Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

```
10
                                                  11
                                                                14
A[q]
              10
                   15
                       19
                            20
                                42
                                     54
                                         67
                                              75
                                                  81
                                                       87
                                                            93
                                                                96
```

```
BSearch (A, p, r, x):
if p = r then
  if A[p] = x then return p
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else
  q \leftarrow \lfloor (p+r)/2 \rfloor
  if x \leq A[q] then BSearch (A, p, q, x)
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First call: BSearch (A, 1, n, x)
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BSearch(*A*, 1, 14, 67)

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A[q]	4	7	10	15	19	20	42	54	67	75	81	87	93	96

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```

BSearch(
$$A$$
, 1, 14, 67)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 7$$

$$67 > 42 = A[7]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A[q]								54	67	75	81	87	93	96

BSearch (A, p, r, x): if p = r then if A[p] = x then return p else return nilelse $q \leftarrow \lfloor (p+r)/2 \rfloor$ if $x \leq A[q]$ then BSearch (A, p, q, x) else BSearch (A, q + 1, r, x)First call: BSearch (A, 1, n, x)

BSearch(
$$A$$
, 8, 14, 67)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 11$$

$$67 \le 81 = A[11]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A[q]								54	67	75	81			

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First call: BSearch (A, 1, n, x)
```

BSearch(
$$A$$
, 8, 11, 67)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 9$$

$$67 \le 67 = A[9]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

BSearch (A, p, r, x): if p = r then if A[p] = x then return p else return nilelse $q \leftarrow \lfloor (p+r)/2 \rfloor$ if $x \leq A[q]$ then BSearch (A, p, q, x) else BSearch (A, q+1, r, x)First call: BSearch (A, 1, n, x)

BSearch(
$$A$$
, 8, 9, 67)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 8$$

$$67 > 54 = A[8]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

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First call: BSearch (A, 1, n, x)
```

BSearch(
$$A$$
, 9, 9, 67)
$$p = r \text{ AND } 67 = A[p] = A[9]$$

Success

Previous example was of a successful search, in which the search key was in the the array.

What happens if the search key is not in the array?

```
BSearch (A, p, r, x):
if p = r then
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else
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  if x \leq A[q] then BSearch (A, p, q, x)
  else BSearch (A, q + 1, r, x)
First call: BSearch (A, 1, n, x)
```

BSearch(*A*, 1, 14, 66)

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A[q]	4	7	10	15	19	20	42	54	67	75	81	87	93	96

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  else BSearch (A, q + 1, r, x)
```

BSearch(
$$A$$
, 1, 14, 66)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 7$$

$$66 > 42 = A[7]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
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BSearch(
$$A$$
, 8, 14, 66)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 11$$

$$66 \le 81 = A[11]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

q =	1	2	3	4	5	6	7	8	9	10	11	12	13	14
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BSearch(
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, 8, 11, 66)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 9$$

$$66 \le 67 = A[9]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate

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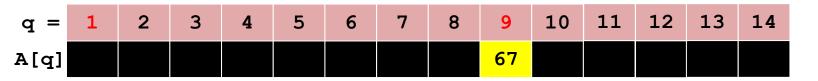
BSearch(
$$A$$
, 8, 9, 66)
$$q \leftarrow \lfloor (p+r)/2 \rfloor = 8$$

$$67 > 54 = A[8]$$

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate



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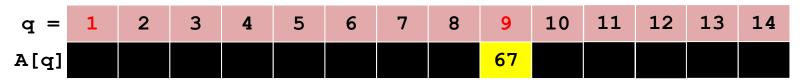
BSearch(
$$A$$
, 9, 9, 66)
$$p = r \text{ AND } 66 \neq A[p] = A[9]$$
 Failure

Failure Return *nil*

Input: An array A of elements in sorted order, and an element x.

Output: Return the position of x if it exists; otherwise output nil.

Idea: Set q = middle item. Check if $x \le A[q]$, and search either to left or right of q as appropriate



BSearch(*A*, 9, 9, 66)

Failure Return *nil*

In fact, algorithm knows where 66 should be. In case of failure it could be modified to return locations of predecessor and successor of x in the array.

```
BSearch (A, p, r, x):
if p = r then
   if A[p] = x then return p
   else return nil
else
    q \leftarrow |(p+r)/2|
     if x \le A[q] then BSearch (A, p, q, x)
     else BSearch (A, q + 1, r, x)
```

Analysis of Binary Search

Analysis: Let T(n) be the number of comparisons needed for running the algorithm on at most n elements. (Assume n a power of 2).

Recurrence: A single comparison eliminates half of the array.

Therefore, we search for the element in the remaining half, which has size n/2. Thus, the recurrence counting the number of comparisons is:

$$T(1) = 1$$
. Otherwise, if $n > 1$, $T(n) = T(\frac{n}{2}) + 1$.

Solve the recurrence by iterating it: (Assume n a power of 2)

$$T(n) = T(n/2) + 1$$

 $= (T(n/2^2) + 1) + 1$
 $= T(n/2^2) + 2$
 $=$
 $= T(n/2^i) + i$
 $=$
 $= T(1) + \log n = \Theta (\log n)$

Proof of correctness more formal analysis and analysis when n is not a power of 2 are developed in practice homework

Note: may sometimes terminate faster than $\Theta(\log n)$, but worst-case running time is $\Theta(\log n)$.