DKE Numerical Mathematics 2019/2020
Resit Exam Questions

Programme: Bachelor Data Science and Knowledge Engineering

Course codes: KEN1540

Examiners: Dr. P.J. Collins, Dr. G. Stamoulis

Date/time: Wednesday July 1st, 2020, 09.00-11.00h

Format: Closed book exam

Allowed aides: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators, formula sheet (provided).

Instructions to students:

- The exam consists of 7 questions on 2 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on every page of answers you submit.
- Number each page of answers you submit in the top left corner.
- Answer every question on a separate piece of paper. Do not mix the answers on different exam sub-questions.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- If you think a question is ambiguous, or even erroneous, explain this in detail in your answer.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.

For on-site exams:

- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- The use of pencils is not allowed.

For online proctored exams:

- The scan / photographs of your submitted answers must be readable.
- Reserve space for your ID in the top right corner of every page of answers you submit.

- Success!

1. (12 points) Apply one step of Newton's method to estimate the root of $f(x) = x - \cos(2x)$ in $[0, 4]$, starting at the midpoint of the interval. Compare your result with that obtained by first applying one step of the bisection method, and then applying Newton's method.

2. (12 points) Use the three-stage Adams-Bashforth method with a step-size of $h = 0.25$ to estimate the solution of the initial value problem $\dot{y} = 1 - t/y$, $y(1) = 1.200$ up to time $t = 2$. Bootstrap your calculation using values of $w_1 \approx y(1.25)$ and $w_2 \approx y(1.5)$ for the most appropriate of the methods below, giving a reason for your answer.

1st-order Euler:	$w_1 = 1.24166667$,	$w_2 = 1.23998881$
2nd-order Ralston:	$w_1 = 1.21974925$,	$w_2 = 1.18718854$
3rd-order Heun:	$w_1 = 1.21830794$,	$w_2 = 1.18324169$
4th-order Runge-Kutta:	$w_1 = 1.21821660$,	$w_2 = 1.18293646$

3. (10 points) Use divided differences to compute the cubic polynomial interpolating the following data:

x_i	1.2	1.4	1.1	1.5
$f(x_i)$	0.600	0.299	0.727	0.174

Estimate the value of $f(x)$ when $x = 1.3$.

4. (14 points) Use the most accurate three-point formulae available to complete the following table:

x	$f(x)$	$f'(x)$	$\sqrt{1 + f'(x)^2}$
0.0	0.85714	-0.02212	1.00681
0.25	0.82759	-0.21428	1.04745
0.5	0.75000	-0.35788	1.06800
0.75	0.64865		
1.0	0.54545		

Use the trapezoid rule to estimate $\int_0^1 \sqrt{1 + f'(x)^2}\, dx$.

Explain the difference between *roundoff* and *truncation* errors in a calculation. At which point in the calculation do you expect significant *roundoff* errors to occur? What would you expect to happen to the truncation error of your calculuation if you were to take twice as many interpolation points?

5. (12 points) Explain the significance of using an orthogonal basis to compute the weighted least-squares polynomial approximation to a function.

The least-squares Chebyshev approximation to a function f is given by $p_n(x) = \sum_{k=0}^{n} c_k T_k(x)$ where the T_k are the Chebyshev polynomials, and the c_k are given by

k	0	1	2	3	4	5
c_k	0.5689	-0.4346	0.049650	0.0308	-0.0191	0.0045

Use the recurrence relation to evaluate $T_k(x)$ for $k = 0, 1, \ldots, 4$ for $x = 0.2160$, and hence compute $p_4(x)$.

Compute the weighted square error $\int_{-1}^{+1} w(x)(p_k(x) - f(x))^2\, dx$ for p_2, assuming $\int_{-1}^{+1} w(x) f(x)^2 dx = 1.319261\,(6\,\text{dp})$, where $w(x) = 1/\sqrt{1 - x^2}$.

6. (12 points) Use two steps of the Gauss-Seidel method to estimate the solution of the equation $Ax = b$, starting at $x^{(0)}$.

$$A = \begin{pmatrix} 3 & 1 & -2 \\ -2 & 2 & -1 \\ 1 & -1 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix}, \quad x^{(0)} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

Compute the 2-norm $\|\cdot\|_2$ of the residual of $x^{(2)}$. Using the fact that $\|A\|_2 = 5.0$ and $\|A^{-1}\|_2 = 0.67$, provide an upper bound on the norm of the error.

7. (8 points) Consider the following equation for a particle in a potential well:

$$m\ddot{x} + \delta\dot{x} + kx(1 + bx^2) = A\cos(\omega t)$$

Show how to solve this system of differential equations in Matlab, including writing the code you would use.