Exercises

1	2	3	4	5	6	7	8	9

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Surname, First name

KEN1130 Discrete Mathematics
Exam

a ~~b~~ c d e f → b
a b ~~c~~ d e f → c
a b c ~~d~~ e f → a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence
Course code: KEN1130
Examiners: dr. Marieke Musegaas and dr. Stefan Maubach
Date/time: Friday 28.10.2022 9h00-11h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1
Consider the following logical proposition.

- $(p \Rightarrow q) \Leftrightarrow ((q \wedge \neg r) \vee \neg p)$

Answer the following questions about (the truth table of) the above proposition. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

1.25p **1a** Suppose p is TRUE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1b** Suppose p is TRUE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1c** Suppose p is TRUE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1d** Suppose p is TRUE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1e** Suppose p is FALSE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1f** Suppose p is FALSE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1g** Suppose p is FALSE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1h** Suppose p is FALSE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

Question 2

12p **2a** Use induction to prove the following statement. For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i \cdot 2^{i} = (n-1) \cdot 2^{n+1} + 2$$

3p **2b** The theorem below is incorrect. What goes wrong in the corresponding proof?

Theorem: For all integers $n \geq 1$, $3n + 2$ is divisible by 3.

Proof: Let $n \in \mathbb{N}$ and assume that $3n + 2$ is divisible by 3. So, $3n + 2 = 3 \cdot k$, where $k \in \mathbb{Z}$. We need to show that $3(n + 1) + 2$ is divisible by 3.

$$3(n+1) + 2 = 3n + 3 + 2 = (3n + 2) + 3 = 3 \cdot k + 3 = 3 \cdot (k + 1).$$

Since $k \in \mathbb{Z}$, we also have $k + 1 \in \mathbb{Z}$. So, $3(n + 1) + 2$ is divisible by 3. □

Question 3

8p **3a** Prove or disprove the following statement. For all sets A and B,

$$A^c \cup B^c \subseteq (A \cup B)^c.$$

Fill in the blanks for the direct proof of the following theorem. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

Theorem: For all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$.

Proof: Let A and B be two sets. Assume $A \subseteq B$. We must show that $\underline{\quad (i) \quad}$. Let $x \in \underline{\quad (ii) \quad}$. We must show that $x \in \underline{\quad (iii) \quad}$. By the definition of $\underline{\quad (iv) \quad}$ we know $x \in A$ or $x \in B$. In case $x \in \underline{\quad (v) \quad}$, then since $A \subseteq B$, $x \in \underline{\quad (vi) \quad}$. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in \underline{\quad (vii) \quad}$ (as was to be shown). □

1p **3b** What must be filled in at position (i)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3c** What must be filled in at position (ii)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3d** What must be filled in at position (iii)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3e** What must be filled in at position (iv)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3f** What must be filled in at position (v)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3g** What must be filled in at position (vi)?

- (a) $A \cup B \subseteq B$
- (b) $A \cup B$
- (c) $A \subseteq B$
- (d) A
- (e) B
- (f) intersection
- (g) union

1p **3h** What must be filled in at position (vii)?

- a $A \cup B \subseteq B$
- b $A \cup B$
- c $A \subseteq B$
- d A
- e B
- f intersection
- g union

Question 4

7p **4a** Let $A = \mathbb{P}(\{1,2\})$. Let R be the relation on A defined as follows: XRY means "$X \cap Y = \varnothing$". Draw the relation diagram.

8p **4b** Let $A = \{1, 2, 3\} \times \{1, 2, 3\}$. Let R be the relation on A defined as follows: $(x_1, x_2)R(y_1, y_2)$ means "$x_1 + x_2 = y_1 + y_2$". This is an equivalence relation. (You do not need to prove this.) How many equivalence classes does R have? For each equivalence class, list explicitly which elements of A belong to the equivalence class.

Question 5

All the following questions are about *counting*. For each of the questions below an explanation/derivation is not required; you only need to state the final answer. Please give an exact number as final answer (i.e. don't just leave your answer as a counting equation).

5p **5a** A palindrome is a word that can be read the same way in either direction (such as RACECAR, MADAM or ABCBA). How many 9-letter palindromes (not necessarily meaningful) can be formed using the letters A-Z?

5p **5b** The four women Anne, Betsie, Charlotte and Dolores and the six men Eric, Frank, George, Harry, Ian and James are friends. Each of the women wants to marry one of the six men. In how many ways can this be done?

5p **5c** You go to get 7 cans of soda. There are 5 types of soda. How many different possibilities are there?

Question 6

Prove or disprove the following statements.

6p **6a** $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})((x+1)^2 - x^2 = 2y - 1)$.

6p **6b** $(\forall x \in \mathbb{Z})((x^2 \text{ is divisible by } 3) \Rightarrow (x \text{ is divisible by } 3))$.

Question 7

Let $f : \mathbb{R} \setminus \{-1, 1\} \to \mathbb{R}$ be the function defined as follows:

$$f(x) = \frac{1}{x^2 - 1}.$$

Let $\mathbb{R}^+ = \{x \in \mathbb{R} \;:\; x \geq 0\}$ and let $g : \mathbb{R}^+ \setminus \{0\} \to \mathbb{R}$ be the function defined as follows:

$$g(x) = \frac{1}{x^2}.$$

3p **7a** Prove or disprove that f is injective.

3p **7b** Prove or disprove that f is surjective.

3p **7c** Prove or disprove that g is injective.

3p **7d** Prove or disprove that g is surjective.

Question 8

True or False? For each of the statements below: state whether the statement is true or false. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

1p **8a** $\{1,2\} = \{2,1\}$

(a) True (b) False

1p **8b** $(1,2) = (2,1)$

(a) True (b) False

1p **8c** $\mathbb{P}(\{\{a,b,c\}\})$ contains eight elements

(a) True (b) False

1p **8d** $\{1\} \subseteq \{\{1\}\}$

(a) True (b) False

1p **8e** $\{1\}, \{1,2\}$ and $\{3\}$ are three different subsets of $\{1,2,3\}$

(a) True (b) False

1p **8f** $\{\{1\}, \{1,2\}, \{3\}\}$ forms a partition of $\{1,2,3\}$

(a) True (b) False

Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

9c

9d