

## Exercises

1	2	3	4	5	6	7	8
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Surname, First name

## Calculus Test

Example exam

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

☐ a ☒ b ☐ c ☐ d ☐ e ☐ f → b  
☐ a ☐ b ☒ c ☐ d ☐ e ☐ f → c  
☒ a ☐ b ☐ c ☒ d ☐ e ☐ f → a

Answer multiple-choice questions as shown in the example.

## Program:

Course code: KEN/BCS 1440

Examiners: Otti D'Huys, Gijs Schoenmakers

Date/time:

Format: Written, closed book

Allowed aids: A formula sheet is attached to the exam.

## Instructions to students:

- The exam consists of 9 questions on 12 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

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**Question 1**

- 6p **1a** For  $f(x)$  and  $g(x)$  differentiable functions, let  $f(1) = 1$ ,  $f(-2) = -2$ ,  $f'(1) = -3$ ,  $f'(-2) = 3$ ,  $g(1) = -2$  and  $g'(1) = 1$ .

Find  $\lim_{x \rightarrow 1} \frac{f(g(2-x)) - g(f(x^2))}{x-1}$

- ☐ a  $-\infty$
- ☐ b  $-1$
- ☐ c  $0$
- ☐ d  $3$
- ☐ e  $-6$
- ☐ f  $+\infty$
- ☐ g  $-9$
- ☐ h The limit does not exist
- ☐ i None of the above

- 6p **1b** Calculate  $\lim_{x \rightarrow 0^-} \frac{x}{\sqrt{x^2 - x}}$

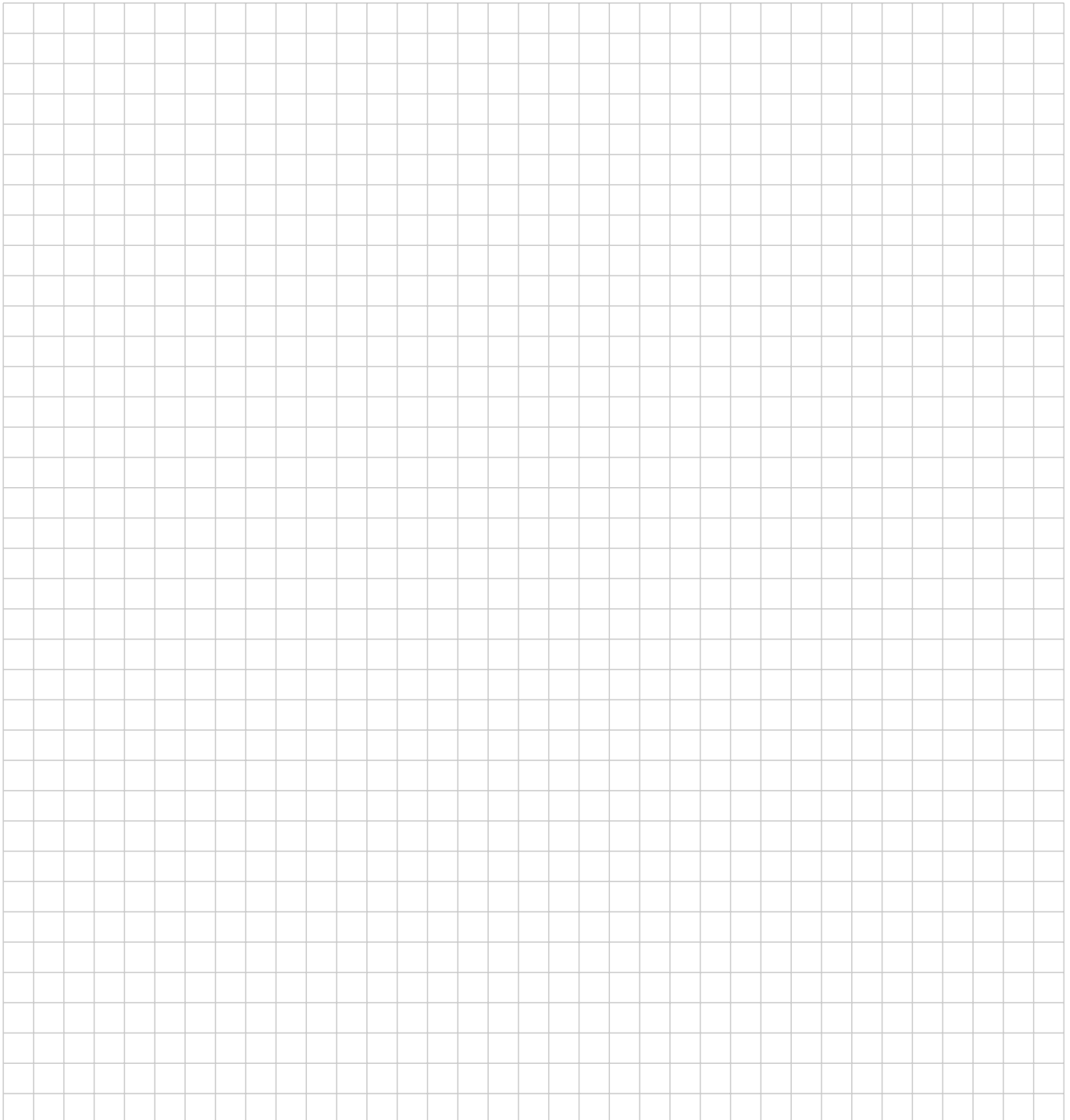
- ☐ a  $4$
- ☐ b  $-\frac{1}{2}$
- ☐ c  $\frac{1}{2}$
- ☐ d  $-1$
- ☐ e  $-\infty$
- ☐ f  $0$
- ☐ g  $+\infty$
- ☐ h  $1$
- ☐ i The limit does not exist
- ☐ j None of the above

- 6p **1c** Calculate  $\int_0^1 x \ln(x) dx$

- ☐ a  $-\frac{1}{2}$
- ☐ b This integral diverges to  $-\infty$
- ☐ c  $0$
- ☐ d This integral diverges to  $+\infty$
- ☐ e None of the above

**Question 2**

- 16p **2** Find all the asymptotes of the function  $f(x) = \frac{1}{\sqrt{x^2+3x+x}}$ . For the horizontal and oblique asymptotes (if any), specify whether they are one-sided or two-sided. For the vertical asymptotes (if any), specify how the function approaches the asymptotes (i.e. whether the function goes to  $+\infty$  or  $-\infty$  on the left or right side of the asymptotes.)

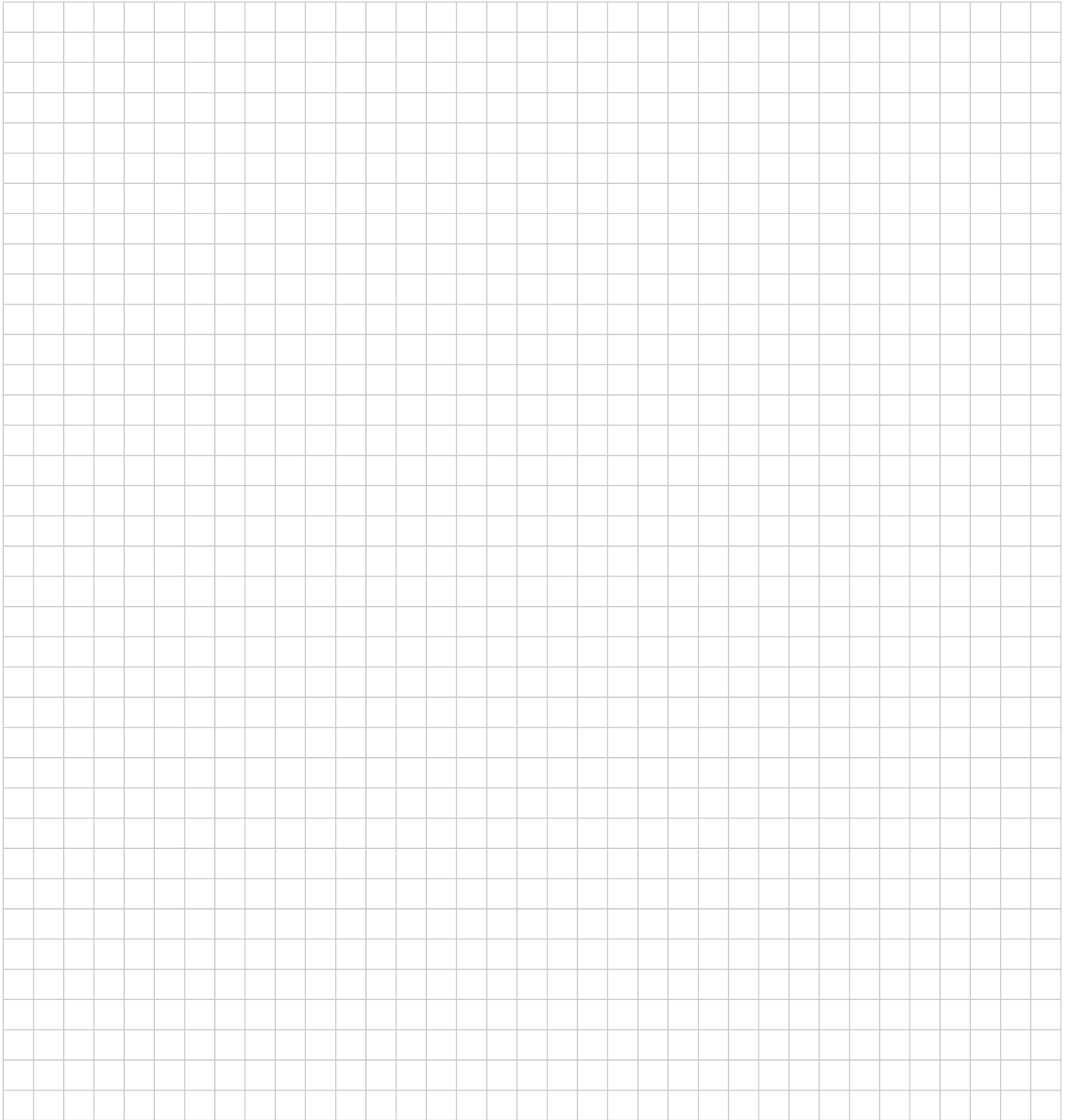


**Question 3**

10p **3** Give an example of a real function  $f(x)$  with the following properties:

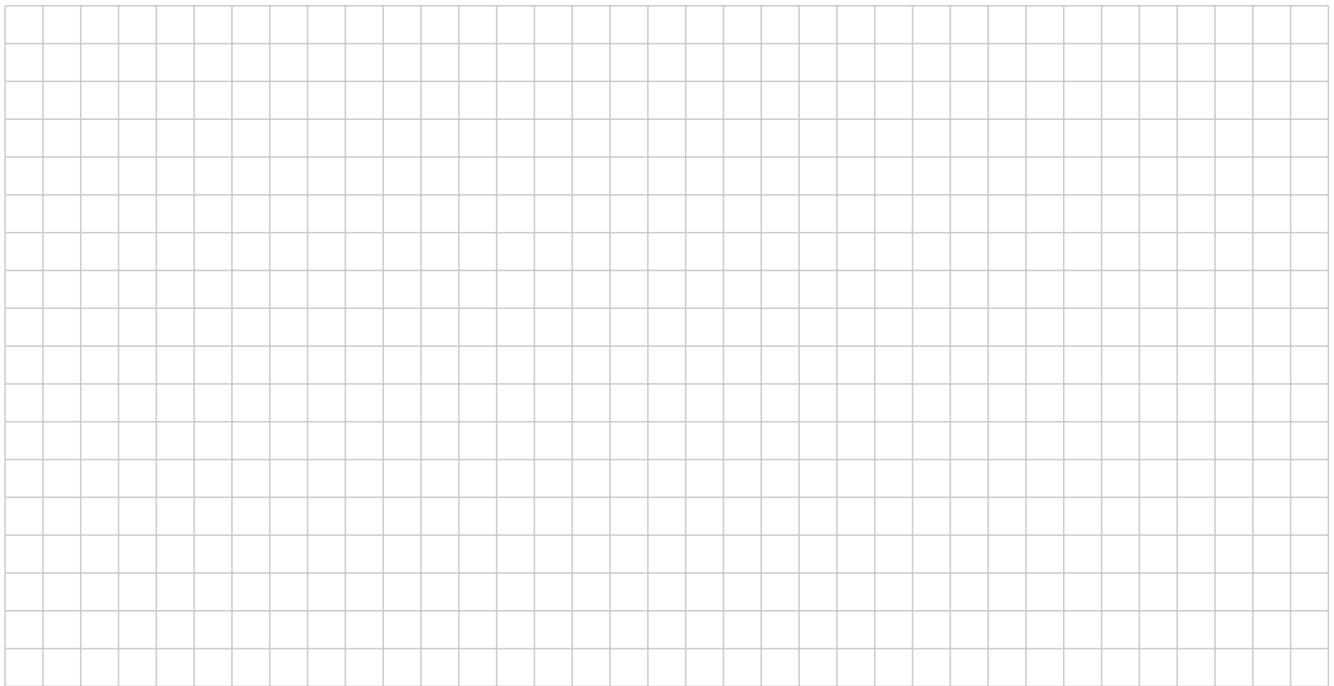
- The domain of  $f(x)$  includes  $(0, 1)$
- $f(x)$  is continuous and differentiable on its domain.
- $\lim_{x \rightarrow 0^+} f(x) = +\infty$ ,  $\lim_{x \rightarrow 1^-} f(x) = -\infty$

You only need to give a formula for  $f(x)$ , you do not need to show explicitly that your solution fulfills all the requirements.



**Question 4**

7p **4a** Evaluate the following integral:  $\int_0^{\frac{\pi}{2}} (\sin(x))^3 dx$

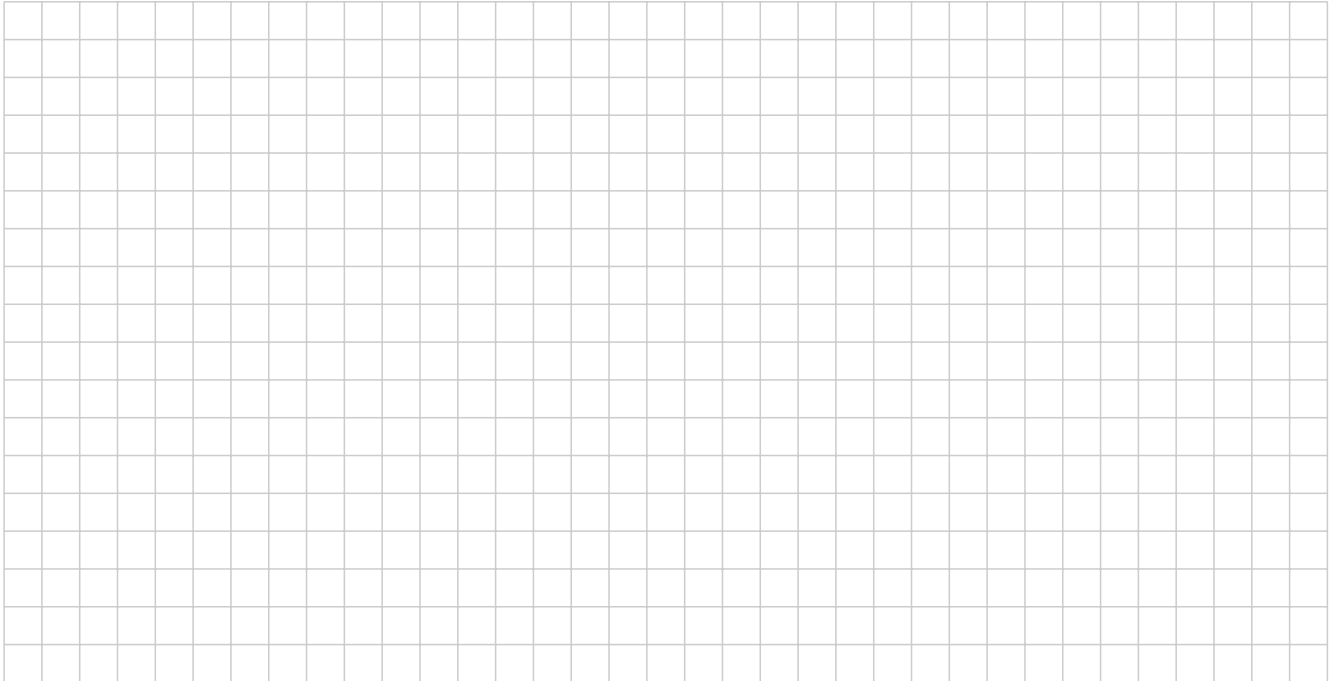


7p **4b** Evaluate the following integral, or explain why it diverges:  
 $\int_0^1 \frac{dx}{x^2-1}$



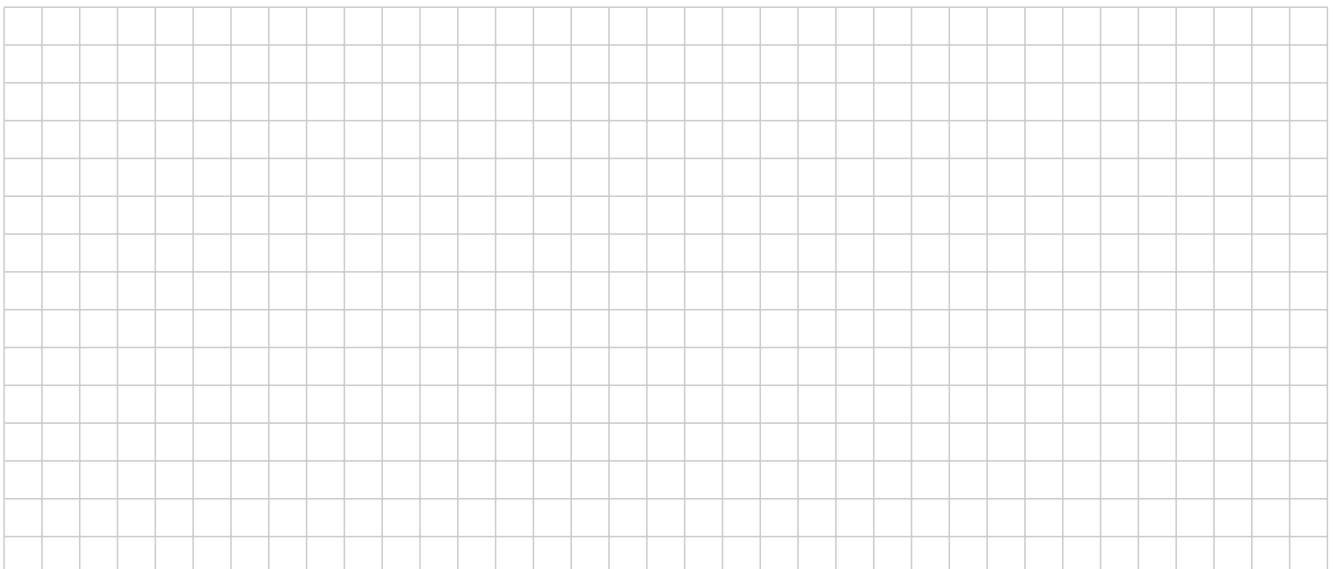
7p **4c** Evaluate the following indefinite integral:

$$\int \frac{\ln(x)}{x^2} dx$$

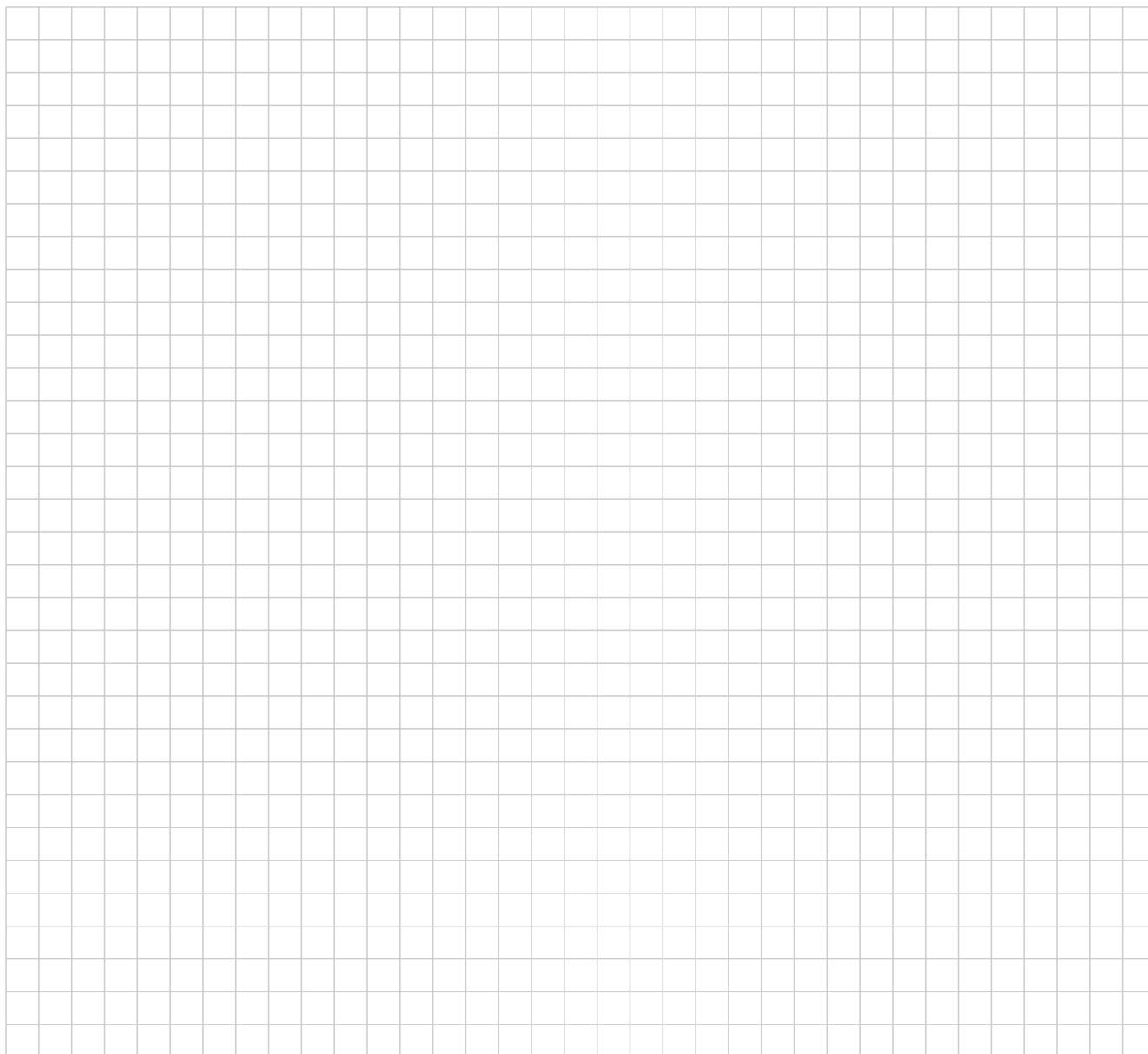


### Question 5

6p **5a** Consider the series  $\sum_{n=1}^{\infty} a_n$ , where  $a_n = \frac{(-1)^n}{3^n}$ . Prove or disprove that this series converges. If possible, compute the sum  $\sum_{n=1}^{\infty} a_n$



- 9p **5b** Consider the sequence  $\{a_n\}$ , with  $a_n = \sqrt{n+1} - \sqrt{n}$ .
- calculate  $\lim_{n \rightarrow \infty} a_n$ , or explain why the limit does not exist.
  - Is the sequence is bounded? Explain
  - Is the sequence is increasing, decreasing, alternating, or none of these. Give a proof.



**Question 8**

10p **6** For the function  $f(x, y) = \sin(2x - y)$ , give the equation of the tangent plane at  $(0, 0, f(0, 0))$

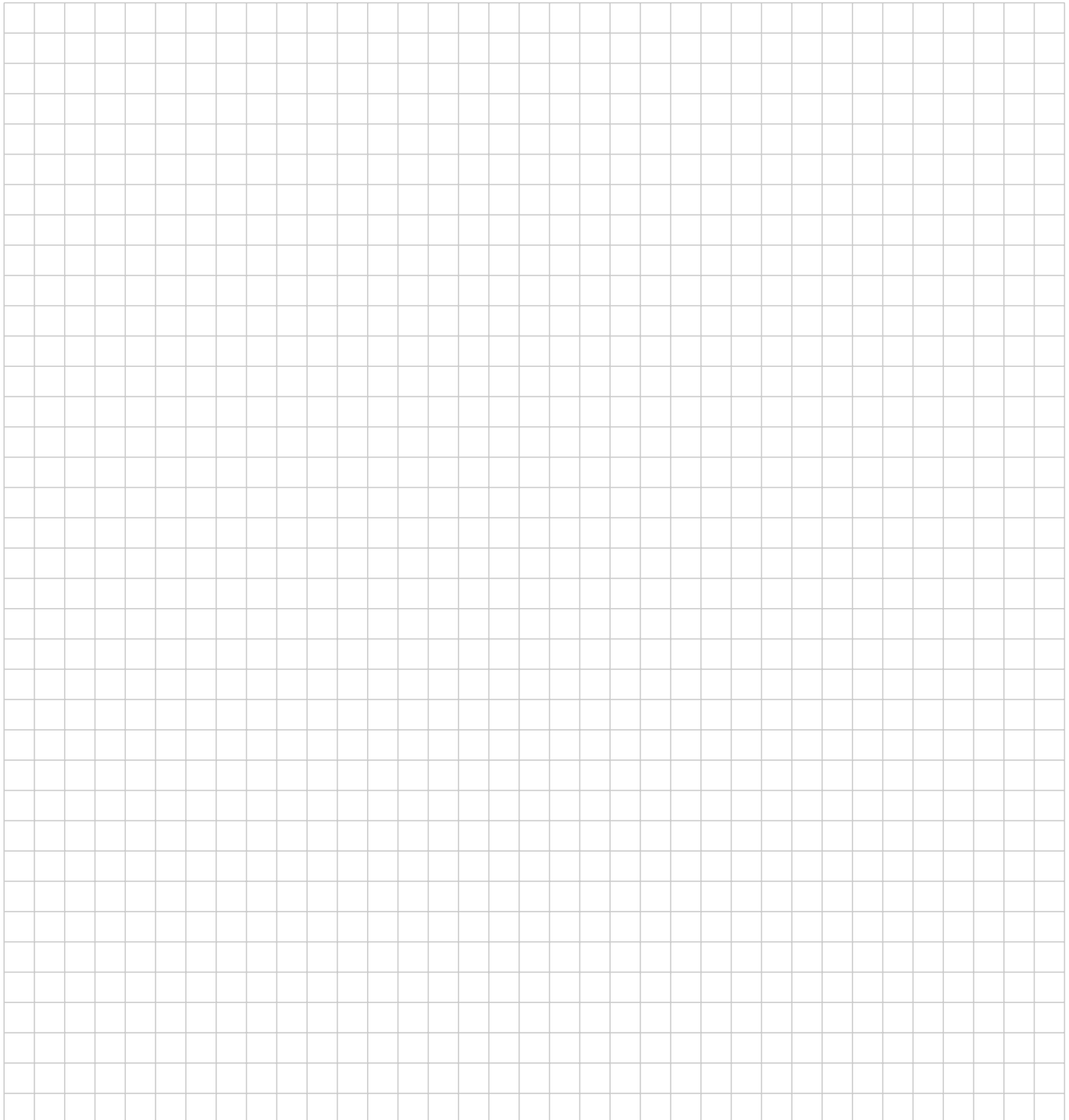




**Question 9**

10p **7** Evaluate the double integral over a triangle  $T$  with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$ :

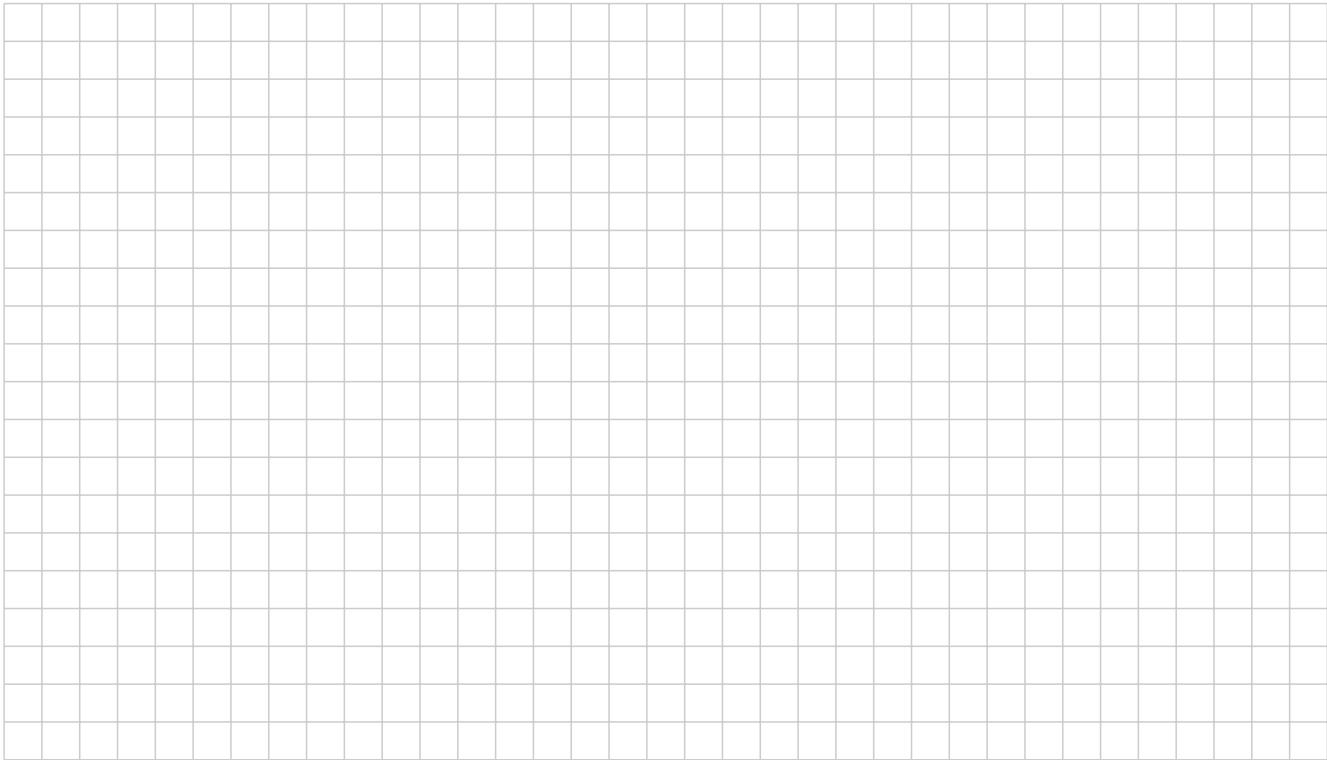
$$\iint_T e^{2x+y} dA$$



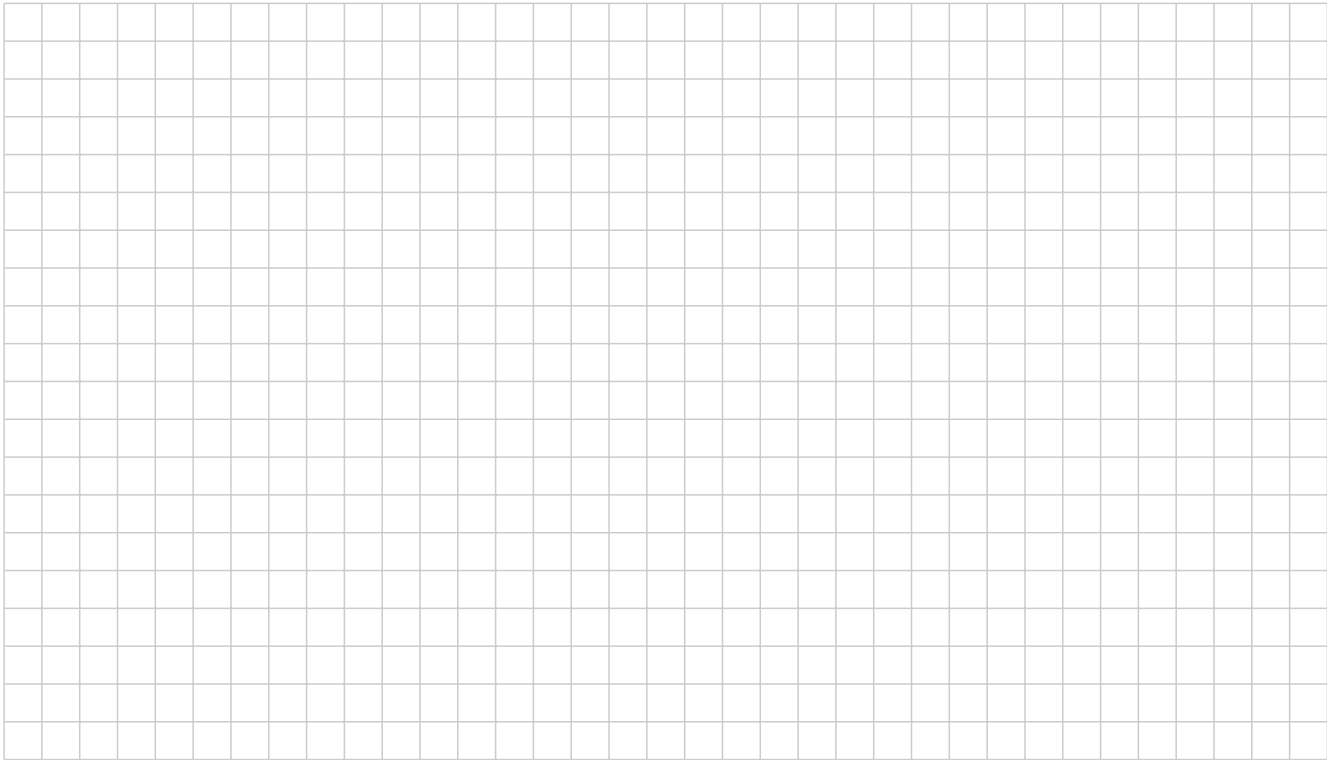
**8a**



8c



8d



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