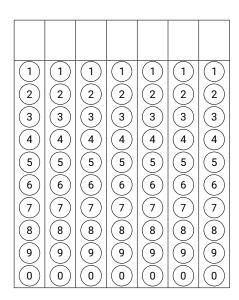
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Surname, First name

**Numerical Mathematics (KEN1540)** 

Exam



**Program:** Bachelor Data Science and Knowledge Engineering

Course code: KEN1540

**Examiners:** Pieter Collins & Martijn Boussé **Date/time:** Monday 30th May 13:00-15:00

Format: Written Exam

**Allowed aids:** DKE-approved calculator; Formula sheet (provided)

**Instructions to students:** 

- The exam consists of 7 questions on 18 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- · You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

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### **Algebraic Equations**

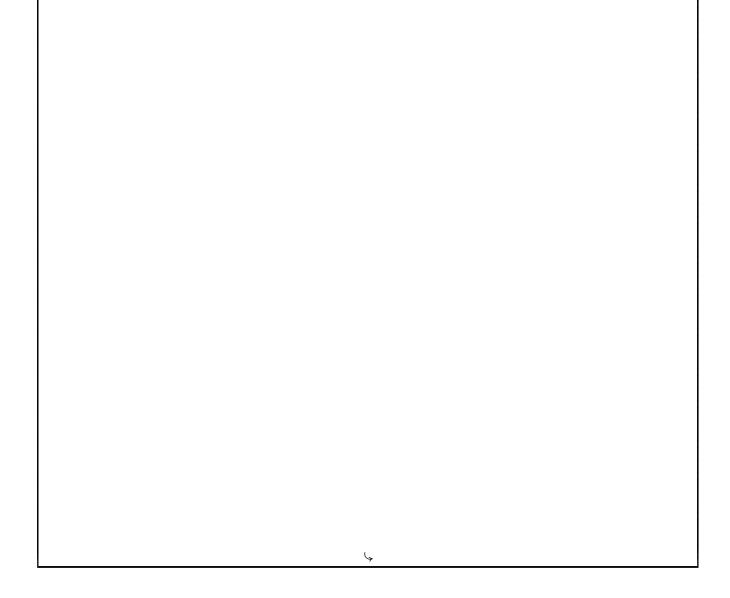
12p **1** Explain the similarity between decimal arithmetic rounded to 4 significant figures and binary double-precision floating-point arithmetic.

Apply one step of the bisection method, followed by one step of Newton's method, to estimate a root of the function

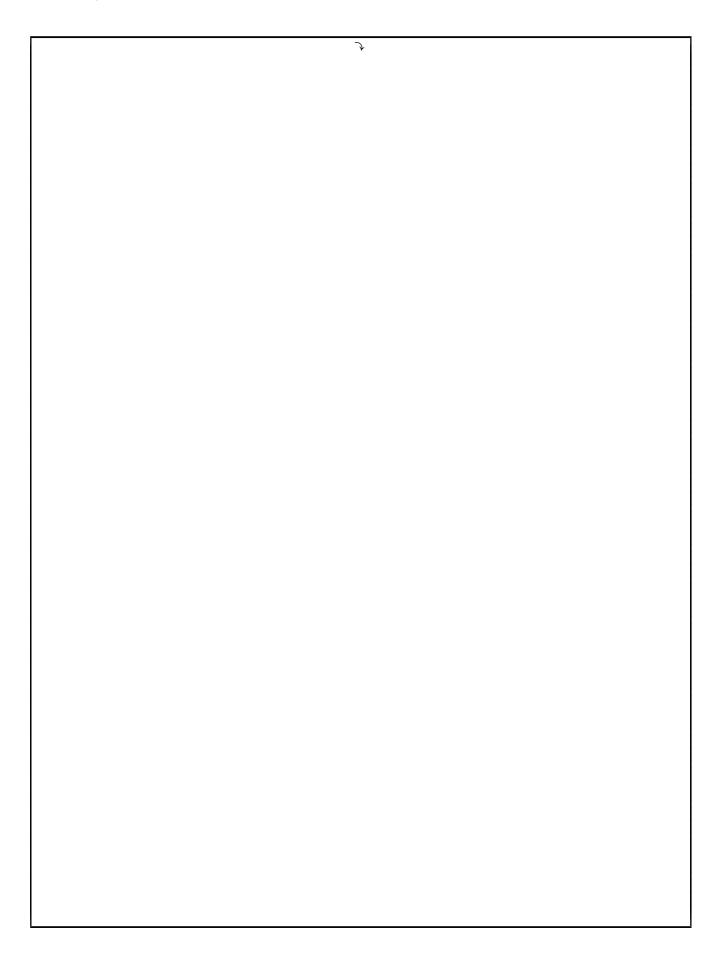
$$f(x) = x(x^2 - 2) - 1$$

in the interval [0,2].

When applying Newton's method, start in the midpoint of the interval under consideration, and use rounded arithmetic to 4 significant figures throughout your calculation.









#### **Differential Equations**

Use the two-stage Adams-Bashforth method to find the solution of the initial value problem  $\dot{y} = 1/(1+ty), y(1) = 2.0000, t \in [0,1.5]$ , with h = 0.5.

Bootstrap your calculation using values of  $w_1 \approx y(0.5)$  for the most appropriate of the methods below, giving a reason for your answer.

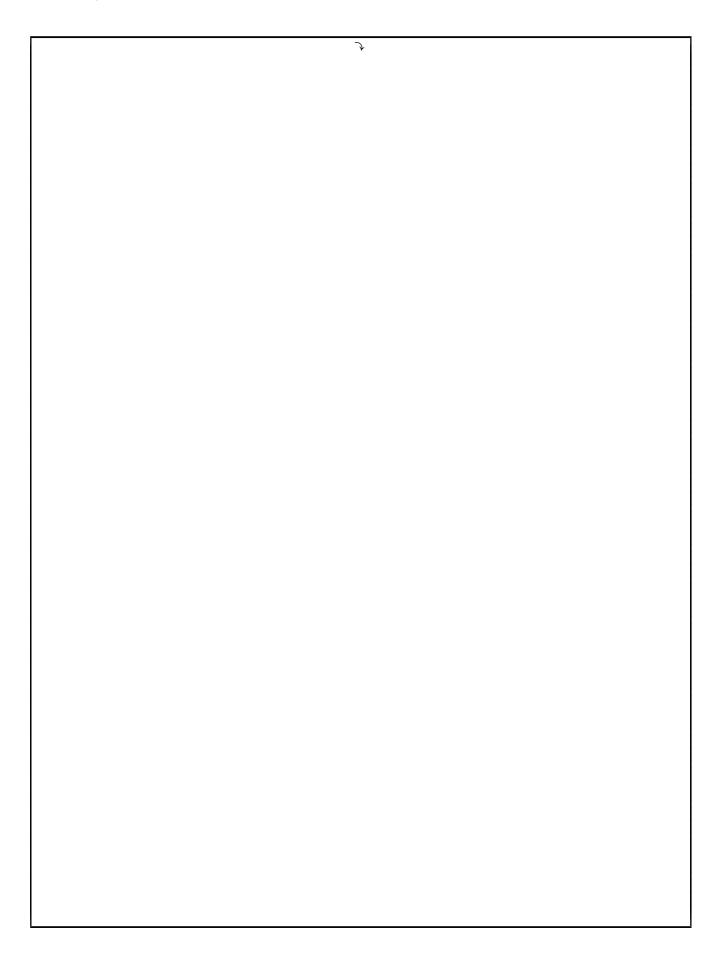
1st-order Euler:	$w_1 = 2.50000000$
2nd-order Ralston:	$w_1$ = 2.33593750
3rd-order Heun:	$w_1 = 2.33949416$
4th-order Runge-Kutta:	$w_1 = 2.33676365$

Calculate the absolute and relative error of y(1.5), given that the exact value is  $2.63577585 \, (8dp)$ . Roughly what would you expect the absolute error to be if you were to use h = 0.25?

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## **Polynomial Interpolation**

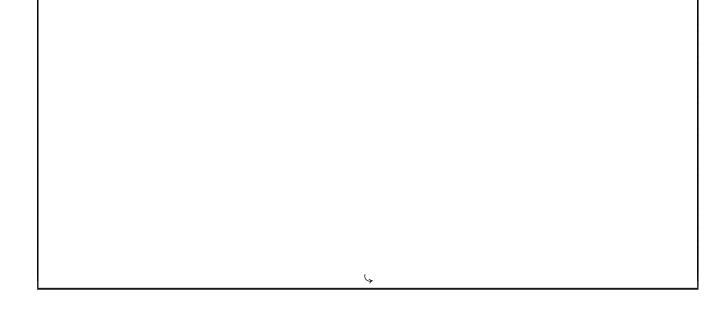
12p **3** Use divided differences to compute the cubic polynomial interpolating the following data:

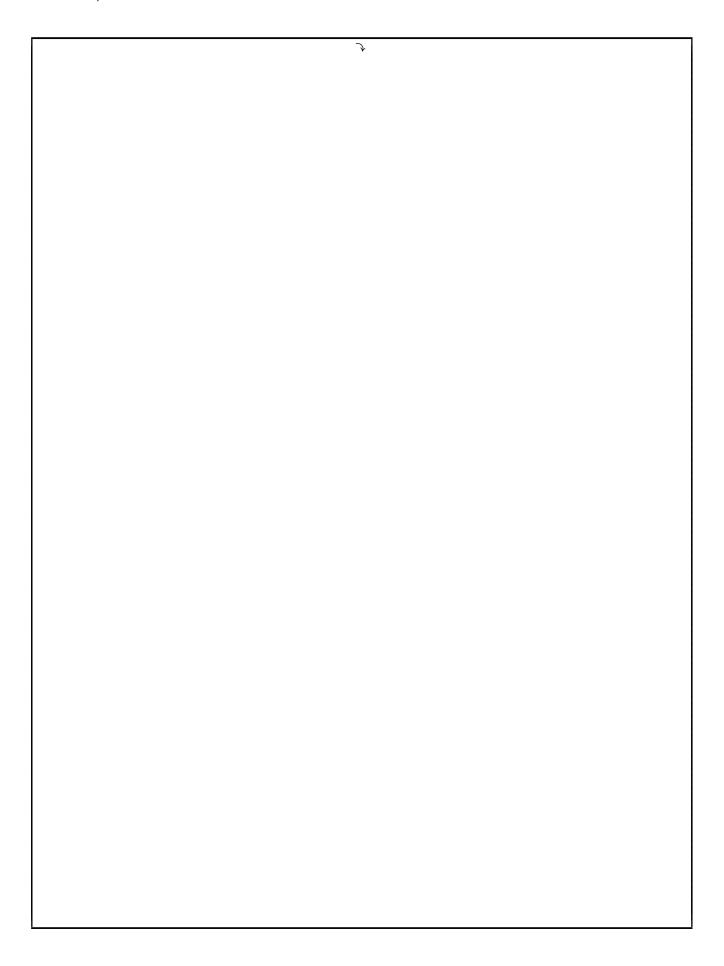
i	0	1	2	3	4
$x_i$	2.0	2.25	2.5	2.75	3.0
$f(x_i)$	0.838	0.509	0.180	0.041	0.020

Estimate the value of f(x) when x = 2.333.

Provide a bound on the error of your estimate if the data comes from a function f satisfying  $f''''(x) \le 140$  for  $x \in [2,3]$ .

Hint: You may assume  $f[x_0, x_1] = f[x_1, x_2] = -1.3160$ ,  $f[x_2, x_3] = -0.5560$ ,  $f[x_0, x_1, x_2] = 0.0000$  and  $f[x_1, x_2, x_3] = 1.5200$ .





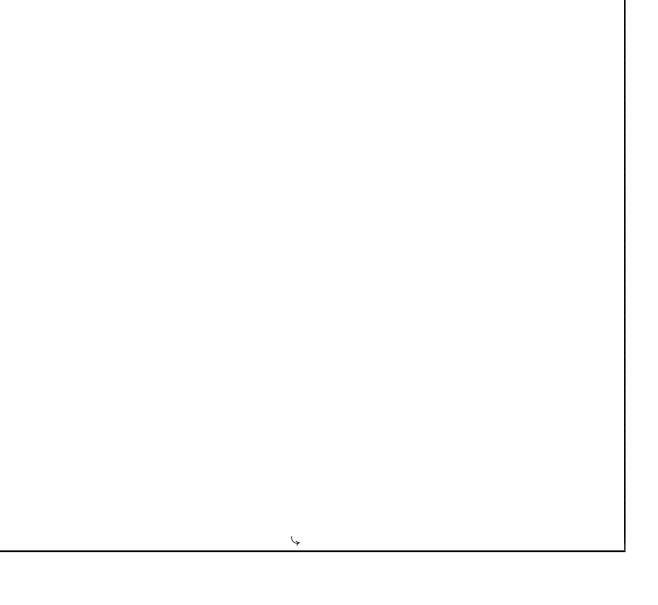


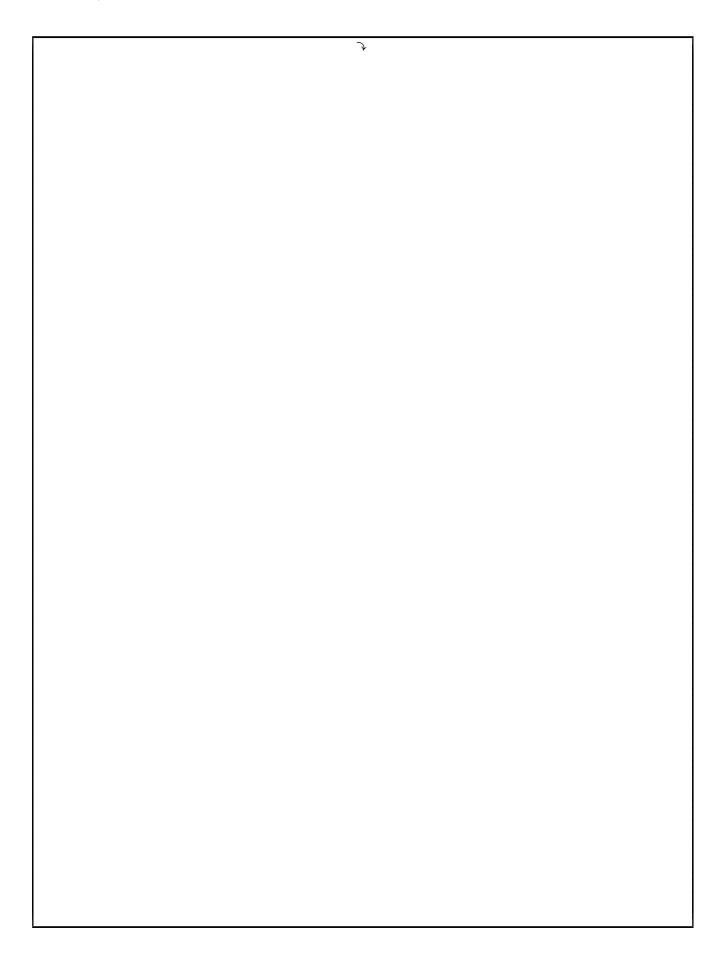
### **Differentiation & Integration**

12p **4** Use the trapezoid rule with n=4 partitions to approximate

$$\int_2^3 \frac{x}{x^3 - 6} dx.$$

Use the trapezoid rule error estimate to estimate the error of the trapezoid rule on each of the intervals [2.0, 2.5] and [2.5, 3.0]. What is your estimate of the total error? On which interval should you subdivide in order to reduce the error the most?





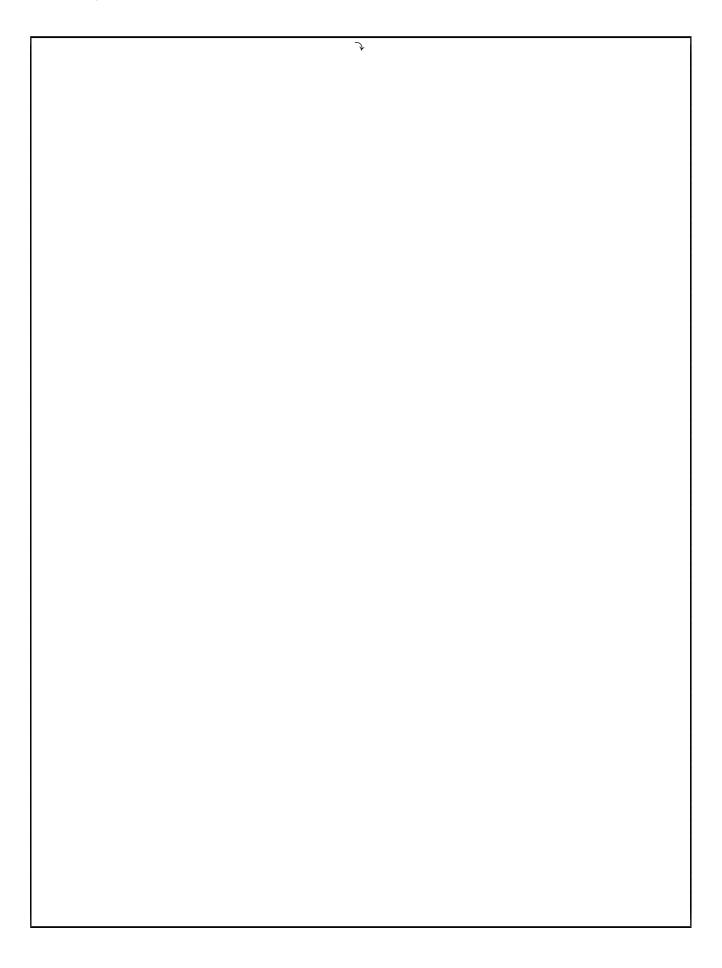
### **Least-Squares Approximation**

12p **5** Compute the first approximation  $s_1$  of the discrete Fourier transform for the following n=6 data points over the time interval  $[0,2\pi]$ :

Hence estimate the value of y when  $t = \pi/2$ , and the value of  $\int_0^{2\pi} f(t)^2 dt$ .

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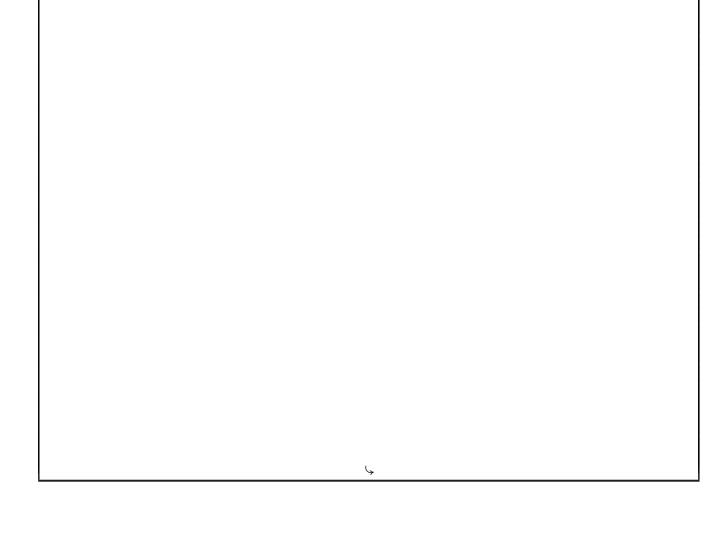
## Linear Algebra

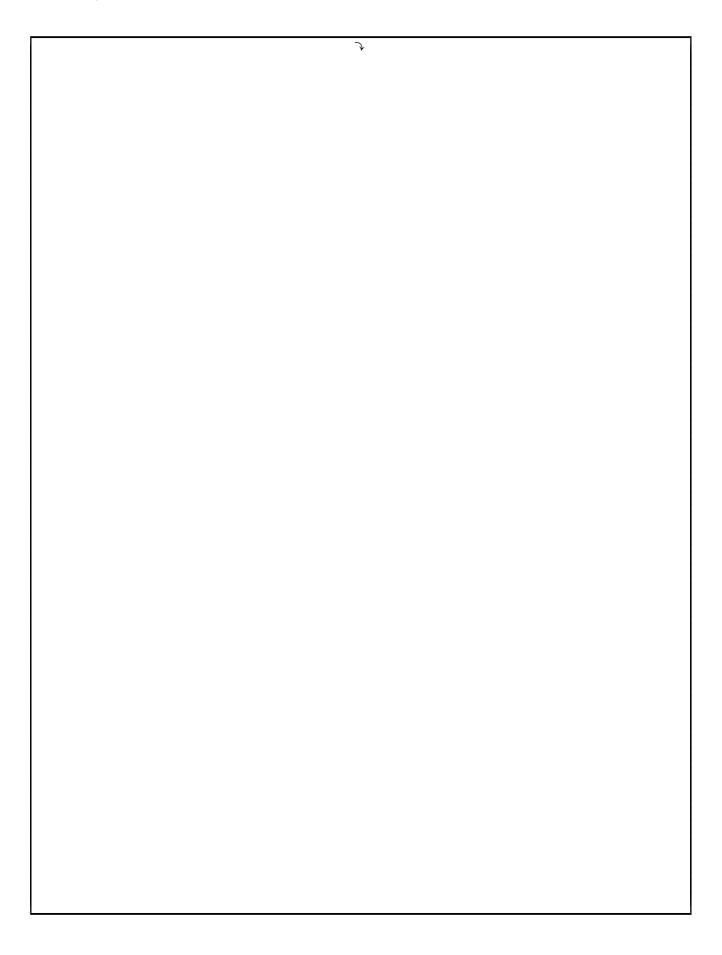
12p **6** Let

$$A = \begin{pmatrix} 5 & 1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$

Is  ${\cal A}$  diagonally dominant? What does this tell you about the convergence of the Gauss-Seidel method?

Apply two steps of the Gauss-Seidel method to find an approximate solution  $\mathbf{x}_2$  of  $A\mathbf{x} = \mathbf{b}$ . What is the residual of  $\mathbf{x}_2$ ?



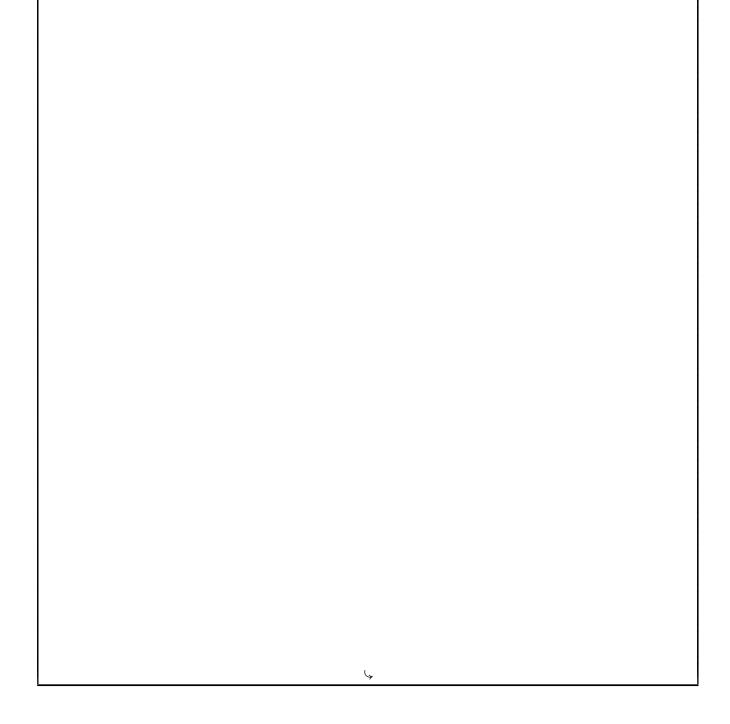


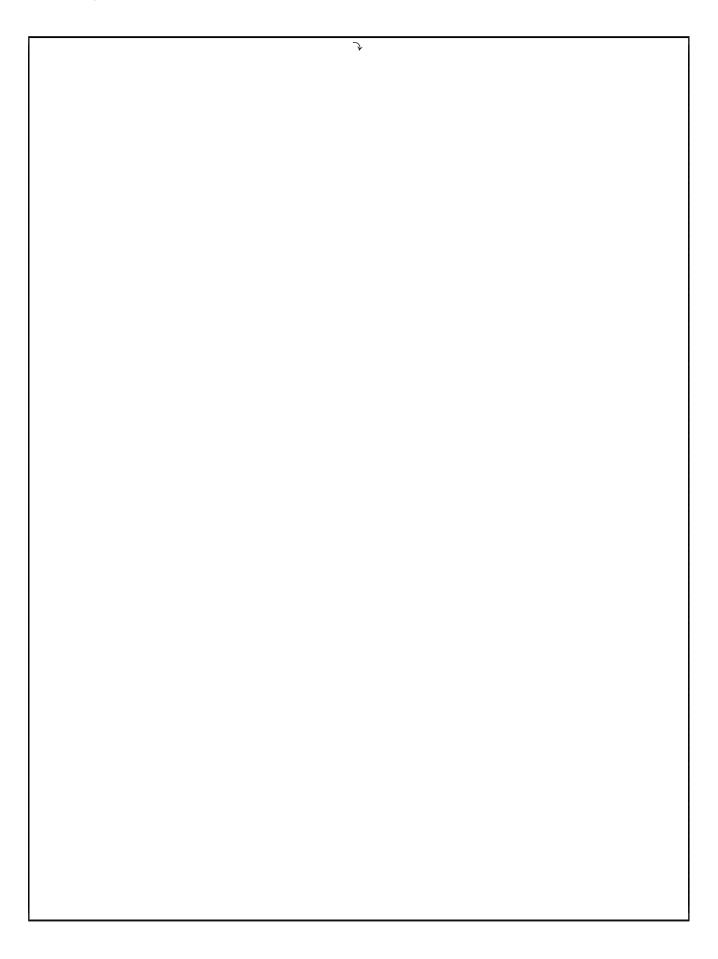
# Modelling

8p **7** Consider the following equation for the *forced Van der Pol oscillator*:

$$\ddot{x} + \mu(1 - x^2)\dot{x} + x = A\cos(\omega t)$$

Show how to solve this differential equation in Matlab, including the code you would use.





Extra Paper	
8	



