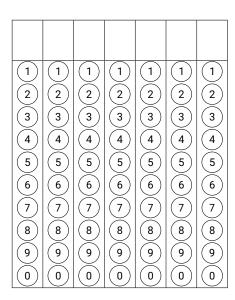
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Surname, First name

**Numerical Mathematics (KEN1540)** 

Resit



**Program:** Bachelor Data Science and Knowledge Engineering

Course code: KEN1540

Examiners: Pieter Collins & Martijn Boussé Date/time: Friday 1st July 2022; 13:00-15:00

Format: Closed Book Exam

Allowed aids: DKE-approved calculator; Formula sheet (provided)

Instructions to students:

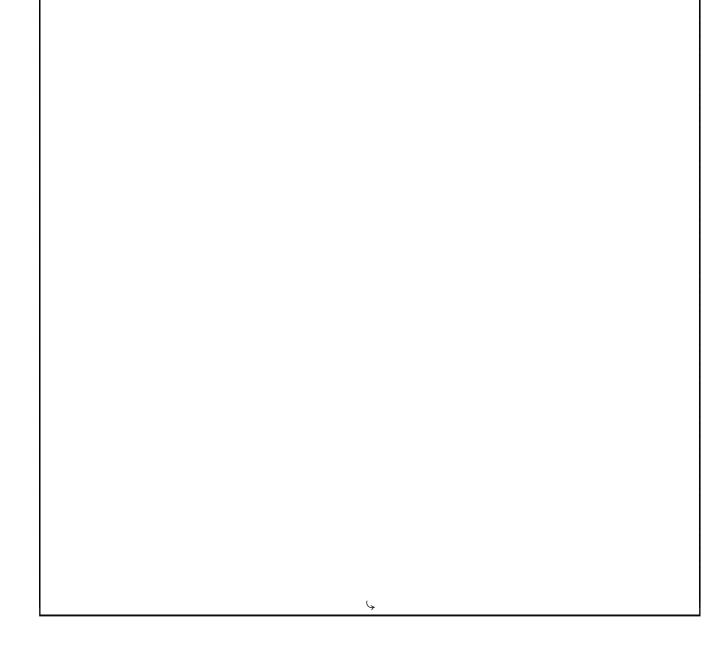
- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- · Answer every question in the reserved space below the question. Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

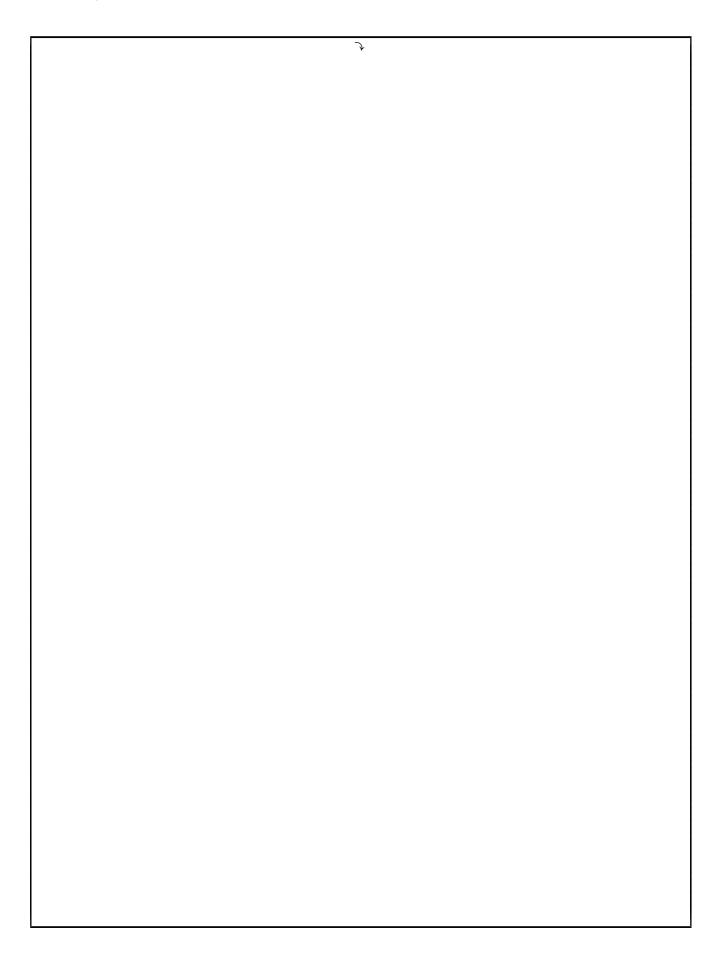
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#### **Algebraic Equations**

14p **1** Explain the difference between the error and the residual of an approximate solution  $\tilde{x}$  to an equation of the form f(x) = 0.

Next, use two steps of the secant method to estimate the root of  $f(x) = e^x - 3x - 1$  in the interval [1,2], starting at  $p_0 = 1$  and  $p_1 = 2$ . Afterwards, compute  $f(p_3)$ , give the best bracket for the root that you have found, and give an estimate of the error.







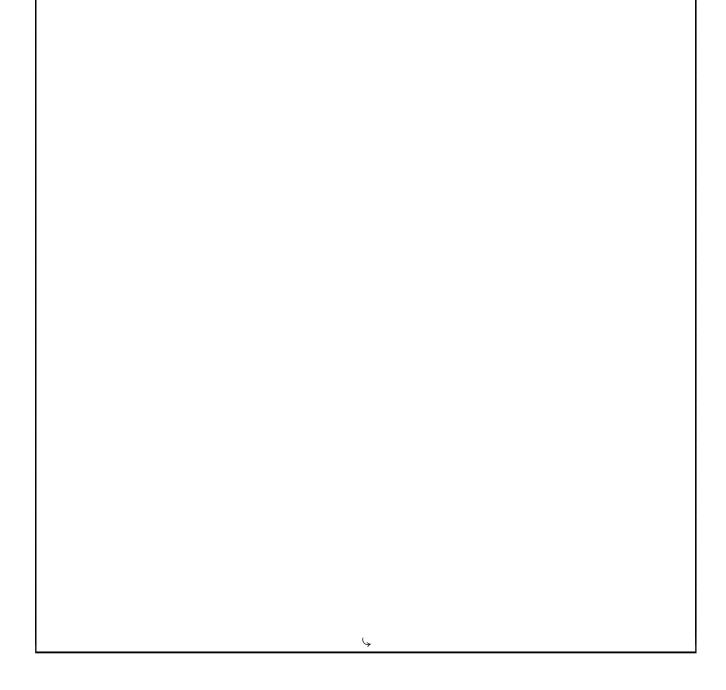
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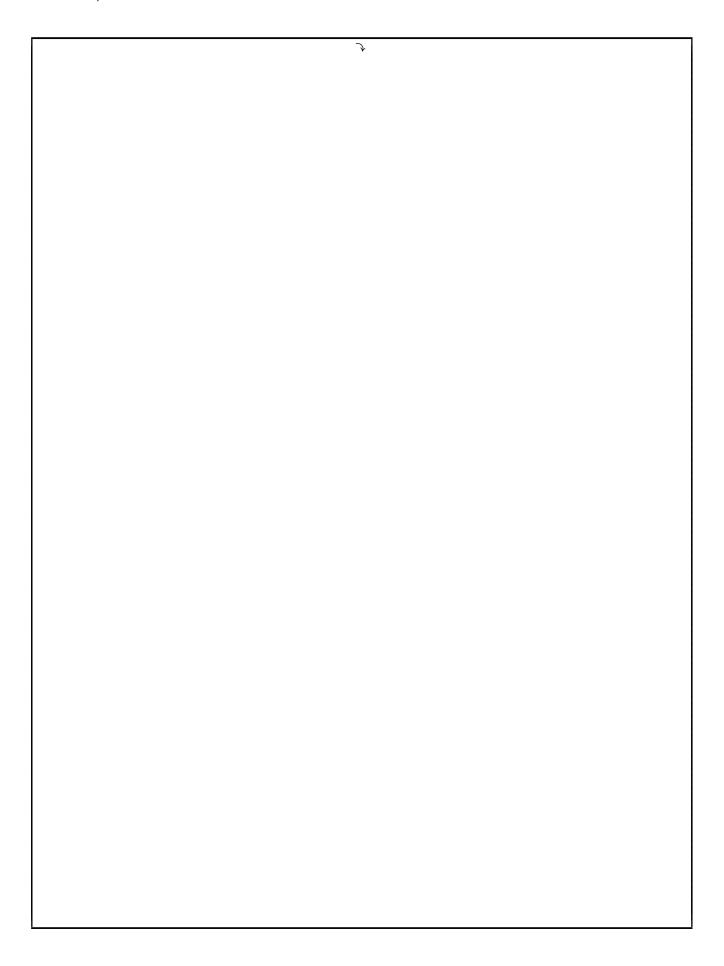
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#### **Differential Equations**

Use two steps of Heun's third order method to estimate the solution of the initial value problem  $\dot{y} = t - y^2$ , y(0.5) = 2.500 up to time t = 1.0. You should aim to compute the solution using Heun's method to an accuracy of at least 3 decimal places, and use sufficient precision in your working to do this.

Compare your answer with the exact solution, which has  $y(1.0) = 1.32733490\,(8\mathrm{dp})$ . What would you expect the absolute error to be if you were to use Heun's method with 10 steps?





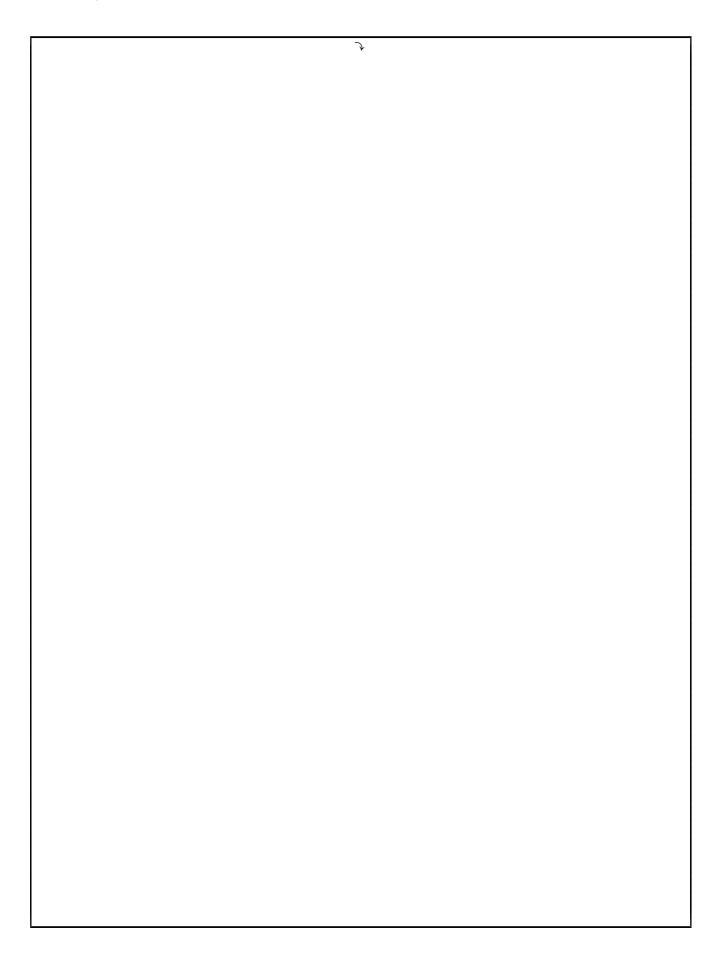
# **Polynomial Interpolation**

12p **3** Use divided differences to compute the cubic polynomial interpolating the following data:

i	0	1	2	3
$x_i$	2.3	2.7	2.0	3.0
$y_i$	0.60	0.23	0.98	0.06

Estimate the value of y when x = 2.5. How does the ordering of the data points affect the interpolating polynomial?

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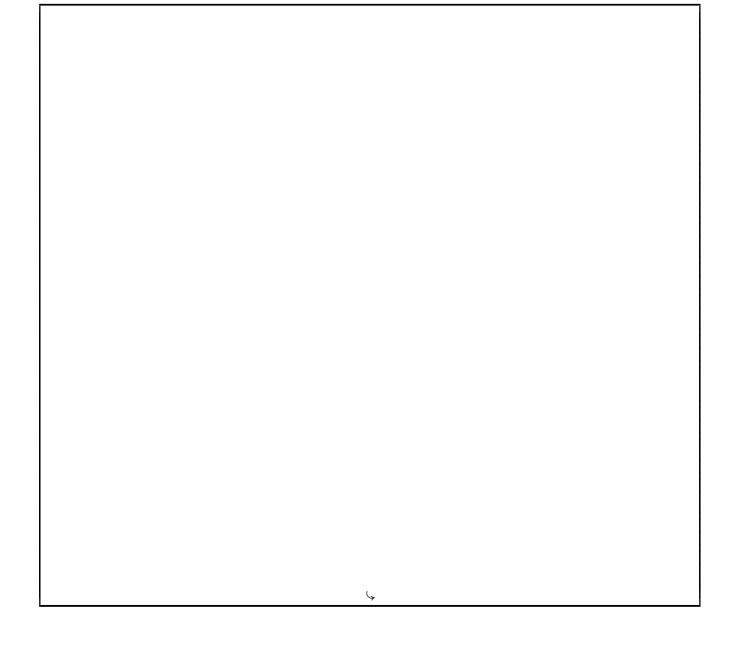


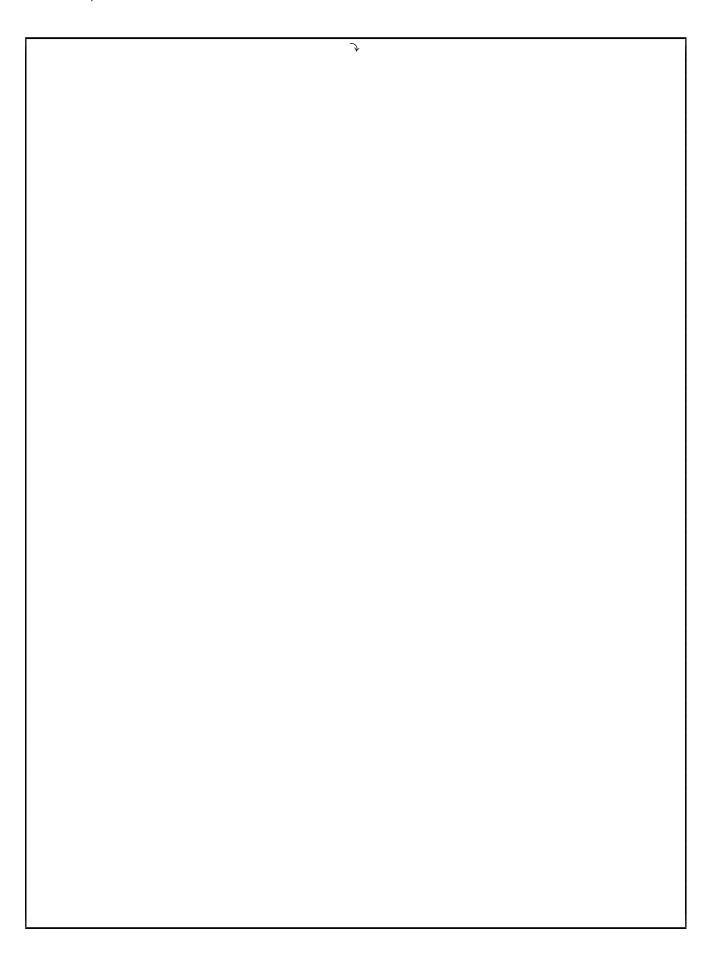
# **Differentiation & Integration**

12p **4** Use the most accurate three-point formulae available to complete the following table:

$\underline{x}$	f(x)	f'(x)	$\sqrt{1+f'(x)^2}$
2.0	-0.57844		
2.2	-0.96792	-0.8093	1.2865
2.4	-0.90217	1.3895	1.7119
2.6	-0.41212	2.9348	3.1005
2.8	0.27176		
3.0	0.82783		

Use the trapezoid rule to estimate  $\int_2^3 \sqrt{1+f'(x)^2}\,dx$ .





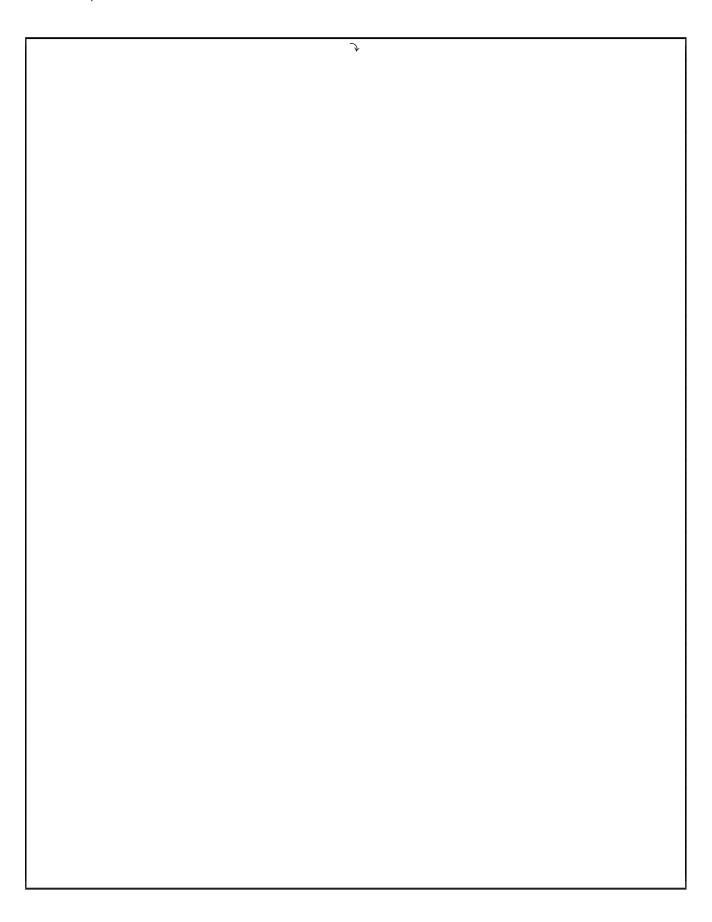
#### **Least-Squares Approximation**

The least-squares approximation to a function f is given by  $q_n(x) = \sum_{k=0}^n c_k P_k(x)$  where the  $P_k$  are the Legendre polynomials, and the  $c_k$  are given by

Use the recurrence relation to evaluate  $P_k(x)$  for  $k = 0, 1, \dots, 4$  for x = 0.3, and hence compute  $q_4(x)$ .

Compute the total square error  $\int_{-1}^{+1} (q_2(x) - f(x))^2 dx$ , assuming  $\int_{-1}^{+1} f(x)^2 dx = 0.117242$  (6dp).

4



### Linear Algebra

12p **6** Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 2 \end{pmatrix}.$$

After applying one step of the QR method we obtain

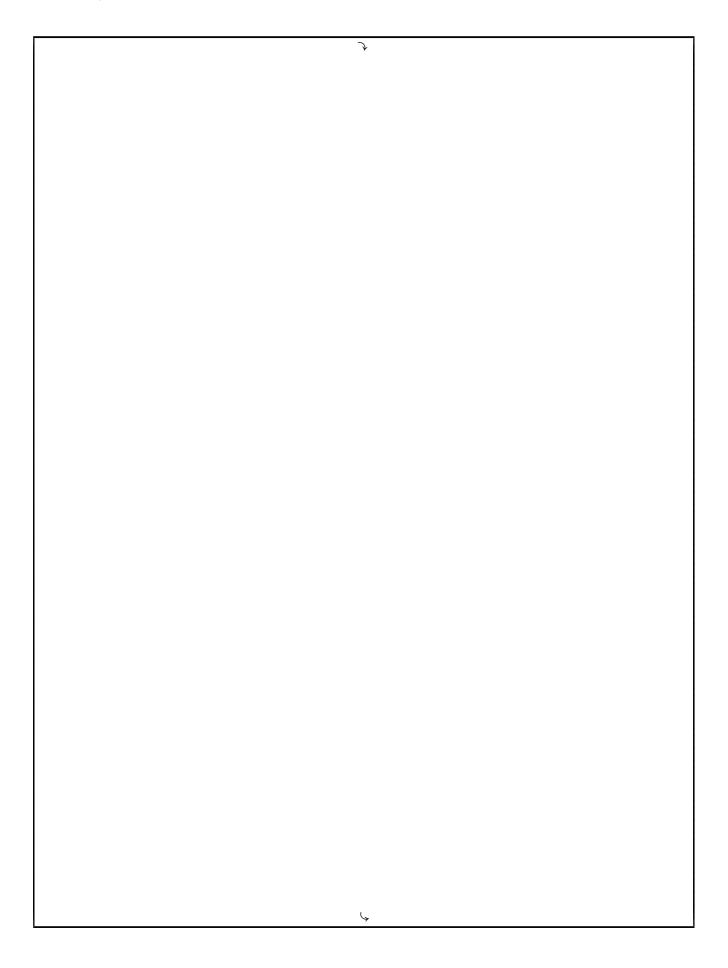
$$A = QR = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.4472 & -0.8944 \\ 0 & -0.8944 & 0.4472 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2.2361 & -2.6833 \\ 0 & 0 & -0.8944 \end{pmatrix}.$$

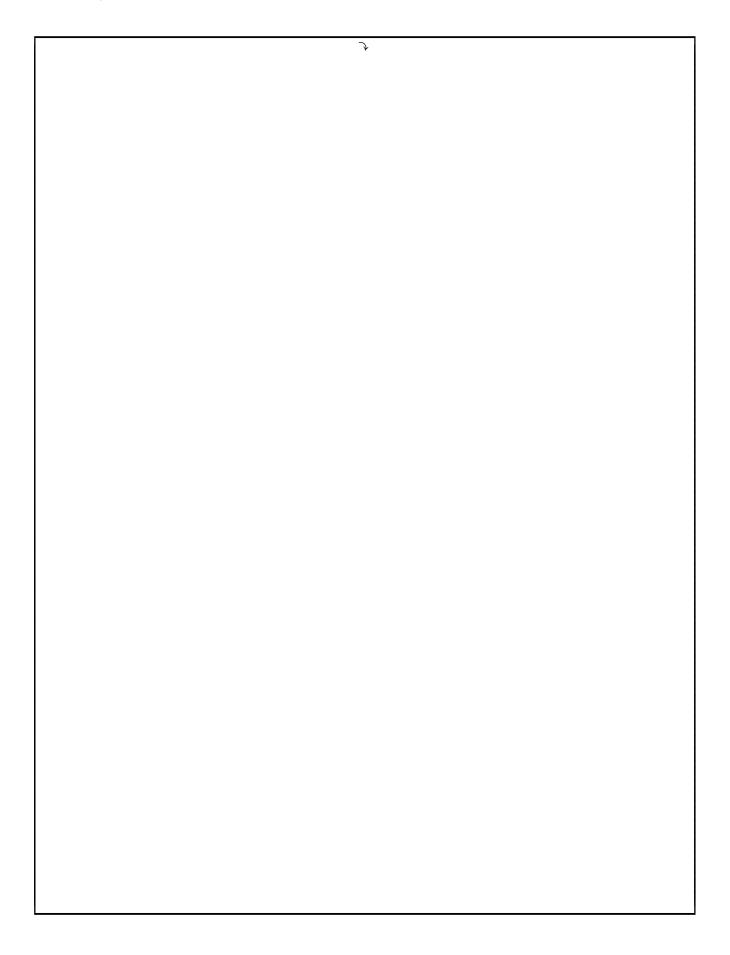
and

$$A^{(1)} = R^{(0)}Q^{(0)} = RQ = \begin{pmatrix} 3 & 0 & 0\\ 0 & 3.4000 & 0.8000\\ 0 & 0.8000 & -0.4000 \end{pmatrix}.$$

Apply another step of the QR method to estimate the eigenvalues of  $\it A$ . Give bounds on the eigenvalues using the Gersgorin circle theorem.

**\** 





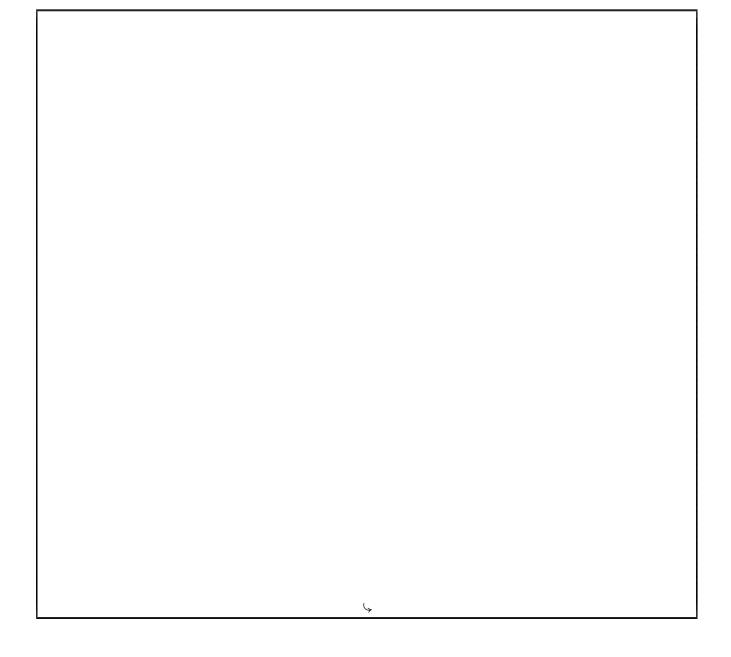
## Modelling

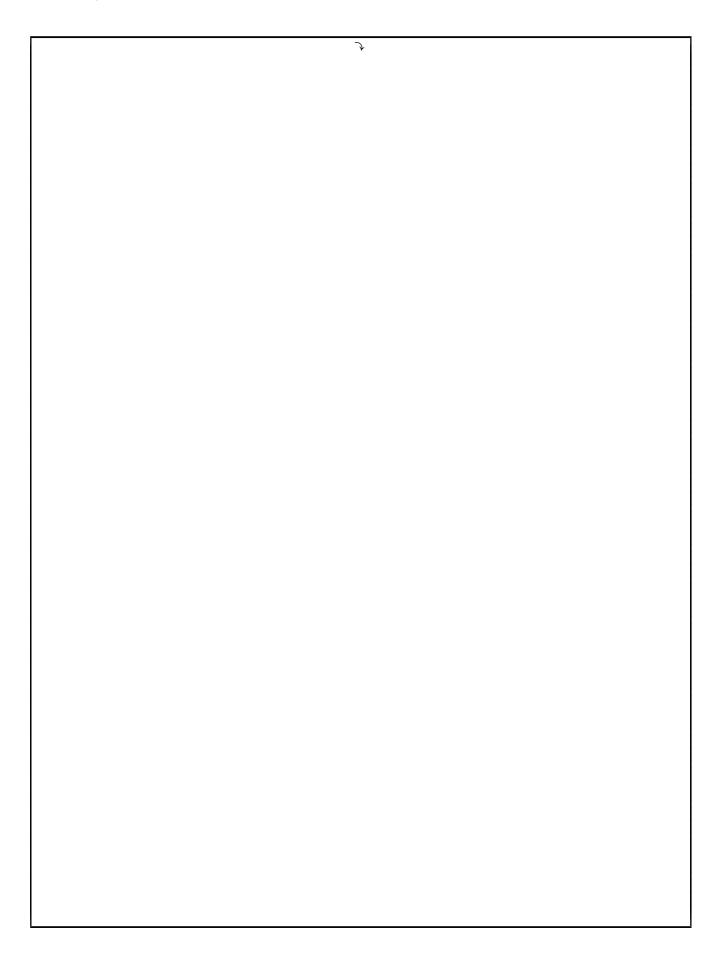
In a compartmental model of an infectious disease with two variants, individuals are either 7 6р susceptible (S), infections with the first variant  $(I_1)$ , or with the second variant  $(I_2)$ . The progress of the disease is modelled by

$$\dot{S} = -(\alpha_1 I_1 + \alpha_2 I_2)S/N, \quad \dot{I}_1 = \alpha_1 I_1 S/N - \beta_1 I_1, \quad \dot{I}_2 = \alpha_2 I_2 S/N - \beta_2 I_2.$$

Assume initially 340 individuals infected with variant 1 and 1 individual infected with variant 2, in a population with size N = 17000000.

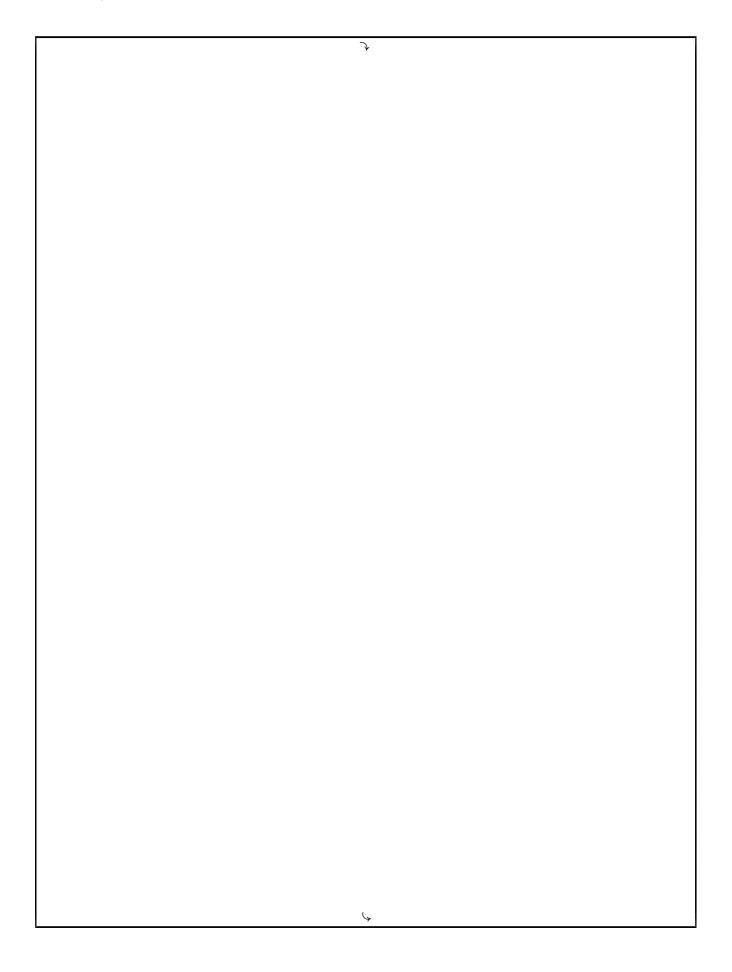
Show how to solve this system of differential equations in Matlab, including writing the code you would use.

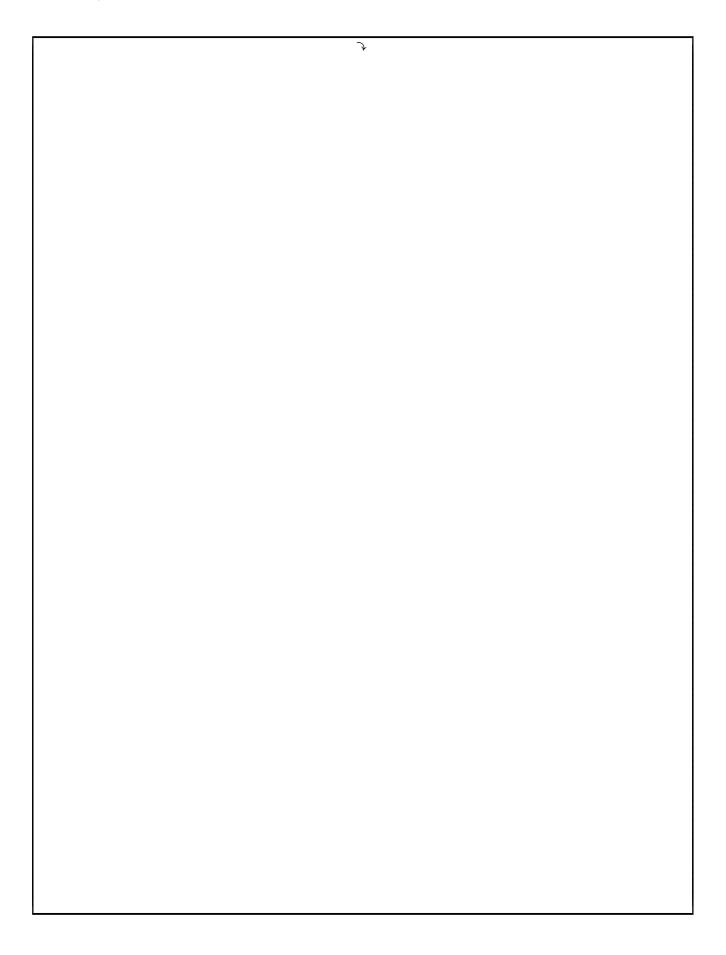




Extra Paper			
8			







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