DKE/MSP Numerical Mathematics 2018/2019
Exam Questions

— Do not turn this page before the official start of the exam! —

First Name, Surname: ______________________________

Student ID: ______________________________

Programmes: Bachelor Data Science and Knowledge Engineering / Maastricht Science Programme

Course codes: KEN1540 / MAT3005

Examiner: Dr. P.J. Collins

Date/time: Monday June 3^{rd}, 2019, 9.00-12.00h

Format: Closed book exam

Allowed aides: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators, formula sheet (provided).

Instructions to students:

- The exam consists of 8 questions (DKE) or 9 questions (MSP) on 9 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Answer every question at the reserved space below the questions. If you run out of space, continue on the back side, and if needed, use the extra blank page.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- **Good luck!**

The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	8	9 MSP	Total DKE/MSP
Points:	8	10	8	6	12	12	12	12	10	80 / 90
Score:										

1. (8 points) Explain how the result of subtracting two almost-equal quantities can result in a loss of (relative) accuracy using rounded arithmetic.

   Consider the polynomial

   $$p(x) = 1.18 \times x^2 - 5.82 \times x + 1.93 = 0 \; .$$

   Use decimal arithmetic rounded to 3 significant figures (i.e. round after every operation) to evaluate p at $x = 4.82$. Compare your results to the exact answer 1.291832, giving the relative error.

   Identify the step(s) in which your calculation lost accuracy. How could you re-write p to improve the accuracy of the calculation?

2. (10 points) Explain the notion of a *bracket* for the root of a function f.

   Use the secant method starting with $p_0 = 3$, $p_1 = 2$ to find the root of $f(x) = x^3 - x - 9$ to within an estimated accuracy of 10^{-1}. (You should stop computing if you have not achieved sufficient accuracy after 3 steps.) State your estimate of the error, and the best bracket for the root you have found.

3. (8 points) Consider the initial value problem

$$\frac{dy}{dt} = t/y - 1; \quad y(0) = \tfrac{3}{5}.$$

Use two steps of Heun's 2nd-order method to approximate y at $t = 0.5$.

Compare your answer to the exact value $y(0.5) = 0.40402385$ (8dp), giving the absolute error. Roughly what would you expect the error to be if you were to use 20 steps instead of 2?

4. (6 points) The equation of motion of the *damped pendulum* is given by

$$\ddot{\theta} + \delta\dot{\theta} + \rho \sin(\theta) = 0,$$

where δ, ρ are parameters. Describe how to simulate this system using Matlab, with initial condition $\theta(0) = \pi$ and $\dot{\theta}(0) = 1$, including giving the code you would write.

5. (12 points) Compute the divided differences $f[x_i, \ldots, x_j]$ for the following data.

i	0	1	2	3
x	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2
y	1.14	0.24	0.04	0.72

Hence write down the cubic polynomial interpolation p, giving your answer in nested form. Estimate the value of y when $x = 1\frac{1}{2}$.

How would you use builtin Matlab commands to compute the polynomial interpolation to the data, and to evaluate the interpolant?

6. (12 points) Use the trapezoid rule with $n = 4$ subdivisions to estimate the integral

$$\int_2^3 \frac{\sin(x)}{x} dx.$$

Use the error estimate formula for the trapezoid rule to estimate the error on $[2, 2\frac{1}{2}]$ and on $[2\frac{1}{2}, 3]$. Obtain an improved value for the integral by subdividing the interval on which you expect the result to be less accurate, and provide an estimate of the total error.

7. (12 points) Let $f(x) = 1/(3 + \cos(x) + 2\sin(x))$, which takes the following values:

x	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0.500	1.138	1.000	0.436	0.250	0.195	0.200	0.270	0.500

Is this function either even or odd? If so, how does this property affect the Fourier coefficients?

Compute the Fourier coefficients a_i, b_i for $i \leq 2$ using the discrete Fourier transform, and write down the Fourier approximation s_2.

Use your Fourier coefficients to compute $\int_{-\pi}^{\pi} s_2(x)^2 \, dx$. How would you expect this to compare with $\int_{-\pi}^{\pi} f(x)^2 dx$?

8. (12 points) Let

$$A = \begin{pmatrix} 3 & 0 & -2 \\ 0 & 4 & -1 \\ 1 & 2 & 5 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}.$$

Is A diagonally-dominant? What does this tell you about the convergence of the Gauss-Seidel method? Apply two steps of the Gauss-Seidel method find an approximate solution $\mathbf{x}_2$ of $A\mathbf{x} = b$. What is the residual of $\mathbf{x}_2$?

9. (10 points) **MSP Students ONLY**

Let

$$A = \begin{pmatrix} 4 & -2 \\ -1 & -3 \end{pmatrix}, \quad \mathbf{x}_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Let $\mu = \mathbf{x}_0^T A \mathbf{x}_0$, an approximation to an eigenvalue of A. Apply one step of the inverse power method to obtain an approximation to the eigenvalue of A closest to μ, and the corresponding eigenvector.