Differential Equations Computational practicum

Danila Korneenko B20-04

Variant №21:

$y' = y/x - y - x$

$y(x_0) = 0$

$x_0 = 1$

$X = 10$

Exact solution of the ODE:

$y' = y/x - y - x$; $y(x0) = 0$; $x0 = 1$ $D(y): x \in (-\infty; 0) \cup (0; +\infty)$

$$dy = \left(\frac{y}{x} - y - x\right)dx \quad |:x$$

$$\frac{dy}{x} + \left(\frac{y}{x} - \frac{y}{x^2} + 1\right)dx = 0$$

$$\frac{dy \, x - dx \, y}{x^2} + \frac{y \, dx}{x} + dx = 0$$

$$d\left(\frac{y}{x}\right) + \frac{y \, dx}{x} + dx = 0$$

Substitution: $u = \frac{y}{x} \quad du = d\left(\frac{y}{x}\right)$

$$du + u \, dx + dx = 0$$

$$du = dx \, (-u - 1)$$

$$\int dx = \int -\frac{du}{u+1} \quad (u = -1 \text{ is also sol.} \Rightarrow y = -x$$

$$C - x = Ln(u+1)$$

$$e^{C-x} = u + 1$$

$$e^{C-x} = \frac{y}{x} + 1$$

$y = \frac{C}{e^x} - x$ for $x \in (-\infty; 0) \cup (0; +\infty)$ is General solution

Solution for initial value problem $x_0 = 1, y(x_0) = 0$:

$$0 = \frac{C}{e} - 1 \Rightarrow C = e$$

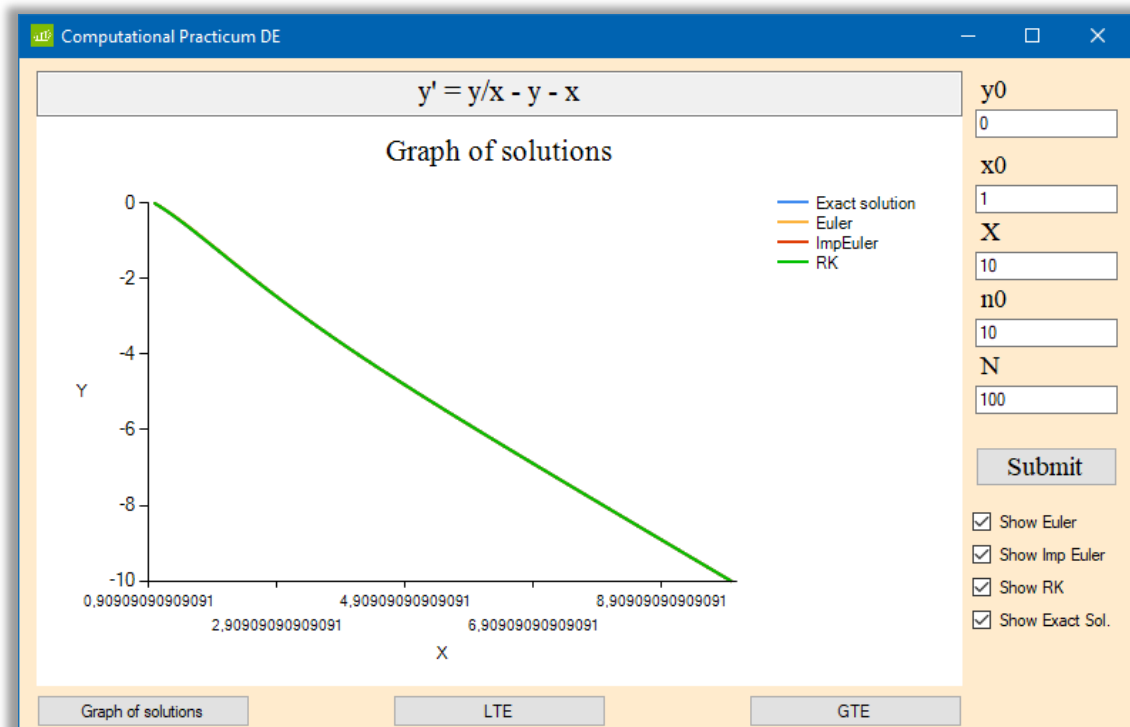$y = x \cdot e^{1-x} - x$ is particular solution for initial value problem.

X = 0 is the point of the removable (first type) discontinuity

UML, GUI and the results of the computing:

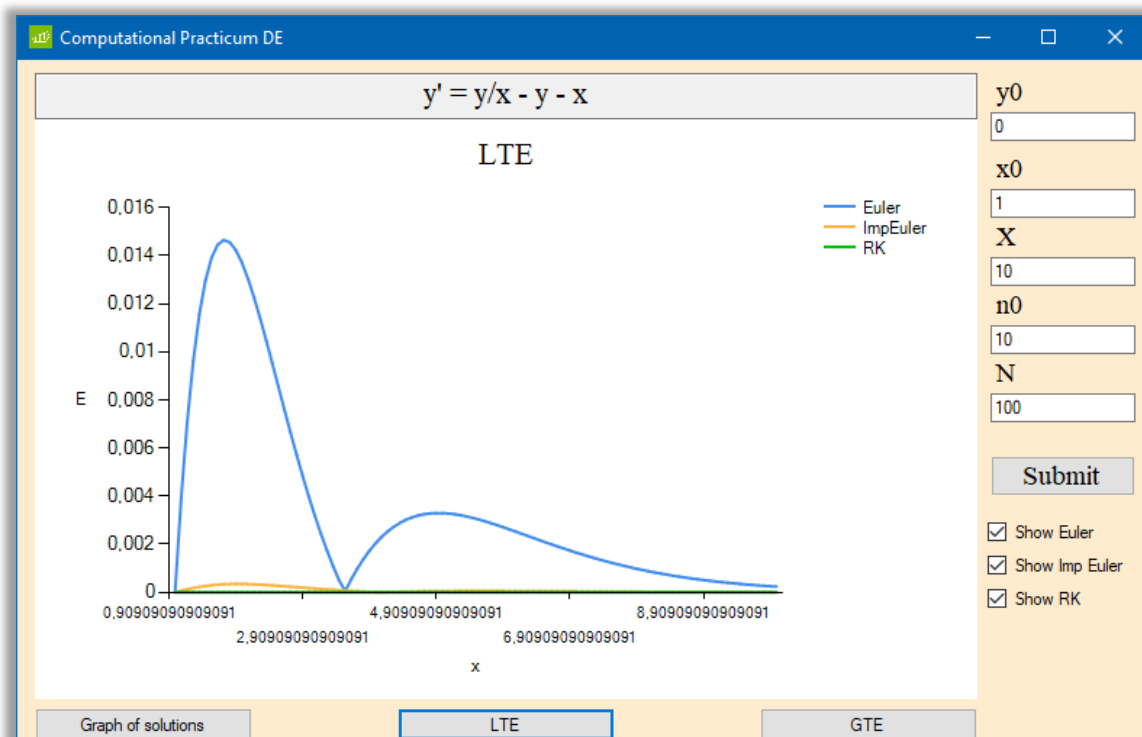This GUI was built by the Windows Forms (.NET Framework) on C# v7.3

The approximation of the solution of a given initial value problem: (x0 = 1, y0 = 0, X = 10, n0 = 20, N = 100):
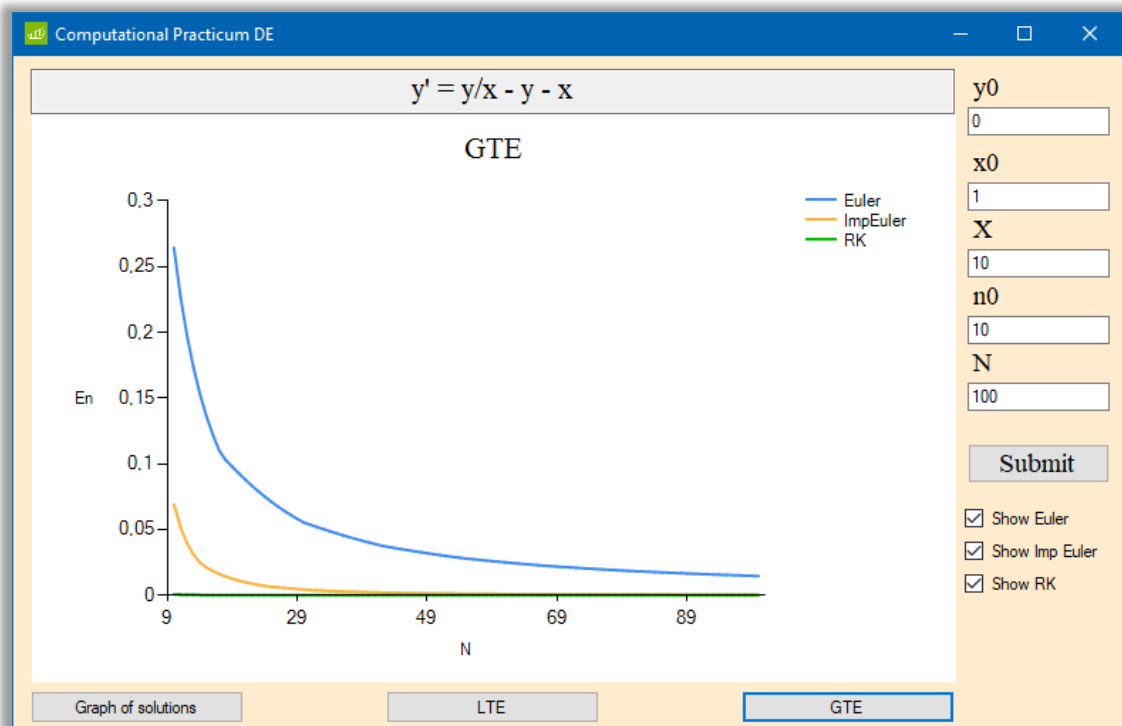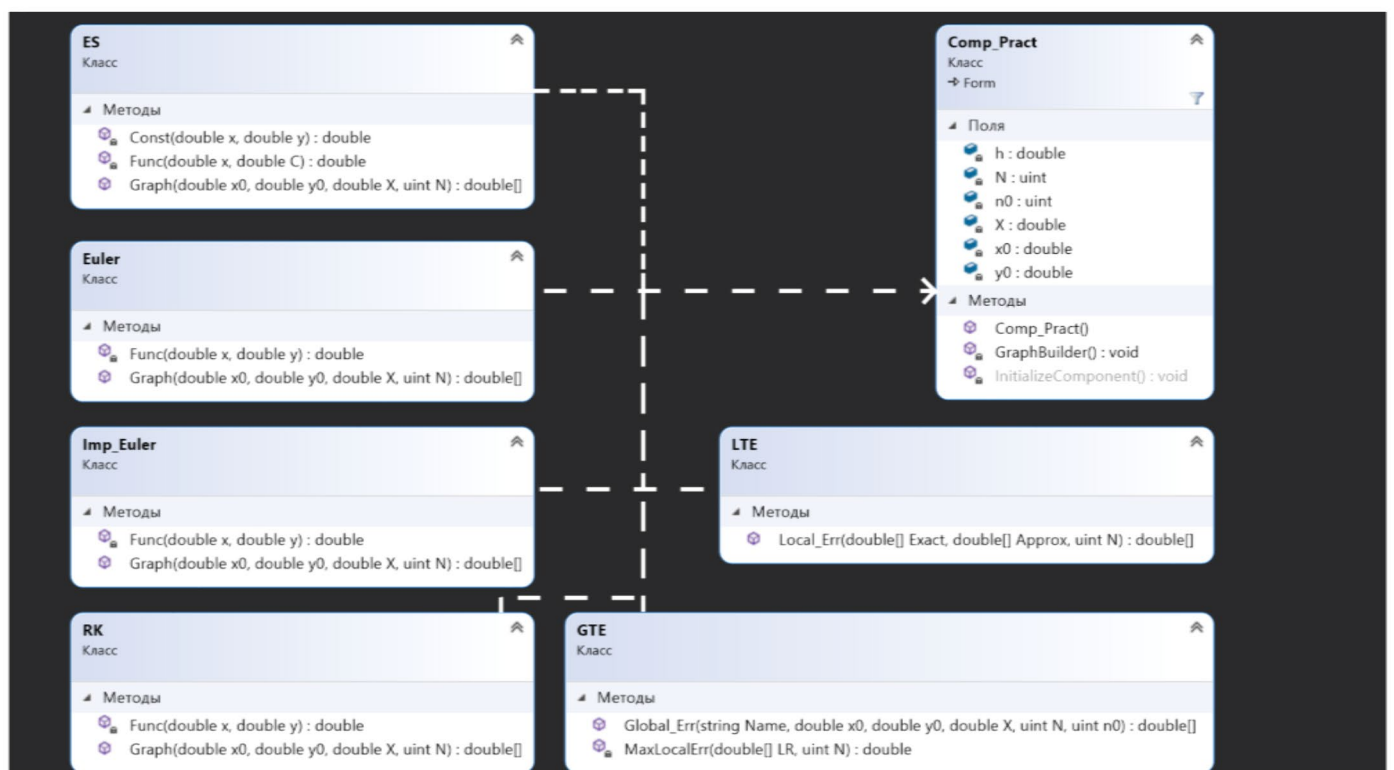
Graph of solutions:

Graph of LTE (Local errors):

Graph of GTE (Global errors):

The UML diagram of the program:

P.S. In the original version of the structure, there should have been an interface "Abs_method", however, since "the interface implementation component is not available by default in C# 7.3", any abstraction of method classes was a rewrite of the code, so the abstract class was not implemented in this application.

Interesting code parts

1) Each class of numerical methods is **<u>static</u>**, so the program does not use a lot of memory on displaying graphs, because the data destroys after calculations. This is done to optimize large-volume calculations.

```
//Initial function for computing
ссылка: 1
private static double Func(double x, double y)
```

```
//Output X and Y of the Euler method
Ссылок: 2
public static double[] Graph(double x0, double y0,double X, uint N)
{
```

```
Ссылок: 6
public class LTE
{
    //Output X and Y of the Local Errors of the method
    Ссылок: 6
    public static double[] Local_Err(double[] Exact, double[] Approx, uint N)
```
and etc.

2) When you click on the "send data" button, program process **<u>exceptions</u>** so the methods work correctly

```
//Exceptions
if (x0 <= 0)
{
    throw new Exception("x0 can\'t be less or equal zero");
}
else if (X <= x0)
{
    throw new Exception("The value of \"X\" is less then \"x0\"");
}
else if ((X - x0) / N > 1)
{
    throw new Exception("The length of the interval exceeds the number of iterations. \"X - x0\" must be less than \"N\"");
}
else if ((X - x0) / n0 > 1)
{
    throw new Exception("The length of the interval exceeds the number of minimal iterations. \"X - x0\" must be less than \"n0\"");
}
```

3) For each graph, there is an option to **enable/disable the visibility** of certain methods to make it easier to compare certain data.

```csharp
//Change visibility of Euler Graphs
ссылка: 1
private void checkBox_E_GoS_CheckedChanged(object sender, EventArgs e)
{
    GS_chart.Series["Euler"].Enabled = !GS_chart.Series["Euler"].Enabled;
}
ссылка: 1
private void checkBox_E_LTE_CheckedChanged(object sender, EventArgs e)
{
    LTE_chart.Series["Euler"].Enabled = !LTE_chart.Series["Euler"].Enabled;
}
ссылка: 1
private void checkBox_E_GTE_CheckedChanged(object sender, EventArgs e)
{
    GTE_chart.Series["Euler"].Enabled = !GTE_chart.Series["Euler"].Enabled;
}


//Change visibility of Improved Euler Graphs
ссылка: 1
private void checkBox_IE_CheckedChanged(object sender, EventArgs e)
{
    GS_chart.Series["ImpEuler"].Enabled = !GS_chart.Series["ImpEuler"].Enabled;

}
ссылка: 1
private void checkBox_IE_LTE_CheckedChanged(object sender, EventArgs e)
{
    LTE_chart.Series["ImpEuler"].Enabled = !LTE_chart.Series["ImpEuler"].Enabled;
}
ссылка: 1
private void checkBox_IE_GTE_CheckedChanged(object sender, EventArgs e)
{
    GTE_chart.Series["ImpEuler"].Enabled = !GTE_chart.Series["ImpEuler"].Enabled;
}
```
and etc.

4) For GTE was used **switch-case** to find particular GTE graph.

```csharp
//Output X and Y of the Global Errors of the method
Ссылок: 3
public static double[] Global_Err(string Name,double x0, double y0, double X, uint N, uint n0)
{
    double[] Err = new double[N - n0 + 1];
    for (uint En = n0; En <= N; En++)
    {
        switch (Name) {
            case "E":
                Err[En - n0] = MaxLocalErr(LTE.Local_Err(ES.Graph(x0, y0, X, En), Euler.Graph(x0, y0, X, En),En), En);
                break;
            case "IE":
                Err[En - n0] = MaxLocalErr(LTE.Local_Err(ES.Graph(x0, y0, X, En), Imp_Euler.Graph(x0, y0, X, En),En), En);
                break;
            case "RK":
                Err[En - n0] = MaxLocalErr(LTE.Local_Err(ES.Graph(x0, y0, X, En), RK.Graph(x0, y0, X, En),En), En);
                break;
        }
    }
    return Err;
}
```