Exercises

1	2	3	4	5	6	7	8	9

Surname, First name

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KEN1130 Discrete Mathematics
KEN1130 Discrete Mathematics Resit

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a b c d e f → b
a b c d e f → c
a b c d e f → a

Answer multiple-choice questions as shown in the example.

Program: Data Science and Artificial Intelligence
Course code: KEN1130
Examiners: dr. Marieke Musegaas and dr. Otti D'Huys
Date/time: Friday 02.02.2024 09h00-11h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.
Instructions to students:

- The exam consists of 8 questions on 16 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1
Consider the following logical proposition.

$$((p \Rightarrow q) \vee (q \Rightarrow p)) \wedge (\neg(p \Leftrightarrow q))$$

Answer the following questions about (the truth table of) the above proposition.

2.5p **1a** Suppose p is TRUE and q is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1b** Suppose p is TRUE and q is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1c** Suppose p is FALSE and q is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1d** Suppose p is FALSE and q is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

Question 2

12p **2a** Use induction to prove the following statement. For every natural number $n \in \mathbb{N}$,

$$3 + \sum_{i=1}^{n} (3 + 5i) = \frac{(n+1)(5n+6)}{2}.$$

3p **2b** Disprove the following statement. For every natural number $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} \left(2^{2i} - 1\right) = 6n - 3.$$

Question 3

6p **3a** Prove or disprove the following statement. For all sets A, B and C,

$$(A \cap B \neq \varnothing) \wedge (B \cap C \neq \varnothing) \quad \Rightarrow \quad A \cap C \neq \varnothing.$$

6p **3b** Prove or disprove the following statement. For all sets A, B and C,

$$(A \neq \varnothing) \wedge (B \neq \varnothing) \wedge (A \times B \subseteq B \times C) \quad \Rightarrow \quad A \subseteq C.$$

Question 4

12p **4a** Let R be the relation on the set $\mathbb{Z}$ defined as follows: xRy means "$|x - y| \leq 2.5$". Is R reflexive? Is R symmetric? Is R transitive? Is R anti-symmetric? For each of these properties, prove or disprove that it has that property.

5p **4b** Let R be an equivalence relation on $\{a, b, c, d, e\}$ such that

$$\{(a, d), (b, c), (e, a), (c, e)\} \subseteq R.$$

How many equivalence classes does R have?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4
- (f) 5
- (g) 6
- (h) None of the above.

Question 5

The following questions are about *counting*. For each of the questions below an explanation/derivation is not required; you should only state the final answer. Please give an exact number as final answer (i.e. don't just leave your answer as a counting equation).

5p **5a** How many 8-digit binary strings have exactly four ones or end with a one (or both)?
(Examples of such 8-digit binary strings are: 1 0 1 0 1 0 1 0, 0 0 0 0 1 0 1 1 and 0 0 1 0 1 0 1 1.)

5p **5b** Suppose that in a certain country, all license plates have four capital letters (from the alphabet, which contains 26 letters) followed by three digits. How many license plates could begin with an A and end with a 0? (An example of such a license plate is: AZYZ990.)

5p **5c** A set X has exactly 56 subsets with three elements. How many elements does X have?

Question 6

6p **6a** Prove or disprove the following statement.

$$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(y \neq 0 \Rightarrow xy = 1)$$

4p **6b** Let $A = \{0, 2, 4\}$ and $B = \{1, 9, 12\}$. Prove or disprove the following statement.

$$(\forall a \in A)(a \text{ is odd } \Rightarrow a \in B)$$

5p **6c** Let $x \in \mathbb{Z}$. What is the contrapositive of the statement “If x is even, then $3x + 7$ is odd”?

- (a) If x is even, then $3x + 7$ is even.
- (b) If x is odd, then $3x + 7$ is odd.
- (c) If x is odd, then $3x + 7$ is even.
- (d) If $3x + 7$ is even, then x is odd.
- (e) If $3x + 7$ is even, then x is even.
- (f) If $3x + 7$ is odd, then x is odd.
- (g) If $3x + 7$ is odd, then x is even.
- (h) None of the above.

(Note: only one answer is the correct answer.)

Question 7

6p **7** Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined as follows:

$$f(x) = \left\lfloor \frac{x+2}{3} \right\rfloor.$$

That means, $f(x)$ equals the greatest natural number less than or equal to $\frac{x+2}{3}$.
For example, $f(6) = \left\lfloor \frac{6+2}{3} \right\rfloor = \left\lfloor 2\frac{2}{3} \right\rfloor = 2$.
Prove or disprove that f is surjective.

Question 8

5p **8a** Let A be a set with $|A \times A| = 9$ and $\{(\{1,2\},3),(3,\{4,5\}),(\{1,2\},\{4,5\})\} \subseteq A \times A$. Which one of the following statements is true?

- (a) $A = \{1,2,3,4,5\}$
- (b) $A = \{(1,2,3,4,5)\}$
- (c) $A = \{\{1,2\},3,\{4,5\}\}$
- (d) $A = \{\{1,2\},\{3\},\{4,5\}\}$
- (e) $A = \{\{1,2,3\},\{3,4,5\},\{1,2,4,5\}\}$
- (f) $A = \{(1,2,3),(3,4,5),(1,2,4,5)\}$
- (g) None of the above.

(Note: only one answer is the correct answer.)

5p **8b** Let A and B be sets such that $A \cap B = A$ and $A \neq B$. Which one of the following statements is true?

- (a) $A \cup B = A$
- (b) $A \cup B = B$
- (c) $A \cup B = \varnothing$
- (d) None of the above.

(Note: only one answer is the correct answer.)

Question 9

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

9c

9d