A Comparison of the FRQI and NEQR Quantum Image Representations for Astrophysical Digital Image Processing

Dylan Zhang

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Abstract

The rapid analysis of the growing number of images acquired by telescopes and interferometers is critical to discovery in modern astrophysics. While this analysis can be conducted using classical computational infrastructure, quantum computation promises to substantially accelerate image processing by being able to analyse many images in parallel, store large amounts of data across relatively small numbers of qubits, and rapidly apply quantum machine learning algorithms to this data. We analyze two techniques for representing quantum images, Flexible Representation of Quantum Images (FRQI) and Novel Enhanced Quantum Representation (NEQR), as well as quantum algorithms that can be used to process the data in these quantum images. We show that both the FRQI and NEQR representations could be useful for astrophysical image processing on future quantum computing hardware. We specifically highlighted the noise issues of FRQI on existing quantum computing hardware and the higher qubit counts of NEQR, which could also be limiting on current hardware. My results underscore how quantum image processing can substantially accelerate the rate of scientific discovery in astrophysics once more fault-tolerant quantum computers become more freely available.

Introduction

Modern astrophysics is highly dependent on the rapid interpretation and processing of images from telescopes and interferometers. Modern and next-generation scientific instruments and surveys, such as the James Webb Space Telescope, have exponentially increased the volume of astronomical imaging data that requires processing, analysis, and de-noising.[1] In order to extract meaningful information and insights from astronomical images in an efficient manner, current image processing methods, such as classical Fourier Transforms, will not suffice as they require large amounts of expensive computational resources.[10] This has led to the exploration of using quantum computing to process astronomical images.

Quantum computing leverages concepts such as superposition, entanglement, and quantum parallelism to perform certain tasks more efficiently than classical computers.[2] In quantum image processing, images are encoded into quantum states, allowing for multiple image processing tasks to be performed simultaneously. This leads to image processing that is more efficient and has better scaling than conventional image processing.[4] However, there are many significant drawbacks to using quantum computing and quantum image processing algorithms to process astronomical images. The most prominent issue in relying on quantum computers to process astronomical images is the lack of current hardware technology in quantum computers. Specifically, modern quantum computers lack the required number of qubits (quantum bits) needed to process large data sets and images.

Several quantum image processing algorithms have been proposed in recent years to address this issue regarding qubits, such as Flexible Representation of Quantum Images (FRQI)[6] and Novel Enhanced Quantum Representation (NEQR);[7] both of these algorithms can significantly reduce the complexity of encoding images compared to conventional image processing algorithms, as well as reduce the number of qubits necessary to encode these images.[3] Additionally, quantum algorithms such as QRMW (Quantum Representation of Multi-Wavelength Images), which builds on the NEQR model but expands it to multiple wavelength channels, can reduce the complexity of processing multi-wavelength images as well as the number of qubits necessary to process multi-wavelength images by allowing for multiple image processing tasks to be performed simultaneously.[5]

One of the earliest and most well-known of these algorithms is FRQI, in which the intensity of each pixel of an image is encoded in a rotation angle of the amplitudes of the quantum state, allowing for easier application of the Quantum Fourier Transform (QFT) algorithm to the amplitudes of the image.[6] QFT is the quantum analogue of the Fourier Transform, which is often used for classical image processing. For an image with dimensions $2^n \times 2^n$, FRQI would require $2n + 1$ qubits to encode the image. NEQR, on the other hand, represents pixel intensity in the basis states rather than the amplitudes. It requires $2n + q$ qubits (where q increases with the number of grayscale levels) for an image with dimensions $2^n \times 2^n$.

The quantum image processing algorithms FRQI and NEQR are evaluated on simplified astrophysical images. The aim is to determine the potential advantages of each quantum image processing algorithm when used for common astrophysical tasks, as well as to determine the short-term practicality of using these algorithms for astrophysical image processing. We encode and then retrieve astrophysical images compressed to 8×8 dimensions with both FRQI and NEQR, and then compare the integrity of the retrieved image as well as the qubit requirement of the two quantum image representations.

Algorithm Background

Flexible Representation of Quantum Images (FRQI)

FRQI represents a grayscale image of dimensions $2^n \times 2^n$ in $2n + 1$ qubits, with $2n$ qubits used to encode pixel location and an additional qubit for the grayscale value. The quantum state for FRQI is given by the following equation.

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2n-1} (cos(\theta_i)|0\rangle + sin(\theta_i)|1\rangle)|i\rangle \tag{1}$$

In this equation, θ_i is an angle derived from the intensity of pixel i.[3] This amplitude-based representation allows for better compatibility with QFT, which allows for quantum versions of classical image processing operations such as filtering, edge detection, and frequency-domain analysis. FRQI has been implemented for small images (e.g., 2×2); however, it is more suitable for fault-tolerant hardware due to the rotation gates that are used for encoding intensity being prone to error.

Novel Enhanced Quantum Representation (NEQR)

In contrast to FRQI, NEQR stores pixel values in binary form using basis states rather than through amplitude rotations.[7] For a grayscale image of dimensions $2^n \times 2^n$, NEQR requires $2n + q$ qubits (where 2^q is the number of grayscale values in the image), with $2n$ qubits for pixel location and an additional q qubits for the grayscale value. NEQR is expressed mathematically as follows:

$$|I\rangle = \frac{1}{2^n} \sum_{X=0}^{2^n-1} \sum_{Y=0}^{2^n-1} |f(X,Y)\rangle |XY\rangle, \tag{2}$$

where $f(X, Y)$ represents the pixel intensity at (X, Y). By avoiding rotation operations for intensity encoding and instead storing intensity values in binary form with basis states, NEQR is less susceptible to noise compared to FRQI.[7] This allows for better implementation of several important quantum algorithms for image analysis, for example, quantum machine learning algorithms like Quantum Support Vector Machines (QSVM), which can be used for image classification tasks. However, NEQR requires an extra $q - 1$ qubits compared to FRQI, where 2^q is the number of grayscale values in the image.[7] The greater number of required qubits could be limiting, given the current constraints on quantum computing hardware.

Methods

In order to evaluate and compare the capabilities of FRQI and NEQR for processing astronomical images, we utilized IBM's Qiskit quantum computing framework. The computations were done on the Qiskit Aer quantum simulator. To simulate an astrophysical use case, we used simplified, grayscale versions of astrophysical images from the Hubble Space Telescope, rescaled to dimensions 8×8 (which allowed for a manageable 6 qubits to encode pixel position). The original astrophysical images were loaded in JPEG format and converted to grayscale. To downscale the images to 8×8 dimensions, we used the INTER_AREA interpolation method in OpenCV to map the average pixel intensity in an area and map it to a single pixel. The intensity values were then divided by 255 to scale them to the [0, 1] range. These normalized and resized images were then used as inputs for encoding with FRQI and NEQR.

The codes for FRQI and NEQR were taken from open-source implementations on GitHub[9] and adapted for astrophysical images.[11] For FRQI, the intensity of each pixel was encoded into the rotation angle (θ_i). For NEQR, on the other hand, the intensity of the pixels was encoded into an 8-bit binary string. Both FRQI and NEQR were run with 20,000 shots on the normalized and resized 8×8 grayscale astronomical images with 256 pixel intensities (0 to 255). The two original astronomical images are shown below:

Figure 1: (Left) NGC 4921 captured by the Hubble Space Telescope. **(Right)** Hoag's Object captured by the Hubble Space Telescope.

For both images, we wanted to recover the overall shape of the galaxy in the retrieved images from FRQI and NEQR. For the image of NGC 4921, we looked for the retrieved images to have a brighter center and a lower pixel intensity near the edge of the galaxy. For the image of Hoag's Object, we hoped for both retrieved images to have a clear bright spot in the middle and a circular shape overall.

After retrieving the images, we compared the results of two quantum image processing algorithms, FRQI and NEQR. We compared them in terms of how well the original image was reconstructed and recovered from the quantum state and how many qubits each algorithm took. Because the images we used were relatively small (being compressed to 8×8 pixels and grayscale), the results we got will likely tend to be more accurate than similar results for larger images with more potential pixel intensities and colors.

Results

To evaluate the performance of FRQI and NEQR on astrophysical images, we used both representations to encode and retrieve an 8×8 pixel 256 intensity value grayscale image of an astrophysical body with 20000 shots. Below are the original images, the resized 8×8 pixel grayscale image, the image retrieved after encoding with FRQI, and the image retrieved after encoding with NEQR.

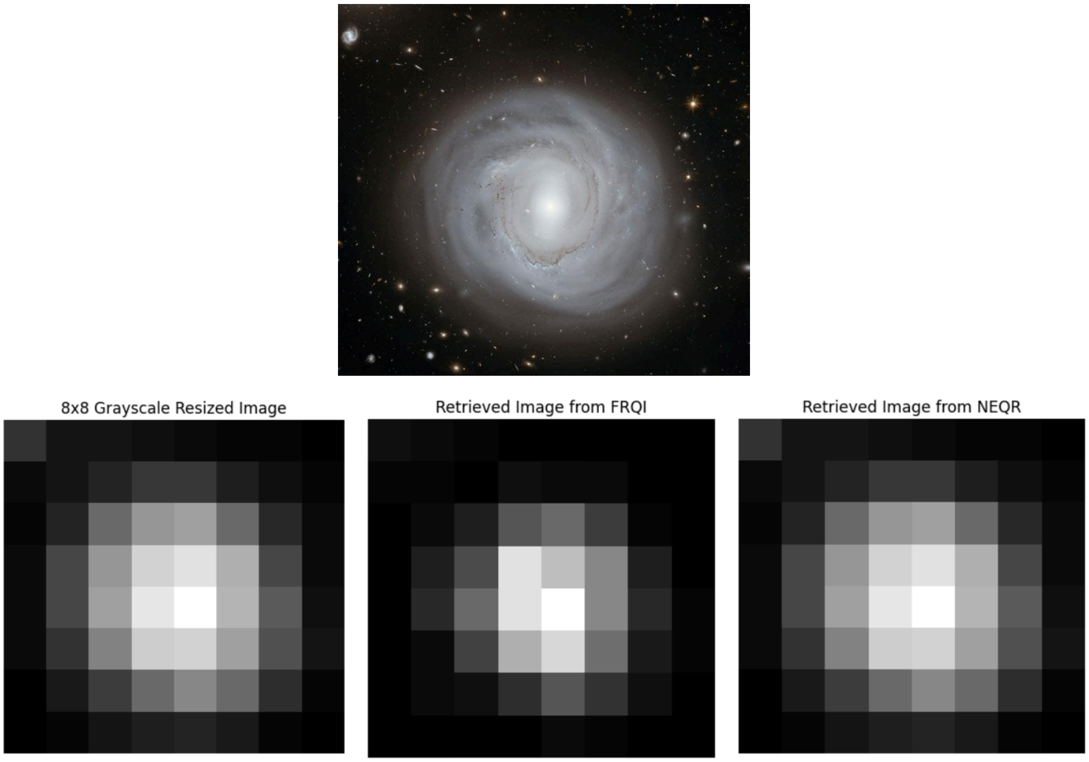

Figure 2: (Top) NGC 4921 captured by the Hubble Space Telescope. **(Bottom, from left to right)** 8×8 resized grayscale image, image retrieved from FRQI, image retrieved from NEQR.

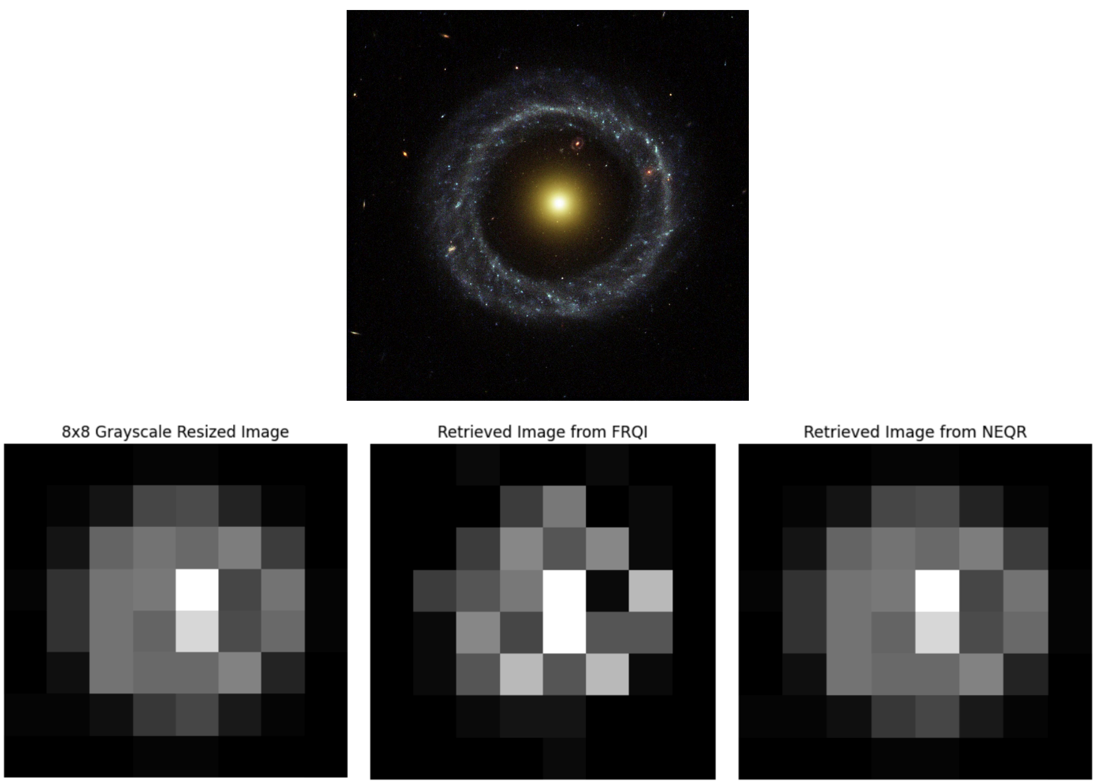

Figure 3: (Top) Hoag's Object captured by the Hubble Space Telescope. **(Bottom, from left to right)** 8×8 resized grayscale image, image retrieved from FRQI, image retrieved from NEQR.

Image Fidelity

Visually, one can see that the image encoded and retrieved with FRQI is noticeably different from the original figure based on pixel intensities and overall image fidelity. The image that was encoded and retrieved with NEQR is a much more accurate reconstruction of the original image, without any noticeable differences between the original and retrieved image in terms of the overall shape of the object in the image or any pixel intensity values. This difference is likely because of NEQR's use of base states and binary to encode pixel values, which is more free from error and noise than FRQI's rotation-based encoding. Since both quantum representations were run with the same number of shots (20000), it is apparent that NEQR is freer from error than FRQI for encoding and decoding images.

Number of Qubits

In terms of the number of qubits needed to run FRQI and NEQR, NEQR used substantially more qubits than FRQI. FRQI only required

$$2n + 1$$
$$2^3 = 8,\ n = 3$$
$$2 \times 3 + 1 = 7 \text{ qubits},$$

while NEQR required

$$2n + q$$
$$2^3 = 8,\ n = 3$$
$$256 = 2^8,\ q = 8$$
$$2 \times 3 + 8 = 14 \text{ qubits}.$$

The extra qubits NEQR requires compared to FRQI can be attributed to NEQR requiring 8 qubits to represent pixel intensity on top of the 6 qubits for pixel position, as NEQR stores each grayscale value in an 8-bit binary number. NEQR required twice the number of qubits that FRQI required, which is a very substantial increase, though these extra qubits do allow for better image retrieval and less susceptibility to noise and error.

	FRQI	NEQR
Number of qubits	7	14
Visual fidelity	Imprecise, noise	Exact grayscale values
Circuit depth	Deeper (multi-controlled rotations)	Shallower (mostly Hadamard and controlled NOT gates)

Table 1. Comparison of FRQI and NEQR

Discussion

The results clearly show a trade-off between the lower qubit counts and reduced visual fidelity of FRQI, with the relatively higher qubit counts and higher visual fidelity with less noise and errors of

NEQR. Of course, with more shots, the visual fidelity of FRQI would start to approach the level of NEQR, so future research into this topic could focus on larger datasets of images and more shots for each algorithm.

With the limitations on qubit counts in modern quantum computers, FRQI is undoubtedly very useful for current quantum image representation when qubit count or quantum computing hardware is a major limitation. If a large image is being encoded, FRQI will be able to use fewer qubits and therefore be easier to run on modern quantum computers with limited numbers of qubits. However, when noise and errors are of greater concern than the limit on the number of qubits, NEQR will provide more accurate and less noisy results when encoding image data due to its binary encoding of pixel intensity values.

For astrophysics, FRQI could be more useful than NEQR for things analyzed in the frequency domain (such as pulsar timings or gravitational waves), as these often rely on Fourier Transforms, and FRQI's reliance on amplitude encoding works well with Fourier Transforms.[12] NEQR can also be used extensively in cosmology, as it can be extended into models such as QRMW. QRMW is a framework that allows for the encoding and processing of multi-wavelength channel images, such as the composite images often used for star-forming regions and nebulae.[13] Also, due to NEQR encoding pixel intensity data into basis states, it is better suited for quantum machine learning algorithms, which could be useful for detecting and classifying astronomical objects through image data. Because you can directly access the definite intensity values of each pixel with NEQR, quantum machine learning algorithms that require discrete and definite inputs, such as Quantum SVM, can be implemented much easier, as NEQR can provide those inputs directly.

However, it should be made clear that neither FRQI nor NEQR can be implemented on a large scale or at any scale that would be of real use in cosmology and astrophysics research with current quantum computing technology. Current hardware restrictions on the number of qubits available and the problem of noise in modern quantum computers make implementing quantum image representations like FRQI and NEQR on a scale larger than very small images unfeasible, hence why we tested them on images with dimensions 8×8 pixels. While they both have their use cases and both have the potential to massively improve image encoding and processing for astrophysics and cosmology research, we may have to wait for future innovations in quantum computing hardware to see them implemented at a large scale for image processing for research.

Conclusions

We compared two quantum image processing algorithms, FRQI and NEQR, on simplified astronomical images. Both of these algorithms aim to utilize quantum computing to more efficiently process images by encoding image data into quantum states. Due to FRQI's amplitude-based encoding, the retrieved astronomical images came out far noisier compared to the ones retrieved after being encoded with NEQR; however, FRQI did use fewer qubits than NEQR. NEQR's use of basis states for intensity data, on the other hand, allowed for far less noise when compared with the image retrieved after being encoded with FRQI, but also required more qubits to encode intensity values.

The results clearly show that FRQI is more suitable for hardware where the number of qubits is a major limitation, and NEQR is more suited for situations where noise is more of an issue than any qubit limitation.. These results suggest that, for astrophysical image processing, NEQR may be more practical on current noisy quantum computers due to the reduced noise, though if there are many grayscale values or if qubits are a large limiting factor, FRQI could be more practical.

While both of these algorithms show promise and have their own specific use cases in astrophysics research, they are both limited by the hardware of current quantum computers, both in terms of the number of qubits and in terms of noise and error, as highlighted by the usage of images of dimensions 8×8 pixels. As quantum computing hardware improves, both of these quantum image processing algorithms will prove to be useful for processing astrophysical image data and offer a more efficient alternative to traditional image processing algorithms for astrophysics research.

Data and Software Availability

All code for quantum image processing, encoding, and image reconstruction with FRQI and NEQR was adapted from publicly available repositories that can be found on GitHub at: https://github.com/Sharma-Siddhartha/FRQI-and-NEQR

The astrophysical images used in this study were obtained from a dataset of Hubble Space Telescope images at: https://www.kaggle.com/datasets/mightyglow/astronomical-image-and-csv-dataset?resource=download

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References

1. Garofalo, M., Botta, A., and G. Ventre. Astrophysics and Big Data: Challenges, Methods, and Tools. *Proceedings of the International Astronomical Union*. 12(S325):345-348 (2016). doi:10.1017/S1743921316012813
2. J. Preskill. Quantum Computing and the Entanglement Frontier. arXiv:1203.5813 (2012).
3. Anand, A., Lyu, M., Baweja, P.S., and V. Patil. Quantum Image Processing. arXiv:2203.01831 (2022).
4. Dendukuri, A. and K. Luu. Image Processing in Quantum Computers. arXiv:1812.11042 (2019).
5. Engin, S. and I. Yilmaz. QRMW: Quantum Representation of Multi-Wavelength Images. *Turkish Journal of Electronical Engineering and Computer Sciences*. 26(2): 768-779 (2018).
6. Le, P.Q., Dong, F., and K. Hirota. A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. *Quantum Inf Process*. 10, 63–84 (2011). https://doi.org/10.1007/s11128-010-0177-y
7. Zhang, Y., Lu, K., Gao, Y., and Mo Wang. NEQR: A novel enhanced quantum representation of digital images. *Quantum Information Processing*. 12, 10.1007/s11128-013-0567-z (2013).
8. Roscani, V., Tozza, S., Castellano, M., Merlin, E., Ottaviani, D., Falcone, M., and A. Fontana. A comparative analysis of denoising algorithms for extragalactic imaging surveys. *A&A*. 643 A43 (2020). https://doi.org/10.1051/0004-6361/201936278
9. Sharma, S. (n.d.). *FRQI and NEQR* [GitHub repository]. GitHub. https://github.com/Sharma-Siddhartha/FRQI-and-NEQR *(Accessed July 2025)*
10. D. C. Price. Reduced-resolution beamforming: lowering the computational cost for pulsar and technosignature surveys. arXiv:2402.12723v2 (2024)
11. *Astronomical Image and CSV Dataset* [Data set]. Kaggle. https://www.kaggle.com/datasets/mightyglow/astronomical-image-and-csv-dataset?resource=download *(Accessed July 2025)*
12. Coles, W., Hobbs, G., Champion, D. J., Manchester, R. N., and J. P. W. Verbiest. Pulsar timing analysis in the presence of correlated noise. *Monthly Notices of the Royal Astronomical Society*. 418(1):561-570 (2011). https://doi.org/10.1111/j.1365-2966.2011.19505.x
13. Umana, G., Buemi, C., Trigilio, C., Leto, P., Hora, J., and G. Fazio. The nebulae around LBVs: a multiwavelength approach. arXiv:1011.3730 (2010).