DKE Numerical Mathematics 2020/2021
Resit Exam Questions

Programme: Bachelor Data Science and Knowledge Engineering

Course codes: KEN1540

Examiners: Dr. P.J. Collins, Dr. K. Staňková

Date/time: UNKOWN

Format: Closed book exam

Allowed aides: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators, formula sheet (provided).

Instructions to students:

- The exam consists of 7 questions on 1 pages (excluding the 1 cover page(s)).
- You can obtain maximally 80 points for this exam.
- Fill in your name and student ID number on the top-right corner of each side of paper you submit.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in with your answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- **Good luck!**

The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	Total
Points:	14	14	10	12	10	12	8	80
Score:								

1. (14 points) Explain the difference between the error and the residual of an approximate solution $\tilde{x}$ to the equation $f(x) = 0$.

   Apply one step of Newton's method to estimate the root of $f(x) = x^3 - 2x - 1$ in $[0, 2]$, starting at the midpoint of the interval. Compare your result with that obtained by first applying one step of the bisection method, and then applying Newton's method starting at the midpoint of the new interval.

2. (14 points) Use two steps of Heun's third order method to estimate the solution of the initial value problem $\dot{y} = y/t - 1/y$, $y(1.0) = 2.400$ up to time $t = 1.5$. You should aim to compute to solution using Heun's method to an accuracy of at least 3 decimal places, and use sufficient precison in your working to do this.

   Compare your answer with the exact solution, which has $y(1.5) = 4.24607313\,(8\,\mathrm{dp})$. What would you expect the absolute error to be if you were to use Heun's method with 10 steps?

3. (10 points) Use divided differences to compute the cubic polynomial interpolating the following data:

x_i	1.3	1.7	1.0	2.0
$f(x_i)$	0.600	0.299	0.727	0.174

   Estimate the value of $f(x)$ when $x = 1.5$.

4. (12 points) Use the trapezoid rule with $n = 4$ partitions to approximate

$$\int_{0.4}^{1.2} \frac{x}{1 + x^3}\,dx.$$

   Use the trapezoid rule error estimate to estimate the error on each of the intervals $[0.4\!:\!0.8]$ and $[0.8\!:\!1.2]$. What is your estimate of the total error? On which interval should you subdivide in order to reduce the error the most?

5. (10 points) The least-squares approximation to a function f is given by $q_n(x) = \sum_{k=0}^{n} c_k P_k(x)$ where the P_k are the Legendre polynomials, and the c_k are given by

k	0	1	2	3	4	5
c_k	0.5536	−0.4536	0.0807	0.0454	−0.0354	0.0099

   Use the recurrence relation to evaluate $P_k(x)$ for $k = 0, 1, \ldots, 4$ for $x = 0.5$, and hence compute $q_4(x)$.

   Compute the total square error $\int_{-1}^{+1} (q_2(x) - f(x))^2\,dx$, assuming $\int_{-1}^{+1} f(x)^2 dx = 0.753574\,(6\,\mathrm{dp})$.

6. (12 points) Let

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 8 \end{pmatrix}, \quad \mu^{(0)} = 3, \quad \mathbf{x}^{(0)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

   Use two steps of the inverse power method to estimate the smallest eigenvalue of A and its corresponding eigenvector.

7. (8 points) Consider the following equation for a forced dampted pendulum:

$$m\ddot{x} + \delta\dot{x} + k\sin(x) = A\cos(\omega t)$$

   Show how to solve this differential equation in Matlab, including writing the code you would use.