COMP 3711 Design and Analysis of Algorithms
Spring 2016 Midterm Exam Solution

Question 1: B, A, U, B, A

Question 2: (a) $\Theta(n \log n)$, (b) $\Theta(n^2 \log n)$, (c) $\Theta(n)$, (d) $\Theta(\log^2 n)$

Question 3:

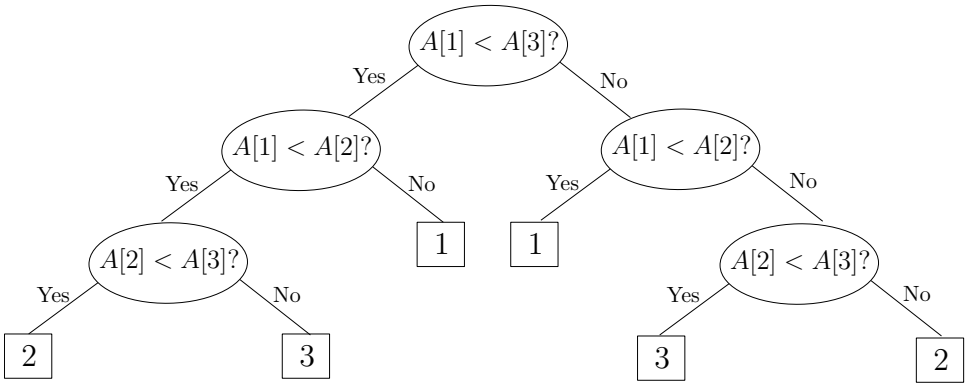
```
k-sort(array A, int s, int e, int block_size)
if e − s + 1 > block_size then
    Find median x of A[s..e] by linear-time selection algorithm;
    Partition A[s..e] with pivot x;
    m = ⌊(s + e)/2⌋;
    k-sort(A, s, m, block_size);
    k-sort(A, m+1, e, block_size);
end
```

Initial call: k-sort$(A, 1, n, n/k)$.
Running time: $T(n/k) = 1$, $T(m) = 2T(m/2) + m$ if $m > n/k$. So, $T(n) = O(n \log k)$.

Question 4:

Array index	1	2	3	4	5	6	7	8	9	10
Initial content	4	5	1	12	9	10	16	7	3	8
Build-Max-Heap	16	12	10	7	9	4	1	5	3	8
Increase-Key(10, 11)	16	12	10	7	11	4	1	5	3	9
Decrease-Key(2, 2)	16	11	10	7	9	4	1	5	3	2
Extract-Max	11	9	10	7	2	4	1	5	3	
Insert(13)	13	11	10	7	9	4	1	5	3	2

Question 5:

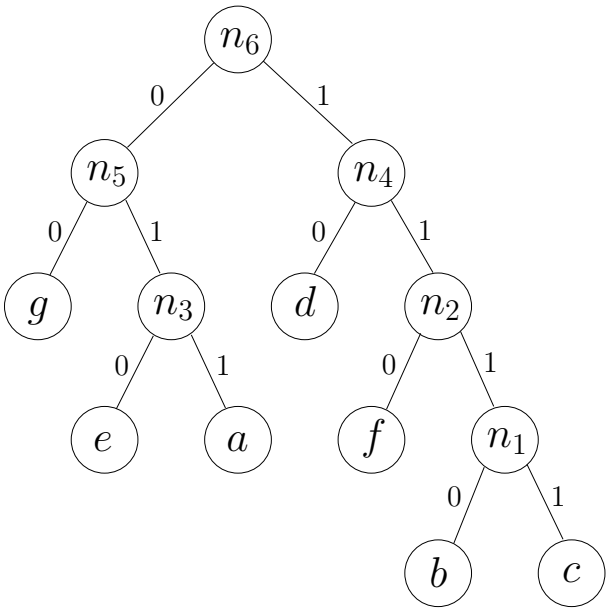


Question 6:

```
RandomNumber()
first_bit = FairCoin()
second_bit = FairCoin()
while first_bit = 1 AND second = 1 do
    first_bit = FairCoin()
    second_bit = FairCoin()
end
if first_bit = 0 AND second_bit = 0 then output '0'
if first_bit = 0 AND second_bit = 1 then output '1'
if first_bit = 1 AND second_bit = 0 then output '2'
```

The expected number of calls to FairCoin() is $2 \cdot 1/(\frac{3}{4}) = 8/3$.

Question 7:



Huffman code:

$a = 011$

$b = 1110$

$c = 1111$

$d = 10$

$e = 010$

$f = 110$

$g = 00$