

Exercises

1	2	3	4	5	6	7	8	9
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Surname, First name

KEN1130 Discrete Mathematics
Exam

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a	<input checked="" type="radio"/>	c	d	e	f	→ b
a	b	<input checked="" type="radio"/>	d	e	f	→ c
<input checked="" type="radio"/>	b	c	<input checked="" type="radio"/>	e	f	→ a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence

Course code: KEN1130

Examiners: dr. Marieke Musegaas and dr. Stefan Maubach

Date/time: Friday 28.10.2022 9h00-11h00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

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Question 1

Consider the following logical proposition.

$$\bullet (p \Rightarrow q) \Leftrightarrow ((q \wedge \neg r) \vee \neg p)$$

Answer the following questions about (the truth table of) the above proposition. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

1.25p **1a** Suppose p is TRUE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1b** Suppose p is TRUE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1c** Suppose p is TRUE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1d** Suppose p is TRUE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1e** Suppose p is FALSE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1f** Suppose p is FALSE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

1.25p **1g** Suppose p is FALSE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

- ☐ (a) TRUE ☐ (b) FALSE

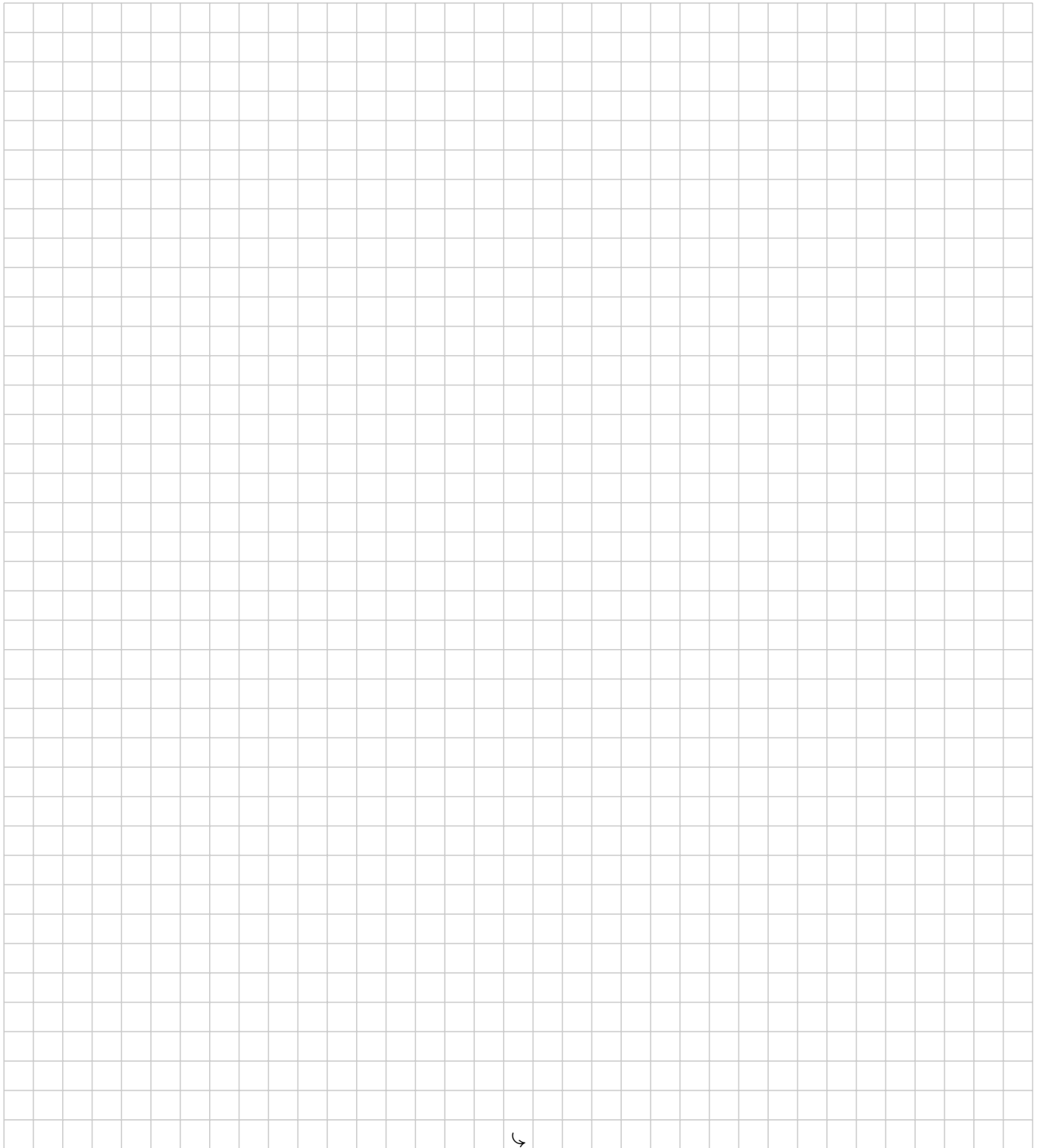
1.25p **1h** Suppose p is FALSE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

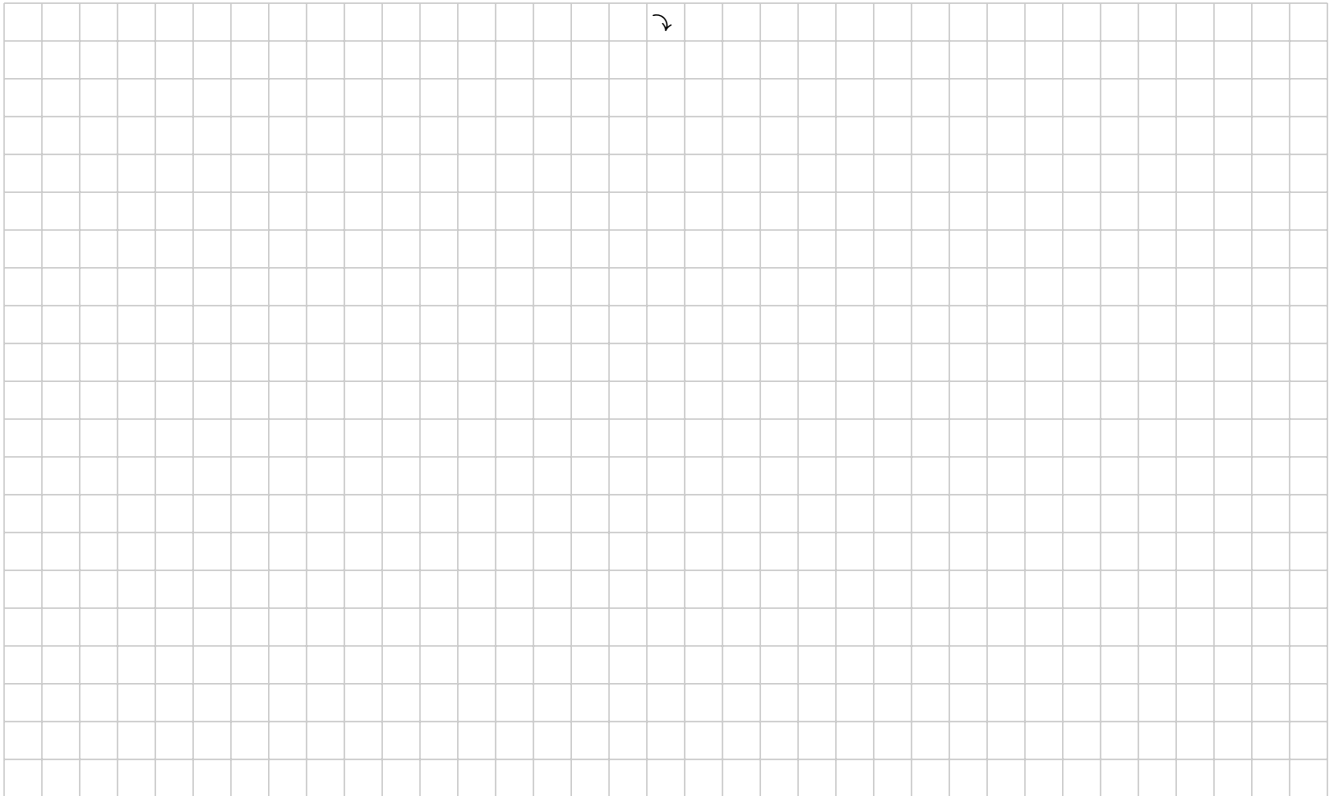
- ☐ (a) TRUE ☐ (b) FALSE

Question 2

12p **2a** Use induction to prove the following statement. For all integers $n \geq 1$,

$$\sum_{i=1}^n i \cdot 2^i = (n-1) \cdot 2^{n+1} + 2$$





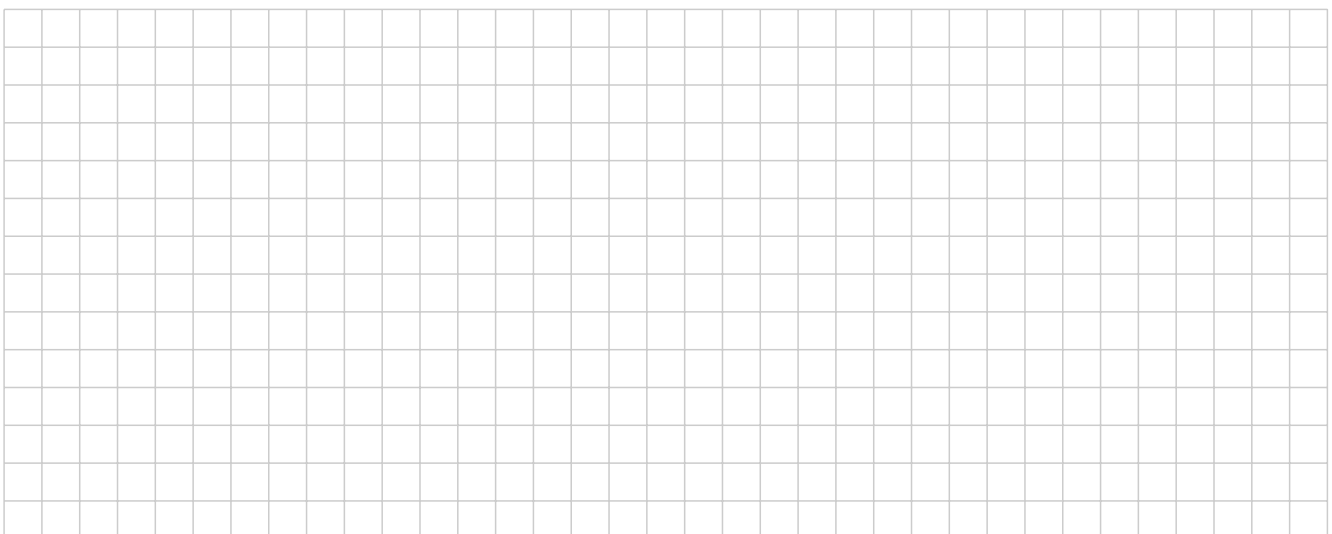
3p **2b** The theorem below is incorrect. What goes wrong in the corresponding proof?

Theorem: For all integers $n \geq 1$, $3n + 2$ is divisible by 3.

Proof: Let $n \in \mathbb{N}$ and assume that $3n + 2$ is divisible by 3. So, $3n + 2 = 3 \cdot k$, where $k \in \mathbb{Z}$. We need to show that $3(n + 1) + 2$ is divisible by 3.

$$3(n + 1) + 2 = 3n + 3 + 2 = (3n + 2) + 3 = 3 \cdot k + 3 = 3 \cdot (k + 1).$$

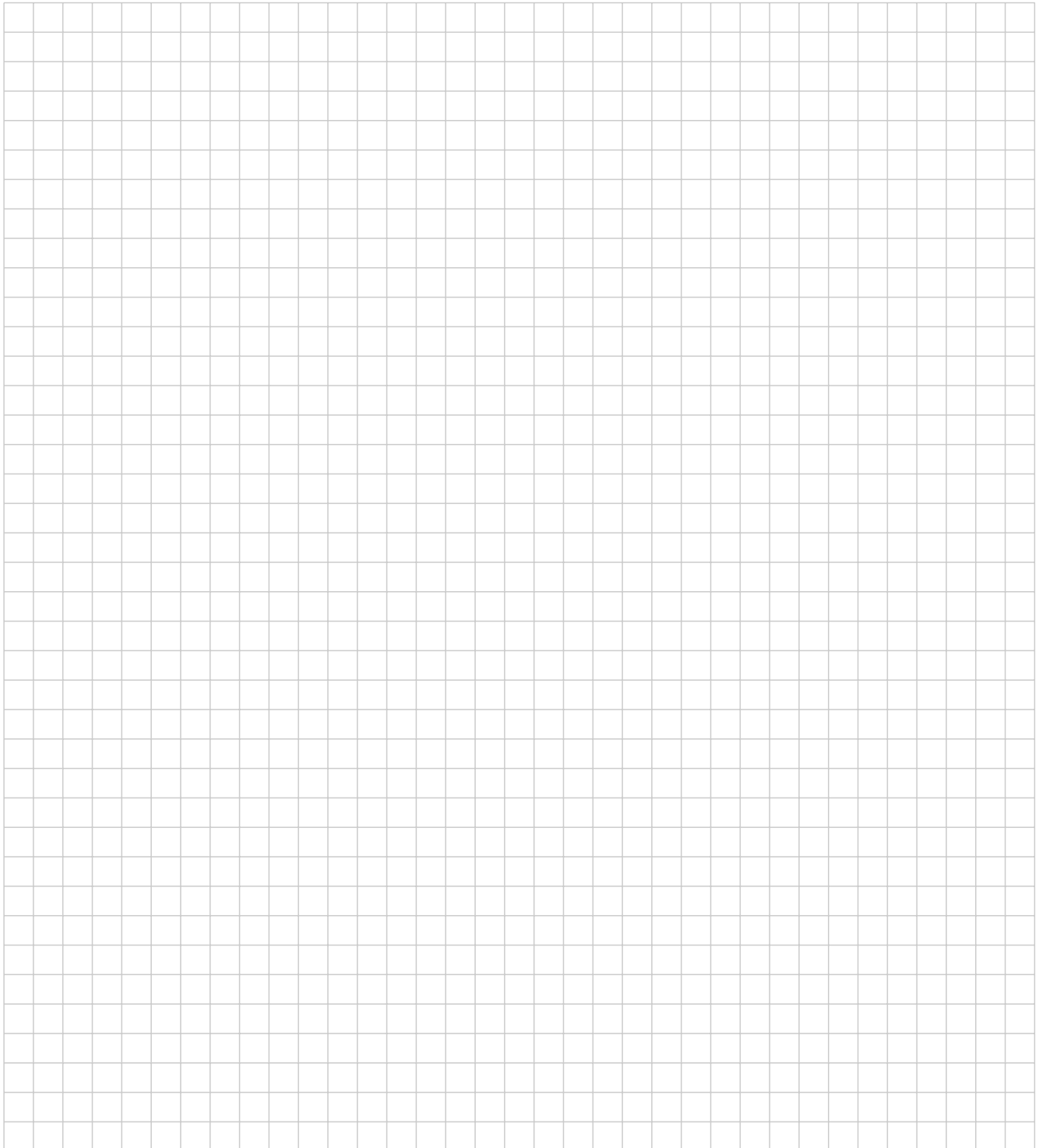
Since $k \in \mathbb{Z}$, we also have $k + 1 \in \mathbb{Z}$. So, $3(n + 1) + 2$ is divisible by 3. \square



Question 3

8p **3a** Prove or disprove the following statement. For all sets A and B ,

$$A^c \cup B^c \subseteq (A \cup B)^c.$$



Fill in the blanks for the direct proof of the following theorem. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

Theorem: For all sets A and B , if $A \subseteq B$, then $A \cup B \subseteq B$.

Proof: Let A and B be two sets. Assume $A \subseteq B$. We must show that _____. Let $x \in$ _____. We must show that $x \in$ _____. By the definition of _____ we know $x \in A$ or $x \in B$. In case $x \in$ _____, then since $A \subseteq B$, $x \in$ _____. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in$ _____ (as was to be shown). \square

1p **3b** What must be filled in at position (i)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

1p **3c** What must be filled in at position (ii)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

1p **3d** What must be filled in at position (iii)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

1p **3e** What must be filled in at position (*iv*)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

1p **3f** What must be filled in at position (*v*)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

1p **3g** What must be filled in at position (*vi*)?

- ☐ a $A \cup B \subseteq B$
- ☐ b $A \cup B$
- ☐ c $A \subseteq B$
- ☐ d A
- ☐ e B
- ☐ f intersection
- ☐ g union

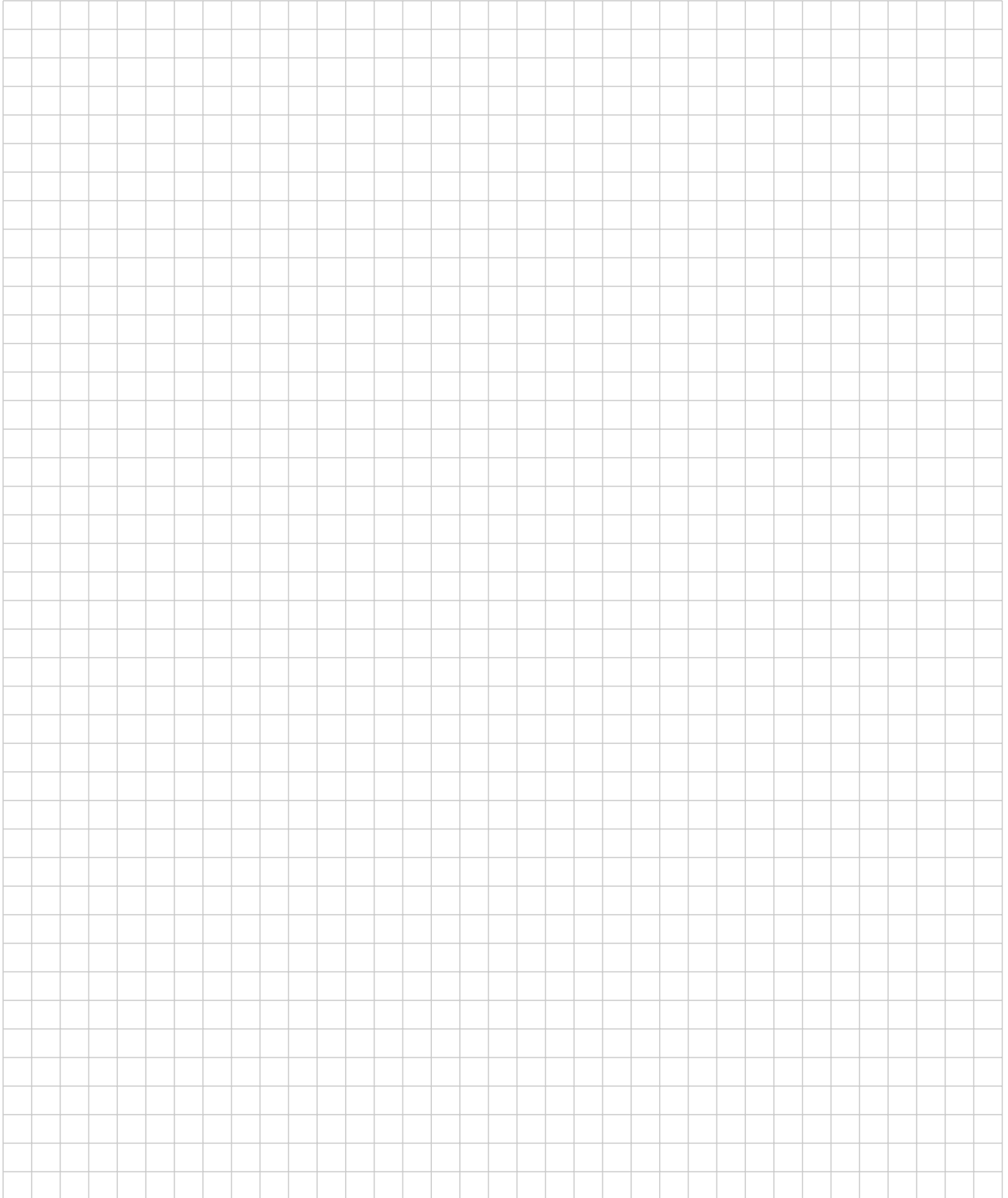
1p **3h** What must be filled in at position (vii)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

Question 4

- 7p **4a** Let $A = \mathbb{P}(\{1, 2\})$. Let R be the relation on A defined as follows: XY means " $X \cap Y = \emptyset$ ". Draw the relation diagram.

- 8p **4b** Let $A = \{1, 2, 3\} \times \{1, 2, 3\}$. Let R be the relation on A defined as follows: $(x_1, x_2)R(y_1, y_2)$ means " $x_1 + x_2 = y_1 + y_2$ ". This is an equivalence relation. (You do not need to prove this.) How many equivalence classes does R have? For each equivalence class, list explicitly which elements of A belong to the equivalence class.

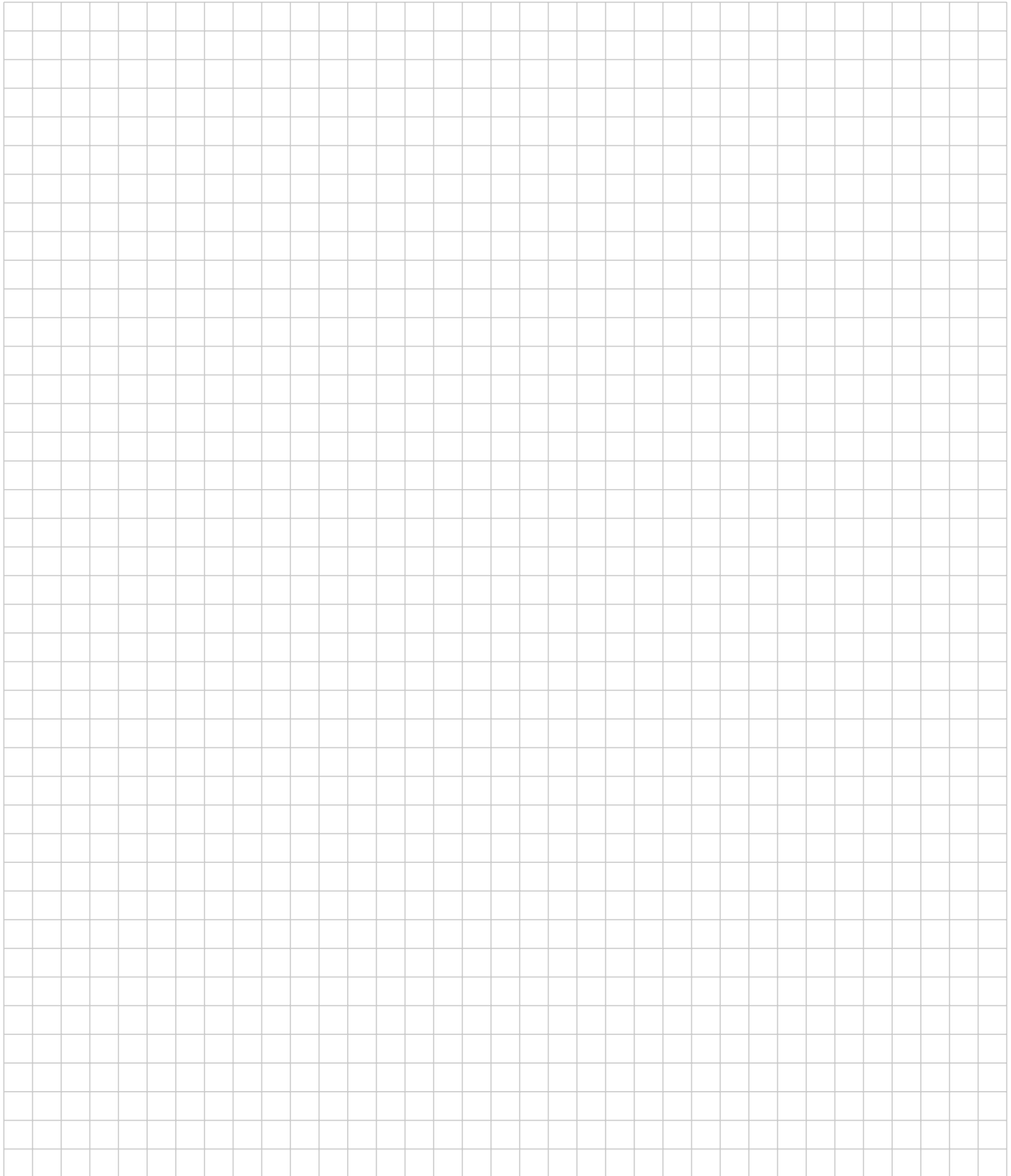


All the following questions are about *counting*. For each of the questions below an explanation/derivation is not required; you only need to state the final answer. Please give an exact number as final answer (i.e. don't just leave your answer as a counting equation).

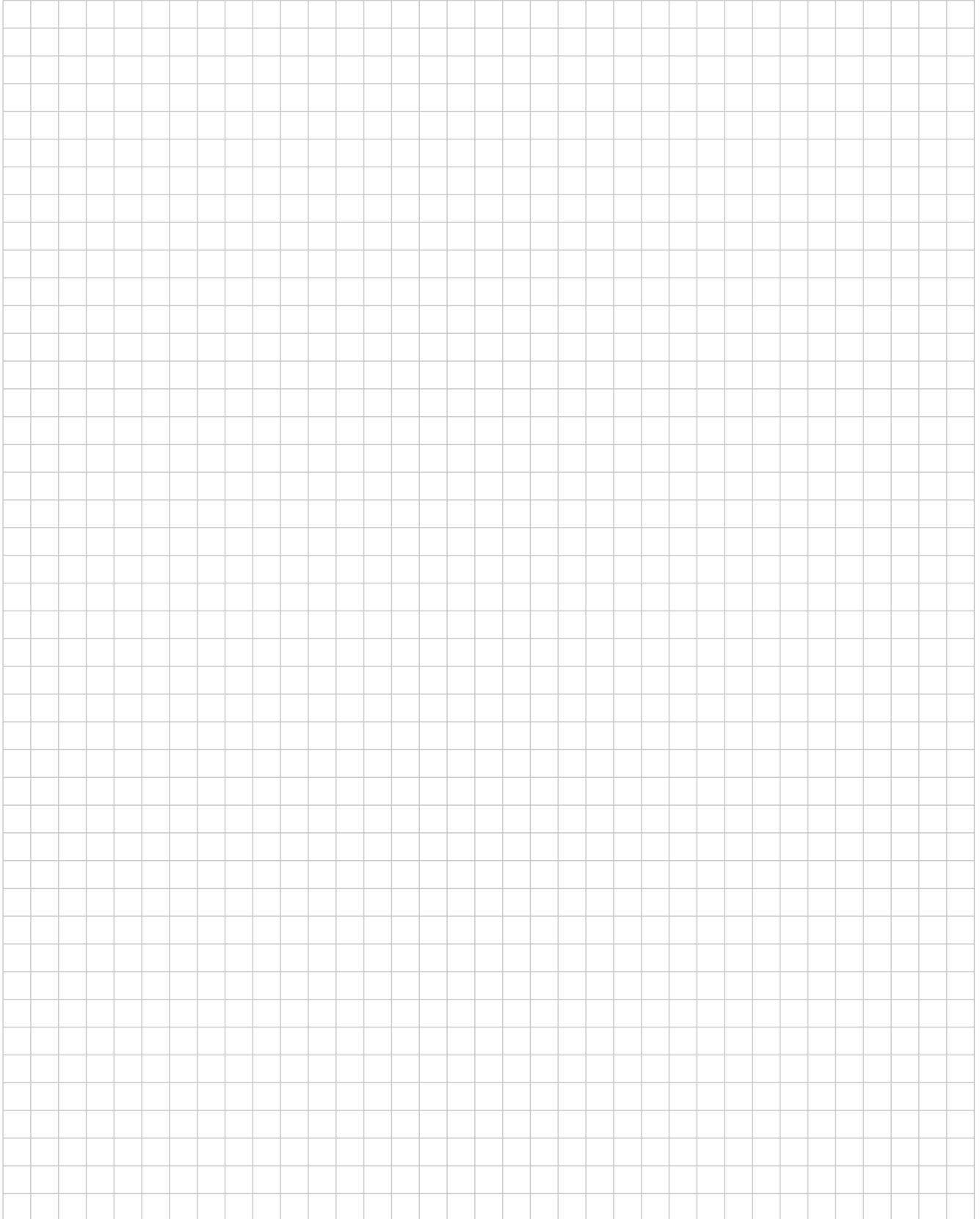
Question 6

Prove or disprove the following statements.

6p **6a** $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})((x+1)^2 - x^2 = 2y - 1).$



6p **6b** $(\forall x \in \mathbb{Z})((x^2 \text{ is divisible by } 3) \Rightarrow (x \text{ is divisible by } 3)).$



Question 7

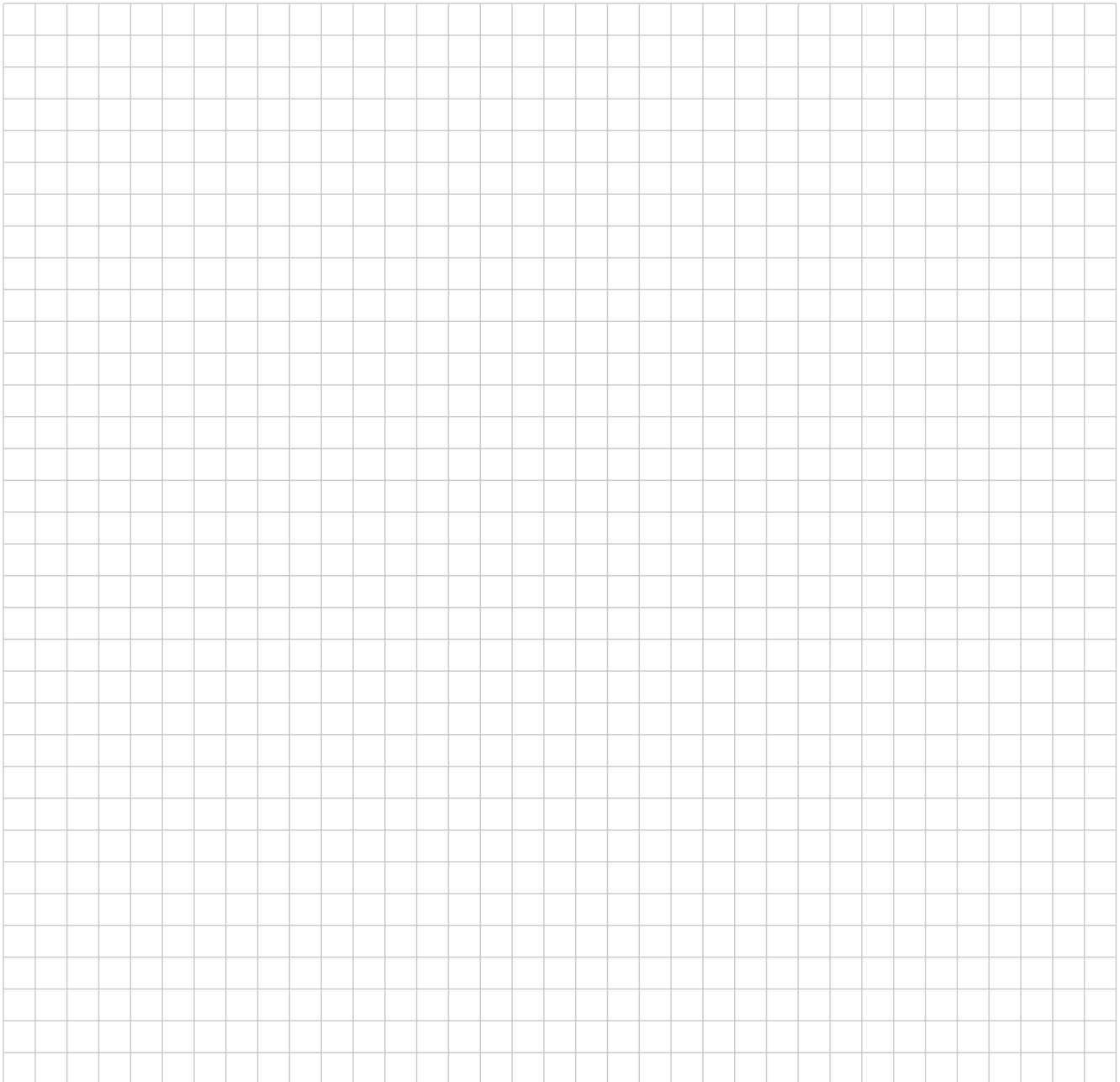
Let $f : \mathbb{R} \setminus \{-1, 1\} \rightarrow \mathbb{R}$ be the function defined as follows:

$$f(x) = \frac{1}{x^2 - 1}.$$

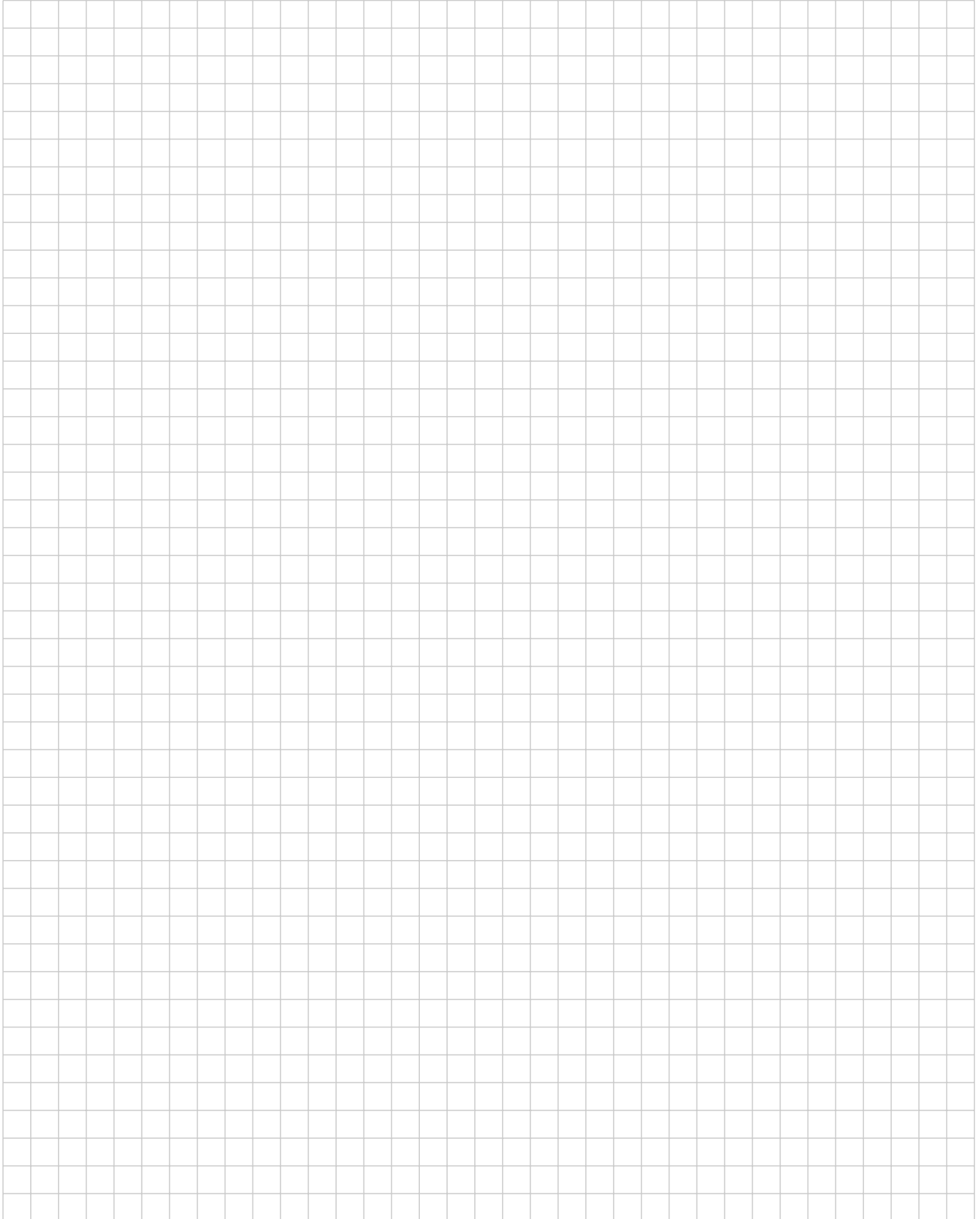
Let $\mathbb{R}^+ = \{x \in \mathbb{R} : x \geq 0\}$ and let $g : \mathbb{R}^+ \setminus \{0\} \rightarrow \mathbb{R}$ be the function defined as follows:

$$g(x) = \frac{1}{x^2}.$$

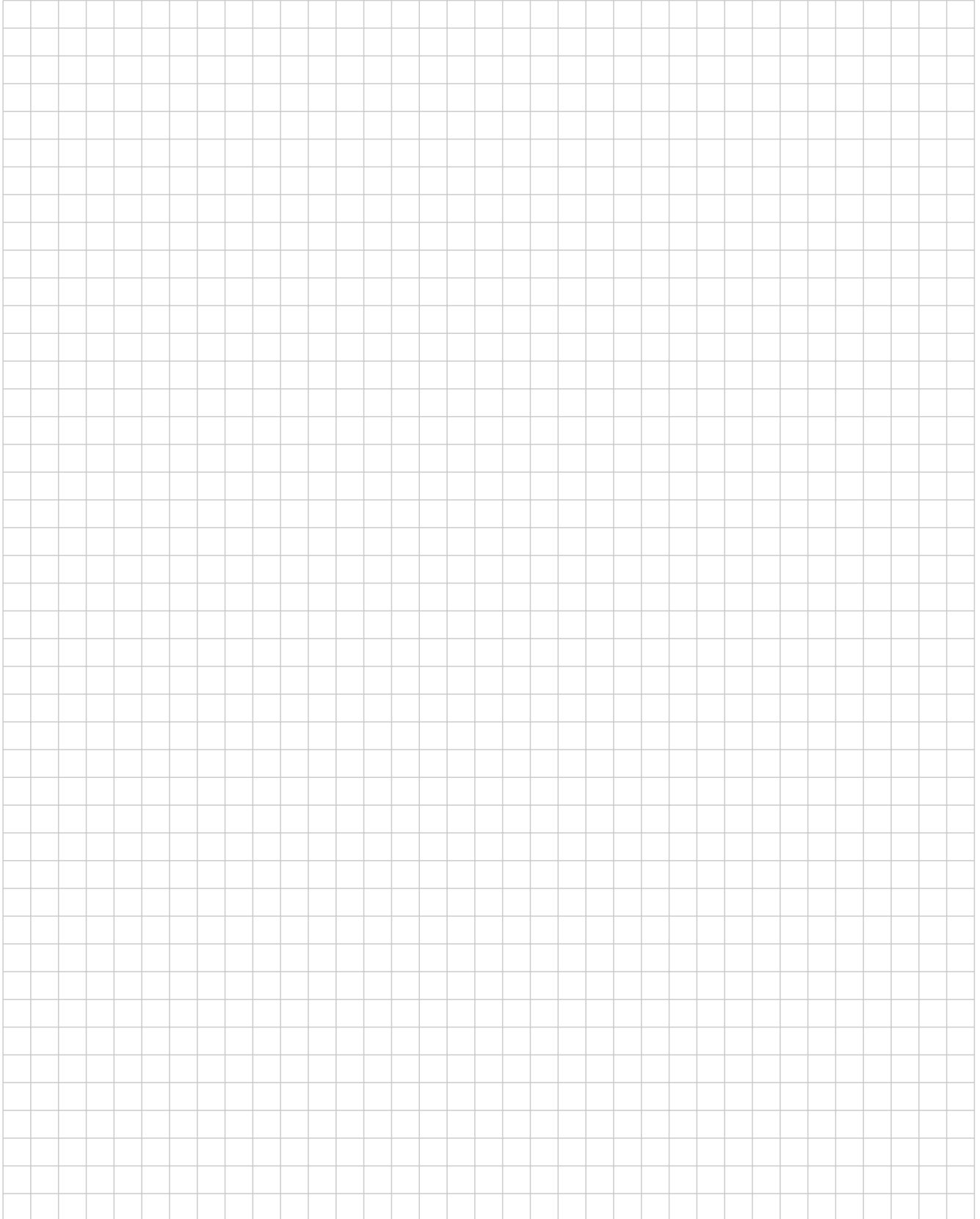
3p **7a** Prove or disprove that f is injective.



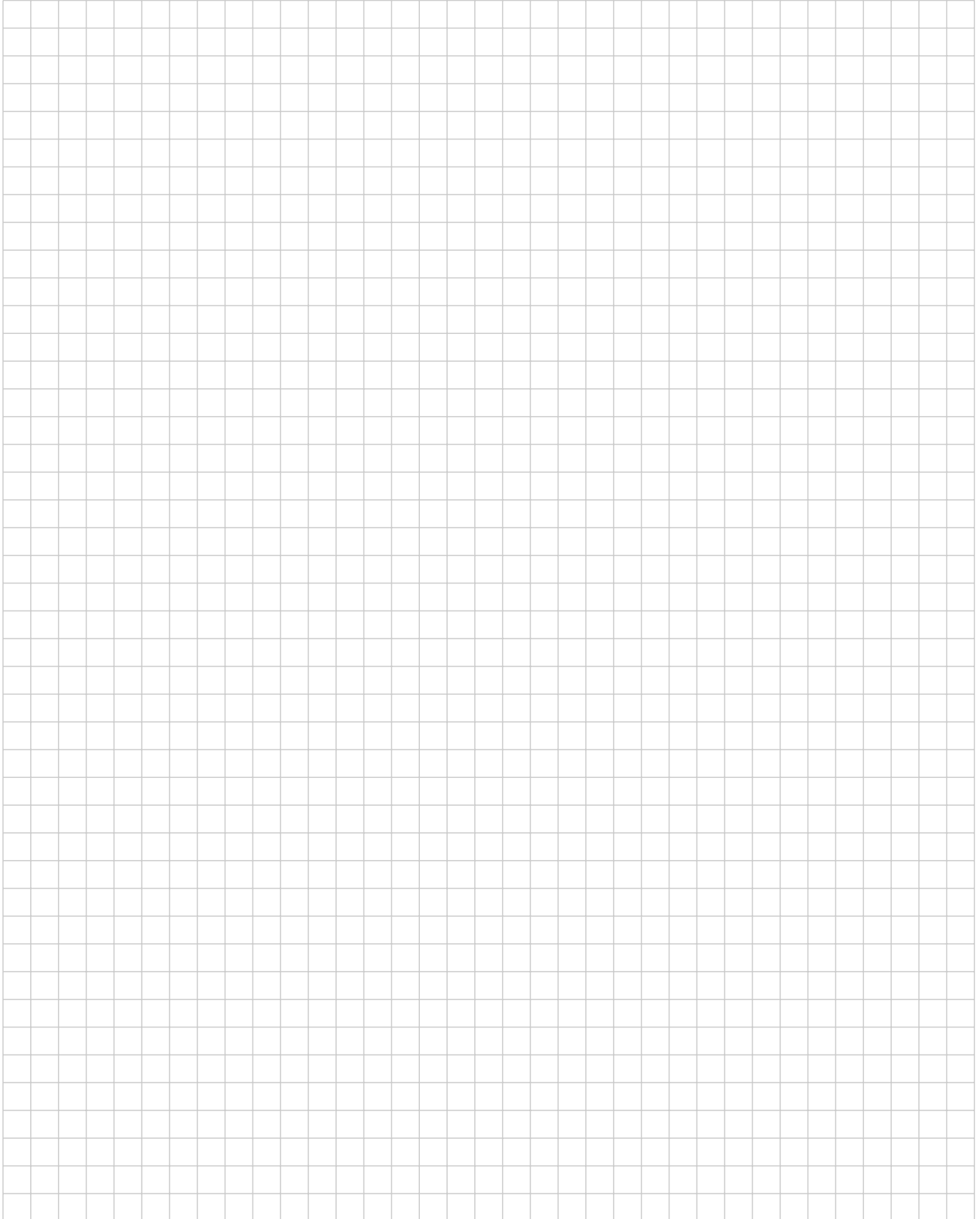
3p **7b** Prove or disprove that f is surjective.



3p **7c** Prove or disprove that g is injective.



3p **7d** Prove or disprove that g is surjective.



Question 8

True or False? For each of the statements below: state whether the statement is true or false. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

- 1p **8a** $\{1, 2\} = \{2, 1\}$
☐ a True ☐ b False
- 1p **8b** $(1, 2) = (2, 1)$
☐ a True ☐ b False
- 1p **8c** $\mathbb{P}(\{\{a, b, c\}\})$ contains eight elements
☐ a True ☐ b False
- 1p **8d** $\{1\} \subseteq \{\{1\}\}$
☐ a True ☐ b False
- 1p **8e** $\{1\}, \{1, 2\}$ and $\{3\}$ are three different subsets of $\{1, 2, 3\}$
☐ a True ☐ b False
- 1p **8f** $\{\{1\}, \{1, 2\}, \{3\}\}$ forms a partition of $\{1, 2, 3\}$
☐ a True ☐ b False

Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

