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Surname, First name

Numerical Methods (KEN1540) KEN1540 Numerical Methods exam

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2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
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Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN1540

Examiners: Dr. Ir. Martijn Boussé and Dr. Pieter Collins **Date/time:** Monday 03 June 2024; 09:00-11:00hr

Format: Closed Book Exam

Allowed aids: DACS-approved calculator; Formula sheet (provided)

Instructions to students:

- The exam consists of 6 questions on 22 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- · You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

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Computer Arithmetic and Algebraic Equations

	nsider the problem for finding a solution of the algebraic equation $e^x - 6x - 2 = 0$ in the interval [0 Compute one step of Newton's algorithm, starting from the midpoint of the interval.
1b	What happens, and why?



Зр

2p

С	Perform one step of the bisection method to find a smaller bracket for the solution.
I	Perform one more step of Newton's method, starting from the midpoint of the interval obtained in
	part c .

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1f Write a Matlab statement which performs a single step of Newton's n	ch performs a single step of Newton's method for a function f , start	1e	Do you expect your new approximation to be within 0.1 of the true solution? Give a reason for your answer.						
1f Write a Matlab statement which performs a single step of Newton's n	ch performs a single step of Newton's method for a function f , star								
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write a matiab statement which performs a single step of Newton's n	ich performs a single step of Newton's method for a function f , stal		Mais - Madala at the second which confirms a simple atom of New tout and the different formation of a tout						
from point x .		1†	Write a Matlab statement which performs a single step of Newton's method for a function f , star from point x .						

Differential Equations

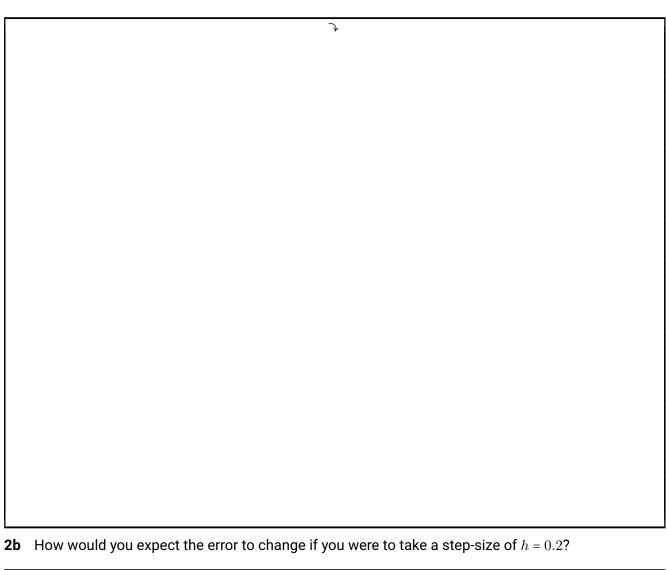
Consider the problem of finding the value of y(3) for the initial value problem

$$\frac{dy}{dt} = \frac{1}{t-1} - y^2, \ y(2) = 1.4.$$

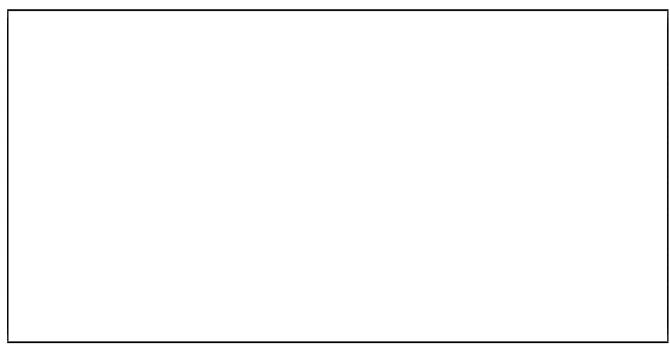
10p **2a** Solve the problem using Ralston's method with a step size of h = 0.5.



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2p





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2c	Write a Matlab function ode_ralston to solve a general differential equation $dy/dt = f(t,y)$ over time interval $[t_{\text{init}}, t_{\text{final}}]$ with intial value y_{init} using n steps of Ralston's method.						

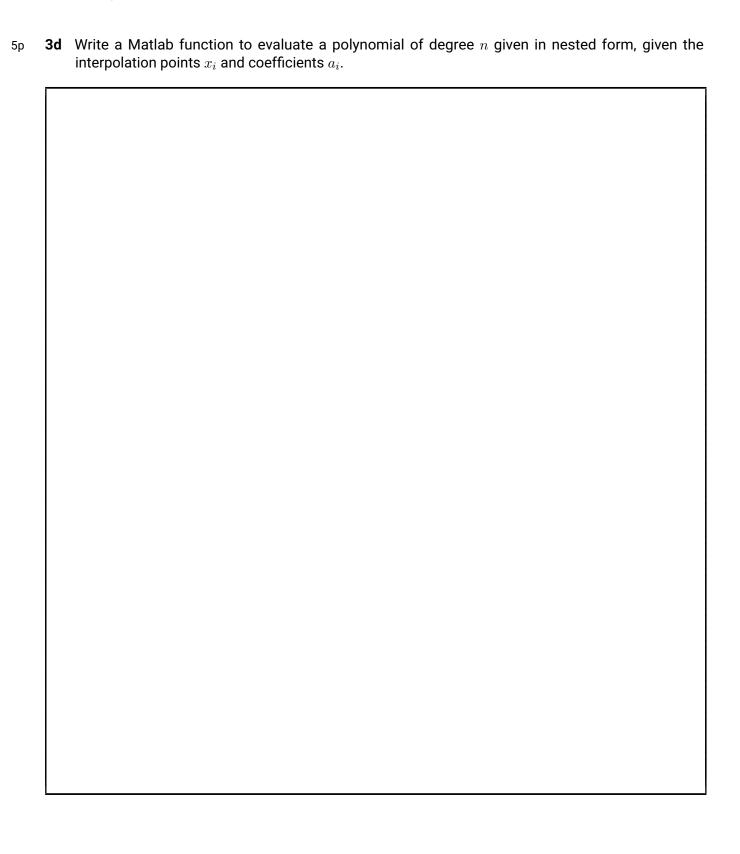
Polynomial Interpolation

Consider the data

6p **3a** Compute the divided differences $f[x_i,\ldots,x_j]$ for $0\leq i\leq j\leq 3$. You may assume $f[x_0,x_1]=-1.200$ and $f[x_1,x_2]=-1.640$.

2p	3b	b Write down the nested form of the cubic polynomial p_3 interpolating the given data.								
3р	3с	Estimate the value of y when x = 2.5 , showing intermediate steps in your computations.								
	_									





Integration & Differentiation

Consider the problem of computing to an accuracy of $0.01\ \mathrm{the}$ integral

$$I = \int_{0.0}^{1.2} f(x)dx$$
 for $f(x) = \cos(x^2)$.

5p **4a** Use the trapezoid rule with n=4 subdivisions to estimate the value of I..

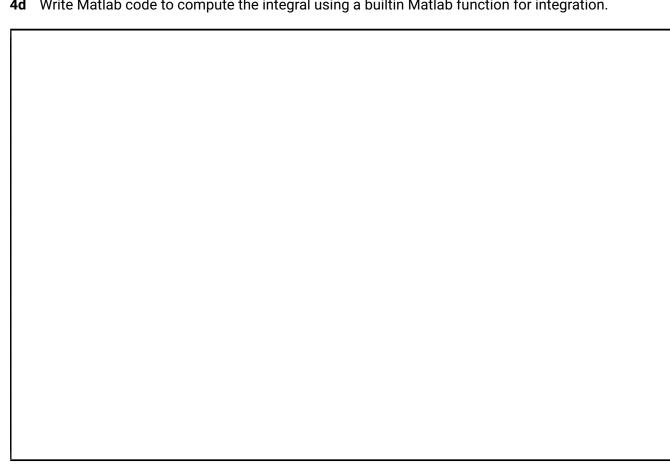




4c	Explain how to use the adaptive trapezoid method to obtain a better estimate to the integral. do not have to perform the computation.

	7	
Write Matlab code to com	npute the integral using a builtin N	Matlab function for integration.

Зр

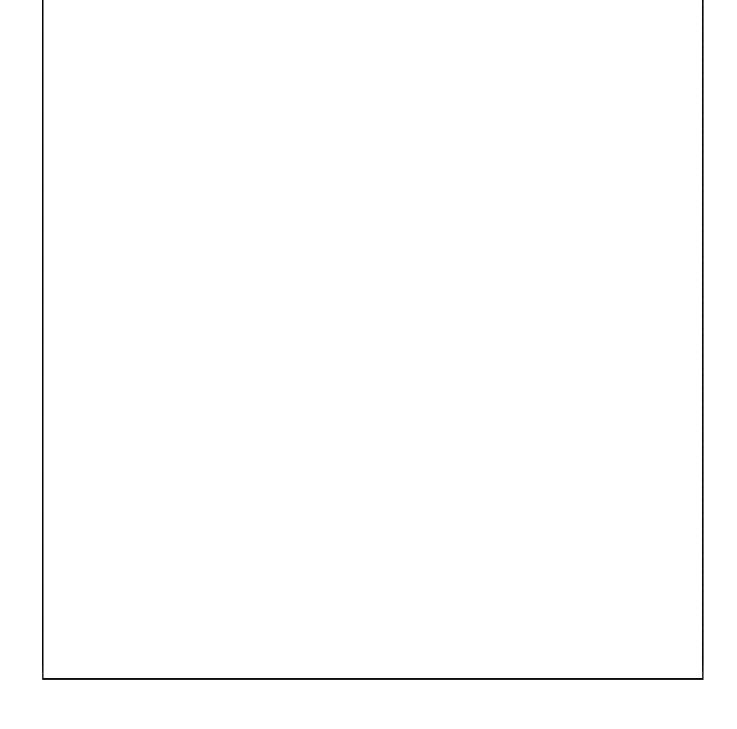


Least-Squares Approximation

Consider the data:

	x_i	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π
j	$f(x_i)$	0.550	0.300	-0.033	0.300	0.800	0.300	-0.033	0.300	0.550

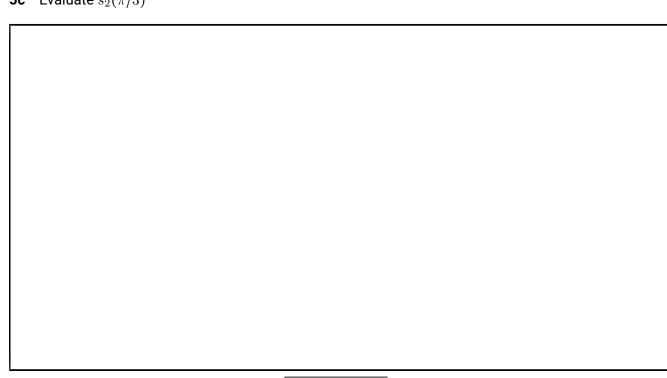
5a Compute the coefficients a_0, a_1, a_2 of the discrete Fourier transform.



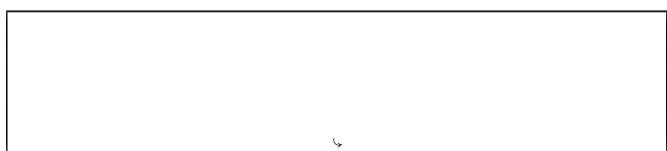
2p	5b	Assuming the coefficients b_i equal 0 for all i , write down the second Fourier approximation s_2 to f .	
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2p **5c** Evaluate $s_2(\pi/3)$



3p **5d** What is the root-mean-square value $\sqrt{\frac{1}{2\pi} \int_0^{2\pi} s_2(x)^2 dx}$?





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5e Write Matlab code to compute the Fourier coefficient a_2 for n equally-spaced data points in $[0,2\pi)$. 3р



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Linear Algebra

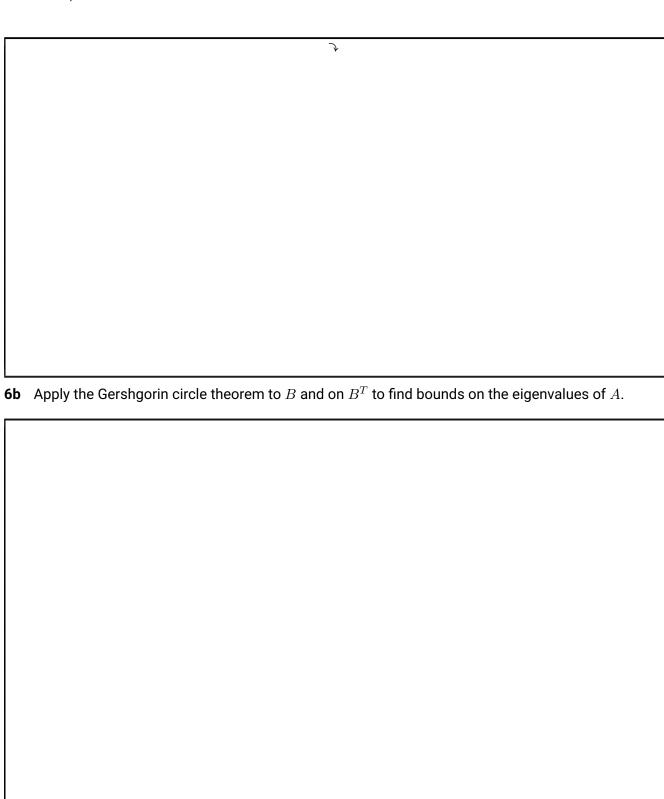
Let A be the matrix

$$\begin{pmatrix} 7 & 5 \\ 1 & 4 \end{pmatrix}$$

9p **6a** Apply one step of the QR-method to find a matrix B with the same eigenvalues as A.



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4p





Extra Paper

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