Name: NIM: Class:

Assignment CLO 4: Elementary Number Theory
Discrete Mathematics (CII1G3/CPI1G3)
Second Term 2022-2023

Instructions:

1. This assignment is due **Thursday, June 15, 2023 at 5:00 p.m.**. Please submit your work to the corresponding submission slot in LMS CeLOE. You need to submit a readable .pdf file of this assignment to the provided submission slot in CeLOE. You can contact your class instructor for more detailed information. **Please make sure that your file size does not exceed the maximum file size allowed**.
2. Please upload your assignment to the LMS CeLOE under the file name: `A4-<student ID>.pdf`, for example: `A4-1301228888.pdf`.
3. You may submit this assignment in one of the following forms:
   a. You print this assignment and write your answer using HB/2B pencil or pen with blue/black ink (handwritten answer). You may add additional A4-sized papers. Afterward, you submit the scan/ photograph of this assignment.
   b. You use a .pdf editing tools and write your answer directly using blue/black colored writing.
   c. You copy the problem from this assignment to a text/word processing program and type your answer neatly.
   d. You rewrite the problem from this assignment on an A4-sized paper and submit the scan/photograph of your work.
4. All problems in this assignment are adapted from the textbooks. **The problems are written in English**. If you are a student in a regular class, you may answer the problems in Bahasa Indonesia. However, if you are a student in the international class, your answers must be written in English—otherwise, your assignment will not be graded. You may ask your class instructor or teaching assistant for helping you understand the problem, but you should not ask them to give the solution to any problem.
5. Be neat and write legibly. You will be graded not only on the correctness of your answers but also on the clarity with which you express them.
6. This assignment consists of **10 problems** and each problem is worth **10 points**.
7. Please retain yourself from copying answers from elsewhere without understanding the steps. Such an attitude will not enhance your knowledge. This assignment is an individual evaluation.
8. You may discuss this assignment with your friends or individually. If you discuss this assignment with anyone else, please put the name of your collaborators on the second page.
9. This assignment is worth at most 2.5% of your final grade. You can still get an A without submitting this assignment.
10. **Important:** late submission without reasonable explanation will not be graded.

Write the name of your collaborators on this page. You may write up to five collaborators. Collaborators are people who discuss this assignment with you. If you work alone, just write your name as collaborator 1.

(a). Collaborator 1:

(b). Collaborator 2:

(c). Collaborator 3:

(d). Collaborator 4:

(e). Collaborator 5:

Problem 1 Fill each of the blanks either with $\equiv$ or $\not\equiv$ and justify your answers.

(a). **[2.5 points]** $20 \ldots -12 \pmod 8$

(b). **[2.5 points]** $-20 \ \ldots \ -12 \pmod 8$

(c). **[2.5 points]** $21 \ \ldots \ 5 \pmod 8$

(d). **[2.5 points]** $21 \ldots -5 \pmod 8$

For example, we have $7 \equiv 22 \pmod 3$ because $7 - 22 = -15$ and $3 \mid -15$ (or because $7 \bmod 3 = 22 \bmod 3 = 1$); we have $8 \not\equiv 4 \pmod 3$ because $8 - 4 = -4$ and $3 \nmid -4$ (or because $8 \bmod 3 = 2$ while $4 \bmod 3 = 1$). No partial credits for wrong answer!

ANSWER:

Problem 2 Find the smallest integer x that satisfies each of the following conditions:

(a). **[2.5 points]** $x \equiv 40 \pmod{45}$ and $-22 \leq x \leq 22$.

(b). **[2.5 points]** $x \equiv 40 \pmod{45}$ and $-144 \leq x \leq -100$.

(c). **[2.5 points]** $x \equiv -40 \pmod{45}$ and $-22 \leq x \leq 22$.

(d). **[2.5 points]** $x \equiv -40 \pmod{45}$ and $-144 \leq x \leq -100$.

ANSWER:

Problem 3 Convert each of these values into its corresponding decimal representation

(a). **[2.5 points]** $(22011)_3$

(b). **[2.5 points]** $(3130)_4$

(c). **[2.5 points]** $(334)_8$

(d). **[2.5 points]** $(DC)_{16}$

ANSWER:

Problem 4 Convert the decimal number 2045 into:

(a). **[2.5 points]** ternary expansion (base 3),

(b). **[2.5 points]** quaternary expansion (base 4),

(c). **[2.5 points]** octal expansion (base 8),

(d). **[2.5 points]** hexadecimal expansion (base 16).

ANSWER:

Problem 5 Andrea owns a candy parlor. One day, she has produced 1635 cinnamon candies, 2725 licorice candies, and 3815 caramel candies. She wants to divide the candies into identical packages so that each package has the same number of candies, and she must use all of the candies (no candies are left over).

(a). **[5 points]** Determine the maximum number of identical packages Andrea can make.

(b). **[5 points]** If the number of identical packages is maximum, determine the total number of candies in a package.

ANSWER:

Problem 6 There are three alarms in a room. The first alarm is set to ring every 2 minutes and 22 seconds, the second one is set to ring every 3 minutes and 33 seconds, and the last one is set to ring every 4 minutes and 44 seconds. All alarms ring simultaneously for the **first time** at $07:45:00$ a.m..

(a). **[5 points]** Determine the least period (in seconds) in which all alarms ring simultaneously.

(b). **[5 points]** When will these alarm rings simultaneously for the **third time**? (We assume that all alarms ring simultaneously for the first time at $07:45:00$ a.m..)

ANSWER:

Problem 7 Suppose $\mathbb{Z}_7$ is the set of integers modulo 7, that is, $\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$.

(a). **[5 points]** Construct the addition table for $\mathbb{Z}_7$. We have $a +_7 b = (a + b) \bmod 7$.

(b). **[5 points]** Construct the multiplication table for $\mathbb{Z}_7$. We have $a \cdot_7 b = (a \cdot b) \bmod 7$.

ANSWER:

Problem 8 Use the addition and multiplication table in Problem 7 to solve each of the following congruences for $x \in \mathbb{Z}_7$.

(a). **[2.5 points]** $x + 6 \equiv 3 \pmod 7$,

(b). **[2.5 points]** $4x \equiv 1 \pmod 7$,

(c). **[2.5 points]** $3x \equiv 2 \pmod 7$,

(d). **[2.5 points]** $2x + 5 \equiv 2 \pmod 7$.

ANSWER:

Problem 9 Use Euclid's algorithm to find the greatest common divisors of the following pairs of number as well as the integers s and t that satisfy each of the following expressions:

(a). **[5 points]** find $\gcd(997, 40)$ and integers $s, t \in \mathbb{Z}$ such that $\gcd(997, 40) = s \cdot 997 + t \cdot 40$,

(b). **[5 points]** find $\gcd(997, 80)$ and integers $s, t \in \mathbb{Z}$ such that $\gcd(997, 80) = s \cdot 997 + t \cdot 80$.

ANSWER:

Problem 10 Use the result in Problem 9 to determine whether each of the following congruences has a solution. If so, determine the smallest positive value of x that satisfies the corresponding congruence.

(a). **[5 points]** $40x + 90 \equiv 45 \pmod{997}$,

(b). **[5 points]** $80x + 90 = 45 \pmod{997}$.

ANSWER: