Surname, First name

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1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

KEN1410 Linear Algebra
KEN1410 Linear Algebra Exam

a ~~b~~ c d e f → b

a b ~~c~~ d e f → c

● b c ~~d~~ e f → a

Answer multiple-choice questions as shown in the example.

Program: Data Science and Artificial Intelligence
Course code: KEN1410
Examiners: Dr. Marieke Musegaas and Dr. Philippe Dreesen
Date/time: Tuesday 02.04.2024 9h00-11h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.
Instructions to students:

- The exam consists of 10 questions on 16 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1
Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & 4 & -1 & 0 \\ -3 & -6 & 1 & 0 \end{bmatrix}.$$

6p **1** What is the reduced row echelon form of A?

(a) $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(f) None of the above.

Question 2

Consider the following system of linear equations:

$$\begin{aligned} 5x_1 - x_2 + x_3 &= 0, \\ 4x_1 - 3x_2 + 7x_3 &= 0. \end{aligned}$$

6p **2** Which one of the following statements is true?

- a This system has no solution.
- b This system has only the trivial solution.
- c This system has only the solution $x_1 = 4, x_2 = 31, x_3 = 11$.
- d This system has infinitely many solutions.
- e None of the above.

Question 3

Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 2 \\ a \end{bmatrix}$.

6p **3** If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$, then we must have

- a $a = 0$
- b $a = 1$
- c $a = 2$
- d $a = 3$
- e None of the above.

Question 4

Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ x-y+z \end{bmatrix},$$

and let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ x+y \end{bmatrix}.$$

Consider the composed linear transformation $S \circ T : \mathbb{R}^3 \to \mathbb{R}^2$, i.e., first transformation T is applied and after that transformation S is applied.

6p **4** What is the standard matrix for the composed linear transformation $S \circ T$?

(a) $\begin{bmatrix} -1 & 3 & -3 \\ 2 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

(e) None of the above.

Question 5
Consider the following matrix:

$$A = \begin{bmatrix} 3 & 2 & -2 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}.$$

It is given that the inverse of A is as follows:

$$A^{-1} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 1 & 1 & b \end{bmatrix}.$$

10p **5** Determine a and b.

Question 6

Consider the following subset of $\mathbb{R}^3$:

$$W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid a \leq b \leq c \right\}.$$

10p **6** Is W a subspace of $\mathbb{R}^3$? If yes, prove it. If not, explain, why not.

Question 7

6p **7** Which one of the following sets of vectors forms a basis for $\mathbb{R}^3$?

(a) $\left\{\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}\right\}$

(d) $\left\{\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}\right\}$

(e) None of the above.

Question 8
Consider the following matrix:

$$A = \begin{bmatrix} 2 & -4 & 1 & 1 & 5 \\ 3 & -6 & -2 & 5 & -7 \\ 5 & -10 & 3 & 2 & 4 \end{bmatrix}.$$

It is given that A is row equivalent to the following matrix:

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

4p **8a** Determine dim(Nul(A)).

4p **8b** Determine dim(Col(A)).

4p **8c** Determine dim(Row(A)).

4p **8d** Determine whether the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ belongs to Nul(A), i.e., determine whether the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is a solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

4p **8e** Determine a basis for Row(A).

Question 9
Consider the following matrix:

$$A = \begin{bmatrix} 1 & 4 \\ 4 & 7 \end{bmatrix}.$$

8p **9a** Show that A has eigenvalues -1 and 9.

12p **9b** Diagonalize A, i.e. find a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

Question 10

Let $\mathbf{u} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$.

10p **10** Determine two vectors of length 1 that are orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

Extra space
If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

11a

11b

11c

11d