

ECE 599 / CS 519 Convex Optimization - Homework 4

Fall 2017

School of Electrical Engineering and Computer Science
Oregon State University

Due: Nov. 28, 2017

Q1 Consider the problem

$$\begin{aligned} & \text{minimize } \|y - Ax\|_2^2 + \lambda \sum_{g=1}^G \|x_g\|_2 \\ & \text{subject to } x \geq 0, \mathbf{1}^T x = 1, \end{aligned}$$

where we have $x = [x_1^T, \dots, x_g^T, \dots, x_G^T]^T$ and $x_g \in \mathbb{R}^{n/G}$ (assuming n/G is an integer) for $g = 1, \dots, G$, and $\lambda = 1$. The formulation promotes group sparsity of x . Such structured sparsity is of great interest in many applications.

- (a) (Design) Design an ADMM algorithm for the above. (50%)
- (b) (Implementation and Discussion) Generate data using the following code:

```
% data generation
n = 100;
m = 30;
A = randn(m,n);
x = [rand(3,1); zeros(30,1);rand(3,1);zeros(30,1);rand(3,1);zeros(31,1)];
x = diag(1./sum(x))*x;
v = 0.1*randn(m,1);
y = A*x + v;
```

Set $n/G = 3$ and implement your ADMM algorithm. Note: There are strategies for stopping the algorithm and tuning ρ in Chapter 3.3 and 3.4 of [Boyd et al. 2011]. Read this part carefully and implement these strategies in your algorithm. Compare the solutions that you obtain from ADMM and CVX (for any random trial, your solution should be close to what CVX outputs, as sanity check). Record the runtime when you can obtain a solution that is fairly close to the solution given by CVX; e.g., stop your algorithm when

$$\|x_{\text{admm}} - x_{\text{cvx}}\|_2 \leq \epsilon,$$

and record the runtime. Make a table using $\epsilon = 0.1, 0.01, 0.001, 0.0001, 0.00001$ and record the time that ADMM and CVX use, respectively (you may take average of 10 random trials for each ϵ). What do you observe from this experiment? (40%)

- (c) Plot your estimation error $\|x_{\text{admm}} - x_{\text{g}}\|_2^2$, where x_{g} denotes the ground truth x that you generate, under a set of λ 's such as $\lambda \in \{0.1, 1, 2, \dots, 10\}$. Try to identify a good λ for our problem setup. (10%)

[Boyd et al. 2011] Stephen Boyd et al. “Distributed optimization and statistical learning via the alternating direction method of multipliers.” Foundations and Trends in Machine Learning. 2011