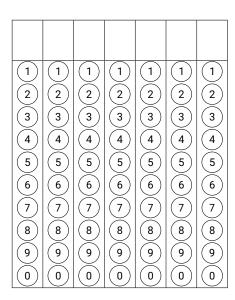
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Surname, First name

KEN1540 Numerical Mathematics

KEN1540 Exam



Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN1540

Examiners: Dr. Ir. Martijn Boussé and Dr. Pieter Collins

Date/time: Monday 5 June 2023; 13:00-15:00

Format: Closed Book Exam

Allowed aids: DACS-approved calculator; Formula sheet (provided)

Instructions to students:

- The exam consists of 7 questions on 22 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- · Answer every question in the reserved space below the question. Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

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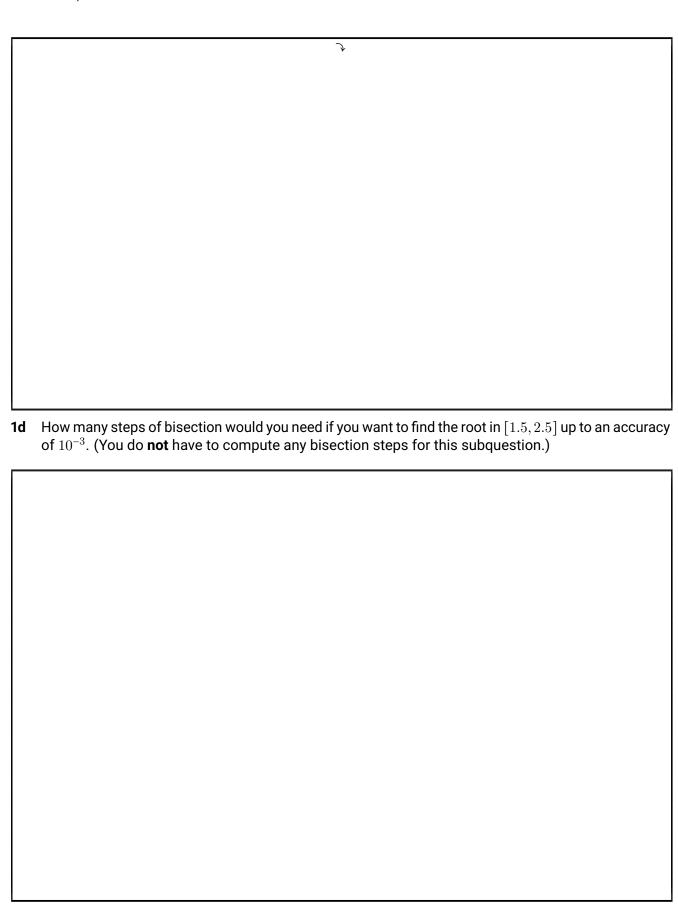
Computer Arithmetic and Algebraic Equations Consider the polynomial $p(x) = 1.03x^4 - 5.34x^2 - 0.0156x + 2.78$. Direct evaluation of p(x) at x = 1.51 using 3-digit rounded arithmetic gives the value -4.08.

1a	Evaluate $p(x)$ at x = 1.51 in nested (Horner) form using 3-digit rounded arithmetic. Give a detailed answer.



1b	Compute the absolute and relative error for direct evaluation and evaluation using nested (Hoform at $x = 1.51$, given that the exact value is -4.05446831 (8 dp). Which method performs be
1c	Perform two steps of bisection to find the root of $p(x)$ in $[1.5, 2.5]$. Then, compute the residuthe estimate in the second step.
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1c	

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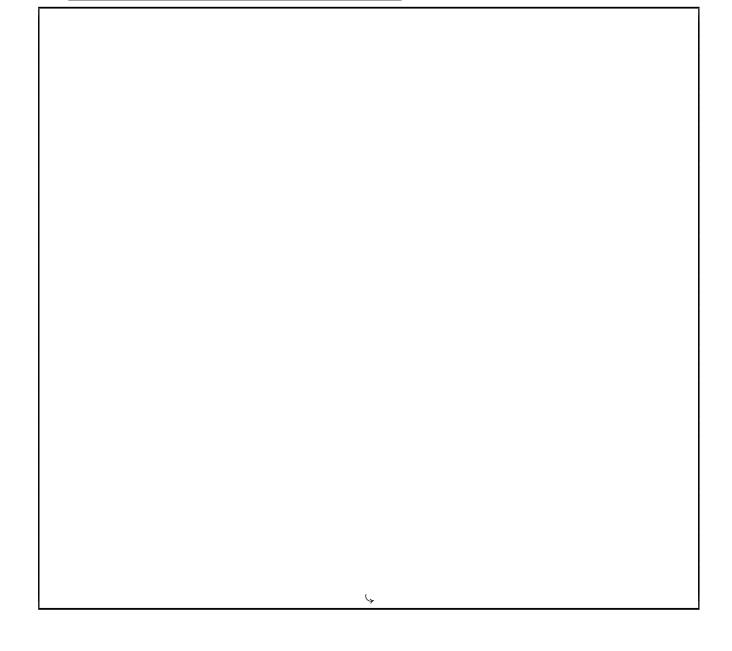
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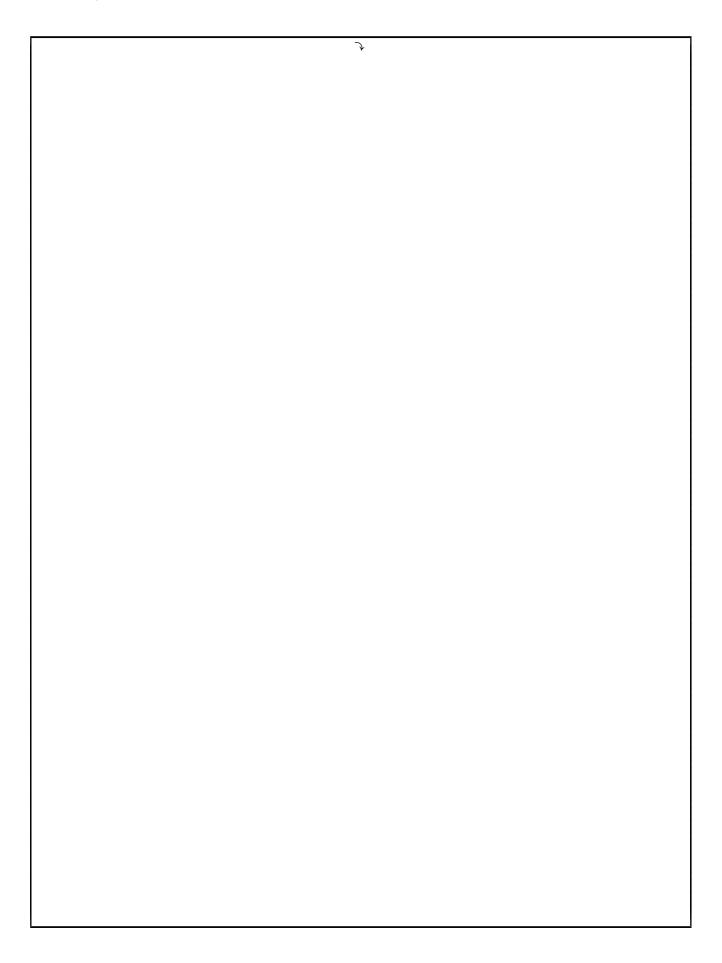
Differential Equations

Consider the initial value problem $y' = \cos(ty) - t^2$ with y(0) = 0.0000.

9p 2a Use a predictor-corrector method to find the solution for $t \in [0, 1.5]$ and h = 0.5. Use the two-stage Adams-Bashforth method for the prediction step and the most appropriate Adams-Moulton method for the correction step. Bootstrap your calculations using values of $w_1 \approx y(0.5)$ for the most appropriate of the methods given in the table below. Motivate your choices for the Adams-Moulton method and the bootstrapping.

Euler's method	$w_1 = 0.50000000$
2nd-order Runge-Kutta method	$w_1 = 0.45602090$
3rd-order Runge-Kutta method	$w_1 = 0.45614919$
4th-order Runge-Kutta method	$w_1 = 0.45545299$





1р

2p

					s 0.2400 3580 (8 dp)
Roug	Jhly, what would	you expect the	absolute error to b	e if you were to us	se $h = 0.25$ instead?

Polynomial Interpolation

Consider the function f on [0,3] given by $f(x) = \frac{1}{1+x}$.

The nth-order derivative of f(x) is given by $f^{(n)}(x) = \frac{(-1)^n n!}{(x+1)^{n+1}}$.

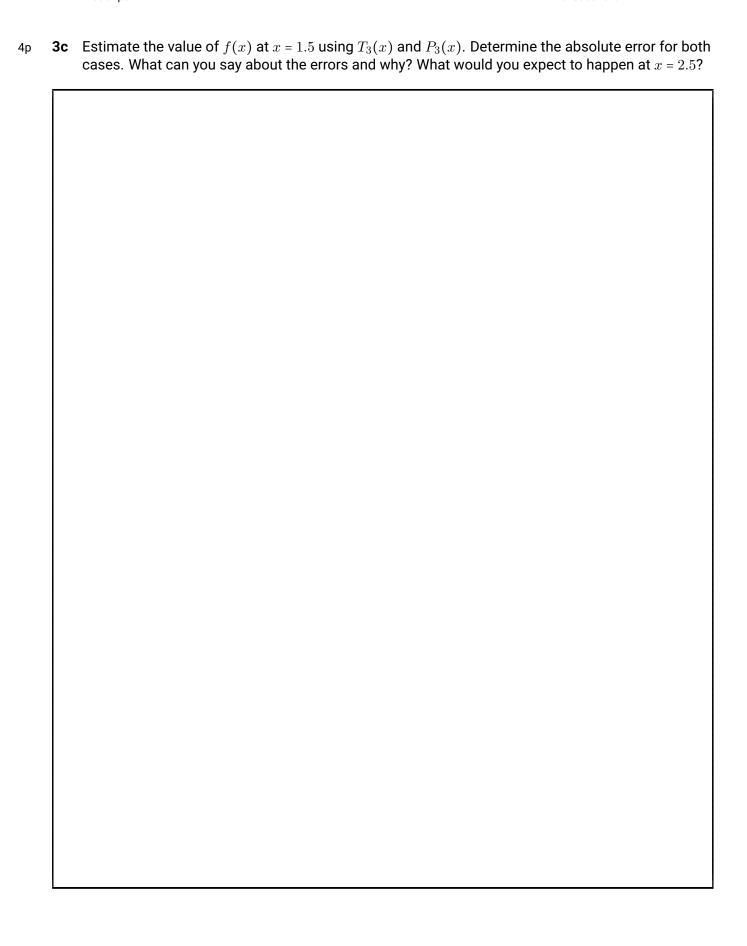
3p **3a** Find the **cubic** Taylor polynomial $T_3(x)$ around $x_0 = 1$ for f(x).





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5p **3b** Use divided differences to compute the **cubic** polynomial $P_3(x)$ interpolating f(x) at the values $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, and $x_3 = 3$. You may assume that $f[x_0, x_1] = -0.5000$, $f[x_1, x_2] = -0.1667$, $f[x_2, x_3] = -0.0833$, $f[x_0, x_1, x_2] = 0.1667$, and $f[x_1, x_2, x_3] = 0.0417$.



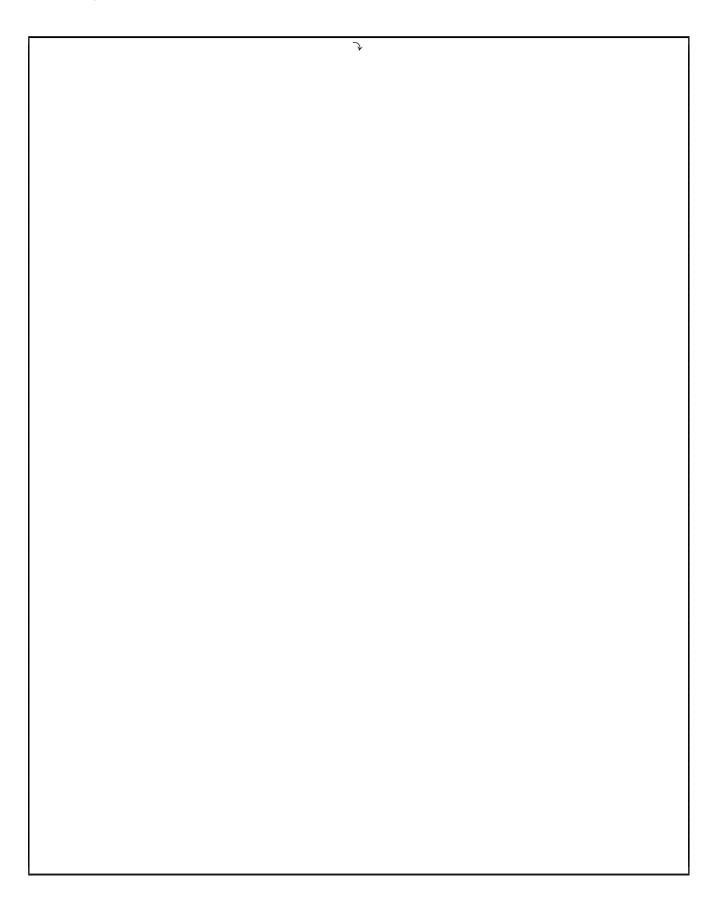
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Numerical Integration & Differentiation

Consider the definite integral $\int_1^3 \sin(x^2) dx$.

10p	4a	Compute the Romburg estimate $R_{3,3}$ using four decimal places for the computed values. You may assume that $R_{2,0}$ = 0.3074 and $R_{3,0}$ = 0.4281 .





2p

4b Give an error estimate for $R_{3,2}$.

Least-Squares Approximation

The least-squares approximation to a function f over [-1,+1] is given by

$$q_n(x) = \sum_{k=0}^{n} c_k P_k(x)$$

where the P_k are the Legendre polynomials.

Suppose the c_k are given by

$k \mid$	0	1	2	3	4	5
c_k	0.3086	0.2193	-0.0842	-0.1233	-0.0267	0.0262

4p **5a** Use the recurrence relation to evaluate $P_k(0.7)$ for $k = 0, 1, \dots, 4$.

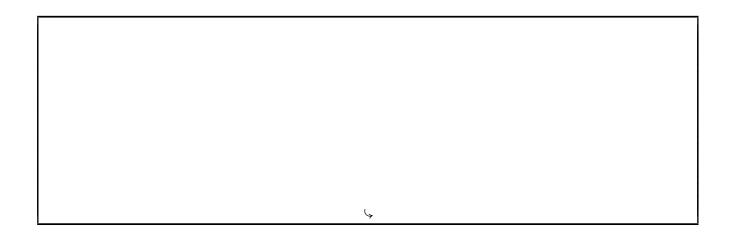


5b Compute $q_4(0.7)$. 2p

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Ea. Evaloin how orthogone	Pr. C.I D : .	 	

5c Explain how orthogonality of the P_k gives rise to a simple formula for the square integral Зр

$$\int_{-1}^{+1} q_n(x)^2 \, \mathrm{d}x.$$

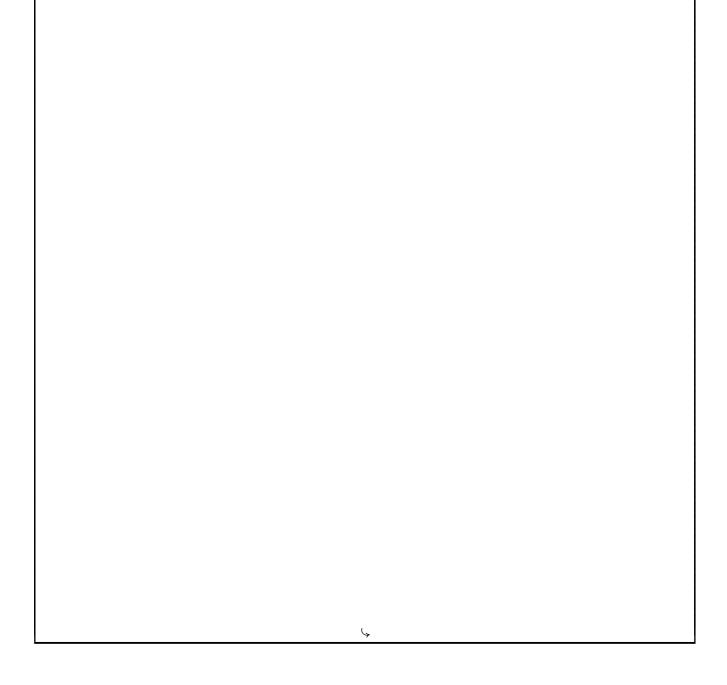


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Linear Algebra

8p **6a** Use two steps of the Gauss-Seidel method to estimate the solution of the equation $A \mathbf{x} = \mathbf{b}$, starting at $\mathbf{x}^{(0)}$.

$$A = \begin{pmatrix} 5 & 2 & 0 \\ 3 & 6 & -1 \\ -2 & 3 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}, \quad \mathbf{x}^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}.$$



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	What is the difference between the command the residual of an annual state of the s
6D	What is the difference between the \textit{error} and the \textit{residual} of an approximate solution $\tilde{\mathbf{x}}$ to the linear
	algebraic equation $Ax = b$?

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Compute the norm of the	e residual of your ansv	wer to .	

2p

Modelling and Matlab

Consider the following equation for the forced Duffing oscillator:

$$\ddot{x} + \delta \dot{x} + \alpha x + \beta x^3 = A \cos(\omega t)$$

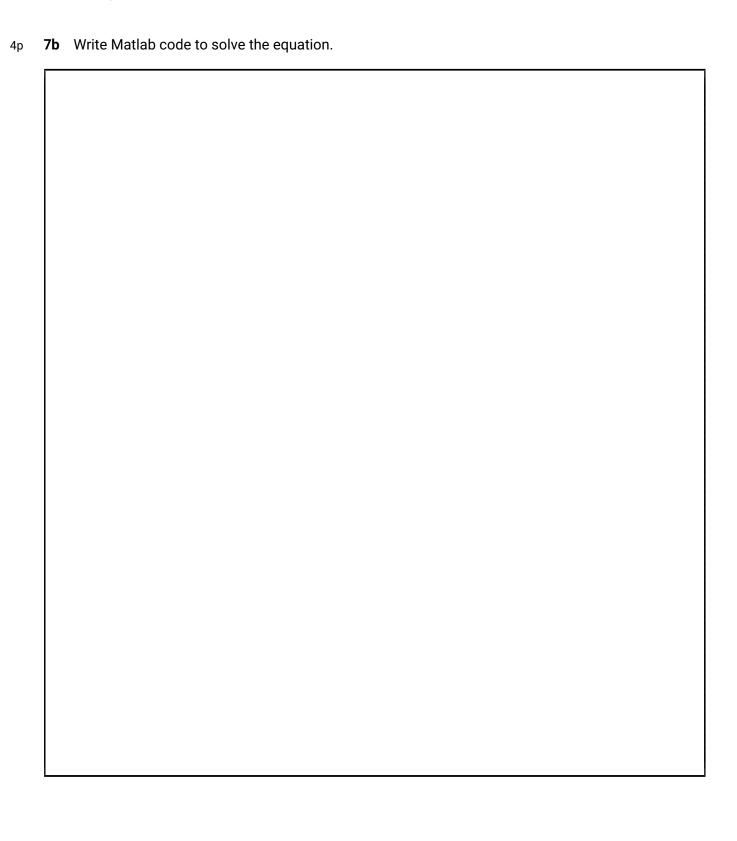
with parameter values

$$\delta = 0.5, \ \alpha = 0.07, \ \beta = 0.1, \ A = 2.3, \ \omega = 1.9.$$

and initial conditions x(0) = 2, $\dot{x}(0) = 1$.

4p **7a** Show how to re-write this second-order equation as a system of first-order equations.

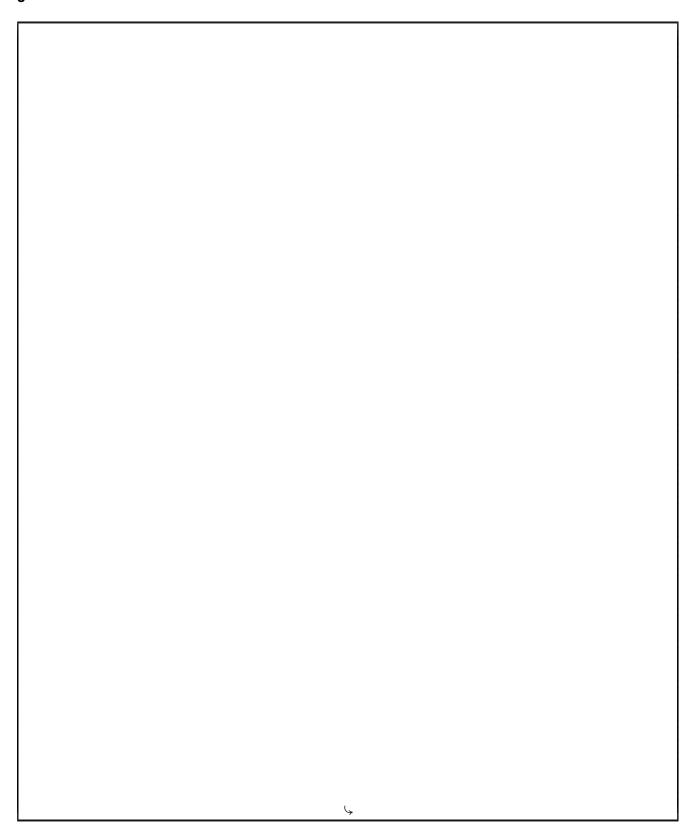




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Extra paper

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