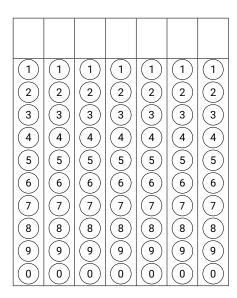
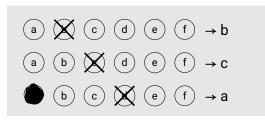
#### Surname, First name

**KEN1410 Linear Algebra** KEN1410 Linear Algebra Exam





Answer multiple-choice questions as shown in the example.

**Program: Data Science and Artificial Intelligence** 

Course code: KEN1410

Examiners: Dr. Marieke Musegaas and Dr. Philippe Dreesen

Date/time: Tuesday 02.04.2024 9h00-11h00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators. Instructions to students:

- The exam consists of 10 questions on 16 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- · You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

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Consider the following matrix:

$$A = \left[ \begin{array}{rrrr} 1 & 2 & -1 & -1 \\ 2 & 4 & -1 & 0 \\ -3 & -6 & 1 & 0 \end{array} \right].$$

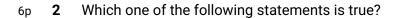
6p **1** What is the reduced row echelon form of A?

(f) None of the above.

Consider the following system of linear equations:

$$5x_1 - x_2 + x_3 = 0,$$

$$4x_1 - 3x_2 + 7x_3 = 0.$$



- (a) This system has no solution.
- (b) This system has only the trivial solution.
- (c) This system has only the solution  $x_1 = 4, x_2 = 31, x_3 = 11$ .
- (d) This system has infinitely many solutions.
- (e) None of the above.

#### **Question 3**

Let 
$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
,  $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{w} = \begin{bmatrix} 4 \\ 2 \\ a \end{bmatrix}$ .

 $_{6p}$  3 If w is a linear combination of u and v, then we must have

- (a) a=0
- (b) a=1
- $\bigcirc$  a=2
- $(\mathbf{d})$  a=3
- (e) None of the above.

Let  $T: \mathbb{R}^3 \to \mathbb{R}^2$  be a linear transformation such that

$$T\left(\left[\begin{array}{c} x \\ y \\ z \end{array}\right]\right) = \left[\begin{array}{c} x+y-z \\ x-y+z \end{array}\right],$$

and let  $S:\mathbb{R}^2 \to \mathbb{R}^2$  be a linear transformation such that

$$S\left(\left[\begin{array}{c} x \\ y \end{array}\right]\right) = \left[\begin{array}{c} x - 2y \\ x + y \end{array}\right].$$

Consider the composed linear transformation  $S \circ T : \mathbb{R}^3 \to \mathbb{R}^2$ , i.e., first transformation T is applied and after that transformation S is applied.

**4** What is the standard matrix for the composed linear transformation  $S \circ T$ ?

$$\left(\begin{array}{cccc}
a & \begin{bmatrix}
-1 & 3 & -3 \\
2 & 0 & 0
\end{array}\right)$$

6р

(b) 
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{c|c}
\hline
\begin{array}{c}
1 & 1 \\
2 & -1 \\
-1 & 2
\end{array}
\end{array}$$

$$\begin{array}{c}
 \text{d} \\
 \hline
 & 1 & -2 \\
 \hline
 & 1 & 1
\end{array}$$

(e) None of the above.



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### **Question 5**

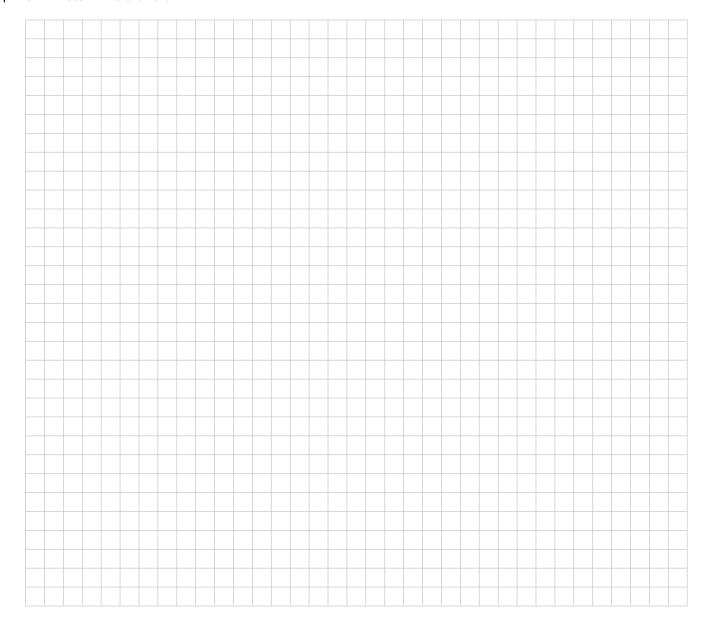
Consider the following matrix:

$$A = \left[ \begin{array}{ccc} 3 & 2 & -2 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{array} \right].$$

It is given that the inverse of  $\boldsymbol{A}$  is as follows:

$$A^{-1} = \left[ \begin{array}{ccc} 1 & 0 & a \\ 0 & 1 & 0 \\ 1 & 1 & b \end{array} \right].$$

10p **5** Determine a and b.





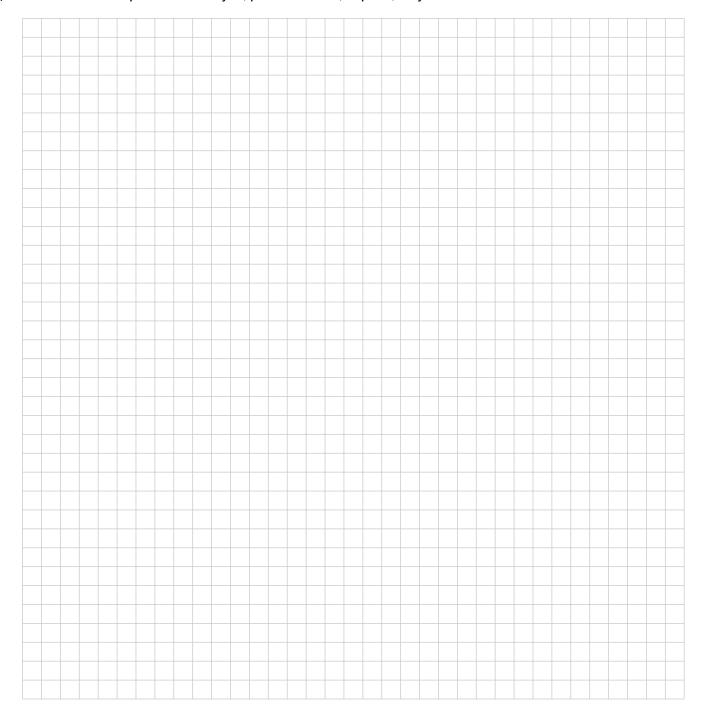
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### **Question 6**

Consider the following subset of  $\mathbb{R}^3$ :

$$W = \left\{ \left[ \begin{array}{c} a \\ b \\ c \end{array} \right] \in \mathbb{R}^3 \mid a \le b \le c \right\}.$$

10p **6** Is W a subspace of  $\mathbb{R}^3$ ? If yes, prove it. If not, explain, why not.



6p 7 Which one of the following sets of vectors forms a basis for  $\mathbb{R}^3$ ?

$$\left\{ \begin{bmatrix} -2\\1\\3 \end{bmatrix}, \begin{bmatrix} 3\\5\\-1 \end{bmatrix} \right\}$$

$$\begin{array}{ccc}
\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}
\end{pmatrix}$$

(e) None of the above.

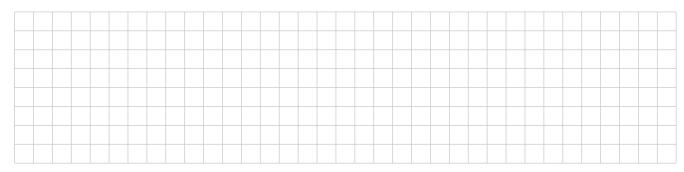
Consider the following matrix:

$$A = \left[ \begin{array}{ccccc} 2 & -4 & 1 & 1 & 5 \\ 3 & -6 & -2 & 5 & -7 \\ 5 & -10 & 3 & 2 & 4 \end{array} \right].$$

It is given that A is row equivalent to the following matrix:

$$\left[\begin{array}{ccccc} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right].$$

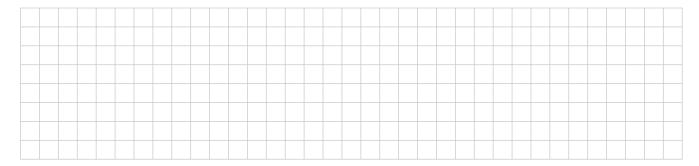
4p **8a** Determine dim(Nul(A)).



4p **8b** Determine  $\dim(Col(A))$ .



4p **8c** Determine  $\dim(Row(A))$ .



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4p

**8d** Determine whether the vector  $\begin{bmatrix} 1\\1\\1\\1\\1 \end{bmatrix}$  belongs to Nul(A), i.e., determine whether the vector  $\begin{bmatrix} 1\\1\\1\\1\\1 \end{bmatrix}$  is a

solution of the homogeneous equation Ax = 0.



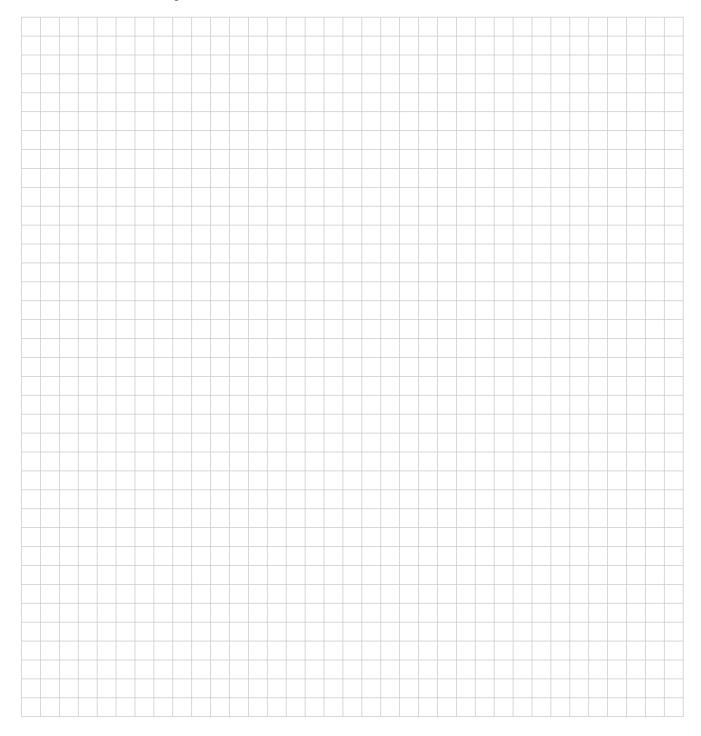
4p **8e** Determine a basis for Row(A).



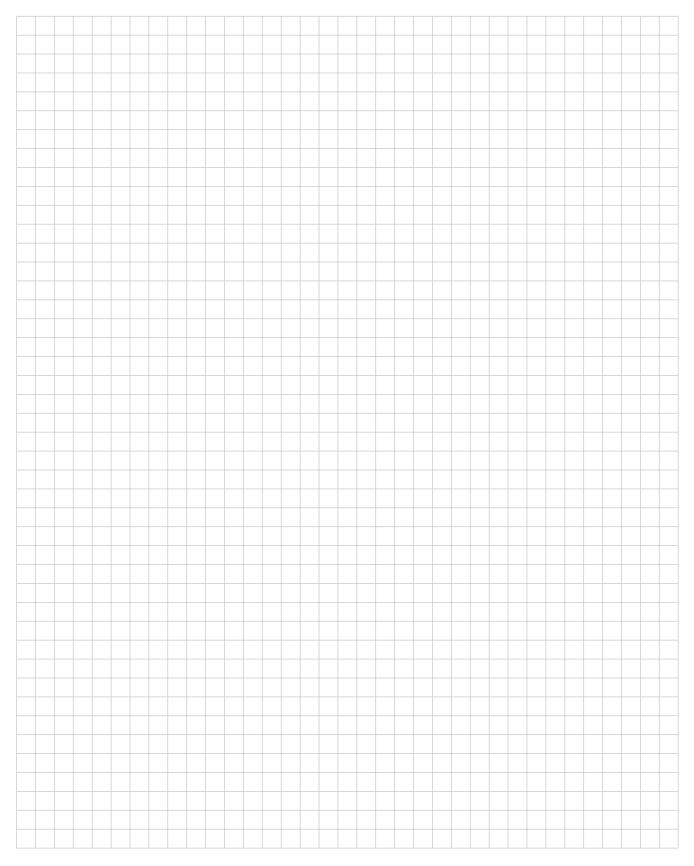
Consider the following matrix:

$$A = \left[ \begin{array}{cc} 1 & 4 \\ 4 & 7 \end{array} \right].$$

8p **9a** Show that A has eigenvalues -1 and 9.



12p **9b** Diagonalize A, i.e. find a matrix P and a diagonal matrix D such that  $A = PDP^{-1}$ .



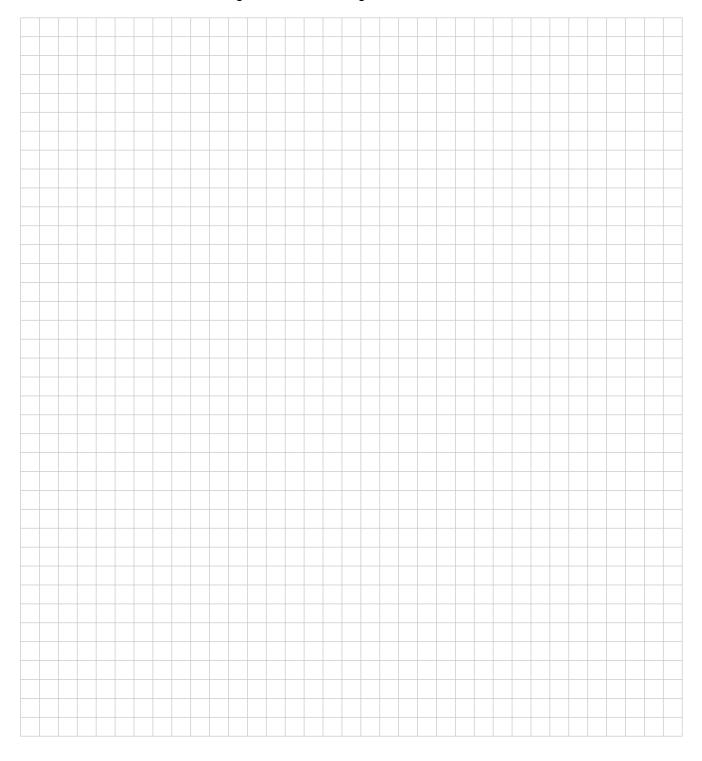


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# **Question 10**

Let 
$$\mathbf{u} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$
 and  $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ .

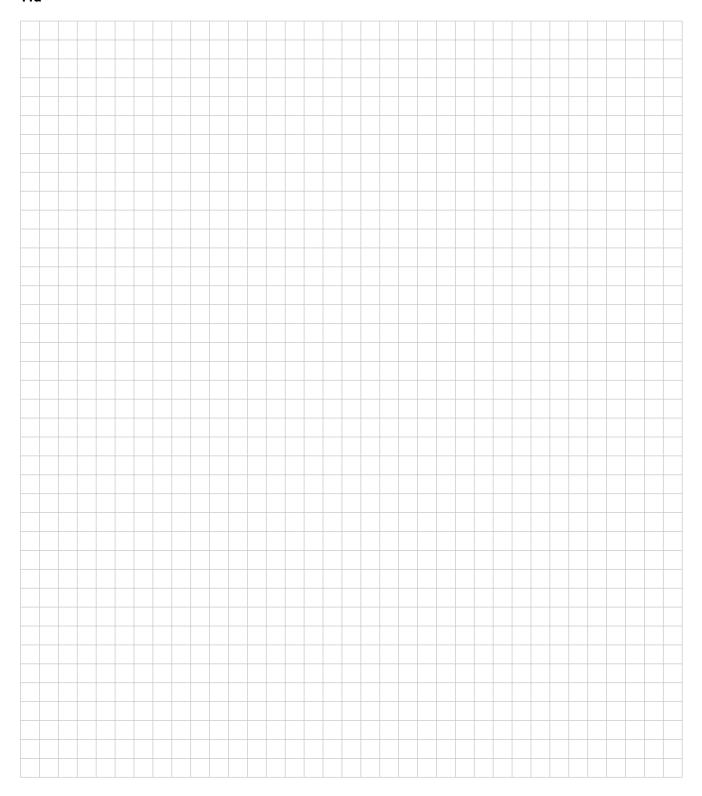
10p  $\,$  10 Determine two vectors of length 1 that are orthogonal to both  ${\bf u}$  and  ${\bf v}$ .



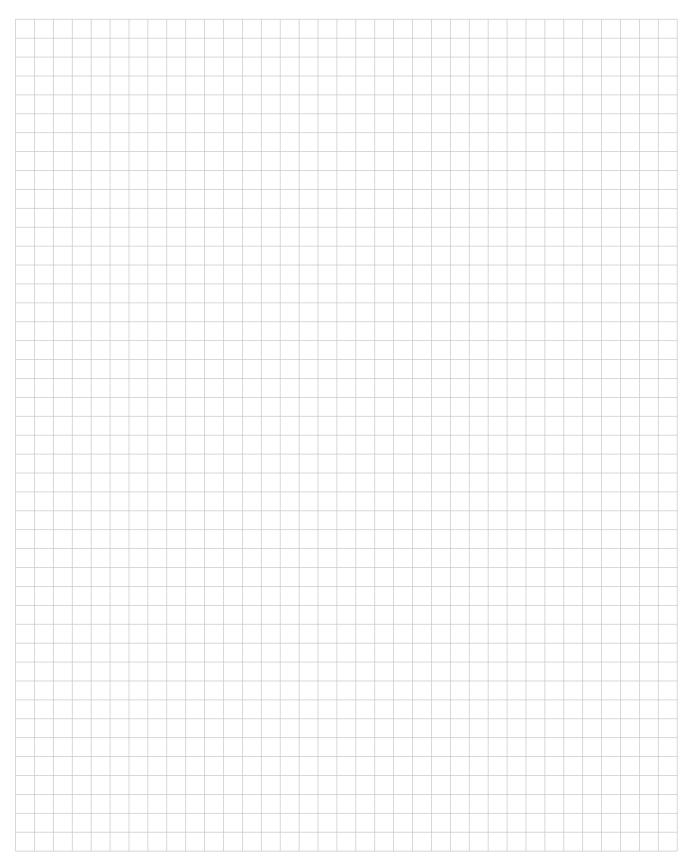
## Extra space

If you use these extra answer boxes, please mention clearly in your main answer that part of your answer can be found here!

#### 11a

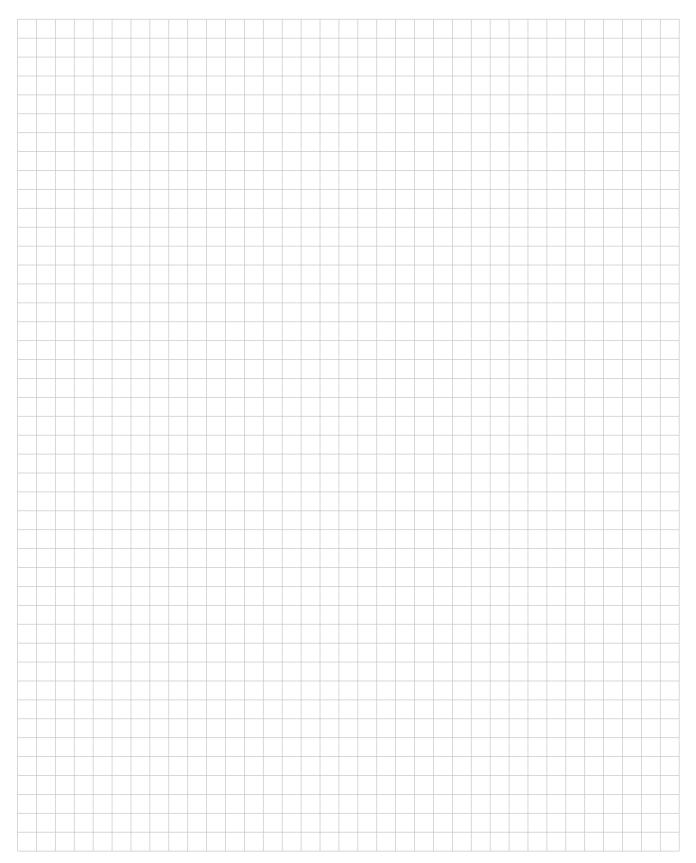


## 11b





## 11c





## 11d

