

## Team DJ: Project 1

### 1. Lawnmower:

#### Pseudo code:

```
def lawnmower(before):  
    disk_state disk = before; |  
  
    assert(disk.is_initialized()) |  
  
    counter=0 |  
    loopAmnt = disk.total_count()/2 |  
  
    for x = 0 to loopAmnt do |  
        for i = 0 to loopAmnt do |  
            if disk[i] EQUAL DISK_LIGHT AND disk[i+1] EQUAL DISK_DARK | ] 4  
                disk.swap(i) |  
                counter = counter + 1 |  
            endif  
        endfor  
        for j = disk.total_count() - 1 to 0 do |  
            if disk[j] EQUAL DISK_DARK AND disk[j-1] EQUAL DISK_LIGHT | ] 4  
                disk.swap(j-1) |  
                counter = counter + 1 |  
            endif  
        endfor  
    endfor  
    return disk, counter |
```

$$\begin{aligned}
& \sum_{x=0}^{n/2} \left( 1 + \sum_{i=0}^{n/2} 4 + \sum_{j=n-1}^0 4 \right) \\
& \sum_{x=0}^{n/2} \left( 1 + 4 \left( \frac{n}{2} - 0 + 1 \right) + 4 \left( 0 - (n-1) + 1 \right) \right) \\
& \sum_{x=0}^{n/2} \left( 1 + 2n - 0 + 4 + 8 - 4n \right) \\
& \sum_{x=0}^{n/2} -2n + 13 \\
& \sum_{x=0}^{n/2} -2n + \sum_{x=0}^{n/2-1} 13 \\
& -2n \left( \frac{n}{2} - 0 + 1 \right) + 13 \left( \left( \frac{n}{2} - 1 \right) - 0 + 1 \right) \\
& \quad \underline{-n^2 - 2n + \frac{13}{2}n + 5} \\
& \quad -n^2 \rightarrow O(n^2)
\end{aligned}$$

### Mathematical Analysis:

For the for loop from  $x = 0$  to  $n/2$  there are two nested for loops. The first for loop goes from  $i = 0$  to  $n/2$  and the second one starts as  $j = n-1$  to  $0$ . The nested for loops both share the same amount of inner steps (4) and are treated as such. After adding an additional 1 for initial step count we solve up to the outer for loop with  $-2n+13$  steps on the inside. We distribute the sigma to both sides to solve and we end with  $-n^2 - 2n + 13/2 n$ . We add 5 to the end to represent the steps outside the loops.

We drop the dominated terms and are left with  $-n^2$ . Lastly we drop the multiplicative constant of  $-1$  and are left with  $n^2$ , giving us  $O(n^2)$ . Therefore the lawnmower algorithm is initialized with an efficiency of  $O(n^2)$ .

## 2. Alternating:

### Pseudo code:

```
def sort_alternate(disk):
```

```
    count=0 |  
    pos=0 |  
    n=disk.total_count() ✓  
    assert.(disk.is_initialized()) ↓
```

```
    for i to n do ↓
```

```
        for f=0 to disk.dark_count() do |  
            if disk[pos] == disk_light AND disk[pos+1] == disk_dark |  
                disk.swap(pos) ✓  
                count = count + 1 |
```

```
            endif
```

```
            pos = pos + 2 |
```

```
        endfor
```

```
        pos = 1 ↓
```

```
        for g=0 to disk.dark_count()-1 do |  
            if disk[pos] == disk_light AND disk[pos+1] == disk_dark |  
                disk.swap(pos) |  
                count = count + 1 |
```

```
            endif
```

```
            pos = pos + 2 |
```

```
        endfor
```

```
        pos = 0 ↓
```

```
    endfor
```

```
    return disk, count |
```

} 5

} 5

$$\sum_{i=0}^n \left( 3 + \sum_{f=0}^n 5 + \sum_{g=0}^{n-1} 5 \right)$$

$$\sum_{i=0}^n \left( 3 + 5(n-0+1) + 5(n-1-0+1) \right)$$

$$\sum_{i=0}^n \left( 3 + (5n+5) + 5n \right)$$

$$\leq (10n + 8) \rightarrow \sum_{i=0}^n 10n + \sum_{i=0}^{n-1} 8$$

$$10n(n-0+1) + 8(n-1-0+1)$$

$$10n^2 + 10n + 8n \rightarrow 10n^2 + 18n$$

$$10n^2 + 18n + 5 \quad (5 \text{ from outer})$$

$$10n^2 + 18n$$

$$10n^2 \rightarrow O(n^2)$$

### Mathematical Analysis:

For loop goes from 0 to n with two inner loops. First inner loop goes f=0 to n. The second loop starts g=0 to n-1. Both loops have inner step count and overhead totaling to 5. Add 3 for the outer loop step counts. Evaluate the sigmas using  $C(L-F+1)$  and add 3 to get  $10n+8$ . Distribute the sigma to both sides to get and add 5 for the outer step count to get  $10n^2 + 18n + 5$ .

Drop the dominant term 5 and  $18n$  to get  $10n^2$ . Then drop the multiplicative constant 10 according to the properties of  $O(n)$  to get n. Therefore, the efficiency of `sort_alternate()` is  $O(n^2)$ .

### 3. is\_initialized()

Pseudo code:

```
def is_initialized():  
    disk_color colorCheck = DISK_LIGHT  
    n = _colors.size()  
  
    for i=0 to n do  
        if _colors[i] NOT EQUAL colorCheck  
            return false  
        endif  
        if colorCheck EQUAL DISK_LIGHT  
            colorCheck = DISK_DARK  
        endif  
        else  
            colorCheck = DISK_LIGHT  
        end  
    endfor  
    return true
```

A blue bracket on the right side of the loop body (from the first 'if' to the 'end' statement) is labeled with a blue '6', indicating 6 operations per iteration.

$$3 + \sum_{i=0}^n 6$$

$$C(L-F+1)$$

$$6(n-0+1)$$

$$6n+6$$

$$6n+6+3 = 6n+9$$

$$6n \rightarrow O(n)$$

#### Mathematical Analysis:

For loop starts  $i=0$  to  $n$ . Evaluate the sigma to  $C(L-F+1)$  and add the outer step count 3 to get  $6n+9$ . Drop the dominant term 9 to get  $6n$ . Then drop the multiplicative constant 6 according to the properties of  $O(n)$  to get  $n$ . Therefore, the efficiency of `is_initialized()` is  $O(n)$ .

#### 4. is\_sorted()

Pseudo code:

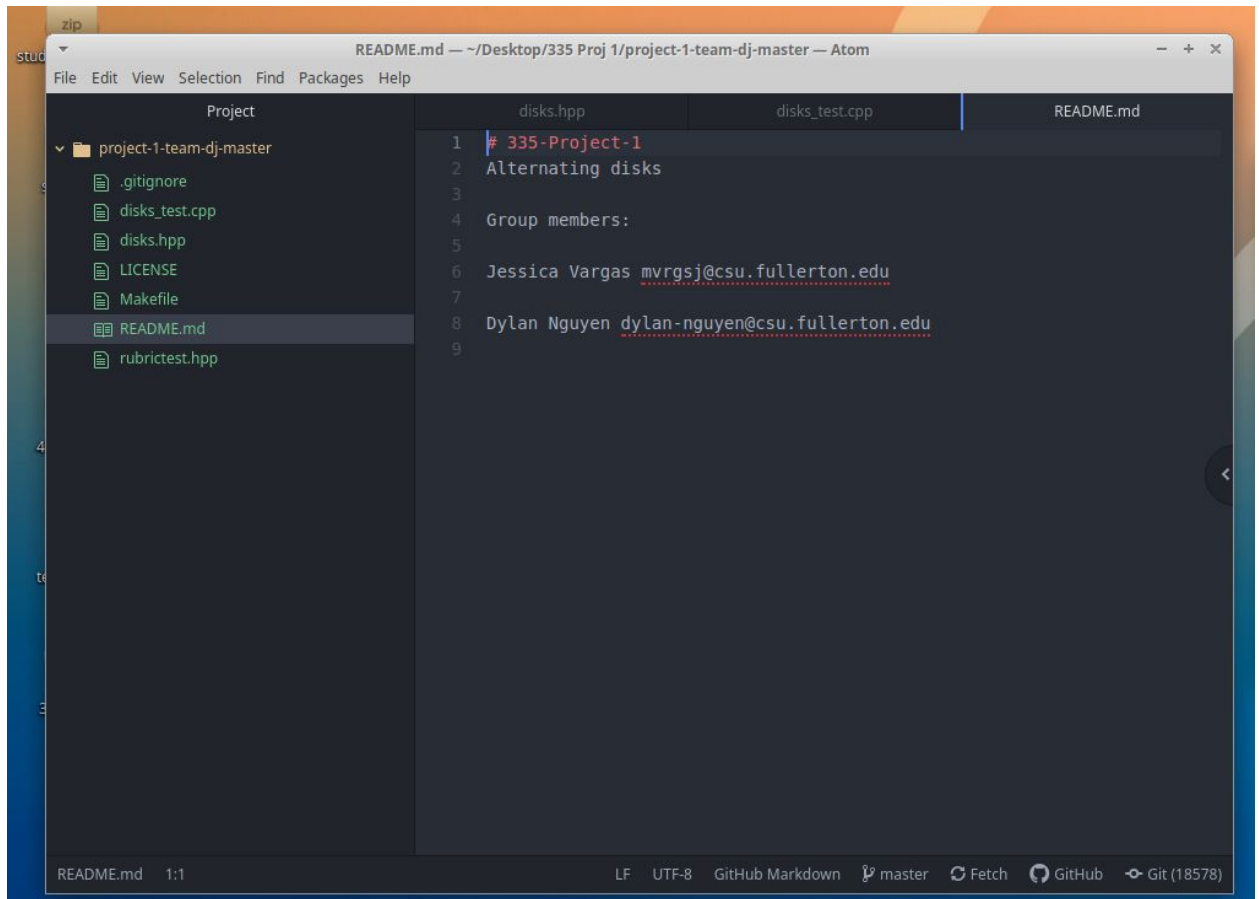
```
def is_sorted():  
    colorsHalf = total_count()/2  
  
    for j=0 to colorsHalf do  
        if _colors[j] EQUAL DISK_LIGHT  
            return false  
        endif  
    endfor  
  
    for i = colorsHalf to total_count() do  
        if _colors[i] EQUAL DISK_DARK  
            return false  
        endif  
    endfor  
    return true
```

$$\sum_{j=0}^{n/2} 3 + \sum_{i=n/2}^0 3 + 2$$
$$3\left(\frac{n}{2} - 0 + 1\right)$$
$$\left(\frac{3n}{2} + 3\right)$$
$$O(n)$$

#### Mathematical Analysis:

For the algorithm `is_sorted`, we have two for loops of equal length and steps. To calculate big O notation easier we only need to choose one of the loops to calculate. As such we chose the first loop. We add one initial count to the body count and multiply it by 3. After that we are left with  $3n/2 + 3$ , we drop dominated terms and the result is  $3n/2$ . When we drop the multiplicative constant we are left with simply  $n$ , resulting in  $O(n)$ .

## Screenshots:



```
student@tuffix-vm:~/Desktop/335 Proj 1/project-1-team-dj-master$ make
g++ -std=c++11 -Wall disks_test.cpp -o disks_test
./disks_test
disk_state still works: passed, score 1/1
sorted_disks still works: passed, score 1/1
disk_state::is_initialized: passed, score 1/1
disk_state::is_sorted: passed, score 1/1
alternate, n=3: passed, score 1/1
alternate, n=4: passed, score 1/1
alternate, other values: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, n=4: passed, score 1/1
lawnmower, other values: passed, score 1/1
TOTAL SCORE = 10 / 10
```