Maastricht University

Department of Data Science and Knowledge Engineering

# Logic 2020/2021
# Resit Exam Questions

*— Do not turn this page before the official start of the exam! —*

First Name, Surname: ______________________________

Student ID: ______________________________

**Program:** Bachelor Data Science and Knowledge Engineering

**Course code:** KEN1530

**Examiners:** dr. Otti D'Huys, dr. ir. ing. Nico Roos

**Date/time:** July 1$^{st}$, 2021, 9:30-11:30h

**Format:** Closed book exam.

**Allowed aides:** Pens.

**Instructions to students:**

General instructions:

- The exam consists of 10 questions on 12 pages (excluding the 1 cover page(s)).
- Answer every question on a separate piece of paper. Do not mix the answers on different exam questions.
- **Number each page of answers you submit in the top left corner.**
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- If you think a question is ambiguous, or even erroneous, explain this in detail in your answer.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.

For on-site exams:

- Fill in your name and student ID number on every page of answers you submit.
- Please make sure that communication devises and watches are not within reach.
- The use of pencils is not allowed.

For online proctored exams:

- The scan / photographs of your submitted answers must be readable.
- **Reserve space for your ID in the top right corner of every page of answers you submit.**

**Success!**

**The following table will be filled by the examiner:**

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Points: | 10 | 10 | 15 | 10 | 10 | 10 | 10 | 5 | 10 | 10 | 100 |
| Score: | | | | | | | | | | | |

**Question 1** (10 points)

- Alice likes running or Bob likes cycling.
- If Chris likes dancing, then Bob does not like cycling.
- It is not the case that ( Alice likes running and Chris likes dancing ).
- If Bob likes cycling and Alice does not like running, then Chris likes dancing.

Determine the answers of the following questions using a truth table. (Draw the full table!) Answer with: *Yes*, *No* or *Unknown*!

- Does Alice like running?
- Does Bob like cycling?
- Does Chris like dancing?

Use the atomic propositions $a$, $b$ and $c$, use the truth-value 0 representing false and the truth-value 1 representing true, and enumerate the valuations of $a$, $b$ and $c$ as binary numbers from 0 to 7.

**Solution:**

| | $a$ | $b$ | $c$ | $a \vee b$ | $c$ | $\rightarrow$ | $\neg$ | $b$ | $\neg$ | $(a \wedge c)$ | $(b$ | $\wedge$ | $\neg$ | $a)$ | $\rightarrow$ | $c$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | | 1 | 1 | | 1 | 0 | | 0 | 1 | | 1 | |
| | 0 | 0 | 1 | 0 | | 1 | 1 | | 1 | 0 | | 0 | 1 | | 1 | |
| | 0 | 1 | 0 | 1 | | 1 | 0 | | 1 | 0 | | 1 | 1 | | 0 | |
| | 0 | 1 | 1 | 1 | | 0 | 0 | | 1 | 0 | | 1 | 1 | | 1 | |
| $\Rightarrow$ | 1 | 0 | 0 | 1 | | 1 | 1 | | 1 | 0 | | 0 | 0 | | 1 | |
| | 1 | 0 | 1 | 1 | | 1 | 1 | | 0 | 1 | | 0 | 0 | | 1 | |
| $\Rightarrow$ | 1 | 1 | 0 | 1 | | 1 | 0 | | 1 | 0 | | 0 | 0 | | 1 | |
| | 1 | 1 | 1 | 1 | | 0 | 0 | | 0 | 1 | | 0 | 0 | | 1 | |

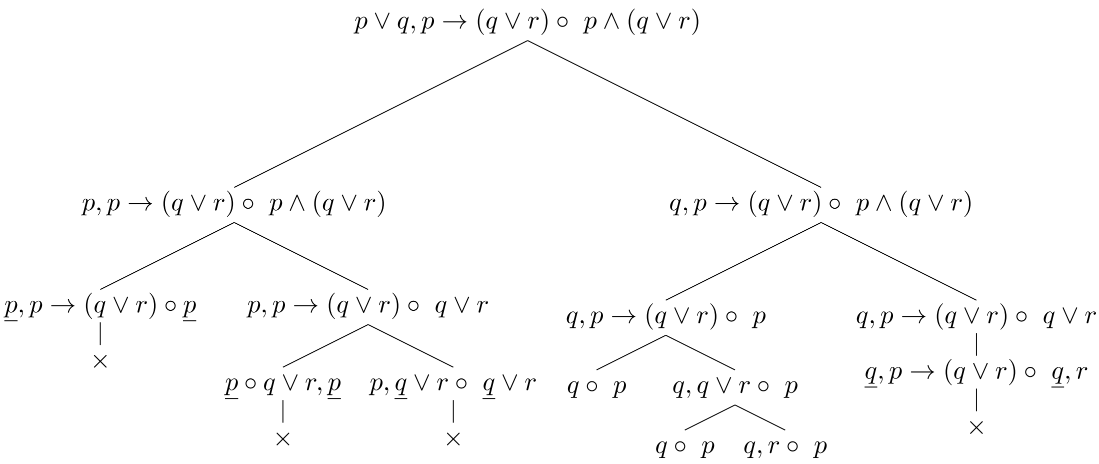
- Does Alice like running? **yes**
- Does Bob like cycling? **unknown**
- Does Chris like dancing? **no**

**Question 2** (10 points)

Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give **all** the counterexamples.

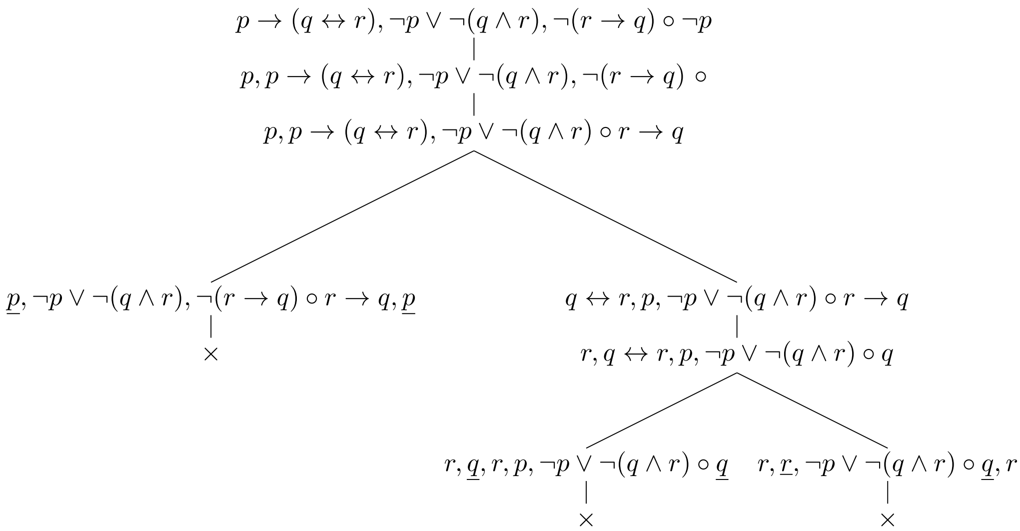
- $\{p \vee q, p \to (q \vee r), \neg(r \wedge p) \to \neg q\} \models p \wedge (q \wedge r)$
- $\{p \to (q \leftrightarrow r), \neg p \vee \neg(q \wedge r), \neg(r \to q)\} \models \neg p$

**Solution:**




Counter example 1: $V(p) = 0$, $V(q) = 1$, $V(r) = 0$.

Counter example 2: $V(p) = 0$, $V(q) = 1$, $V(r) = 1$.



**Question 3** (15 points)

Prove by means of natural deduction:

- From the premises $p \to \neg(r \to q)$ derive $p \to (\neg q \wedge r)$
- From the premise $\neg(\neg p \vee \neg q)$ derive $p \wedge q$

**Solution:**

| | | |
|---|---|---|
| 1 | $\mid \underline{p \to \neg(r \to q)}$ | |
| 2 | $\mid \mid \underline{p}$ | |
| 3 | $\mid \mid \neg(r \to q)$ | $\to$E, 1, 2 |
| 4 | $\mid \mid \mid \underline{\neg(\neg q \wedge r)}$ | |
| 5 | $\mid \mid \mid \mid \underline{r}$ | |
| 6 | $\mid \mid \mid \mid \mid \underline{\neg q}$ | |
| 7 | $\mid \mid \mid \mid \mid \neg q \wedge r$ | $\wedge$I, 5, 6 |
| 8 | $\mid \mid \mid \mid \mid \bot$ | $\bot$I, 4, 7 |
| 9 | $\mid \mid \mid \mid q$ | $\neg$E, 6–8 |
| 10 | $\mid \mid \mid r \to q$ | $\to$I, 5, 9 |
| 11 | $\mid \mid \mid \bot$ | $\bot$I, 3, 10 |
| 12 | $\mid \mid \neg q \wedge r$ | $\neg$E, 4–11 |
| 13 | $\mid p \to (\neg q \wedge r)$ | $\to$I, 2, 12 |

| | | |
|---|---|---|
| 1 | $\mid \underline{\neg(\neg p \vee \neg q)}$ | |
| 2 | $\mid \mid \underline{\neg p}$ | |
| 3 | $\mid \mid \neg p \vee \neg q$ | $\vee$I, 2 |
| 4 | $\mid \mid \bot$ | $\bot$I, 1, 3 |
| 5 | $\mid p$ | $\neg$E, 2–4 |
| 6 | $\mid \mid \underline{\neg q}$ | |
| 7 | $\mid \mid \neg p \vee \neg q$ | $\vee$I, 6 |
| 8 | $\mid \mid \bot$ | $\bot$I, 1, 7 |
| 9 | $\mid q$ | $\neg$E, 6–8 |
| 10 | $\mid p \wedge q$ | $\wedge$I, 5, 9 |

**Question 4** (10 points)

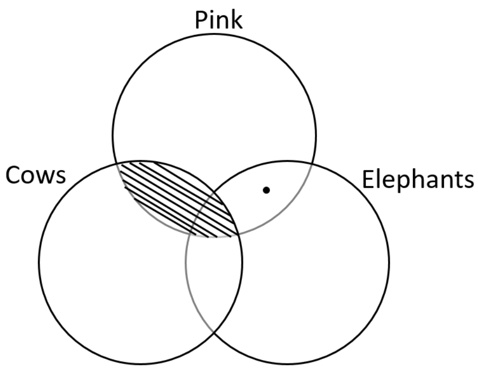
Consider the following two premises of a syllogism:

No cow is pink.
Some elephants are pink.
??

Complete the syllogism with a conclusion that makes it valid.

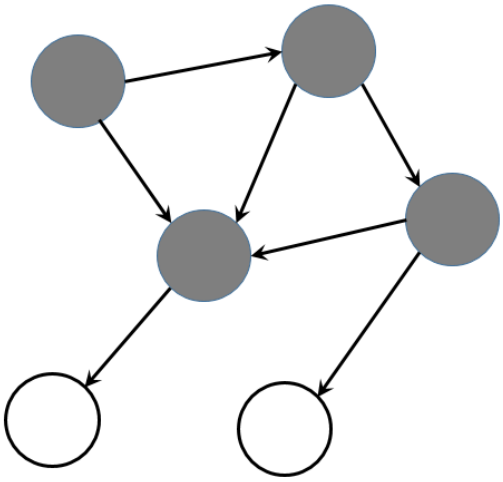
Demonstrate the validity of your syllogism using the method with Venn diagrams.

**Solution:** Some elephants are not cows.



**Question 5** (10 points)

Consider the predicate logic model shown below. The model has a unary predicate $P$ and a binary predicate $S$. Shaded objects have property $P$, a $\rightarrow$ from $a$ to $b$ meant that $Sab$ is true in this model.



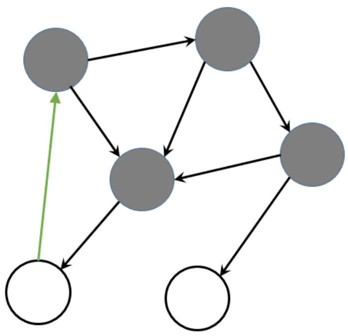
(a) State two closed formulas that are true in this model. Your formulas should include both predicates $P$ and $S$.

(b) Add one pair to $I(S)$ (one arrow to the model representation), such that the statement $\forall x(Px \rightarrow \exists y Syx)$ becomes true.

**Solution:**

(a) Examples include $\forall x(\exists y Sxy \rightarrow Px), \neg\exists x(Px \wedge Rxx), \forall x \forall y((Syx \wedge \neg Px) \rightarrow Py), \exists y \exists x(Sxy \wedge Py \wedge Px)$

(b) Any arrow arriving at the upper left object makes the formula true. An example is below



**Question 6** (10 points)

Are the following formulas a tautology? Show by means of a semantic tableau.

- $\forall x \exists y\,(Ryx \to Rxx)$
- $\forall x\,((\exists y Ryx) \to Rxx)$

**Solution:**

$$\begin{array}{c}
\circ \underline{\forall x \exists y (Ryx \to Rxx)} \\
| \\
\circ^{+} \underline{\exists y (Rya \to Raa)} \\
| \\
\circ \underline{Raa \to Raa} \\
| \\
Raa \circ Raa \\
| \\
\times
\end{array}$$

This statement is a tautology

$$\begin{array}{c}
\circ \underline{\forall x ((\exists y Ryx) \to Rxx)} \\
| \\
\circ^{+} \underline{(\exists y Rya) \to Raa} \\
| \\
\exists y Rya \circ Raa \\
| \\
Rba \odot^{+} Raa
\end{array}$$

This tableau has an open branch: we find a countermodel with two objects $a$ and $b$, with $b$ related to $a$, but $a$ not related to itself.

**Question 7** (10 points)

Prove by means of natural deduction:

- From the premises $\{\forall x(Px \to \exists y Rxy), \forall x(\exists y Ryx \to \neg Px), \exists x Px\}$ derive $\exists x \neg Px$
- From the premises $\{\forall x(Ax \to \neg Bx), \exists x(Bx \wedge Cx)\}$ derive $\exists x(Cx \wedge \neg Ax)$

**Solution:**

| | | |
|---|---|---|
| 1 | $\forall x(Px \to \exists y Rxy)$ | |
| 2 | $\forall x(\exists y Ryx \to \neg Px)$ | |
| 3 | $\exists x Px$ | |
| 4 | $a$ \| $Pa$ | |
| 5 | \| $Pa \to \exists y Ray$ | $\forall$E, 1 |
| 6 | \| $\exists y Ray$ | $\to$E, 5, 4 |
| 7 | \| $b$ \| $Rab$ | |
| 8 | \| \| $\exists y Ryb$ | $\exists$I, 7 |
| 9 | \| \| $\exists y Ryb \to \neg Pb$ | $\forall$E, 2 |
| 10 | \| \| $\neg Pb$ | $\to$E, 9, 8 |
| 11 | \| \| $\exists x \neg Px$ | $\exists$I, 10 |
| 12 | \| $\exists x \neg Px$ | $\exists$E, 6, 7–11 |
| 13 | $\exists x \neg Px$ | $\exists$E, 3, 4–12 |

**Solution:**

| | | |
|---|---|---|
| 1 | $\forall x(Ax \to \neg Bx)$ | |
| 2 | $\exists x(Bx \wedge Cx)$ | |
| 3 | $a$ $\quad Ba \wedge Ca$ | |
| 4 | $\quad Ca$ | $\wedge$E, 3 |
| 5 | $\quad Ba$ | $\wedge$E, 3 |
| 6 | $\quad\quad Aa$ | |
| 7 | $\quad\quad Aa \to \neg Ba$ | $\forall$E, 1 |
| 8 | $\quad\quad \neg Ba$ | $\to$E, 6, 7 |
| 9 | $\quad\quad \bot$ | $\bot$I, 5, 8 |
| 10 | $\quad \neg Aa$ | $\neg$I, 6–9 |
| 11 | $\quad \neg Aa \wedge Ca$ | $\wedge$I, 4, 10 |
| 12 | $\quad \exists x(\neg Ax \wedge Cx)$ | $\exists$I, 11 |
| 13 | $\exists x(\neg Ax \wedge Cx)$ | $\exists$E, 2, 3–12 |

Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

**Question 8** (5 points)

Translate the following sentence into a formula of epistemic logic, using appropriate translation keys.

- Bob does not know whether Chris knows whether Alice likes sailing.

Use the modal operators $\Box_A$, $\Box_B$ and $\Box_C$, and the atomic proposition $a$.

**Solution:**

- $\neg(\Box_B(\Box_C a \vee \Box_C \neg a) \vee \Box_B \neg(\Box_C a \vee \Box_C \neg a))$
  or
  $\neg\Box_B(\Box_C a \vee \Box_C \neg a) \wedge \neg\Box_B \neg(\Box_C a \vee \Box_C \neg a)$

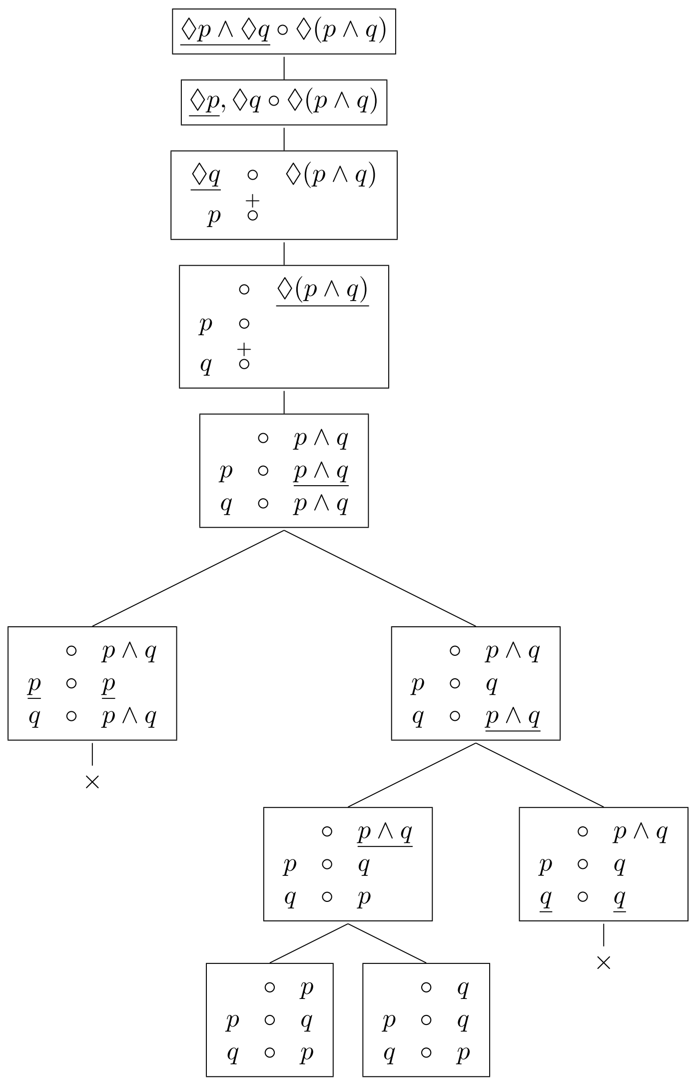
Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

**Question 9** (10 points)

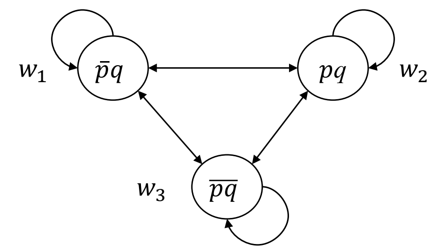
Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give a counterexample.

- $\Diamond p \wedge \Diamond q \models \Diamond(p \wedge q)$
- $\neg\Diamond(p \vee q) \models \Box\neg p \wedge \Box\neg q$

**Solution:**

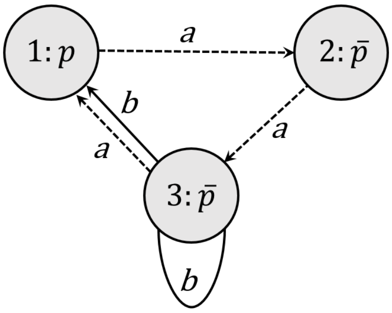



One of the three counter examples described by the open branches:


$w_1$
$\bar{p}q$
$pq$
$w_2$
$w_3$
$\overline{pq}$

**Question 10** (10 points)

Consider the labelled transition system below, with 3 states and actions $a$ and $b$.




(a) In which states are the following formula true?

- $\langle a^* \rangle p$.
- $[b]\neg p$

(b) Give all the elements of the relation defined by the action $(?\neg p; a \cup b)^*$.

**Solution:**

(a) In which states are the following formula true?

- $\langle a^* \rangle p$: true in all states.
- $[b]\neg p$: true in states 1 and 2.

(b) The elements of the relation defined by the action $(?\neg p; a \cup b)^*$ are
$(1,1), (2,2), (3,3), (2,3), (3,1), (2,1)$