Exercises

1	2	3	4	5	6	7	8

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Surname, First name

KEN1540 Numerical Mathematics
KEN1540 Resit

Program: Bachelor Data Science and Artificial Intelligence
Course code: KEN1540
Examiners: Dr. Ir. Martijn Boussé & Dr. Pieter Collins
Date/time: Friday 7 July 2023; 09:00-11:00
Format: Closed Book Exam
Allowed aids: DACS-approved calculator; Formula sheet (provided)

Instructions to students:

- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Computer Arithmetic & Algebraic Equations

Consider the quadratic polynomial

$$p(x) = ax^2 + bx + c,$$

where $a = 0.51$, $b = 1.96$, and $c = -0.05$.

4p **1a** Compute the positive root of $p(x)$ using the formula $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ in **3-digit rounded arithmetic**.

3p **1b** Compute the positive root of $p(x)$ using the alternative formula $x = \frac{-2c}{\sqrt{b^2-4ac}+b}$ in **3-digit rounded arithmetic**.

2p **1c** Compute the relative error for the estimates in subquestions (a) and (b), given that the exact value is $0.02534308 (8\text{dp})$.

3p **1d** Which formula do you prefer for this root, and why?

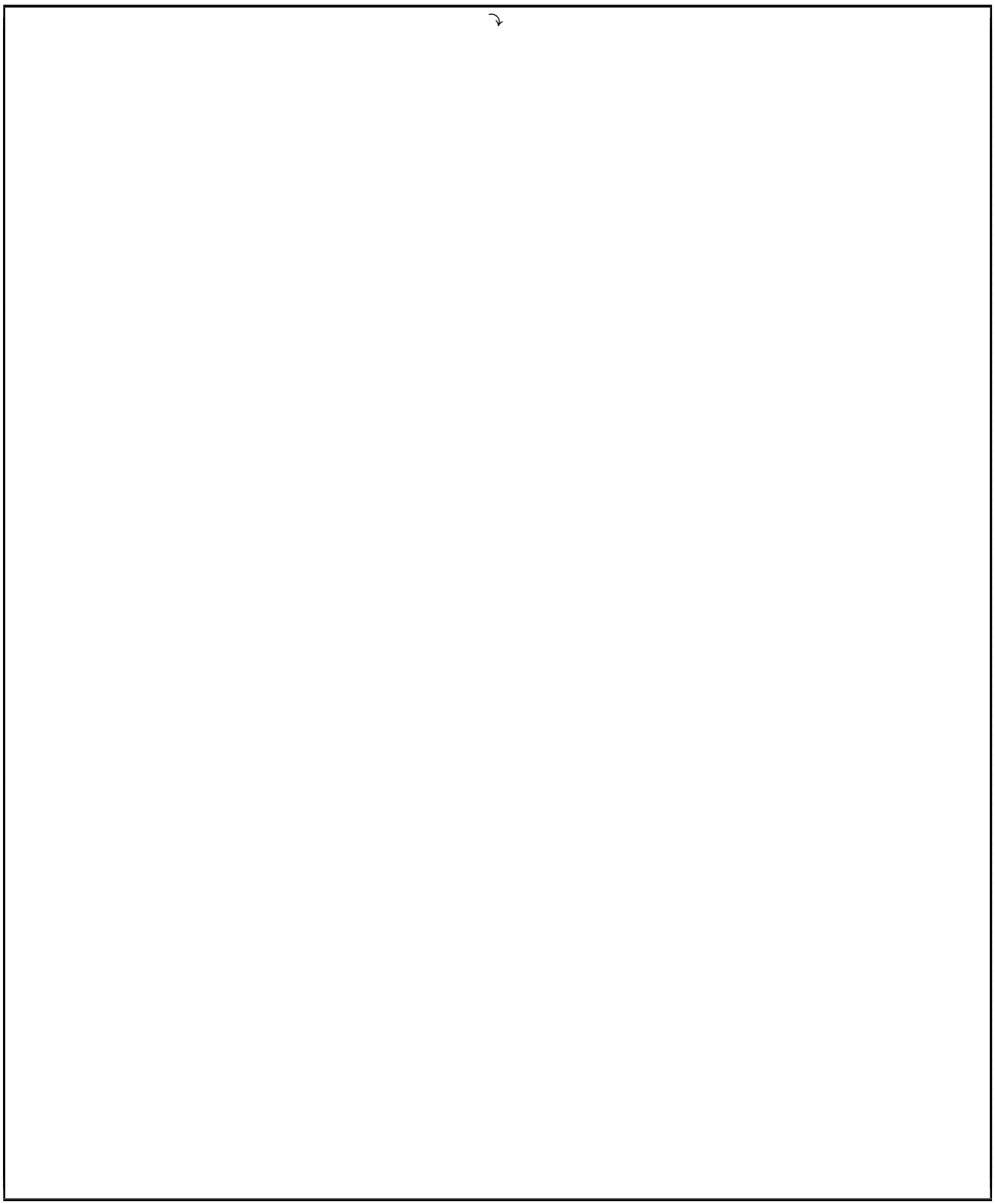
Differential Equations

Consider the initial value problem

$$\frac{dy}{dt} = \frac{\cos(ty)}{t}$$

with $y(2.0) = 1.000$.

10p **2** Perform two steps of **Ralston's method**, using step size $h_0 = 0.6$ for the first step and $h_1 = 0.4$ for the second step.



Polynomial Interpolation

Consider the function f on $[0, 3]$ given by

$$f(x) = \frac{1}{1+x}.$$

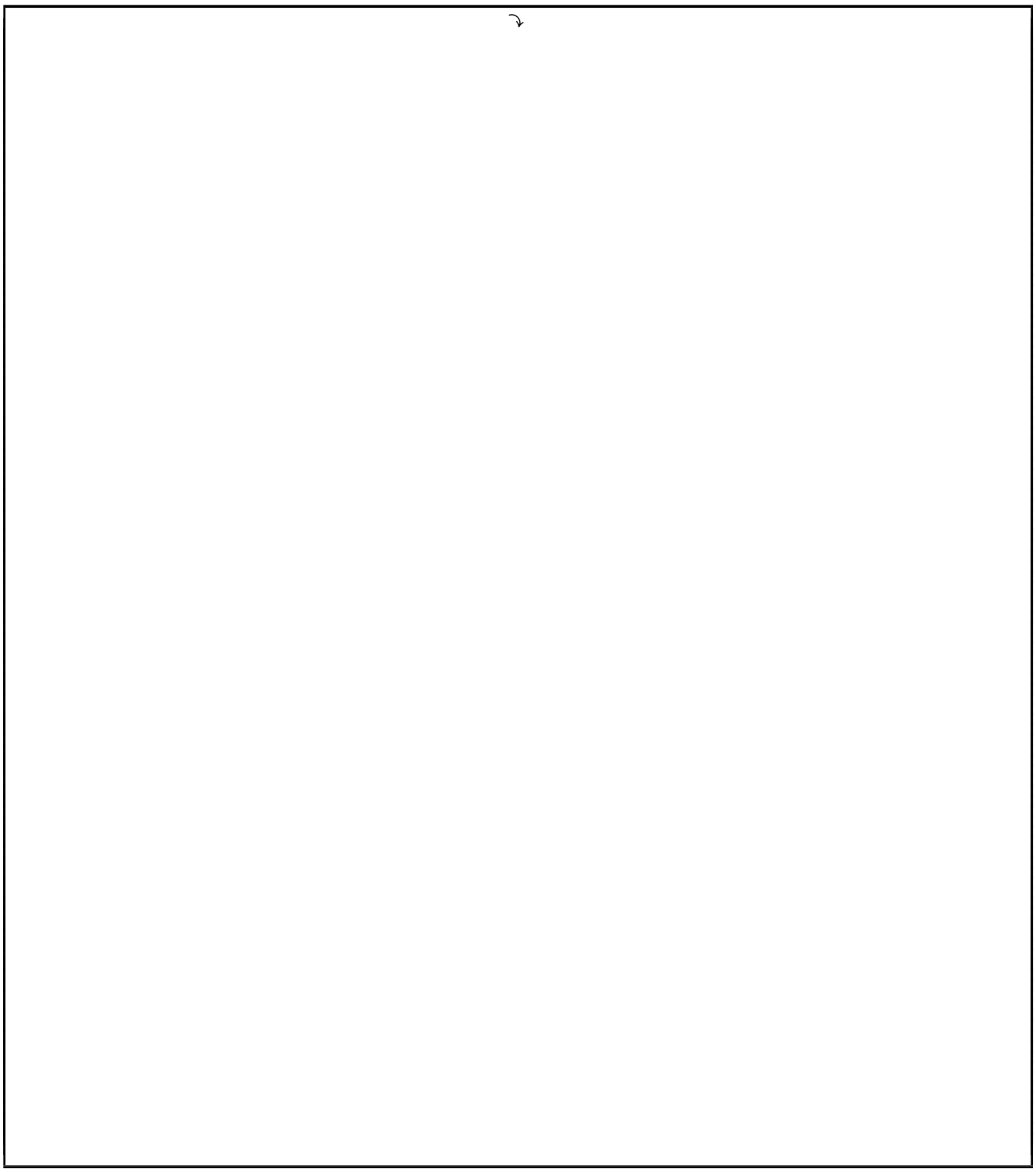
3p **3a** For $n = 4$, write down the $n + 1 = 5$ Chebyshev nodes $x_0, \ldots, x_n$ in $[0, 3]$.

6p **3b** Compute a **fourth-degree** polynomial $p(x)$ approximating $f(x)$ using **Chebyshev nodes** by means of divided differences. The following values are given:

$$f[x_0, x_1] = 1.088129, \quad f[x_1, x_2] = 0.691811, \quad f[x_2, x_3] = 0.391463, \quad f[x_3, x_4] = 0.296950,$$
$$f[x_0, x_1, x_2] = -0.416713, \quad f[x_1, x_2, x_3] = -0.255492, \quad f[x_2, x_3, x_4] = -0.099377\,.$$

Note there is no need to evaluate f at every x_i to complete this question.

↳



2p **3c** Use $p(x)$ to estimate the value of $f(x)$ at $x = 0.75$.

3p **3d** Determine a bound on the error, assuming $|f^{(5)}(\xi)| \leq 16$, for all $0 \leq \xi \leq 3$.

Numerical Integration & Differentiation
Consider the following data:

x	1.0	1.5	2.0	2.5	3.0
$f(x)$	0.84	1.52	1.82	1.48	0.42

.

5p **4a** Approximate $\int_1^3 f(x)\,dx$ using **Simpson's rule** for the given data.

2p **4b** How would you expect the accuracy of your result to improve if you were to use Simpson's rule with a spacing of 0.1 between the data points?

5p **4c** Using the most accurate 3-point difference formula available, give approximations of $f'(1.0)$ and $f'(2.5)$.

Least-Squares Approximation

2p **5a** Briefly explain the significance of the fast Fourier transform for large-scale signal processing, including a discussion of the computational complexity.

8p **5b** Compute the second approximations s_2 of the discrete Fourier transform for the following $n = 8$ data points over the time interval $[0, 2\pi]$:

t_j	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$
y_j	3.42	3.20	1.16	1.80	4.46	3.83	1.16	1.51

↳

2p **5c** What is the strongest frequency present in the signal?

Linear Algebra

3p **6a** State the Gershgorin circle theorem.

Let

$$A = \begin{pmatrix} 4 & 5 \\ 3 & 7 \end{pmatrix}.$$

7p **6b** Apply one step of the QR method to A.

↳

2p **6c** Give bounds on the eigenvalues of A,

Modelling and Matlab

4p **7a** Write Matlab code to find a solution of the equation $g(x) = 0$ up to a tolerance of e, using the secant method starting at points p and q.

The population (in millions) of a particular species in year depends on the population in the previous year as $x_{n+1} = f(x_n)$, where $f(x) = 7xe^{-x}$.

4p **7b** Write Matlab code to estimate the population at time $n = 4$ to an accuracy of a thousand individuals, assuming the population at time $n = 5$ is 2 million.

Extra paper

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