Exercises

1	2	3	4	5	6	7

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Numerical Methods (KEN1540)
KEN1540 Numerical Methods exam

Program: Bachelor Data Science and Artificial Intelligence
Course code: KEN1540
Examiners: Dr. Ir. Martijn Boussé and Dr. Pieter Collins
Date/time: Monday 03 June 2024; 09:00-11:00hr
Format: Closed Book Exam
Allowed aids: DACS-approved calculator; Formula sheet (provided)
Instructions to students:

- The exam consists of 6 questions on 22 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Computer Arithmetic and Algebraic Equations

Consider the problem for finding a solution of the algebraic equation $e^x - 6x - 2 = 0$ in the interval $[0, 4]$.

4p **1a** Compute one step of Newton's algorithm, starting from the midpoint of the interval.

2p **1b** What happens, and why?

3p **1c** Perform one step of the bisection method to find a smaller bracket for the solution.

2p **1d** Perform one more step of Newton’s method, starting from the midpoint of the interval obtained in part **c**.

2p **1e** Do you expect your new approximation to be within 0.1 of the true solution? Give a reason for your answer.

3p **1f** Write a Matlab statement which performs a single step of Newton's method for a function f, starting from point x.

Differential Equations

Consider the problem of finding the value of $y(3)$ for the initial value problem

$$\frac{dy}{dt} = \frac{1}{t-1} - y^2,\ y(2) = 1.4.$$

10p **2a** Solve the problem using Ralston's method with a step size of $h = 0.5$.

2p **2b** How would you expect the error to change if you were to take a step-size of $h = 0.2$?

8p **2c** Write a Matlab function ode_ralston to solve a general differential equation $dy/dt = f(t, y)$ over the time interval $[t_{\text{init}}, t_{\text{final}}]$ with intial value y_{init} using n steps of Ralston's method.

Polynomial Interpolation

Consider the data

x_i	2.0	1.5	1.0	3.0
$y_i = f(x_i)$	0.40	1.00	1.82	0.03

6p **3a** Compute the divided differences $f[x_i, \ldots, x_j]$ for $0 \leq i \leq j \leq 3$. You may assume $f[x_0, x_1] = -1.200$ and $f[x_1, x_2] = -1.640$.

2p **3b** Write down the nested form of the cubic polynomial p_3 interpolating the given data.

3p **3c** Estimate the value of y when $x = 2.5$, showing intermediate steps in your computations.

5p **3d** Write a Matlab function to evaluate a polynomial of degree n given in nested form, given the interpolation points x_i and coefficients a_i.

Integration & Differentiation

Consider the problem of computing to an accuracy of 0.01 the integral

$$I = \int_{0.0}^{1.2} f(x)dx \text{ for } f(x) = \cos(x^2).$$

5p **4a** Use the trapezoid rule with $n = 4$ subdivisions to estimate the value of I..

5p **4b** Use the error estimate for the trapezoid rule to estimate the error over the sub-intervals $[0.0, 0.6]$ and $[0.6, 1.2]$. Do you expect the total error to be less than the desired accuracy bound?

3p **4c** Explain how to use the adaptive trapezoid method to obtain a better estimate to the integral. (You do not have to perform the computation.

3p **4d** Write Matlab code to compute the integral using a builtin Matlab function for integration.

Least-Squares Approximation

Consider the data:

x_i	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π
$f(x_i)$	0.550	0.300	-0.033	0.300	0.800	0.300	-0.033	0.300	0.550

6p **5a** Compute the coefficients a_0, a_1, a_2 of the discrete Fourier transform.

2p **5b** Assuming the coefficients b_i equal 0 for all i, write down the second Fourier approximation s_2 to f.

2p **5c** Evaluate $s_2(\pi/3)$

3p **5d** What is the root-mean-square value $\sqrt{\frac{1}{2\pi}\int_0^{2\pi} s_2(x)^2\,dx}$?

3p **5e** Write Matlab code to compute the Fourier coefficient a_2 for n equally-spaced data points in $[0, 2\pi)$.

Linear Algebra
Let A be the matrix

$$\begin{pmatrix} 7 & 5 \\ 1 & 4 \end{pmatrix}$$

9p **6a** Apply one step of the QR-method to find a matrix B with the same eigenvalues as A.

4p **6b** Apply the Gershgorin circle theorem to B and on B^T to find bounds on the eigenvalues of A.

3p **6c** Write code to perform one step of the QR-method on the matrix A in Matlab. You may use a builtin command for performing a QR-factorisation.

Extra Paper

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