ECE 599 / CS 519 Convex Optimization - Homework 3

Fall 2017
School of Electrical Engineering and Computer Science
Oregon State Univeristy

Due: Nov. 9, 2017

Q1 The following are either convex, concave, or neither convex nor concave. Identify their convexity/concavity, and provide your answer with a proof.

a) The function
$$f(x) = \max\{\|APx - b\|_2 \mid P \text{ is a permutation matrix}\}$$
with with $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$. Note that a permutation matrix P is a column-permuted version of the identity matrix. (5%)

b) The smallest eigenvalue function
$$f(X) = \lambda_{\min}(X), \quad \mathrm{dom} f = \mathbb{S}^n.$$
(5%)

c) The function
$$f(x) = \int_0^{2\pi} \log p(x,\omega) d\omega,$$
where $p(w,\omega) = x_1 + x_2\cos(\omega) + \ldots + x_n \cos((n-1)\omega)$ and $\mathrm{dom} f = \{x \mid p(x,\omega) > 0, 0 \le \omega < 2\pi\}$ (note that $\log(\cdot)$ here is the natural logarithm). (5%)

d) The difference between the maximum and minimum value of a polynomial on a given interval, as a function of its coefficients:
$$f(x) = \sup_{t\in[a,b]}\ p(t) - \inf_{t\in[a,b]}\ p(t),$$
where $p(t) = x_1 + x_2 t + x_3 t^2 + ... + x_n t^{n-1}$, and a and b are real constants with $a < b$. (5%)

Q2 Verify that $x^\star = (1, 1/2, -1)$ is optimal for the optimization problem
$$\begin{aligned} &\text{minimize } (1/2)x^T P x + q^T x + r \\ &\text{subject to } -1 \le x_i \le 1,\ i = 1,2,3, \end{aligned}$$

where

$$P = \begin{bmatrix} 13, & 12, & -2 \\ 12, & 17, & 6 \\ -2, & 6, & 12 \end{bmatrix}, \; q = \begin{bmatrix} -22.0 \\ -14.5 \\ 13.0 \end{bmatrix}, \; r = 1.$$

(10%)

Q3 Answer the following questions.

a) Consider the linear program

$$\begin{aligned} &\text{minimize } c^T x \\ &\text{subject to } Ax \preceq b \end{aligned}$$

with A square and nonsingular. Show that the optimal value is given by

$$p^\star = \begin{cases} c^T A^{-1} b, & A^{-T} c \preceq 0 \\ -\infty & \text{otherwise} \end{cases}$$

(10%)

b) Formulate the following problems as an LP:

$$\begin{aligned} &\text{miminize } \|x\|_1 \\ &\text{subject to } \|Ax - b\|_\infty \leq 1, \end{aligned}$$

where $A \in \mathbb{R}^{m \times n}$. (10%)

c) Formulate the ℓ_4-norm approximation problem as an equivalent QCQP:

$$\text{minimize } \|Ax - b\|_4,$$

where $A \in \mathbb{R}^{m \times n}$. (10%)

d) Consider a robust variation of the (convex) quadratic program

$$\begin{aligned} &\text{minimize } (1/2)x^T P x + q^T x + r \\ &\text{subject to } \; Ax \preceq b \end{aligned}$$

For simplicity we assume that only the matrix P is subject to errors, and the other parameters (q, r, A, b) are exactly known. The robust quadratic program is defined as

$$\begin{aligned} &\text{minimize } \sup_{P \in \mathcal{E}} (1/2)x^T P x + q^T x + r \\ &\text{subject to } \; Ax \preceq b. \end{aligned}$$

Express the robust QP as an SOCP problem given that

$$P = \{P_0 + \sum_{i=1}^{K} P_i u_i \mid \|u\|_2 \leq 1\},$$

where $P_i \in \mathbb{S}_+^n$ for $i = 0, 1, \ldots, K$. (10%)

Q4 Download the datasets `train_separable.mat` and `test_separable.mat` from the course website. Download `CVX` from `http://cvxr.com/cvx/` (or `http://www.cvxpy.org/en/latest/` if you use Python) and learn how to use it. Implement the following using `CVX`.

a) Apply the C-Hull formulation to train a classifer, i.e.,

$$\begin{aligned} \text{minimize}_{u,v} \ & \|Au - Bv\|_2^2 \\ \text{subject to } & 1^T u = 1, \ u \succeq 0 \\ & 1^T v = 1, \ v \succeq 0 \end{aligned}$$

Visualize the training data together with the classifier. Also visualize the testing data and the classifier in another figure, and report the classification error on the testing data using the true labels provided in `test_separable.mat` . (15%)

d) Repeat the above for `train_overlap.mat` and `test_overlap.mat` using the reduced C-Huall, i.e.,

$$\begin{aligned} \text{minimize}_{u,v} \ & \|Au - Bv\|_2^2 \\ \text{subject to } & \mathbf{1}^T u = 1, \ d\mathbf{1} \succeq u \succeq 0 \\ & \mathbf{1}^T v = 1, \ d\mathbf{1} \succeq v \succeq 0. \end{aligned}$$

Report the classification error on the testing data using $d = 0.9$. (15%)

(Please print out and submit your scripts. In case that your already forgot what is C-Hull, check out the paper by Kristin P. Bennett, and Erin J. Bredensteiner, "**Duality and geometry in SVM classifiers**," ICML 2000. In addition, for background of classification, check out the slides of **Lecture 1**.)