mathcal{L}_{ib}(t)$  is the set of loans  $\ell$  that firm  $i$  still has outstanding with bank  $b$ . Additionally, firm  $i$ 's total debt in period  $t$  is given by the sum of all firm  $i$ 's outstanding loans with each bank  $b$ :

$$D_i(t) = \sum_{b \in \mathcal{B}_i(t)} \sum_{\ell \in \mathcal{L}_{ib}(t)} L_{ib,\ell}(t). \quad (29)$$

The actual loan,  $L_{ib,\ell}(t)$ , given to firm  $i$  from bank  $b$  in period  $t$  is determined on the credit market, see Appendix 2.9, and bank loan supply, Eq. 66.

### 2.2.7 Accounting

Firm  $i$ 's profits are given by their revenue and interest on deposits minus the wage bill and total interest payments:

$$\Pi_i(t) = P_i(t)Q_i(t) + r^M M_i(t) - W_i(t) - IP_i(t) - \delta KE_i(t), \quad (30)$$

where  $P_i(t)$  denotes firm  $i$ 's price,  $Q_i(t)$  is the quantity of C-goods sold,  $r^M$  is the interest rate on deposits,  $M_i(t)$  are deposits at the bank, and  $IP_i(t)$  are total interest payments on all outstanding loans.

Firm  $i$  distributes a percentage of profits as dividends to households:

$$Div_i(t) = \max\{\rho\Pi(t), 0\}, \quad (31)$$

where  $\rho$  is the profit retention rate.

The balance sheet identity of firm  $i$  implies that their assets must be equal to their liabilities and equity:

$$KE_i(t) + M_i(t) = D_i(t) + E_i(t), \quad (32)$$

where  $KE_i(t)$  is firm  $i$ 's total capital expenditure,  $D_i(t)$  is firm  $i$ 's debt, and  $E_i(t)$  is firm  $i$ 's equity. The equity of firm  $i$  is updated based on their profits:

$$E_i(t+1) = E_i(t) + \Pi_i(t) - Div_i(t). \quad (33)$$

Hence, given the above, the deposits of firm  $i$  are derived as:

$$M_i(t+1) = M_i(t) + \Pi_i(t) - Div_i(t) + L_{ib,\ell}(t+1) - \rho D_i(t) - IE_i(t+1) + \delta KE_i(t), \quad (34)$$

where  $L_{ib,\ell}(t+1)$  is the actual  $\ell$ th loan firm  $i$  takes out from bank  $b$ ,  $\rho D_i(t)$  is the cost of all principal repayments, and  $IE_i(t)$  is firm  $i$ 's actual investment expenditure. If firm  $i$ 's deposits  $M_i(t) \leq 0$ , then firm  $i$  becomes bankrupt, see Appendix 2.5 for details on entry and exit dynamics.

## 2.3 Capital Firms

### 2.3.1 Production

The  $j$ th K-firm is assumed only to use labour,  $N_j(t)$ , as a factor of production, with constant returns to scale. Hence, firm  $j$ 's production function is given by:

$$Y_j(t) = a_j(t)N_j(t), \quad (35)$$

where  $a_j(t)$  is firm  $j$ 's labour productivity. As with C-firms, the  $j$ th K-firm updates their own production efficiency, which evolves according to a GBM:

$$a_j(t) = a_j(t-1) \exp \left\{ g - \frac{1}{2} \sigma_a^2 + \sigma_a \varepsilon_j(t) \right\}, \quad (36)$$

where  $g$  is the average growth rate of labour productivity,  $\sigma_a$  is the standard deviation of labour productivity, and  $\varepsilon_j(t) \sim \mathcal{N}(0, 1)$  is a standard normal random variable.

### 2.3.2 Desired Output & Inventories

Capital goods are assumed to be durable; hence, firm  $j$  can keep inventories from one period to the next. Therefore, firm  $j$ 's desired output,  $Y_j^d(t+1)$ , is determined by their expected demand,  $Z_j^e(t+1)$ , and their level of inventories,  $V_j(t)$ :

$$Y_j^d(t+1) = Z_j^e(t+1)(1 + \xi) - V_j(t)(1 - \delta), \quad (37)$$

where  $\xi \in (0, 1)$  is firm  $j$ 's desired excess capacity to meet future variations in demand that were not forecasted,  $V_j(t)(1 - \delta)$  are firm  $j$ 's inventories from the previous period that have depreciated by  $\delta$ , and  $Z_j^e(t+1)$  is firm  $j$ 's expected demand, adaptively updated according to their actual demand:

$$Z_j^e(t+1) = Z_j^e(t) + \gamma_Z(Z_j(t) - Z_j^e(t)), \quad (38)$$

where  $\gamma_Z$  determines the speed of adjustment to actual demand, and  $Z_j(t)$  is firm  $j$ 's actual demand, which is determined on the market for capital goods, see Appendix 2.8.

Due to the assumption that K-goods are durable and depreciate at a rate  $\delta$  each period, firm  $j$  initially sets their inventories to  $V_j(t) = V_j(t-1)(1 - \delta) + Y_j(t)$  in period  $t$ . Then, after C-firms have consumed from K-firms, K-firm  $j$ 's leftover inventories are given by:

$$V_j(t) = V_j(t-1)(1 - \delta) + Y_j(t) - Q_j(t), \quad (39)$$

where  $Q_j(t)$  is firm  $j$ 's actual quantity of sold K-goods, determined on the capital good market, see Section 2.8. Additionally, because firm  $j$ 's desired excess capacity is positive, this implies that firm  $j$  has desired inventories equal to their desired excess capacity:

$$V_j^d(t+1) = \xi Y_j(t). \quad (40)$$

### 2.3.3 Prices

Similarly to C-firms, the  $j$ th K-firm sets their price according to both internal and external factors. In particular, firm  $j$  will adjust their price according to their inventories and the average capital price, given by:

$$P_j(t) = \begin{cases} P_j(t-1)(1 + \sigma_P|\varepsilon_j(t)|) + \gamma_P(\bar{P}^K(t-1) - P_j(t-1)) & \text{if } V_j(t-1) \leq V_j^d(t) \\ P_j(t-1)(1 - \sigma_P|\varepsilon_j(t)|) + \gamma_P(\bar{P}^K(t-1) - P_j(t-1)) & \text{if } V_j(t-1) > V_j^d(t) \end{cases} \quad (41)$$

where  $\sigma_P$  is the standard deviation of firm  $j$ 's price growth,  $\varepsilon_j(t) \sim \mathcal{N}(0, 1)$ , so  $|\varepsilon_j(t)|$  is distributed according to a folded normal distribution,  $\gamma_P \in (0, 1)$  is an adjustment parameter,  $V_j(t-1)$  are firm  $j$ 's inventories in the previous period,  $V_j^d(t)$  are firm  $j$ 's desired inventories,

and  $\bar{P}^K(t)$  is the weighted average of capital good prices:

$$\bar{P}^K(t) = \frac{\sum_{k=1}^{N_K} P_k(t) Y_k(t)}{\sum_{k=1}^{N_K} Y_k(t)}. \quad (42)$$

#### 2.3.4 Labour & Wages

Firm  $j$  uses their desired output and expected productivity in the next period to determine their desired amount of labour, derived from Eq. 35 as:

$$N_j^d(t+1) = \frac{Y_j^d(t+1)}{a_j^e(t+1)}, \quad (43)$$

where  $Y_j^d(t+1)$  is firm  $j$ 's desired output, defined in Eq. 37. Similarly to C-firms, K-firm  $j$ 's expected productivity is derived as the expectation of a discrete-time GBM from Eq. 36:

$$a_j^e(t+1) = a_j(t) \exp\{g\}. \quad (44)$$

Firm  $j$  then decides how many employees they wish to hire or fire in the next period:

$$\eta_j(t+1) = N_j^d(t+1) - N_j(t). \quad (45)$$

As with C-firms, K-firm  $j$  updates their wage rate according to their labour demand and the average wage, given by:

$$w_j(t) = \begin{cases} w_j(t-1)(1 + \sigma_w |\varepsilon_j(t)|) + \gamma_w (\bar{w}(t-1) - w_j(t-1)) & \text{if } \eta_j(t) \geq 0 \\ w_j(t-1)(1 - \sigma_w |\varepsilon_j(t)|) + \gamma_w (\bar{w}(t-1) - w_j(t-1)) & \text{if } \eta_j(t) < 0, \end{cases} \quad (46)$$

where  $\sigma_w$  is the standard deviation of firm  $i$ 's wage growth,  $\varepsilon_j(t) \sim \mathcal{N}(0, 1)$ , hence  $|\varepsilon_j(t)|$  is distributed according to a folded normal distribution,  $\gamma_w \in (0, 1)$  is a speed of adjustment parameter, and  $\bar{w}(t-1)$  is the average wage in the previous period. K-firm  $j$ 's actual labour,  $N_j(t)$  is determined on the labour market, see Appendix 2.6.

#### 2.3.5 Debt

K-firms also experience higher costs for external finance; hence, they adhere to the pecking order theory of finance in which they initially use internal finance and then seek external funding in imperfect credit markets. As with C-firms, K-firms will never completely exhaust their internal finances. Therefore, firm  $j$  keeps a buffer of internal funds for precautionary reasons. This buffer is assumed to be proportional to their wage bill. Thus, firm  $j$ 's desired new loan is given by:

$$L_j^d(t+1) = \max\{W_j(t) + Div_j(t) - \Pi_j(t) - M_j(t), 0\}. \quad (47)$$

where  $W_j(t) = w_j(t)N_j(t)$  is firm  $j$ 's wage bill,  $\Pi_j(t)$  is firm  $j$ 's profits, and  $M_j(t)$  is firm  $j$ 's deposits. K-firms use the same method as C-firms when calculating the amortisation cost of a new loan (Eq. 25), total interest payments (Eq. 28), and total debt (Eq. 29). Additionally, firm

$j$ 's actual loan  $\ell$  given by the bank  $b$ ,  $L_{jb,\ell}(t)$ , is determined on the credit market, see Appendix 2.9 and bank loan supply, Eq. 66. Firm  $j$  updates its debt with the new loan:

$$D_j(t+1) = D_j(t)(1 - \phi) + L_j(t+1), \quad (48)$$

where  $L_j(t+1)$  is the actual new loan taken out from bank  $b$ .

### 2.3.6 Accounting

Firm  $j$ 's profits are given by their revenue and interest on deposits minus the wage bill and total interest payments:

$$\Pi_j(t) = P_j(t)Q_j(t) + r^M M_j(t) - W_j(t) - IP_j(t), \quad (49)$$

where  $P_j(t)$  denotes firm  $j$ 's price,  $Q_j(t)$  is the quantity of C-goods sold,  $r^M$  is the interest rate on deposits,  $M_j(t)$  are deposits at the bank,  $W_j(t)$  is the wage bill, and  $IP_j(t)$  are total interest payments on all outstanding loans.

Firm  $i$  distributes a percentage of profits as dividends to households:

$$Div_i(t) = \max\{\rho\Pi(t), 0\}, \quad (50)$$

where  $\rho$  is the profit retention rate.

The balance sheet identity of firm  $j$  implies that their assets must be equal to their liabilities and equity. However, capital firms do not have any physical assets only monetary assets, hence firm  $j$ 's balance sheet is reduced to:

$$M_j(t) = D_j(t) + E_j(t), \quad (51)$$

where  $M_j(t)$  is firm  $j$ 's deposits,  $D_j(t)$  is firm  $j$ 's debt and  $E_j(t)$  is firm  $j$ 's equity. The equity of firm  $j$  is updated based on their profits:

$$E_j(t+1) = E_j(t) + \Pi_j(t) - Div_j(t). \quad (52)$$

Hence, given the above, the deposits of firm  $j$  are derived as:

$$M_j(t+1) = M_j(t) + \Pi_j(t) - Div_j(t) + L_{jb,\ell}(t+1) - \rho D_i(t), \quad (53)$$

where  $L_{jb,\ell}(t+1)$  is the actual  $\ell$ th loan firm  $j$  takes out from bank  $b$  and  $\rho D_i(t)$  is the cost of all principal repayments. If firm  $j$ 's deposits  $M_j(t) \leq 0$ , then firm  $j$  becomes bankrupt, see Appendix 2.5 for details on entry and exit dynamics.

## 2.4 Banks

### 2.4.1 Accounting

The balance sheet identity of bank  $b$  is given by:

$$R_b(t) + L_b(t) = A_b(t) + M_b(t) + E_b(t), \quad (54)$$

where  $R_b(t)$  are the reserves of bank  $b$ , assumed to be deposited at an unmodelled central bank,  $L_b(t)$  is bank  $b$ 's total stock of loans extended to firms,  $A_b(t)$  are central bank advances,  $M_b(t)$  is the total amount of household and firm deposits held by bank  $b$ , and  $E_b(t)$  is bank  $b$ 's equity.

Bank  $b$ 's reserves are simply derived from the above accounting identity:

$$R_b(t) = A_b(t) + M_b(t) + E_b(t) - L_b(t). \quad (55)$$

For simplicity, it is assumed that the central bank offers an interest free advance to banks when they have negative reserves, therefore, bank  $b$ 's advances from the central bank are given by:

$$A_b(t+1) = \max\{-R_b(t), 0\}. \quad (56)$$

Bank  $b$ 's loans are given by the sum of all outstanding loans extended to both C-firms and K-firms:

$$L_b(t) = \sum_{\iota \in \mathcal{F}_b^L(t)} \sum_{\ell \in \mathcal{L}_b(t)} L_{\iota b}(t), \quad (57)$$

where  $\mathcal{F}_b^L(t)$  is the set of C-firms and K-firms with outstanding loans to bank  $b$  in period  $t$ , which is determined on the credit market, see Appendix 2.9. Bank  $b$ 's deposits are given by the sum of all deposits held by C-firms, K-firms and households at bank  $b$ :

$$M_b(t) = \sum_{\iota \in \mathcal{F}_b^M(t)} M_{\iota}(t) + \sum_{h \in \mathcal{H}_b^M(t)} M_h(t), \quad (58)$$

where  $\mathcal{F}_b^M(t)$  is the set of C-firms and K-firms with deposits,  $M_{\iota}(t)$ , at bank  $b$  in period  $t$ , and  $\mathcal{H}_b^M(t)$  is the set of households with deposits,  $M_h(t)$ , at bank  $b$  in period  $t$ . The equity of bank  $b$  is updated by their profits,  $\Pi_b(t)$ , and bank  $b$  is assumed to absorb all the losses from bad loans  $B_b(t)$ , which is the sum of all outstanding loans extended to insolvent firms. Moreover, when bank  $b$ 's equity becomes negative, they are assumed to be bailed in by their depositors, so that their equity is given by their desired capital ratio times their assets, see Section 2.5 for details. Hence, bank  $b$ 's equity is given by:

$$E_b(t+1) = \begin{cases} E_b(t) + \Pi_b(t) - Div_b(t) - B_b(t) & \text{if } E_b(t) > 0 \\ CR_b^d(t+1)(L_b(t) + R_b(t)) & \text{if } E_b(t) \leq 0, \end{cases} \quad (59)$$

where  $CR_b^d(t+1)$  is bank  $b$ 's desired capital ratio, defined in Eq. 62. The profits of bank  $b$  are given by the difference between the sum of interest received from loans and interest paid on

deposits:

$$\Pi_b(t) = \sum_{\iota \in \mathcal{F}_b^L(t)} \sum_{\ell \in \mathcal{L}_{ib}(t)} IP_{\iota b, \ell}(t) - r^M M_b(t), \quad (60)$$

where  $\mathcal{F}_b^L(t)$  is the set of C-firms and K-firms that have outstanding loans at bank  $b$  in period  $t$ ,  $\mathcal{L}_{ib, \ell}(t)$  is the set of loans  $\ell$  that firm  $\iota$  has outstanding with bank  $b$  in period  $t$ , and  $IP_{\iota b, \ell}(t)$  are firm  $\iota$ 's interest payments on loan  $\ell$  to bank  $b$  in period  $t$ , Eq. 27, and  $r^M$  is bank  $b$ 's interest rate on deposits, which is assumed to be constant and uniform across all banks.

Banks pay dividends to households:

$$Div_b(t) = \max\{\rho_B \Pi_b(t), 0\}, \quad (61)$$

where  $\rho_B$  is the bank payout ratio.

## 2.4.2 Loans

Bank  $b$  is assumed to lend to firm  $\iota$  based on bank  $b$ 's risk tolerance, which is measured by bank  $b$ 's desired capital ratio. It is assumed that if bank  $b$ 's ratio of expected bad loans to total loans increases, then bank  $b$  becomes more risk averse, and vice versa. Therefore, bank  $b$ 's desired capital ratio is given by a linear function of their expected bad loans ratio:

$$CR_b^d(t+1) = \kappa + \beta_b^e(t+1), \quad (62)$$

derived from the fact that banks regulatory capital requirement is given by  $\kappa$  and they also desire to hold additional capital to absorb expected losses.  $\beta_b^e(t+1) = EL_b(t+1)/L_b(t)$  is bank  $b$ 's expected loss (EL) to loans ratio. Expected loss is given by the well known credit risk measure<sup>2</sup>, such that  $EL_b(t+1) = \sum_{\iota \in \mathcal{L}_{ib}(t)} PD_{\iota}(t+1) \times LGD_{ib}(t) \times EAD_{ib}(t)$  (Chatterjee, 2015), where  $\mathcal{L}_b(t)$  is the set of all firms in period  $t$  with outstanding loans at bank  $b$ ,  $PD_{\iota}(t)$  is the probability of default of firm  $\iota$ ,  $LGD_{\iota}(t) = 1$  is the loss given default, percentage of exposure bank  $b$  expects to lose if firm  $\iota$  defaults (assumed to be equal to 100%),  $EAD_{ib}(t)$  is the exposure at default of bank  $b$  if firm  $\iota$  defaults (sum of outstanding loans). Therefore, EL is given by:

$$EL_b(t+1) = \sum_{\iota \in \mathcal{F}_b^L(t)} PD_{\iota}(t+1) \sum_{\ell \in \mathcal{L}_{ib}(t)} L_{\iota b, \ell}(t). \quad (63)$$

where  $PD_{\iota}(t+1)$  is the probability of default of firm  $\iota$  in the next period and the sum of loans ( $L_{\iota b, \ell}(t)$ ) is bank  $b$ 's current EAD to firm  $\iota$ . Similar to Assenza et al. (2015), firm  $\iota$ 's probability of default is estimated each period by bank  $b$  using a logistic regression model of firm  $\iota$ 's expected leverage ratio,  $\lambda_{\iota}^e(t+1)$ :

$$PD_{\iota}(t+1) = \frac{1}{1 + \exp\{-\left(\hat{\theta}_0 + \hat{\theta}_1 \lambda_{\iota}^e(t)\right)\}}, \quad (64)$$

---

<sup>2</sup>Expected loss for a single borrower is a function of their probability of default (PD), loss given default (LGD), and exposure at default (EAD), such that:  $EL = PD \times LGD \times EAD$ .

where  $\hat{\theta}_0$  and  $\hat{\theta}_1$  are estimated using a time series of firm defaults (0 no default, 1 default) and expected leverage ratios, where firm  $\iota$ 's expected leverage ratio is defined as:

$$\lambda_\iota^e(t+1) = \frac{D_\iota^e(t+1)}{M_\iota(t) + \Pi_\iota(t) + D_\iota^e(t+1)}, \quad (65)$$

where  $D_\iota^e(t+1)$  is firm  $\iota$ 's expected debt level,  $D_\iota^e(t+1) = D_\iota(t)(1-\rho) + L_\iota^d(t+1)$ ,  $M_\iota(t)$  is firm  $\iota$ 's deposits, and  $\Pi_\iota(t)$  is firm  $\iota$ 's profits. Hence, the expected leverage of firm  $\iota$ 's is bounded between 0 and 1, making for a more stable measure. Moreover, bank  $b$  separates firms into C-firms and K-firms and estimates the bankruptcy probability of each firm type separately.

Bank  $b$  supplies firm  $\iota$ 's desired loan in full if bank  $b$ 's desired capital ratio is lower than their actual capital ratio; bank  $b$  does not supply the loan if their desired capital ratio is higher than their actual capital ratio, which ensures bank  $b$  does not take on too much risk. Therefore, the loan that firm  $\iota$  receives from bank  $b$  is given by:

$$L_{\iota b, \ell}(t+1) = \begin{cases} L_\iota^d(t+1) & \text{if } CR_b^d(t+1) < CR_b(t) \\ 0 & \text{if } CR_b^d(t+1) \geq CR_b(t), \end{cases} \quad (66)$$

where  $L_\iota^d(t+1)$  is firm  $\iota$ 's desired loan, Eq. 24, and bank  $b$ 's actual capital ratio is given by:

$$CR_b(t) = \frac{E_b(t)}{L_b(t)}, \quad (67)$$

where  $E_b(t)$  is bank  $b$ 's equity and  $L_b(t)$  are bank  $b$ 's total outstanding loans to firms.

### 2.4.3 Interest Rate

Bank  $b$  also uses their desired capital ratio to determine the interest rate set on loans. Bank  $b$  will stochastically increase (decrease) their loan interest rate to make themselves less (more) desirable to firms looking to take out a loan, which will increase (decrease) their capital ratio. Furthermore, bank  $b$  also adjusts their loan interest rate towards an assumed fixed natural rate of interest, which can be thought of as a central bank policy rate. Hence, bank  $b$ 's interest rate on loans is updated as:

$$r_b^L(t+1) = \begin{cases} r_b^L(t)(1 + \sigma_r |\varepsilon_b(t)|) + \gamma_r (r^N - r_b^L(t)) & \text{if } CR_b^d(t+1) \geq CR_b(t) \\ r_b^L(t)(1 - \sigma_r |\varepsilon_b(t)|) + \gamma_r (r^N - r_b^L(t)) & \text{if } CR_b^d(t+1) < CR_b(t), \end{cases} \quad (68)$$

where  $\sigma_r$  is the standard deviation of bank  $b$ 's loan interest rate growth,  $\varepsilon_b(t) \sim \mathcal{N}(0, 1)$ , hence  $|\varepsilon_b(t)|$  is distributed according to a folded normal distribution,  $\gamma_r \in (0, 1)$  is a speed of adjustment parameter, and  $r^N$  is a fixed natural rate of interest, uniform across all banks,  $CR_b^d(t+1)$  is bank  $b$ 's desired capital ratio, and  $CR_b(t)$  is bank  $b$ 's actual capital ratio.

## 2.5 Entry & Exit Dynamics

The number of all agents apart from firms is fixed throughout the simulation. The bankruptcy condition of firm  $\iota$  is assumed to be when their deposits are less than or equal to zero,  $M_\iota(t) \leq 0$ .



There is a one-to-one replacement of firms; thus, if firm  $\iota$  exits the market, a new firm will enter. New entrants are a random copy of incumbent firms with no debt, a single employee (chosen from the pool of unemployed households), and average market values for their price  $P_\iota(t) = \bar{P}^F(t)$  ( $F = \{C, K\}$  for either C-market or K-market respectively) and wage  $w_\iota(t) = \bar{w}(t)$ . It is assumed that bankrupt C-firms' capital is disposed of, and new C-firm entrants are endowed with the capital stock of the firm they imitate.

Furthermore, banks become bankrupt when their equity is less than or equal to zero,  $E_b(t) \leq 0$ , however, unlike firms, banks are bailed in by their depositors (firms and households) and the new equity of the bankrupt bank in the next period is  $E_b(t+1) = CR_b^d(t+1)(L_b(t) + R_b(t))$ , which is the banks desired amount of equity to their assets. Each depositor's deposits are reduced proportionally by the size of their deposit account at the bankrupt bank.

## 2.6 The Market for Labour

The labour market is characterised by a search and matching mechanism similar to the one presented in Russo et al. (2007) and Gaffeo et al. (2008). Initially, firm  $\iota$  determines their employment decision for the next period, to either hire or fire employees depending on their employment variable  $\eta_\iota(t)$ , Eq. 22. If  $\eta_\iota(t) < 0$ , then firm  $\iota$  wants to fire  $|\eta_\iota(t)|$  employees, and when  $\eta_\iota(t) > 0$ , firm  $\iota$  will post vacancies equal to  $\eta_\iota(t)$ . Firm  $\iota$  offers potential employees a single-period labour contract with a guaranteed wage of  $w_\iota(t)$  for the current period. It is assumed that the labour contract is periodically renewed and updated with the current wage offered by the firm until the firm decides to terminate the employee's employment.

The labour market then opens, and firms post their vacancies with the corresponding wage rate. If a household  $h$  is unemployed, they will randomly visit  $n_F$  firms each period. Household  $h$  incurs no travel costs when visiting the initial  $n_F$  firms; however, after this number is reached, travel costs become prohibitively high. The probability that household  $h$  visits firm  $\iota$  is equal to firm  $\iota$ 's labour market share,  $ms_\iota^N(t) = N_\iota(t) / (\sum_{c=1}^{N_C} N_c(t) + \sum_{k=1}^{N_K} N_k(t))$ . Household  $h$  then sorts the firms they visited by descending wage rate and will send an application to all firms on their list which have open vacancies,  $\eta_\iota(t) > 0$ .

Firms then hire or fire employees based on the value of  $\eta_\iota(t)$ . When  $\eta_\iota(t) < 0$ , it is assumed that firm  $\iota$  randomly chooses  $\min\{|\eta_\iota(t)|, N_\iota(t) - 1\}$  workers to fire from their set of employees, hence firm  $\iota$  always keeps at least 1 employee. When  $\eta_\iota(t) > 0$ , it is assumed that firm  $\iota$  randomly hires  $\min\{\eta_\iota(t), |\mathcal{A}_\iota(t)|\}$  new workers from their set of applications, denoted  $\mathcal{A}_\iota(t)$ , where  $|\mathcal{A}_\iota(t)|$  is length of the set  $\mathcal{A}_\iota(t)$ , the number of households in firm  $\iota$ 's set of applications in period  $t$ . Thus, the number of households firm  $\iota$  can hire is constrained by the number of applications they receive each period.

## 2.7 The Market for Consumption Goods

The consumption market uses a search and matching mechanism to match households' consumption demands with C-firms' output, similar to that proposed by Russo et al. (2007) and Gaffeo et al. (2008). At the beginning of period  $t$ , C-firm  $i$  sets its inventories equal to output and discards any inventories from the previous period because C-goods are assumed to be perishable, hence  $V_i(t) = Y_i(t)$  at the start of period  $t$ . Households randomly visit  $n_C$  C-firms each

period, the maximum amount that assumed travel costs permit. The probability that household  $h$  will visit C-firm  $i$  is equal to C-firm  $i$ 's consumption market share,  $ms_i(t) = Y_i(t) / \sum_{k=1}^{\mathbf{N}_C} Y_k(t)$ . Household  $h$  then sorts the C-firms they visited by ascending price and demands to consume C-goods worth  $C_h^d(t) = E_h^d(t) / P_i(t)$  from the first C-firm on their list. If C-firm  $i$ 's inventories run out before household  $h$  exceeds their desired expenditure, household  $h$  purchases the remaining C-goods from C-firm  $i$  and moves to the next C-firm on their list. Households continue to demand C-goods from their list of C-firms to visit until they have reached their desired expenditure or exhausted their list of C-firms. Hence, household  $h$ 's actual expenditure is given by:

$$E_h(t) = \min \left\{ E_h^d(t), \max \left\{ \sum_{i \in \mathcal{C}_h(t)} P_i(t) V_i(t) - \sum_{h' \in \mathcal{H}'_i(t)} E_{h'}^d(t), 0 \right\} \right\}, \quad (69)$$

where  $E_h^d(t)$  is household  $h$ 's desired expenditure,  $\mathcal{C}_h(t)$  is the set of C-firms visited by household  $h$  in period  $t$ ,  $P_i(t)$  is visited C-firm  $i$ 's price,  $Y_i(t)$  is C-firm  $i$ 's output, and  $\mathcal{H}'_i(t)$  is C-firm  $i$ 's set of households that visited before household  $h$  in period  $t$ . Therefore, the actual demand for C-goods faced by C-firm  $i$  is given by:

$$Z_i(t) = \max \left\{ \sum_{h \in \mathcal{H}_i(t)} \frac{E_h^d(t)}{P_i(t)} - \sum_{i' \in \mathcal{C}'_h(t)} V_{i'}(t), 0 \right\}, \quad (70)$$

where  $\mathcal{H}_i(t)$  is the set of households that visited C-firm  $i$  in period  $t$  and  $\mathcal{C}'_h(t)$  is the set of C-firms  $i'$  that household  $h$  visited before C-firm  $i$ . C-firm  $i$  then calculates their actual quantity of sold C-goods as:

$$Q_i(t) = \min \{ Z_i(t), Y_i(t) \}. \quad (71)$$

Then C-firm  $i$ 's involuntary inventories after the consumption market can be derived as:

$$V_i(t) = Y_i(t) - Q_i(t). \quad (72)$$

## 2.8 The Market for Capital Goods

The capital market employs a search and matching mechanism to match C-firm investment demand with K-firm output, similar to the market for consumption goods as in Assenza et al. (2015). At the start of the capital market, K-firm  $j$  sets their inventories given previous inventories, less depreciation, and new production,  $V_j(t) = V_j(t-1)(1-\delta) + Y_j(t)$ . If the  $i$ th C-firm desires to invest in new capital,  $I_i^d(t+1) > 0$ , then C-firm  $i$  will randomly visit  $n_K$  K-firms each period, the maximum amount that assumed travel costs will permit. The probability that C-firm  $i$  will visit K-firm  $j$  is equal to K-firm  $j$ 's capital market share,  $ms_j(t) = Y_j(t) / \sum_{k=1}^{\mathbf{N}_K} Y_k(t)$ . C-firm  $i$  then sorts the K-firms they visited by ascending price and demands orders for K-goods worth  $I_i^d(t+1) = E I_i^d(t+1) / P_j(t)$ , where  $E I_i^d(t+1)$  is C-firm  $i$ 's desired investment expenditure and  $P_j(t)$  is K-firm  $j$ 's price, from the first K-firm on their list for use in the next period's production. It is assumed that capital goods require additional time for delivery. If K-firm  $j$ 's inventories run out before C-firm  $i$  exceeds their desired investment expenditure, then C-firm  $i$  purchases the remaining K-goods from K-firm  $j$  and moves to the next K-firm on their

list. C-firms continue to demand K-goods from their list of K-firms until they have reached their desired investment or exhausted their list of K-firms. Hence, C-firm  $i$ 's actual investment expenditure is given by:

$$IE_i(t+1) = \min \left\{ IE_i^d(t+1), \max \left\{ \sum_{j \in \mathcal{K}_i(t)} P_j(t) V_j(t) - \sum_{i' \in \mathcal{C}'_j(t)} IE_{i'}^d(t+1), 0 \right\} \right\}, \quad (73)$$

where  $\mathcal{K}_i(t)$  is the set of K-firms visited by C-firm  $i$  in period  $t$ ,  $IE_i^d(t+1)$  is C-firm  $i$ 's desired investment expenditure (Eq. 16), and  $\mathcal{C}'_j(t)$  is the set of C-firms  $i'$  that visited K-firm  $j$  before C-firm  $i$  in period  $t$ . C-firm  $i$ 's investment is then derived as:

$$I_i(t+1) = \sum_{j \in \mathcal{K}_i(t)} \min \left\{ \frac{IE_i^d(t+1)}{P_j(t)}, \max \left\{ V_j(t) - \sum_{i' \in \mathcal{C}'_j(t)} \frac{IE_{i'}^d(t+1)}{P_j(t)}, 0 \right\} \right\}. \quad (74)$$

Furthermore, the actual demand for K-goods faced by K-firm  $j$  is given by:

$$Z_j(t) = \max \left\{ \sum_{i \in \mathcal{C}_j(t)} \frac{IE_i^d(t+1)}{P_j(t)} - \sum_{j' \in \mathcal{K}'_i(t)} V_{j'}(t), 0 \right\}, \quad (75)$$

where  $\mathcal{C}_j(t)$  is the set of C-firms that visited K-firm  $j$  in period  $t$  and  $\mathcal{K}'_i(t)$  is the set of K-firms  $j'$  that C-firm  $i$  visited before K-firm  $j$ . K-firm  $j$  then calculates their actual quantity of sold K-goods as:

$$Q_j(t) = \min \{ Z_j(t), Y_j(t) \}. \quad (76)$$

Therefore, due to the durability of K-goods with a depreciation rate  $\delta$ , the inventories of K-firm  $j$  after the capital goods market has closed is given by:

$$V_j(t) = V_j(t-1)(1-\delta) + Y_j(t) - Q_j(t). \quad (77)$$

## 2.9 The Market for Credit

C-firms and K-firms randomly visit a single bank  $b$  each period. The probability that firm  $\iota$  visits bank  $b$  is equal to bank  $b$ 's credit market share,  $ms_b(t) = L_b(t) / \sum_{k=1}^{N_B} L_k(t)$ . Firm  $\iota$  then demands loans worth  $L_\iota^d(t+1)$  from the selected bank  $b$ . Bank  $b$  supplies the loan depending on their risk tolerance, Eq. 66. Hence, firms' actual loans are given by the loan supply of the bank they demand a loan from. When firm  $\iota$  takes out a loan from bank  $b$ , bank  $b$  simultaneously credits firm  $\iota$ 's deposit account, while also increasing bank  $b$ 's stock of loans. Hence, this action simultaneously increases total bank assets and liabilities, thus creating new money. When firm  $\iota$  repays a proportion of a loan, this destroys money, because banks record this transaction as both a reduction in deposits and loans, which simultaneously decreases both assets and liabilities by the size of the loan repayment.

### 3 Parameters

Symbol	Description	Value	Calibration
$T$	Number of years	100	Free
$\Delta t$	Time step delta	1/4	Free
$\mathbf{N}_H$	Number of households	5000	Free
$\mathbf{N}_C$	Number of C-firms	400	Free
$\mathbf{N}_K$	Number of K-firms	100	Free
$\mathbf{N}_B$	Number of banks	20	Free
$n_C$	Number of C-firms visited by households	2	Free
$n_K$	Number of K-firms visited by C-firms	2	Free
$n_F$	Number of job applications	4	Free
$n_B$	Number of banks visited by firms	2	Free
$g$	Average growth rate of labour productivity	$0.02 \times \Delta t$	OECD
$\sigma_\alpha$	Standard deviation of productivity growth	$0.03 \times \sqrt{\Delta t}$	Free
$\sigma_P$	Standard deviation of prices	$0.03 \times \sqrt{\Delta t}$	Free
$\sigma_w$	Standard deviation of wages	$0.03 \times \sqrt{\Delta t}$	Free
$\sigma_r$	Standard deviation of interest rates	$0.03 \times \sqrt{\Delta t}$	Free
$\gamma_Z$	Demand speed of adjustment	$0.1 \times \Delta t$	Free
$\gamma_P$	Price speed of adjustment	$0.1 \times \Delta t$	Free
$\gamma_w$	Wage speed of adjustment	$0.1 \times \Delta t$	Free
$\gamma_r$	Interest rate speed of adjustment	$0.1 \times \Delta t$	Free
$c_Y$	Household marginal propensity to consume income	0.8	Free
$c_M$	Household marginal propensity to consume deposits	0.1	Free
$\nu$	C-firm capital accelerator	3	Jackson and Victor (2015)
$d_0$	C-firm desired debt ratio intercept	0.5	Barrett (2018)
$d_1$	C-firm desired debt ratio productivity growth	3	Barrett (2018)
$d_2$	C-firm desired debt ratio profit share	2	Barrett (2018)
$\delta$	Depreciation rate of capital	$0.07 \times \Delta t$	Jackson and Victor (2015)
$\xi$	K-firm desired excess capacity	0.1	Free
$n$	Bank loan repayment periods	$10/\Delta t$	Free
$\kappa$	Bank regulatory capital ratio	0.06	Basel III
$r^M$	Bank interest rate on deposits	$0.001 \times \Delta t$	Free
$r^N$	Bank natural interest rate on loans	$0.02 \times \Delta t$	Free

**Table 3:** Model parameters for baseline scenario.

Several parameters were chosen based on estimated values from real data, such as the average growth rate of labour productivity,<sup>3</sup> bank desired capital ratio intercept (minimum capital ratio)<sup>4</sup> capital acceleration, and depreciation of capital<sup>5</sup> - see Table 3. Other parameter values were chosen such that the model did not display degenerate dynamics. In particular, hyperinflation, collapse in GDP towards zero, sustained unemployment rate over 50%, and sustained bankruptcy rates over 50%.

<sup>3</sup>Values were taken from the OECD, as in Barrett (2018), see data from: <https://data.oecd.org/lprdy/labour-productivity-and-utilisation.htm#indicator-chart>. 2% was a typical value for our  $g$  parameter, during the economically stable period from 1981-2006.

<sup>4</sup>The Basel III regulatory framework states that minimum tier 1 capital ratio must be above 6%, see [https://www.bis.org/fsi/fsisummaries/defcap\\_b3.htm](https://www.bis.org/fsi/fsisummaries/defcap_b3.htm).

<sup>5</sup>Values for capital acceleration and depreciation were taken from Jackson and Victor (2015).

## 4 Initial Values

Initial values for each agent in the simulation.

### C-firm initial values:

It is assumed that each C-firm in the simulation is initialised with the same initial values for each variable. Labour productivity and prices are assumed to be normalised to 1 at the start of the simulation. Furthermore, all households are assumed to be employed at the start of the simulation, where each household is randomly allocated to a C-firm or K-firm. Hence, given the assumed starting values for labour productivity, prices, and labour, all other initial values can be derived. To get the starting values for the debt ratio and profit share, we assume that C-firms start in equilibrium according to Barrett (2018). Moreover, all desired variables are initialised with their real counterparts, e.g. desired labour will be initialised to the C-firms actual labour.

$$\begin{aligned}
a_i(0) &= 1, \\
P_i(0) &= 1, \\
N_i(0) &= \frac{\mathbf{N}_H}{\mathbf{N}_C + \mathbf{N}_K}, \\
Y_i(0) &= a_i(0)N_i(0), \\
Z_i(0) &= Y_i(0), \\
K_i(0) &= \nu Y_i(0), \\
d_i(0) &= \frac{d_0 + gd_1 + \nu(g + \delta)d_2}{1 + gd_2}, \\
\pi_i(0) &= \nu(g + \delta) - gd_i(0), \\
\omega_i(0) &= 1 - \pi - r_b^L(0)d_i(0), \\
D_i(0) &= d_i(0)Y_i(0), \\
\Pi_i(0) &= \pi_i(0)Y_i(0), \\
w_i(0) &= \omega_i(0), \\
W_i(0) &= w_i(0)N_i(0), \\
M_i(0) &= \Pi_i(0) + D_i(0), \\
E_i(0) &= K_i(0) + M_i(0) - D_i(0),
\end{aligned} \tag{78}$$

for all  $i \in \{1, \dots, \mathbf{N}_C\}$ . Additionally, C-Firm  $i$  is randomly assigned a bank  $b$  at the start of the simulation, where they hold their initial debt as a single loan contract:

$$\begin{aligned}
L_{ib,0}(0) &= D_i(0), \\
A_{ib,0}(0) &= L_{ib,0}(0) \frac{r_b^L(0)(1 + r_b^L(0))^n}{(1 + r_b^L(0))^n - 1}, \\
IP_{ib,0}(0) &= A_{ib,0}(0) - \rho L_{ib,0}(0),
\end{aligned} \tag{79}$$

for all  $i \in \{1, \dots, \mathbf{N}_C\}$ .

### K-firm initial values:

Similarly to C-firms, it is assumed that each K-firm in the simulation is initialised with

the same initial values for each variable. Labour productivity and prices are assumed to be normalised to 1 at the start of the simulation. Households are again randomly allocated to K-firms. Hence, given the assumed starting values for labour productivity, prices, and labour, all other initial values for K-firms can be derived. Again, all desired variables are initialised with their real counterparts, e.g. desired labour will be initialised to K-firms' actual labour.

$$\begin{aligned}
a_j(0) &= 1, \\
P_j(0) &= 1, \\
D_j(0) &= 0, \\
N_j(0) &= \frac{\mathbf{N}_H}{\mathbf{N}_C + \mathbf{N}_K}, \\
Y_j(0) &= a_i(0)N_j(0), \\
Z_j(0) &= Y_j(0), \\
K_j(0) &= \nu Y_j(0), \\
w_j(0) &= w_i(0), \\
W_j(0) &= w_j(0)N_j(0), \\
\Pi_j(0) &= Y_j(0) - W_j(0), \\
M_j(0) &= \Pi_j(0), \\
E_j(0) &= M_j(0),
\end{aligned} \tag{80}$$

for all  $j \in \{1, \dots, \mathbf{N}_K\}$ .

**Household initial values:**

Households are randomly allocated to C-firms or K-firms as employees, hence, the initial income of households is equal to the firms wage rate (uniform across all firms in the initial period). The initial deposits of households is then easily derived as being equal to their initial income.

$$\begin{aligned}
Y_h(0) &= w_t(0), \\
M_h(0) &= Y_h(0),
\end{aligned} \tag{81}$$

for all  $h \in \{1, \dots, \mathbf{N}_H\}$ .

**Bank initial values:**

C-firms and K-firms are initially randomly allocated to banks. The bank calculates their initial loans as the sum of C-firm debt for each C-firm that was allocated to the bank (K-firms can be ignored as they are initialised with zero debt). Banks then derive their initial equity as their desired capital ratio multiplied by their amount of loans. The desired capital ratio reduces to simply the intercept value,  $\kappa_1$ , because the expected bad loans ratio will be equal to zero as no firms have become bankrupt yet for the bank to estimate their amount of bad loans.

$$\begin{aligned}
r_b^L(0) &= r^N, \\
L_b(0) &= \sum_{\iota \in \mathcal{F}_b^L} L_{\iota b,0}(0), \\
E_b(0) &= \kappa L_b(0),
\end{aligned} \tag{82}$$

for all  $b \in \{1, \dots, \mathbf{N}_B\}$ .

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