HKUST – Department of Computer Science and Engineering

COMP3711: Design and Analysis of Algorithms – Fall 2017

#

Midterm Examination

Date: Monday, Oct. 16, 2017 Time: 19:00–21:00 Venue: LTB

Name: _____	Student ID: _____
Email: _____	Lecture L1 / L2

Instructions

- This is a closed book exam. It consists of 14 pages and 6 questions .
- Please write your name, student ID and ITSC email and circle your lecture section (L1 is M,W 9-9:20 and L2 is W, F 15:00-16:20) at the top of this page.
- For each subsequent page that you write on, please write your student ID at the top of the page in the space provided.
- Please sign the honor code statement on page 2.
- Answer all the questions within the space provided on the examination paper. You may use the back of the pages for your rough work. The last 2 pages are scrap paper and may also be used for rough work. Each question is on a separate page and most have at least one extra page for writing answers. These are provided for clarity and are not meant to imply that each question requires all of the blank pages. Many can be answered using much less space.

Questions	1	2	3	4	5	6	Total
Points	10	10	25	20	10	25	100
Score							

Student ID: _____

As part of HKUST's introduction of an honor code, the HKUST Senate has recommended that all students be asked to sign a brief declaration printed on examination answer books that their answers are their own work, and that they are aware of the regulations relating to academic integrity. Following this, please read and sign the declaration below.

I declare that the answers submitted for this examination are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University regulations governing academic integrity.

Student's Name: _____

Student's Signature: _____

Student ID: _____

1. **Time Complexity [10 pts]**

   Please arrange the following functions in increasing order of asymptotic complexity

   $log(2^n)$, $nlogn + 1000$, 2^{logn}, $(logn)^2$, $n^{1.1}$

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2. **Recurrence [10 pts]**

Give asymptotic upper bounds for $T(n)$ under the following recurrences. Make your bounds as tight as possible.

(a) $T(1) = 1; T(n) = T(n/8) + 1$ for $n > 1$.

(b) $T(1) = 1; T(n) = 3T(n/4) + n^2$ for $n > 1$.

(c) $T(1) = 1; T(n) = 2T(n-1) + 1$ for $n > 1$.

(d) $T(1) = 1; T(n) = 2T(n/4) + n$ for $n > 1$.

3. **Divide-and-Conquer and Heap [25 pts]**
   Suppose that you have k sorted arrays, each with n elements, and you want to combine them into a single sorted array of kn elements

   (a) Design a solution using divide and conquer, and analyze its running time.

(b) Design another solution based on the min-heap implementation of priority queues, and analyze its running time

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4. **Greedy Algorithm [20 pts]**
Given a collection of intervals, find the minimum number of intervals you need to remove to make the rest of the intervals non-overlapping.
Note:
1. You may assume the interval's end point is always bigger than its start point.
2. Intervals like [1,2] and [2,3] have borders touching but they don't overlap each other.

Example 1:
Input: [1,2], [2,3], [3,4], [1,3]
Output: 1
Explanation: [1,3] can be removed and the rest of intervals are non-overlapping.

(a) Give the pseudocode for the greedy algorithm. Also, analyze and state your algorithm's running time.

(b) Prove that your algorithm's output is correct.

5. **Huffman Coding [10 pts]**

You are given a message which contains the following six letters with different associated frequencies:

Letter	a	b	c	d	e	f
Frequency	5	25	7	15	4	12

You are asked to use Huffman coding to encode the message.

(a) Please draw the Huffman tree

(b) Please write down the code for each character in the following table?

Letter	**Code**
a	
b	
c	
d	
e	
f	

6. **Sorting and Selection [25 pts]**

You are given two sorted lists of arrays of size m and n. Design an algorithm to find the kth smallest element in the union of the two lists. Please also analyze the time complexity of your algorithm.

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Scrap Paper

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Scrap Paper