Exercises

1	2	3	4	5	6	7	8

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Numerical Mathematics (KEN1540)
Exam

Program: Bachelor Data Science and Knowledge Engineering
Course code: KEN1540
Examiners: Pieter Collins & Martijn Boussé
Date/time: Monday 30th May 13:00-15:00
Format: Written Exam
Allowed aids: DKE-approved calculator; Formula sheet (provided)
Instructions to students:

- The exam consists of 7 questions on 18 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Algebraic Equations

12p **1** Explain the similarity between decimal arithmetic rounded to 4 significant figures and binary double-precision floating-point arithmetic.

Apply one step of the bisection method, followed by one step of Newton's method, to estimate a root of the function

$$f(x) = x(x^2 - 2) - 1$$

in the interval $[0, 2]$.
When applying Newton's method, start in the midpoint of the interval under consideration, and use rounded arithmetic to 4 significant figures throughout your calculation.

Differential Equations

12p **2** Use the two-stage Adams-Bashforth method to find the solution of the initial value problem $\dot{y} = 1/(1+ty)$, $y(1) = 2.0000$, $t \in [0, 1.5]$, with $h = 0.5$.
Bootstrap your calculation using values of $w_1 \approx y(0.5)$ for the most appropriate of the methods below, giving a reason for your answer.

1st-order Euler:	$w_1 = 2.50000000$
2nd-order Ralston:	$w_1 = 2.33593750$
3rd-order Heun:	$w_1 = 2.33949416$
4th-order Runge-Kutta:	$w_1 = 2.33676365$

Calculate the absolute and relative error of $y(1.5)$, given that the exact value is $2.63577585\,(8\text{dp})$.
Roughly what would you expect the absolute error to be if you were to use $h = 0.25$?

Polynomial Interpolation

12p **3** Use divided differences to compute the cubic polynomial interpolating the following data:

i	0	1	2	3	4
x_i	2.0	2.25	2.5	2.75	3.0
$f(x_i)$	0.838	0.509	0.180	0.041	0.020

Estimate the value of $f(x)$ when $x = 2.333$.

Provide a bound on the error of your estimate if the data comes from a function f satisfying $f''''(x) \leq 140$ for $x \in [2, 3]$.

Hint: You may assume $f[x_0, x_1] = f[x_1, x_2] = -1.3160$, $f[x_2, x_3] = -0.5560$, $f[x_0, x_1, x_2] = 0.0000$ and $f[x_1, x_2, x_3] = 1.5200$.

Differentiation & Integration

12p **4** Use the trapezoid rule with $n = 4$ partitions to approximate

$$\int_2^3 \frac{x}{x^3 - 6} dx.$$

Use the trapezoid rule error estimate to estimate the error of the trapezoid rule on each of the intervals $[2.0, 2.5]$ and $[2.5, 3.0]$. What is your estimate of the total error? On which interval should you subdivide in order to reduce the error the most?

Least-Squares Approximation

12p **5** Compute the first approximation s_1 of the discrete Fourier transform for the following $n = 6$ data points over the time interval $[0, 2\pi]$:

t_j	0	$\frac{1}{3}\pi$	$\frac{2}{3}\pi$	π	$\frac{4}{3}\pi$	$\frac{5}{3}\pi$
y_j	2.08	2.26	1.63	0.18	0.43	1.53

Hence estimate the value of y when $t = \pi/2$, and the value of $\int_0^{2\pi} f(t)^2\,dt$.

Linear Algebra

12p **6** Let

$$A = \begin{pmatrix} 5 & 1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$

Is A diagonally dominant? What does this tell you about the convergence of the Gauss–Seidel method?

Apply two steps of the Gauss–Seidel method to find an approximate solution $\mathbf{x}_2$ of $A\mathbf{x} = \mathbf{b}$. What is the residual of $\mathbf{x}_2$?

Modelling

8p **7** Consider the following equation for the *forced Van der Pol oscillator*:

$$\ddot{x} + \mu(1 - x^2)\dot{x} + x = A\cos(\omega t)$$

Show how to solve this differential equation in Matlab, including the code you would use.

Extra Paper

8