Surname, First name

KEN1410 Linear Algebra
KEN1410 Linear Algebra Resit

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a ~~b~~ c d e f → b

a b ~~c~~ d e f → c

a b c ~~d~~ e f → a

Answer multiple-choice questions as shown in the example.

Program: Data Science and Artificial Intelligence
Course code: KEN1410
Examiners: Dr. Marieke Musegaas and Dr. Philippe Dreesen
Date/time: Monday 01.07.2024 9h00-11h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.
Instructions to students:

- The exam consists of 13 questions on 18 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1

Each of the following matrices needs only one elementary row operation to be in the reduced (row) echelon form. For each matrix, indicate which operation is required and give the reduced (row) echelon form of the matrix that results from performing that operation.

2p **1a** $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2p **1b** $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & 1 \end{bmatrix}$

2p **1c** $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \end{bmatrix}$

Question 2
Consider the following system of linear equations:

$$\begin{aligned} x_1 \qquad\; + x_3 &= 3 \\ x_1 - x_2 - x_3 &= 1 \\ -x_1 + x_2 \qquad\; &= 4 \end{aligned}$$

6p **2** Which one of the following statements is true?

- (a) This system has no solution.
- (b) This system has a unique solution.
- (c) This system has infinitely many solutions.
- (d) None of the above.

Question 3

Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ -4 \\ a \\ b \end{bmatrix}$.

10p **3** Determine a and b such that $\mathbf{w} \in \mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$.

Question 4

6p **4** Which one of the following sets of vectors is linearly independent?

(a) $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$

(d) $\left\{\begin{bmatrix}1\\-1\\4\end{bmatrix}, \begin{bmatrix}2\\-2\\0\end{bmatrix}\right\}$

(e) $\left\{\begin{bmatrix}-1\\4\end{bmatrix}, \begin{bmatrix}3\\-12\end{bmatrix}\right\}$

(f) None of the above.

Question 5

6p **5** Which one of the following transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation?

(a) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0 \\ x \end{bmatrix}$

(b) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{\sqrt{x^2+y^2}} \begin{bmatrix} x \\ y \end{bmatrix}$

(c) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+1 \\ y+1 \end{bmatrix}$

(d) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2+y^2 \\ 2xy \end{bmatrix}$

(e) None of the above.

Question 6

Let $AB = \begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$.

10p **6** Determine A.

Question 7

Consider the following subset of $\mathbb{R}^3$:

$$W = \left\{ \begin{bmatrix} a \\ a \\ a \end{bmatrix} \in \mathbb{R}^3 \mid a \in \mathbb{R} \right\}.$$

10p **7** Is W a subspace of $\mathbb{R}^3$? If yes, prove it. If not, explain, why not.

Question 8
Let A be a 3×8 matrix.

4p **8a** What is the smallest possible dimension of Nul A? Explain.

4p **8b** What is the largest possible dimension of Col A? Explain.

Question 9
Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

It is given that A is row equivalent to the following matrix:

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

10p **9** Determine a basis for Nul A, a basis for Col A and a basis for Row A.

Question 10
Consider the following matrix:

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}.$$

4p **10a** Is $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ an eigenvector of A? Explain.

4p **10b** It is given that $\begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$ is an eigenvector of A. Determine the corresponding eigenvalue.

Question 11
Consider the following matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

It is given that $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is an eigenvector of A with corresponding eigenvalue 1.

6p **11** What is $A^{50}\mathbf{x}$?

a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

c) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

d) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

e) None of the above.

Question 12
Consider the following matrix:

$$A = \begin{bmatrix} 4 & -2 & 4 \\ -2 & 7 & 2 \\ 4 & 2 & 4 \end{bmatrix}.$$

It is given that both $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are eigenvectors corresponding to the eigenvalue 8.

10p **12** Determine two eigenvectors, corresponding to the eigenvalue 8, that are orthogonal to each other. (Hint: you might want to use that the formula for the orthogonal projection of a vector $\mathbf{y}$ onto a vector $\mathbf{u}$ is given by $\frac{\mathbf{y}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\mathbf{u}$.)

Question 13

4p **13** Construct a 4×4 matrix that is orthogonally diagonalizable.

Extra space
If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

14a

14b

14c

14d