

Model hierarchies for understanding atmospheric circulation

Penelope Maher¹, Edwin P. Gerber², Brian Medeiros³, Timothy M. Merlis⁴, Steven Sherwood⁵, Aditi Sheshadri^{6,7}, Adam H. Sobel^{6,8}, Geoffrey K. Vallis¹, Aiko Voigt^{8,9}, Pablo Zurita-Gotor¹⁰

¹Department of Mathematics, University of Exeter, UK.

²Courant Institute of Mathematical Sciences, New York University, USA.

³National Center for Atmospheric Research, USA.

⁴McGill University, Canada

⁵Climate Change Research Centre, University of New South Wales, Australia.

⁶Department of Applied Physics and Applied Mathematics, Columbia University, USA.

⁷Department of Earth System Science, Stanford University, USA.

⁸Lamont-Doherty Earth Observatory, Columbia University, USA

⁹Inst. of Meteorol. and Climate Res., Dept. Troposphere Res., Karlsruhe Inst. of Technology, Germany

¹⁰Universidad Complutense de Madrid, and Instituto de Geociencias, Spain

Key Points:

- Model hierarchies help address open research questions. We focus on how they have improved our understanding of atmospheric circulation.
- Key benchmark models are identified that have helped to advance our understanding of the atmospheric circulation.
- The model hierarchies are commonly referred to but remain poorly defined. We identify three principles to organize models into hierarchies.

22 **Abstract**

23 In this review, we highlight the complementary relationship between simple and complex models
 24 in addressing key scientific questions to describe Earth's atmospheric circulation. The systematic
 25 representation of models in steps, or hierarchies, connects our understanding from idealized systems
 26 to comprehensive models, and ultimately the observed atmosphere. We define three interconnected
 27 principles that can be used to characterize the model hierarchies of the atmosphere. We explore
 28 the rich diversity within the governing equations in *dynamical hierarchies*, the ability to isolate
 29 and understand atmospheric processes in *process hierarchies*, and the importance of choices in the
 30 physical domain and resolution in *hierarchies of scale*.

31 We center our discussion on the large scale circulation of the atmosphere and its interaction with
 32 clouds and convection, focusing on areas where simple models have had a significant impact. Our
 33 confidence in climate model projections of the future is embedded in our efforts to ground the climate
 34 predictions in fundamental physical understanding. This understanding is, in part, possible due to
 35 the hierarchies of idealized models that afford the simplicity required for understanding complex
 36 systems.

37 **1 Introduction**

38 In this review, we showcase idealized models which have enabled a deeper understanding of
 39 the large-scale circulation of the atmosphere and provide a set of principles for organizing them into
 40 hierarchies. We regard a *hierarchy* to be a sequence that connects our most simple models to our most
 41 complex, with the ultimate goal of explaining and predicting the behavior of Earth's atmosphere.
 42 The *simplicity*, or idealization, of a model is thus defined relative to other members of the hierarchy,
 43 where a simpler model seeks to reduce the problem to its most fundamental components at the cost
 44 of quantitative accuracy and realism.

45 We use simple models to ask fundamental science questions, which are ideally validated against
 46 observations of the real atmosphere. In practice, simple models are often validated against more
 47 complex models in the hierarchy. This is necessary when observations are sparse, such as in the
 48 upper stratosphere or Southern Hemisphere storm tracks, or not available, such as projecting future
 49 climates with different emission rates.

50 We are conscious of the subtle differences between a 'theory' and a 'model'. Here we consider
 51 a *model* to be a set of equations which seek to capture the behavior of the system in question, without
 52 necessarily regarding the model as representing the truth or having any general applicability. (A
 53 model is sometimes also taken to imply the *implementation* of an idea.) *Theory* may be regarded as
 54 the assumptions and, if needed, equations needed to economically describe or predict the behaviour
 55 of some phenomena or system. Still, the distinction is blurry, for a simple, testable model will
 56 have many of the attributes of a theory. Further, the behavior of a complex system may not be
 57 directly explainable by a simple theory in the conventional sense, and a model hierarchy itself then
 58 becomes a theory, or at least a hypothesis, for the system; some of these issues are discussed further
 59 in *Vallis* [2016]. In this review, we focus on models that may be deliberately simplified, and which
 60 implement a set of (usually time-dependent) equations in a more or less complex fashion, sometimes
 61 independently of any specific theory. Having differing degrees of complexity, connected to each
 62 other in some way, is the key step in sorting models into a hierarchy.

63 While the spectrum of available models has increased in the last decade or two, the idea of a
 64 'hierarchy of climate models' in itself is not new. *Schneider and Dickinson* [1974] may have been
 65 the first to explicitly discuss the hierarchy in the sense we understand today, commenting that 'solid
 66 progress in understanding . . . climate change will require steady development of an almost continuous
 67 spectrum or hierarchy of models of increasing physical or mathematical complexity'. A decade later
 68 *Hoskins* [1983] noted the 'unhealthy' trend toward building models which are disconnected from one
 69 another and the real world, advocating, like Schneider and Dickinson, for a spectrum of connected
 70 models to provide a complete and balanced approach. *Nof* [2008] criticized the trend in climate
 71 modeling for higher resolution over increased understanding, and pointed out the danger of regarding

72 comprehensive models as ‘truth’. *Polvani et al.* [2017] noted that ‘Earth system models may be good
73 for simulating the climate system but may not be as valuable for understanding it’.

74 This gap between our understanding of the atmospheric circulation and the increasing complexity
75 of global circulation models was the focus of *Held* [2005] and *Held* [2014]. In these essays,
76 Held echoed earlier concerns about relying too much on comprehensive models that we do not fully
77 understand. He argued, however, that we should be equally concerned that our simpler models are
78 capable of addressing our key scientific questions. He called for more study of ‘elegant’ models that
79 are sufficiently complex to capture key elements of the real atmosphere, but still simple enough to
80 provide understanding.

81 There is certainly no single unique hierarchy. Instead, a suitable model hierarchy may be
82 constructed based on the key scientific questions of interest, as not all models are suitable for all
83 purposes. Even for a given scientific problem different scientists will make different, perhaps equally
84 defensible, choices. Nevertheless, we can attempt to produce a classification system to describe
85 models as being simple or complex within the spectrum of available models. *Bony et al.* [2013]
86 intuitively describe the complexity of climate models, see Figure 1a, as a balance between simplicity
87 of the model and complexity of the system that is being modelled. More recently, *Jeevanjee et al.*
88 [2017] describe the climate model hierarchy, see Figure 1b, in terms of dynamics, boundary layer
89 forcing, and bulk forcing. In section 2 we propose an alternative, but complementary, description
90 based on organizing the model hierarchies in terms of three principles.

91 In this discussion of the large-scale circulation of the atmosphere, we focus our review on the
92 science questions that have been addressed using key idealized models. We structure our review
93 to start with the most simple models and build up toward the more complicated models used
94 to investigate the large-scale circulation within the mid-latitudes, middle atmosphere and tropics,
95 Sections 3, 4 and 5, respectively. We then discuss the important role moisture plays in setting the
96 atmospheric circulation in Section 6 and how the hierarchies have helped improve the representation
97 of, and theory for, the Madden-Julian Oscillation in Section 7. We then summarize in Section 8.

98 We do not attempt to review all models. Instead, we describe a subset of simple models,
99 discuss their broad use and then make connections from the simple models through to the coupled
100 atmosphere-ocean General Circulation Models (GCMs). (The role of GCMs are discussed more in
101 *Ghil and Robertson* [2000].) We will not discuss Earth System Models, the very complex models
102 that include more processes than typical GCMs (e.g., biogeochemistry), but we do acknowledge that
103 Earth System Models form an end point (if only by definition) in modelling processes that affect
104 Earth’s climate and biogeochemistry.

105 2 Three principles guiding model hierarchies

106 There is no single or ubiquitous model hierarchy for the atmosphere. Many model hierarchies
107 are possible, depending in part on the science questions to be addressed. Nevertheless, a broad
108 classification of the hierarchies is useful and here we define three principles that can be used to guide
109 the categorization of the model hierarchies.

110 The first principle is the *dynamical* hierarchies of the atmospheric fluid flow. The dynamical
111 hierarchies allow us to isolate and explore the importance of different temporal and spatial scales on
112 the governing equations.

113 The second principle is the *process* hierarchies of the atmosphere. The process hierarchies allow
114 for the stepwise integration of important atmospheric processes into the governing equations of the
115 fluid flow. We systematically advance terms with the thermodynamic equation to form a sequence of
116 models that make a smaller ‘diabatic hierarchy’. An additional aspect of the process hierarchies are
117 the boundary conditions, such as surface properties such as aquaplanets, topography or orography.

118 The third principle is the *hierarchies of scale*, implicit to both the dynamical and process
119 hierarchies, where the choice of physical domain and numerical resolution allows for the systematic

120 exploration of different dynamical and physical processes across all time and spacial scales. There
 121 are practical trade offs between scale and complexity due to the computation expense. Perhaps unlike
 122 the first two, this is not so much a hierarchy of complexity, but it does describe the practical decisions
 123 about space and time scales which are required when building a model.

124 Almost all theory and modeling efforts can be classified into a hierarchy of some form, so
 125 attempting to catalogue *all* the hierarchies is not helpful. In the remainder of this paper, we
 126 selectively highlight examples of model hierarchies, specifically those that include simple models
 127 and that have advanced our understanding of the large scale circulation of the atmosphere. We focus
 128 on these models not because they necessarily optimally cover the complexity of available models, but
 129 rather because they have been extensively studied, thus establishing their impact. In the remainder of
 130 this review we incrementally build upon the different aspects of the process hierarchy, starting with
 131 the circulation within the mid-latitudes.

132 3 The Mid-Latitude Circulation

133 The large-scale extratropical circulation provides one of the best success stories for hierarchical
 134 climate modeling: some key aspects of the underlying dynamics are now reasonably well understood
 135 and part of modern textbooks [e.g. *Vallis*, 2017]. Other aspects are still areas of active research,
 136 such as the non-linear dynamics related to eddy-mean flow interaction. Idealized simulations have
 137 played an instrumental role in this progress, providing key insights on the non-linear behavior
 138 of extratropical disturbances. Since the early days of climate modeling, theorists recognized the
 139 great power of numerical computing as a means to overcome the stringent limitations of analytical
 140 work. Idealized simulations aimed at understanding the atmosphere were performed in parallel with
 141 comprehensive simulations. Some of the insight gained with these early simulations constitute the
 142 basis of prevalent paradigms on the extratropical circulation.

143 We begin by highlighting two models that have allowed us to isolate the key elements of the mid-
 144 latitude circulation. The first is a class of *barotropic vorticity equation models*, where collapsing the
 145 vertical dimension allow us to focus on feedbacks between the zonal mean flow, Rossby waves, and
 146 the spherical geometry of the planet. The second is the *two-layer quasi-geostrophic (QG) channel*
 147 *model*, which provides perhaps the most simple context for understanding baroclinic instability. We
 148 then discuss an idealized approach to combining elements of the baroclinic and barotropic dynamics
 149 together in eddy life cycle experiments.

150 3.1 Rossby-wave dynamics: The barotropic vorticity equations on the sphere

151 Rossby-wave propagation plays a fundamental role in both upper-troposphere synoptic vari-
 152 ability and the remote atmospheric response to forcing. The barotropic model provides a simple
 153 framework for studying these processes. In addition to providing the first numerical weather sim-
 154 ulations [*Charney et al.*, 1950], the barotropic model served as a test bed to understand the influence
 155 of topography and localized heating on the general circulation [*Grose and Hoskins*, 1979; *Hoskins*
 156 and *Karoly*, 1981]. These experiments revealed the important role played by the mean flow structure
 157 for Rossby wave refraction in the upper troposphere. The widely used concepts of waveguides and
 158 propagation windows are based on these ideas, which are key to our understanding of the extratropical
 159 response to El Niño.

160 So-called ‘stirred’ barotropic models [e.g., *Vallis et al.*, 2004] have seen a resurgence in recent
 161 years for understanding upper-troposphere synoptic variability and the dynamics of eddy momentum
 162 fluxes and eddy-driven jets without the complexity of baroclinic dynamics. In this model, the impact
 163 of baroclinic instability is approximated by a prescribed forcing (the stirring) in the vorticity equation
 164 at the synoptic scales. As a result, there are explicitly no feedbacks of the barotropic circulation
 165 on eddy generation. The model has been used as a conceptual model of annular mode variability
 166 to explain the dependence of zonal index persistence on latitude [*Barnes et al.*, 2010] and to study
 167 the interaction between the tropical and subtropical jets [*O’Rourke and Vallis*, 2013], among other
 168 problems.

169 As a further simplification, when the model is linearized it is possible to obtain a set of closed
 170 solutions (for simple forms of stirring) using stochastic theory [DelSole, 2001]. Lorenz [2014] has
 171 devised a very sophisticated method to calculate the eddy momentum flux given the full space-time
 172 characteristics of the stirring, which can play an important role due to the impact of wave phase
 173 speeds on refraction indices and wave propagation [Barnes and Hartmann, 2011]. The barotropic
 174 model can be a useful tool for exploring ‘eddy-momentum-flux closures’, i.e., the sensitivity of
 175 the direction of wave propagation to the mean state and/or model configuration. This remains a
 176 challenging open question in general circulation theory.

177 3.2 Baroclinic instability: The two-layer quasi-geostrophic model

178 To capture the essence of the eddy generation process, the *two-layer quasi-geostrophic model*
 179 on the β -plane stands out as a benchmark, indeed classical, model of the extratropical baroclinic
 180 circulation [Phillips, 1956]. It vies with the Eady model [Eady, 1949] as the simplest model that can
 181 produce baroclinic instability in a fashion relevant to the real world. There is only one baroclinic
 182 mode and the stratification and radius of deformation are prescribed.

183 The model also provides a simplified framework for studying the nonlinear extratropical circulation
 184 in a forced-dissipative configuration, in which the flow is typically forced by thermal relaxation
 185 to a baroclinic jet and the lower layer wind is damped using Rayleigh friction [e.g. Zurita-Gotor,
 186 2007]. The β -plane approximation and constant deformation radius make upper-troposphere dynamics
 187 simpler than in the spherical case (the symmetry of the model makes northward and southward
 188 propagation equally likely). In this sense, the model is complementary to the barotropic model in
 189 that it is devoid of the barotropic feedbacks associated with sphericity that play an important role in
 190 the dynamics of that model. The two-layer model not only reproduces qualitatively the main features
 191 of the observed extratropical circulation but it also captures more subtle aspects of extratropical
 192 dynamics like the clustering of eddies in wavepackets [Lee and Held, 1993], the driving of low-
 193 frequency baroclinicity variability [Zurita-Gotor et al., 2014] or the character of lower-troposphere
 194 eddy momentum fluxes [Lutsko et al., 2017].

195 In its forced configuration, the two-layer model provides the lower end of a dynamical hierarchy
 196 of forced-dissipative dry models, in which the mean climate is determined by the competition between
 197 the eddy fluxes and very idealized forms of forcing. These models can be formulated at different
 198 levels of complexity along the dynamical hierarchy depending on the scientific problem of interest
 199 [e.g. Zurita-Gotor and Vallis, 2009; Lachmy and Harnik, 2014; Jansen and Ferrari, 2013].

200 At the high end of this dynamical hierarchy, the model of Held and Suarez [1994] has been
 201 widely used to study various aspects of the extratropical circulation and its sensitivity to climate
 202 change [e.g. Lorenz and DeWeaver, 2007; Butler et al., 2010; Yuval and Kaspi, 2016] due to its
 203 realistic circulation. This model uses a primitive-equation formulation and a spherical domain and
 204 is forced by relaxation towards a state approximating radiative convective equilibrium (described in
 205 Section 6.1), with near moist-neutral stratification in the vertical but strong meridional temperature
 206 gradients. Above the tropopause, the atmosphere is simply relaxed towards an isothermal state. A
 207 variant of this model better suited for the tropical circulation combines relaxation to pure radiative-
 208 equilibrium with an idealized convection scheme designed to mimic the stabilizing effect of latent
 209 heating by moist convection [Schneider and Walker, 2006].

210 3.3 Connecting eddy growth, propagation, and decay: The eddy life-cycle paradigm

211 Even in the very idealized physical setting described above, the time-dependent evolution of
 212 forced-dissipative models is inherently nonlinear and turbulent. As a key simplification to the full
 213 non-linear problem, the series of experiments systematized by Hoskins and collaborators in the
 214 1970’s, building on pioneering numerical work by Edelmann [1963] and others, provided insight
 215 on the nonlinear evolution of baroclinic modes. The analysis of an eddy lifecycle by Simmons and
 216 Hoskins [1978] introduced the notions of baroclinic growth and barotropic decay as an idealized
 217 conceptual model for the nonlinear evolution of extratropical disturbances. Similar ideas, but in the

more general context of a statistical steady state and using quasi-geostrophic theory to interpret the simulations, were introduced independently by *Salmon* [1980]. This simple paradigm has survived to today and plays a fundamental role for our understanding of wave–mean-flow interaction and the maintenance of the mean circulation. Additional analysis [*Simmons and Hoskins*, 1980] uncovered the sensitivity of the decay stage in the lifecycle to the mean state, identifying two distinct patterns of evolution.

As theoretical advancements clarified the relation between eddy propagation and wave-mean flow interaction [*Andrews and McIntyre*, 1978; *Edmon et al.*, 1980] and the focus on Potential Vorticity (PV) dynamics highlighted the important role of wave breaking [*McIntyre and Palmer*, 1983], *Thorncroft et al.* [1993] proposed a conceptual model for understanding the two idealized lifecycles based on the direction of propagation and the typology of wave breaking. Idealized simulations were also useful for demonstrating the relevance of critical layer theory for eddy dissipation and wave-mean flow interaction in eddy lifecycles [*Feldstein and Held*, 1989]. The critical layer is a powerful concept for constraining upper-troposphere propagation [*Randel and Held*, 1991] and plays an important role for extratropical variability and climate sensitivity [*Lee et al.*, 2007; *Chen and Held*, 2007; *Cepi et al.*, 2013].

The association between the direction of propagation, the topology of wave breaking and the sign of the eddy momentum flux uncovered by the idealized studies is central to our understanding of jet shifts and phenomena like the North Atlantic Oscillation [*Rivière and Orlanski*, 2007]. On the sphere, equatorward propagation and poleward momentum fluxes dominate [*Thorncroft et al.*, 1993; *Balasubramanian and Garner*, 1997] so that we might expect extratropical jets to shift poleward as they strengthen if the stirring does not move. However, idealized studies show that the direction of propagation is affected by many other factors, such as the latitude and scale of the eddies, the barotropic shear and the low-level baroclinicity [*Simmons and Hoskins*, 1980; *Hartmann and Zuercher*, 1998; *Rivière*, 2009], among others. Due to this complexity, we are still far from a complete theory for the eddy momentum flux closure.

3.4 Case Study: Eddy feedbacks and the variability of the jet stream

To illustrate the use of hierarchical modeling in the extratropics, we discuss its application to the analysis of eddy feedbacks in unforced jet variability. We have chosen this example because it lends itself well to the hierarchical approach and because it is a topic of current research.

The leading (and more persistent) mode of extratropical zonal wind variability consists of a meridional shift of the eddy-driven jet concomitant with annular mode variability [*Thompson and Wallace*, 2000]. *Lorenz and Hartmann* [2001] found a positive correlation between the jet anomalies and their eddy momentum driving in the Southern Hemisphere when the jet leads by a few days, see Figure 2a, which implies that the anomalous eddy momentum fluxes tend to extend the duration of the jet anomalies. They interpreted this positive correlation as depicting the sensitivity of the anomalous eddy momentum flux on the state of the jet, or a positive eddy feedback (but see *Byrne et al.* [2016] for an alternative interpretation).

Climate models are known to be too persistent [*Gerber et al.*, 2008], see Figure 2b, particularly idealized models [*Gerber and Vallis*, 2007]. This is mostly associated with too slow decay of the autocorrelation function at lags beyond 5 days, see Figure 2c, suggesting an excessive eddy feedback. Two different types of mechanisms have been proposed in the literature for this feedback: barotropic and baroclinic. Barotropic mechanisms rely on changes in upper-troposphere propagation due to changes in refraction in the presence of the anomalous jet, which may involve a number of different mechanisms [*Lorenz*, 2014; *Burrows et al.*, 2017]. In contrast, baroclinic mechanisms attribute the eddy momentum flux changes to changes in the stirring driven by the changes in the barotropic flow [*Robinson*, 2000].

Idealized models provide a useful framework for studying these two aspects of the problem in isolation. Using the stirred barotropic model, *Barnes et al.* [2010] investigated the sensitivity of the eddy momentum fluxes to the anomalous jet with fixed stirring. They showed that on the sphere, the

268 eddy momentum flux becomes more asymmetric (equatorward propagation is enhanced) when the
 269 jet moves poleward, leading to a positive feedback. This may be understood in terms of changes in
 270 the turning latitude/reflecting level [Lorenz, 2014].

271 In the opposite direction, *Zurita-Gotor et al.* [2014] analyzed the dynamics of jet variability
 272 in idealized two-layer QG simulations and showed that the enhanced persistence in that model
 273 was consistent with the baroclinic feedback mechanism of *Robinson* [2000]. They found evidence
 274 of baroclinicity driving the barotropic flow and very large coherence between the eddy heat and
 275 momentum fluxes at low frequency, with the momentum fluxes leading the variability, see Fig 2e.
 276 The co-variability between the barotropic and baroclinic components of the wind is also a robust
 277 result in observations [*Blanco-Fuentes and Zurita-Gotor*, 2011] and comprehensive climate models.
 278 In Figure 2d the large correlation between the long-lag decay rates of (barotropic) jet anomalies
 279 and baroclinicity is shown for a selection of CMIP5 models, so that models with more persistent jet
 280 variability also tend to have more persistent baroclinicity.

281 Stirred barotropic models can capture some aspects of the observed jet variability, like the sensitivity
 282 of persistence to latitude [*Barnes et al.*, 2010]. On the other hand, the baroclinic mechanism
 283 may help explain the excessive persistence bias in comprehensive climate models (which cannot
 284 be corrected by eliminating the jet latitude bias; *Simpson et al.* [2013]) or in idealized baroclinic
 285 models. Finally, diabatic effects may also play a role for annular mode persistence [*Xia and Chang*,
 286 2014]. The jet persistence problem underscores the importance of making connections across the
 287 full model hierarchy, as the mechanisms at work may not be the same in all steps of the hierarchy, in
 288 comprehensive climate models and in the real atmosphere.

289 4 The Middle Atmosphere Circulation

290 Work over the last two decades has established the highly coupled nature of the circulation in the
 291 troposphere and stratosphere. Many comprehensive atmospheric models now treat the stratosphere-
 292 troposphere as one system [e.g. *Gerber et al.*, 2012], recognizing the consequences of underresolving
 293 the middle atmosphere for weather and climate prediction [e.g., *Sigmond et al.*, 2013; *Manzini*
 294 *et al.*, 2014]. Historically, however, the middle atmospheric research proceeded on a different track
 295 after *Charney and Drazin* [1961] showed that a detailed representation of the stratosphere was not
 296 necessary to capture the basic structure of synoptic variability in the troposphere.

297 Wave-mean flow theory was developed, in part, to explain and understand the stratospheric
 298 circulation. The gross structure of the stratosphere can not be explained without understanding the
 299 essential role of waves in the transport of momentum, mass, and tracers. We highlight three models
 300 that capture these interactions, and the more sophisticated steps in the hierarchy they have inspired.

301 4.1 Sudden Stratospheric Warming Events: The *Holton and Mass* [1976] Model

302 Cooling during the polar night generates a strong westerly jet in the winter stratosphere, often
 303 referred to as the stratospheric polar vortex, where wind speeds can sometimes reach 100 ms^{-1} . In
 304 the early 1950s, however, it was observed that the polar vortex in the boreal hemisphere aperiodically
 305 undergoes a rapid breakdown. The reversal of the westerly winds is associated with a dramatic
 306 warming (40 K or more in the course of a few days) and hence known as a Sudden Stratospheric
 307 Warming (SSW) [*Scherhag*, 1952]. SSWs occur on average once every other year in the Northern
 308 Hemisphere, but only one such event (in 2002) has been observed in the austral hemisphere. *Baldwin*
 309 and *Dunkerton* [2001] showed that SSWs affect the troposphere, shifting the jet stream equatorward
 310 with substantial impacts on weather in Europe and Eastern North America. The tropospheric impact
 311 persists on the 1-2 month time scale that it takes the stratospheric vortex to recover back to its
 312 climatological state.

313 *Matsuno* [1971] proposed a dynamical mechanism for SSWs based on planetary scale wave
 314 propagation from the troposphere. Long before this process could be captured in atmospheric
 315 GCMs, *Holton and Mass* [1976] developed a simple, stratosphere-only, model that captures the

essence of these abrupt events. They constructed a highly truncated baroclinic quasi-geostrophic model, retaining only wavenumber 1 and the mean flow. The mean state is forced by Newtonian relaxation toward a specified state of radiative equilibrium, the wave generated by specifying a forcing amplitude on the bottom boundary. The model exhibits an abrupt transition between subcritical and supercritical behavior depending on the amplitude of the wave forcing: in the subcritical state, westerly winds coexist with a stationary Rossby wave. If the wave amplitude at the lower boundary exceeds a critical threshold, however, the model transitions abruptly to a new equilibrium: the waves grows, weakening the westerlies until they reverse, i.e., a prototypical SSW.

Multiple flow equilibria have also been demonstrated in more complex 3-dimensional stratosphere-only models – again forced by specifying the amplitude of planetary waves at the lower boundary – but permitting arbitrary height and latitude structure above [e.g., *Scott and Haynes*, 2000; *Scott and Polvani*, 2006]. The highly idealized *Holton and Mass* [1976] model, however, has continued to inspire research on the role of gravity waves in SSWs [e.g., *Albers and Birner*, 2014], and the role of the stratosphere on regulating wave activity [e.g., *Sjoberg and Birner*, 2014].

These models suggest that the near absence of SSWs in the austral hemisphere is due to the fact that stationary wave amplitude is weaker, a process explored in full 3-D atmospheric models using a *Held and Suarez* [1994] forcing, albeit with a modified equilibrium temperature profile in the stratosphere to establish a polar vortex. *Taguchi et al.* [2001], *Taguchi and Yoden* [2002], and *Sheshadri et al.* [2015] show how one can transition from a Southern Hemispheric state to a Northern Hemispheric state by increasing the amplitude of surface topography. *Held and Suarez* [1994] type models have also allowed for exploration of the impact of the vortex strength on the troposphere, both in response to forced changes [*Polvani and Kushner*, 2002] or SSWs [*Gerber and Polvani*, 2009].

4.2 The Quasi-Biennial Oscillation: A physical model

While high latitude variability in the stratosphere is dominated by interactions between planetary scale waves and the mean flow, tropical variability is effected by wave-mean flow interactions involving much smaller-scale gravity waves. The Quasi-Biennial Oscillation (QBO) is an oscillation of the zonal mean wind in the tropical stratosphere with a period of approximately 28 months, associated with the slow downward migration of alternative westerly and easterly jets [see *Baldwin et al.*, 2001, for a comprehensive review]. The long time scales of the QBO make it a potential source of predictability in the troposphere. For example, it was recently observed that the QBO is associated with changes in the strength and predictability of the Madden-Julian Oscillation [e.g., *Yoo and Son*, 2016]. The QBO also provides another example of the advances that a simplified system can bring about, well ahead of our ability to simulate the phenomenon in comprehensive models.

Pioneering work by *Lindzen and Holton* [1968] and *Holton and Lindzen* [1972] proposed that the QBO could be explained as an interaction between gravity waves and the mean flow. Selective absorption (breaking) of waves carrying easterly (westerly) momentum on the lower flank of easterly (westerly) jets leads to a momentum tendency that pulls the jet downward, enough to oppose the tendency of the mean tropical upwelling to advect the jet upward. The balance between the two effects leads to the slow, 28 month period of the jets. These models came long before we had the ability to observe (or simulate) the small scale gravity waves implicated in the mechanism. Even today, gravity waves provide a challenge to observe and model [*Alexander et al.*, 2010].

Given the challenges associated with observing or directly simulating the processes involved in the mechanism, *Plumb and McEwan* [1978] developed a novel physical model of the phenomenon. Models of the atmosphere generally refer to numerical models, but *Plumb and McEwan* [1978] is a rare example of an experiment using a physical model. The *Plumb and McEwan* [1978] model consists of an annulus of stratified salt water and internal waves forced by mechanically oscillating the lower boundary. The waves generate spontaneous formation of jets (an azimuthal circulation in the annulus), with slow oscillations and reversal of the flow, similarly to the QBO of the atmosphere.

364 4.3 Stratospheric transport: The leaky pipe

365 Transport and chemistry play key roles in the distribution of trace gases throughout the strato-
 366 sphere, including water vapor, ozone, and the substances that deplete ozone. The meridional over-
 367 turning circulation of the stratosphere, known as the Brewer-Dobson Circulation, was first inferred
 368 from trace gas measurements, decades before we could observe the circulation directly [Brewer,
 369 1949; Dobson, 1956]. Trace gases are advected by the mean Lagrangian circulation of mass and
 370 mixed along isentropic surfaces in the process of wave breaking. The latter mixing process pro-
 371 duces no net transport of mass, but will transport a trace gas if there is a horizontal gradient in its
 372 concentration.

373 Efforts to understand stratospheric transport began with limiting cases in the balance between
 374 transport of tracers across isentropic surfaces by the mean overturning mass circulation vs. the mixing
 375 of tracers along isentropic surfaces. *Plumb and Ko* [1992] consider a circulation where mixing along
 376 isentropic surfaces is extremely efficient. In contrast, *Plumb* [1996] developed the idea of a ‘tropical
 377 pipe’, where upwelling air in the tropics is entirely isolated from the downwelling air in the higher
 378 latitudes and transport is set by the mean mass circulation alone. These two limiting cases were
 379 combined in a benchmark model in our understanding of transport processes, the ‘leaky pipe’ model
 380 of *Neu and Plumb* [1999].

381 The leaky pipe divides the stratosphere into two regions, an upwelling ‘pipe’ in the tropics, and
 382 a downwelling pipe in the extratropics of both hemispheres. Mass is advected up the tropical pipe
 383 by the Lagrangian mean circulation, detraining continually out to the extratropics. The boundary
 384 between the two regions, the edge of the stratospheric surf zone, is a barrier to transport, but the
 385 ‘leaky’ pipe allows for some mixing of mass between the two. The most important parameters are
 386 the net detrainment (or equivalently, the net Lagrangian transport) and total mixing as a function of
 387 height, and can be solved for analytically with appropriate simplifying assumptions. A key result of
 388 the model is that an increase in the net Lagrangian mass transport will tend to freshen the stratosphere,
 389 cycling tracers more quickly through it, while an increase in mixing tends to slow the cycling, as
 390 mixing leads to recirculation of air through the stratosphere.

391 While designed primarily as a conceptual model, the leaky pipe has been applied in a more
 392 realistic context to understand the make up of the stratosphere, and its response to anthropogenic
 393 forcing. *Garny et al.* [2014] use it to interpret changes in the stratospheric circulation in comprehen-
 394 sive models, separating the roles of mixing from the mean Brewer-Dobson Circulation. *Ray et al.*
 395 [2010] build on the leaky pipe to explain the distribution of trace gases, and *Linz et al.* [2016, 2017]
 396 use it to quantify the strength of the Brewer-Dobson Circulation from satellite measurements.

397 5 The Large Scale Circulation of the Tropics

398 Significant progress in understanding the large-scale circulation of the mid-latitudes and middle
 399 atmosphere was possible in the context of “dry dynamics”. Removing the non-linearities associated
 400 with moist processes simplifies the problem, both conceptually and in terms of the numerical
 401 equations, processes, and scales that must be represented or parametrized. Indeed, all the simple
 402 models highlighted in Sections 3 and 4 do not include moist effects. In the tropics, the circulation and
 403 moist processes are more intimately coupled. A key scientific challenge for understanding tropical
 404 circulation has been: How do we deconvolve the tight coupling between circulation, moisture, clouds,
 405 and convection?

406 Nonetheless, there are still “dry” frameworks for understanding the gross features of the tropical
 407 circulation. In Section 5.1 we explore the Matsuno-Gill model, a model that captures the equatorial
 408 zonal overturning circulation, or Walker circulation, using the dry shallow water equations. In
 409 Section 5.2 we then focus on the zonal mean tropical overturning circulation, the Hadley circulation,
 410 again starting the discussion with a dry atmospheric model, but quickly introducing an idealized
 411 GCM that begins to capture moist processes.

412 5.1 The Walker Circulation: The Matsuno-Gill model

413 The Walker circulation describes equatorial atmospheric cells with ascent over the Maritime
 414 Continent (equatorial Western Pacific) and descent in the Eastern Pacific or Indian Oceans. The
 415 number of equatorial circulation cells and the location of ascending/descending branches are coupled
 416 with SSTs and the phase of ENSO [Julian and Chervin, 1978]. Research questions for the Walker
 417 cell include: How does convection and circulation interact within the Walker circulation? How does
 418 the Walker circulation and El-Niño Southern Oscillation influence the onset of the monsoon? How
 419 will the Walker circulation change with global warming?

420 Similarly to the mid-latitudes, many simple models for the tropical circulation hinge on reducing
 421 the dimensions of the atmospheric flow and a key simplification is to vertically truncate the fluid
 422 governing equations. One such model that has been fundamental for understanding the structure
 423 of the Walker circulation is the Matsuno–Gill model [Matsuno, 1966; Gill, 1980], that uses the dry
 424 shallow water equations on an equatorial-beta plane with a stationary heating source [e.g. Vallis,
 425 2017, section 8.5]. This single-layer model provides an analytic solution for the horizontal structure
 426 associated with the first baroclinic mode. This vertical mode captures the circulation driven by
 427 heating associated with tropical deep convection, and is characterized by opposite signed flow in the
 428 upper vs. lower troposphere. As the troposphere does not have a rigid upper boundary, it is not a true
 429 "mode" as in the ocean, but it often behaves like one.

430 The model's solution is generally described as the Matsuno–Gill pattern, in which two steady-
 431 state circulation cells develop in response to the applied heating, with low-level convergence into and
 432 upper-level divergence out of the heating region. This generates eastward propagating Kelvin waves
 433 and westward propagating Rossby waves. Two off-equatorial low pressure systems form as Rossby
 434 waves can not propagate along the equator [see Figure 8.11 of Vallis, 2017]. This equatorially-
 435 symmetric component of the Matsuno–Gill model generally describes the observed structure of the
 436 Walker circulation, with analogous tropical convection in the West Pacific and descent over the cold
 437 SST in the East Pacific (due to deep water upwelling).

438 The Matsuno–Gill model has also been used as the atmospheric component of the first successful
 439 numerical ENSO prediction model, the Cane-Zebiak model [Cane *et al.*, 1986]. a very influential
 440 reduced complexity coupled atmosphere-ocean model. In addition, the Matsuno–Gill model captures
 441 monsoonal circulations, using off-equatorial heating to mimic the seasonal cycle. Gill [1980] showed
 442 that the anti-symmetric Matsuno–Gill pattern (see Figure 3 of Gill [1980]), describes the general
 443 structure of the monsoon flow [Rodwell and Hoskins, 1996]. Furthermore, the Matsuno–Gill model is
 444 important for understanding the propagation of the Madden-Julian Oscillation, as detailed in Section
 445 7.

446 While some aspects of the Walker circulation are captured by the Matsuno–Gill model, there
 447 are still many limitations. The primary limitation of the Matsuno–Gill model is that it does not
 448 interactively include moisture and, as a result, many important moist feedback mechanisms are
 449 absent. One approach to studying the moist Walker circulation is to impose a large-scale gradient
 450 of SST in a two-dimensional atmospheric model domain, creating a steady-state Walker circulation,
 451 commonly called the “mock” Walker circulation.

452 Bretherton *et al.* [2006] studied the moist Walker circulation using an idealized non-rotating 2D
 453 model, that is vertically truncated (one vertical moisture mode) following the approach of the quasi-
 454 equilibrium tropical circulation model [Neelin and Zeng, 2000], which assumes the weak temperature
 455 gradient (discussed more in Section 6.1), and has simple precipitation and cloud schemes; see their
 456 Figure 4 for the resulting circulation. This is a useful prototype model configuration because it
 457 allows explicit cloud resolving model (CRM) and GCM-physics comparisons of a climate relevant
 458 problem [Jeevanjee *et al.*, 2017]. The beauty of this idealized model is that it includes feedbacks
 459 between convection and the large-scale circulation. In comparing to 3D CRMs, Bretherton *et al.*
 460 [2006] showed many interesting features within the two models: similar precipitation but different
 461 humidity distributions, narrowing of the circulation with warming SSTs and the importance of moist

462 static energy in understanding feedbacks between convection and the large-scale circulation within
 463 the Walker circulation.

464 5.2 The Hadley Circulation: Gray Radiation Aquaplanets

465 The Hadley circulation describes the zonally averaged atmospheric circulation cell with net
 466 ascent near the equator, poleward outflow in the upper troposphere, descent in the subtropics, and
 467 an equatorward near-surface return flow. The Hadley circulation separates the moist tropical regions
 468 from the dry subtropical climate zones and as such is important for setting the surface climate. Key
 469 research questions for the Hadley cell include: What controls its strength? What controls the position
 470 of the descending branch (i.e., the tropical edge) and the near-equatorial ascending region? How will
 471 these components change with global warming?

472 Dry models of the atmosphere have been illuminating in studying some of these research ques-
 473 tions. The dry models used to investigate the dynamics of the Hadley cell range from axisymmetric
 474 models amenable to theoretical progress [Held and Hou, 1980; Lindzen and Hou, 1988] through to
 475 idealized dry GCMs with extratropical eddies that interact with the tropical circulation [Kim and Lee,
 476 2001; Walker and Schneider, 2006]. The behavior across this dry model hierarchy has revealed two
 477 important insights. First, that the Hadley circulation has a finite extent—unlike the Brewer-Dobson
 478 circulation in the stratosphere the Hadley cell sinks before reaching the pole—even in the absence of
 479 extratropical eddies [Held and Hou, 1980]. Second, eddies are important for setting the circulation
 480 strength and extent [Held, 2000]. Dry models have set the foundations for our understanding of the
 481 Hadley cell, but moist processes are critical for determining the width of the ascending branch of the
 482 Hadley cell the circulation’s net energy transport.

483 A next logical step in the hierarchy of models to study the Hadley circulation is to include
 484 moist effects. One such model is the idealized moist primitive equation of Frierson [2007]. This
 485 model has a “gray radiation” scheme that neglects cloud and water vapor feedbacks, so that dynamic
 486 moisture feedbacks are decoupled from radiative feedbacks. The model uses an idealized large-scale
 487 precipitation scheme (condensation upon saturation) and a simple convection scheme that relaxes the
 488 atmosphere towards a stable vertical profile. The Hadley cell is very sensitive to the representation
 489 of convection. For example, the convection scheme impacts the energetic stratification. The moist
 490 static energy difference between the upper- and lower-level Hadley circulation in turn plays a key
 491 role in the strength of the mass overturning [Frierson, 2007].

492 The idealized model of Frierson [2007] can be linked to higher levels of the hierarchy by
 493 including more processes. The monsoons and ITCZ, among others problems, can be studied with
 494 greater realism by including the seasonal cycle. A second addition is to include the spatial variability
 495 in the radiative forcing, and feedbacks, for a more realistic atmospheric energy transport with climate
 496 change [Feldl *et al.*, 2017; Merlis, 2015]. A third addition is an idealized ocean heat transport
 497 coupled to the surface wind stress of the Hadley cell [Held, 2001; Levine and Schneider, 2011;
 498 Codron, 2012] that begins to bridge the gap between full-ocean GCMs and slab-ocean boundary
 499 conditions. A further extension is to couple the atmosphere and ocean for more realistic ocean
 500 heat uptake and transport that results in more realistic atmospheric energy transport by the Hadley
 501 circulation [Zelinka and Hartmann, 2010; Feldl and Bordoni, 2016]. Finally, radiative feedbacks can
 502 be introduced to the model by replacing the gray radiation scheme with a more realistic representation
 503 or radiative transfer [e.g. Merlis *et al.*, 2013; Jucker and Gerber, 2017; Vallis *et al.*, 2018].

504 In addition to understanding the fundamental properties of the Hadley circulation, models
 505 such as Frierson [2007] are a valuable step in the model hierarchy to investigate how changes in
 506 atmospheric water vapor with global warming impact the Hadley circulation. A key science question
 507 is: how will the tropical edge change in response to warming? The idealized moist physics GCM
 508 has been used to test and extend theories that were originally developed for dry flows [Held, 2000] to
 509 those that include moisture [O’Gorman, 2011; Levine and Schneider, 2015]. Furthermore, models
 510 such as Frierson [2007] have been important for understanding the forced response of the ITCZ,
 511 which is formed as a result of converging air toward the equator within the Hadley cell [Kang *et al.*,
 512 2009; Byrne *et al.*, 2018].

513 In addition, simplified moist GCMs have been useful to unravel the controls on monsoonal
 514 circulations with the aim to identify the minimal ingredients needed to develop the cross equatorial
 515 tropical overturning circulations that resemble monsoon flow over South Asia. Interestingly, idealized
 516 GCMs can capture aspects of the monsoon without zonally asymmetric land distributions or elevated
 517 orography [Bordoni and Schneider, 2008]. When idealized orography is included, an important
 518 “ventilation” mechanism is revealed: the poleward progression of the monsoon is prevented by mid-
 519 latitude dry air that is blocked by the elevated topography. This mechanism has been found in reduced
 520 vertical structure models, idealized GCMs, and comprehensive GCMs [Chou *et al.*, 2001; Privé and
 521 Plumb, 2007; Boos and Kuang, 2010]. Furthermore, the role of stationary eddies on the monsoons
 522 has been addressed in simulations with idealized lower boundary conditions [Shaw, 2014; Geen
 523 *et al.*, 2018] to assess seasonal circulation transitions in zonally asymmetric GCM configurations.

524 6 Coupling Clouds and Convection to the Large-scale Circulation

525 A key simplification of the idealized moist models discussed in Section 5.2 is to leave out
 526 the impact of clouds microphysics on the circulation. Clouds, visible manifestations of atmospheric
 527 convection, play a vital role in the radiative budget, both locally within a single convective system, and
 528 globally: clouds are a key uncertainty in predicting the global temperature response to greenhouse
 529 gas forcing (see Section 6.3). Individual convective clouds can be isolated and appear as random
 530 noise in an otherwise homogeneous environment, such as patchy, fair weather cumulus, but can also
 531 interact with nearby convection and the environment to form mesoscale convective systems such
 532 as squall lines. Organized convection impacts the radiation budget by changing the distribution of
 533 cloudy and clear sky. This is important as the radiative properties of clouds shape the large-scale
 534 circulation of the atmosphere [Hunt *et al.*, 1980; Slingo and Slingo, 1988; Randall *et al.*, 1989].

535 Clouds and convection are also embedded within the large-scale circulation of the atmosphere.
 536 The ascending branches of the circulation cells create the lifting force required for deep convection to
 537 develop within the ITCZ and the descending branches create suppressed regions in which clear sky
 538 or low-level clouds dominate. This two way interaction is referred to as cloud-circulation coupling.
 539 Understanding cloud-circulation coupling, and representing it in models, is one of the World Climate
 540 Research Program’s “Grand Challenges” on clouds, circulation and climate sensitivity [Bony *et al.*,
 541 2015].

542 In Section 6.1 we focus our discussion on Radiative Convective Equilibrium (RCE), a conceptual
 543 model of the tropical atmosphere that has helped us better understand the organization of convection.
 544 In Section 6.2 we discuss a “cloud locking” approach that decouples cloud radiative effects from the
 545 circulation, forming a bridge from the idealized moist GCMs to full atmospheric models. In Section
 546 6.3 we describe how the more complex models within the hierarchy are used to study Earth’s climate
 547 sensitivity.

548 6.1 Convective Organization: Radiative-Convective Equilibrium

549 In Section 5.2 we painted a picture of broad ascent within the equatorial branch of the Hadley
 550 circulation, associated with latent heating. This view of tropical precipitation is a reasonable
 551 approximation on longer time scales (weeks or more). On shorter time scale (hours-days), however,
 552 the tropical atmosphere is highly variable, with both ascent and descent in most regions. Convection
 553 on these shorter time scales is organized on small spatial scales, as within a single convective system,
 554 and on large scales, as with the Inter-tropical Convergence Zone (ITCZ) and MJO.

555 Convective organization is not well represented in most atmospheric models [Del Genio, 2012].
 556 This deficiency has been partly attributed to convective parametrizations that have a number of
 557 shortcomings. For example, convection is parametrized in the vertical column without any hori-
 558 zontal interactions, models have limited memory of convection from one time step to the next, and
 559 parametrizations generally do not represent interactions with the (unresolved) mesoscale circulation
 560 [Mapes and Neale, 2011]. A number of persistent model biases have been linked to errors in repre-
 561 senting convection [Randall *et al.*, 2016]. For example, models (i) exhibit too much light rain, which

562 results in insufficient extreme rainfall, (ii) trigger convection too early, resulting in the wrong diurnal
 563 cycle, and (iii) often generate a double ITCZ in the central and eastern Pacific [Stephens *et al.*, 2010;
 564 Dai, 2006; Sun *et al.*, 2006; Oueslati and Bellon, 2015].

565 The need to improve comprehensive atmospheric models motivates the use of a hierarchy of
 566 models to understand, and (ultimately) address long-standing model biases. Models can also be used
 567 to improve our theoretical understanding of convection and identify how convection interacts with
 568 both the local environment and larger scales [e.g., Muller and Bony, 2015].

569 Radiative-convective equilibrium (RCE) describes a state in which atmospheric radiative cooling
 570 is balanced by convective heating in a domain with no externally imposed horizontal structure, e.g.,
 571 uniform SST and insolation. RCE was first considered in the 1960s by Manabe and Strickler [1964],
 572 who originally proposed it to explain the vertical structure of the atmosphere. Since then, it has
 573 evolved into a test-bed for understanding convection in the absence of large-scale circulation. RCE
 574 is an important component of a hierarchical approach connecting physical laws to the complex
 575 behaviour of the Earth system [Popke *et al.*, 2013]. High-resolution models in RCE are a useful
 576 starting point for theories of convective organization [Muller and Bony, 2015].

577 Using a non-rotating cloud resolving model (CRM) in RCE, Bretherton *et al.* [2005] showed
 578 that convection can spontaneously self-organize (see Figure 3), a process sometimes known as "self-
 579 aggregation". The integration is initialized from a uniform state, and in the first weeks of integration,
 580 seemingly random convection is observed homogeneously across the domain. After approximately
 581 50 days, however, the system transitions to a single convecting cluster. Self-aggregation is not solely
 582 a spatial reorganization of convection; it dramatically changes the mean climate in CRMs resulting
 583 in a dryer troposphere, more outgoing long wave radiation (OLR), warmer free troposphere and
 584 surface. Please see Wing *et al.* [2017a] for more details and a full list of references, Mapes [2016]
 585 for a broader perspective and Holloway [2017] for a comparison to observations.

586 Convection also organizes in RCE simulations using GCMs with parametrized convection, in
 587 which large convective clusters form spontaneously [Popke *et al.*, 2013; Reed and Chavas, 2015;
 588 Coppin and Bony, 2015; Becker *et al.*, 2017]. Once convection begins to organize, a large-scale
 589 circulation develops and helps maintain the convection. In GCMs with prescribed SST, in RCE and
 590 non-RCE simulations, convection is more clustered in simulations without parametrized convection,
 591 compared to those with active convection parametrizations, and have larger rain rates [Becker *et al.*,
 592 2017; Maher *et al.*, 2018].

593 When planetary rotation is included in RCE simulations, self-aggregation transforms into
 594 tropical cyclones. Aquaplanet simulations in RCE have been particularly useful for understanding
 595 tropical cyclone characteristics [Shi and Bretherton, 2014; Satoh *et al.*, 2016; Reed and Chavas,
 596 2015] and their response to increasing SSTs [Held and Zhao, 2008; Khairoutdinov and Emanuel,
 597 2013; Merlis *et al.*, 2016]. Satoh *et al.* [2016] used a hierarchy of configurations with a global model
 598 to show the multiscale nature of tropical convective systems and how the effects of rotation change
 599 the vertical structure of the systems, see Figure 3.

600 The multiscale structure is also apparent in CRM simulations of RCE. The emergent structures
 601 remain similar across domain sizes, but the response to perturbations (like imposed surface warming)
 602 can vary depending on the domain size [Silvers *et al.*, 2016]. Similar experiments with global models
 603 are computationally expensive, but one alternative is to test the convergence characteristics of a
 604 model's physics by reducing the planetary radius to mimic increased horizontal resolution; Reed and
 605 Medeiros [2016] use this strategy to show how the large-scale convective aggregation seen in GCMs
 606 transitions to CRM-like self-aggregation without the increased computational cost.

607 Convective organization more generally is not well understood [Muller and Bony, 2015]. For
 608 example it is not clear how important self-aggregation is compared to organization by the mean
 609 wind or by waves or other mesoscale disturbances. There are a number of factors that contribute to
 610 organization such as cloud-radiative feedbacks, SST and convective-moisture feedbacks, see Sessions
 611 *et al.* [2016] for a full list. The model hierarchies has provided insight into why convection organizes

612 and how it is maintained. In this section we have focused on RCE and while it is an idealized model,
 613 it is still very complicated.

614 A further useful idealization, complementary to RCE, is the Weak Temperature Gradient
 615 (WTG) approximation. Under WTG the large-scale circulation, specifically the vertical velocity,
 616 is parametrized [Sobel and Bretherton, 2000; Sobel *et al.*, 2001; Raymond and Zeng, 2005]. This
 617 is done by assuming that horizontal temperature gradients and the local time tendency of temperature
 618 are both negligible at synoptic scales in the tropics – an observational fact explained dynamically
 619 by Charney [1963] – thus reducing the otherwise prognostic temperature equation to a diagnostic
 620 equation that can be solved for the large-scale vertical velocity given the diabatic heating. WTG is a
 621 horizontal truncation, as opposed to the vertical truncation in the Matsuno-Gill model described in
 622 Section 5.1.

623 WTG has been used to study a range of phenomena, including the Walker and Hadley circulations
 624 [Bretherton and Sobel, 2002; Polvani and Sobel, 2002; Burns *et al.*, 2006; Bellon and Sobel, 2010;
 625 Kuang, 2012]; ENSO teleconnections [Chiang and Sobel, 2002]; tropical cyclogenesis [Raymond,
 626 2007] and the MJO [Wang *et al.*, 2013, 2016]. Other related parametrizations of large-scale dynamics,
 627 solving the same problem in different ways, have also been developed [Kuang, 2008; Romps, 2012;
 628 Herman and Raymond, 2014], and WTG and one other, the ‘damped wave’ method [Blossey *et al.*,
 629 2009] applied to a wide range of models in a recent intercomparison [Daleu *et al.*, 2015, 2016]. These
 630 parametrizations of large-scale dynamics represent the circulation on scales smaller than the
 631 global scale at which RCE is relevant – the domain-average vertical motion being parametrized must
 632 vanish in RCE by definition – and the domain of a WTG single-column or cloud-resolving simulation
 633 can be thought of as representing a small fraction of an RCE simulation’s domain.

634 In such WTG simulations, more than one statistical equilibrium state can occur, depending on
 635 the initial humidity, with either dry or persistent deep convection states developing from identical
 636 forcing conditions [Sobel *et al.*, 2007; Sessions *et al.*, 2010] depending on initial conditions. These
 637 so-called ‘multiple equilibria’ are analogous to self-aggregation in RCE simulations in which a
 638 convecting cluster is surrounded by dry subsiding air [Sessions *et al.*, 2016], with the different WTG
 639 equilibria representing the convecting and dry regions separately. RCE and WTG together thus form
 640 a hierarchy of their own, providing distinct but qualitatively consistent views of the self-aggregation
 641 phenomenon.

642 Representing convective organization in Earth system models remains problematic. Progress is
 643 being made through a variety of modelling approaches to develop theories of convective organization
 644 and to better represent organization in GCM. These approaches cover broad resolutions with high
 645 resolutions LES and CRM, and GCMs with different treatments of convection. The convection
 646 approaches include: resolved convection in global CRMs [Tomita *et al.*, 2005; Miyamoto *et al.*,
 647 2013; Bretherton and Khairoutdinov, 2015; Judt, 2018], with and with parametrized convection
 648 [Popke *et al.*, 2013; Maher *et al.*, 2018], and super-parametrization [Arnold and Randall, 2015].

649 **6.2 Decoupling clouds and circulation: Cloud locking**

650 A primary challenge in representing clouds and convection in climate models is to adequately
 651 describe the interactions between clouds, convection, and precipitation, which must be parameterized
 652 in global models, with radiation and the resolved circulation. One opportunity to explore the role
 653 of clouds in the climate system is to adapt the diabatic hierarchy to decouple cloud-radiative effects
 654 from the circulation in which they are embedded. A few different approaches have been developed to
 655 achieve this: (i) a dry GCM forced with atmospheric cloud-radiative effects simulated from GCMs
 656 [Voigt and Shaw, 2016], (ii) reduced-complexity physics (e.g., Frierson-like without clouds [Kang
 657 *et al.*, 2009]), (iii) clouds that are transparent to radiation [Stevens *et al.*, 2012], and (iv) prescribing
 658 the cloud fields (cloud locking) [Zhang *et al.*, 2010].

659 The cloud-locking model approach has proven particularly helpful to understand how changes in
 660 the radiative properties of clouds impact the circulation response to global warming or hemispheric
 661 energy perturbations. Cloud-locking removes the coupling between clouds and circulation by pre-

scribing the cloud properties seen by the model's radiation scheme, generally from an earlier model simulation, and this isolates the circulation response to a perturbation as the clouds are invariant [Zhang *et al.*, 2010]. All four modelling approaches listed in the previous paragraph have been used to understand how changes in clouds with increased greenhouse gases will impact the position of the eddy-driven jet [Voigt and Shaw, 2015, 2016; Ceppi and Hartmann, 2016; Ceppi and Shepherd, 2017].

The eddy-driven jet (discussed in Section 3.4) is an interesting example, as its equatorward bias in coupled GCMs [Kidston and Gerber, 2010] is associated with Southern Ocean clouds that reflect too little shortwave radiation [Ceppi *et al.*, 2012]. Coupled GCMs show diverse responses in the eddy driven jet to global warming, especially in the Southern Hemisphere see Figure 4 a. These broad differences persist in aquaplanet simulations (Figure 4 b), making aquaplanets a desirable configuration to understand the eddy-driven jet response.

Cloud-radiative changes lead to a poleward shift in the eddy driven jet in cloud-locking simulations for the MPI-ESM aquaplanet model (Figure 4 c). The cloud-radiative changes with global warming can be attributed to high-level tropical (orange line) and mid-latitude clouds (blue line). Interestingly, the cloud impact is as large as the differences in jet shifts found in coupled GCMs, which suggests that clouds contribute to uncertainty in future jet shifts. The cloud impact is also reproduced in the dry Held-Suarez simulations perturbed with radiative changes from the cloud-locking simulations (Figure 4 d).

A complementary modelling technique to cloud-locking is the transparent-cloud approach, that prevents the radiation scheme from 'seeing' the clouds and hence sets the radiative heating to cloud free conditions [Randall *et al.*, 1989; Merlis, 2015]. This is easier to implement than cloud-locking and is simply achieved by setting the cloud fraction to zero in the radiation scheme. The transparent-cloud approach has helped to demonstrate the importance of cloud radiative effects for the present-day circulation. Such simulations have highlighted that cloud radiative effects strengthen the Hadley cell and eddy driven jet stream, reduce tropical-mean precipitation, and narrow the ITCZ [Li *et al.*, 2015; Harrop and Hartmann, 2016; Popp and Silvers, 2017; Albern *et al.*, 2018].

The primary task for understanding the role clouds play in the climate system is to understand their coupling with the circulation, and the implications of that coupling for the circulation response to climate change. In this regard, the transparent-cloud approach has proven helpful for understanding the role of clouds in the present-day climate, and the cloud-locking approach for understanding changes in clouds and circulation with global warming. While recent work has clearly shown that a quantitative understanding of the circulation must consider the coupling to clouds, this remains a rather young area of research with many open research questions, including for example cloud impacts on internal variability of the extratropical circulation [Li *et al.*, 2014].

6.3 The role of circulation in Earth's equilibrium climate sensitivity

How sensitive is the climate system to greenhouse gas emissions? Clouds are at the heart of this question because they remain the largest source of uncertainty in projections of future climate change. Despite the broad improvements in climate models, early estimates of the equilibrium climate sensitivity – a measure of globally averaged surface temperature change to doubling CO₂ – have not changed since the Charney report in 1979 with a range of 1.5–4.5 K [Stevens *et al.*, 2016].

The representation of clouds in different climate models is diverse. This results in widely varying cloud responses to the same perturbation [Boucher *et al.*, 2013; Chung and Soden, 2018]. Climate models show a relatively robust positive longwave (infrared/greenhouse) cloud feedback [Zelinka and Hartmann, 2010], attributed to the fixed anvil temperature hypothesis [Hartmann *et al.*, 2001; Hartmann and Larson, 2002]. The shortwave (visible/albedo) cloud feedbacks, however, remains highly uncertain despite the fact that most coupled GCMs suggest a weak positive feedback [Ceppi *et al.*, 2017]. Answering the open research questions about climate sensitivity comes down to understanding shortwave feedbacks for low-level clouds which account for much of the model uncertainty in cloud feedbacks. These low-level clouds form below regions of radiative cooling in

712 the descending branches of the Hadley and Walker circulations [Bony and Emanuel, 2005]. As such,
 713 circulation is key in setting their distribution, but cloud effects also feed back on the circulation,
 714 adding complexity to the problem.

715 Single column models (SCMs) have been used to investigate how parametrized physics can
 716 respond to climate sensitivity [Dal Gesso *et al.*, 2015]. Using SCMs with several configurations,
 717 Zhang *et al.* [2013] showed in idealized climate change experiments that the shallow convection and
 718 boundary layer turbulence are key differences among models. Care must be taken to meaningfully
 719 comparing an SCM to a GCM, however, because of the disconnection of cloud-circulation coupling
 720 in SCMs. In addition, physics packages can exhibit different cloud responses in a GCM and SCMs,
 721 creating obstacles for understanding cloud feedback. Progress has been made to understand cloud
 722 feedbacks in the gap between SCMs and GCMs, such as using WTG to parametrize a circulation in
 723 SCMs [Raymond, 2007; Zhu and Sobel, 2012] and GCMs in RCE to simplify the circulation [Bony
 724 *et al.*, 2016; Popke *et al.*, 2013; Wing *et al.*, 2017b] – in a conceptually similar way to Manabe and
 725 Strickler [1964] who used SCMs.

726 To capture the impact of circulation on climate sensitivity, efforts have focused at the top
 727 of the model hierarchy: coupled atmosphere-ocean GCMs (AOGCM) [Otto *et al.*, 2013; Stevens
 728 *et al.*, 2016; Caldwell *et al.*, 2016]. This is because simpler models make severe assumptions about
 729 the system, removing non-linear behavior that may project on to climate sensitivity [Knutti and
 730 Rugenstein, 2015]. From the perspective of the model hierarchy, AOGCMs are a moving target
 731 that evolves in response to both improvements in our understanding of the climate system, and to
 732 increasing computational resources.

733 The complexity of modern climate models, however, make it challenging to interpret their
 734 results, including the relative role of cloud feedbacks in climate change. The challenges in under-
 735 standing climate sensitivity in AOGCM makes a hierarchical approach appealing. The goal then
 736 becomes understanding the response of state-of-the-art AOGCMs in a simpler setting to reveal the
 737 underlying mechanisms and improve our physical understanding of the system. For example, using
 738 a range of boundary conditions and model configurations (ESM, GCM, aquaplanet, SCM) with the
 739 same model parametrizations, Brient and Bony [2013] identified a positive feedback that depends on
 740 how moist static energy is transported between the free troposphere and the boundary layer. Progress
 741 has been made using aquaplanet simulations to identify shallow cumulus clouds as driving the spread
 742 in climate sensitivity [Medeiros *et al.*, 2008; Ringer *et al.*, 2014; Medeiros *et al.*, 2015].

743 7 Case study: The Madden–Julian Oscillation

744 In Sections 5–6 we described the models that have been fundamental for advancing our under-
 745 standing of tropical circulation and the important role that moisture plays in setting the circulation,
 746 specifically how convective organization and clouds impact the radiative structure of the atmosphere.
 747 In this section we will focus our attention the Madden-Julian Oscillation (MJO). The MJO is an
 748 organized convective system and the primary source of tropical intraseasonal variability. The MJO
 749 continues to challenge our understanding of how circulation couples to clouds, convection and
 750 radiation. Progress in being made in our theoretical understanding of the mechanisms that initiation,
 751 propagate and maintain the MJO, however, there is currently no complete theory for the MJO [Ahn
 752 *et al.*, 2017]. As a result, the MJO is generally poorly represented in comprehensive climate models.
 753 In this review, we use the MJO as a case study to highlight how the model hierarchies—in particular
 754 idealized models—have been used to progress our understanding, develop new theories and improve
 755 the representation of the MJO in comprehensive models.

756 The MJO is an envelope of organized tropical convection that drifts eastward from the Indian
 757 Ocean into the Pacific. It is distinct from most convectively coupled equatorial waves in having
 758 a relatively slow speed of propagation ($\approx 4\text{--}8 \text{ m/s}$), longer timescales (about 1–2 months), and a
 759 relatively large scale (planetary wavenumbers 1–3) in comparison to other synoptic disturbances in
 760 the tropics. While it has also been historically difficult to simulate in global models, some recent

761 models do much better. For the first time, some dynamical forecasts are now superior to statistical
 762 ones. This new simulation capability allows theoretical ideas to be tested.

763 Realistic simulations of the MJO require convection to be sensitive to free-tropospheric moisture,
 764 i.e., a positive moisture-convection feedback, where deep convection is favored in regions where free-
 765 tropospheric humidity is higher. CMIP5-class models with the largest moisture sensitivity tend to
 766 have the most realistic MJO [Kim *et al.*, 2014a]. Poor simulations of the MJO — generally those
 767 with weak to non-existent MJOs [Ahn *et al.*, 2017] — can be improved by increasing the sensitivity
 768 of convection to moisture, such as increasing the entrainment and rain re-evaporation. Such tuning to
 769 optimize the MJO generally causes biases in mean climate [e.g., Kim *et al.*, 2011], but there is some
 770 evidence to suggest a realistic MJO and mean state can occur simultaneously even with traditional
 771 convection schemes [Crueger *et al.*, 2013]. There is considerable additional evidence, apart from the
 772 MJO, that deep convection in general is quite sensitive to moisture [e.g., Derbyshire *et al.*, 2004], and
 773 that typical convective schemes have excessive undilute ascent, as opposed to entraining air about
 774 them [e.g., Tokioka *et al.*, 1988; Kuang and Bretherton, 2006].

775 More recent studies have viewed the MJO through the moist static energy budget where surface
 776 fluxes and radiation are the dominant source terms (since moist static energy is conserved under
 777 condensation, which is the dominant source term in the dry static energy budget in deep convective
 778 conditions). Feedbacks between surface turbulent fluxes and convection were emphasized in early
 779 theories [Neelin *et al.*, 1987; Emanuel, 1987] and appear to be important in some GCMs [e.g.
 780 Maloney and Sobel, 2004]. Other work, however, points to a key role for cloud-radiative feedbacks;
 781 for example, there is less longwave cooling by high-clouds in a moist atmosphere [Andersen and
 782 Kuang, 2012; Chikira, 2013]. Process-based diagnostics [Kim *et al.*, 2015] and so-called “mechanism
 783 denial” experiments [Kim *et al.*, 2012; Crueger and Stevens, 2015; Ma and Kuang, 2016] in which
 784 a process is removed in order to test its importance, have lead to progress. This is consistent with
 785 earlier work with more idealized models. Raymond [2001] argued that radiative feedbacks were
 786 important to the MJO based on results from a 3D model of intermediate complexity, while Bony and
 787 Emanuel [2005] did so based on 2D CRM simulations without rotation. In an even simpler context,
 788 Hu and Randall [1994] found radiative feedbacks are critical in a one-dimensional model without
 789 large-scale circulation.

790 The importance of moisture-convection and cloud-radiative feedbacks suggests a view of the
 791 MJO as essentially a form of self-aggregation on the equatorial β -plane, in a domain much larger
 792 than CRMs simulations [e.g. Arnold and Randall, 2015]. In aquaplanet simulations with super-
 793 parametrized convection in RCE, Arnold and Randall [2015] found similar energy budgets and
 794 radiative feedbacks in non-rotating simulations, where self-aggregation dominates, and simulations
 795 with rotation, where MJO-like variability occurs.

796 The importance of moisture-convection and cloud-radiative feedbacks are the core assumptions
 797 in a recent set of highly idealized models of the MJO. These models represent the MJO as a
 798 moisture mode – a mode that would be absent in a dry atmosphere. In these idealized models,
 799 essential information is contained in the moisture field. Truncation to a single vertical mode, as in
 800 the Matsuno–Gill model, allows the dry dynamics to become shallow water-like. The convection
 801 schemes depend strongly, and in some cases exclusively on the moisture field, building in a strong
 802 moisture-convection feedback.

803 Moisture modes emerged in the idealized models of Fuchs and Raymond [Fuchs and Raymond,
 804 2002, 2007; Raymond and Fuchs, 2007, 2009]. The moisture mode was isolated in the simple 1-D
 805 linear model of Sobel and Maloney [2012, 2013] that has a single moisture prognostic variable,
 806 assumes WTG in the temperature equation, and generates winds by assuming a Matsuno–Gill
 807 response to quasi-steady heating (approximately valid as long as the disturbance does not propagate
 808 too quickly). In this model it can be shown explicitly that radiative feedbacks are critical for eastward
 809 propagation in a linearly unstable mode [Sobel and Maloney, 2013]. While the eastward propagation
 810 was initially slower than observations, modifications by Adames and Kim [2016] increased the
 811 propagation speed by accounting for meridional moisture advection. Because the WTG assumption
 812 eliminates the Kelvin waves, the waves that most early theories relied on to explain the eastward

propagation, the propagation of a moisture mode results largely from horizontal moisture advection, which seems to be supported by a number of observational and modeling studies [e.g., *Maloney*, 2009; *Pritchard and Bretherton*, 2014; *Kim et al.*, 2014b; *Inoue and Back*, 2015a].

Moisture mode theory — including the link to self-aggregation in idealized simulations — provides a useful framework for diagnosing models and observations, although whether moisture mode models correctly capture the MJO remains a topic of debate. The moisture mode ideas are quite different from those in earlier MJO theories, most of which excluded both radiative feedbacks and prognostic moisture (e.g., see review by *Wang* [2005]), and also differ from other, more recent models [e.g., *Majda and Stechmann*, 2009; *Yang and Ingersoll*, 2013]. Now that some comprehensive models at the top of the model hierarchy can simulate the MJO with reasonable fidelity, it is a question of linking them to our theories of MJO behavior. A connection to the moisture mode hypothesis, for example, can be traced through a hierarchical chain from self-aggregation in idealized simulations to more realistic simulations where moisture-convection and radiative feedbacks are allowed.

8 Synthesis and Outlook

All models are wrong but some are useful. The statistician George Box succinctly made two points at a workshop on statistical robustness four decades ago [Box, 1978]. First is the reminder that all of our models, even the most sophisticated, are inherently simplified – and so in Box’s sense “wrong” – and thus unable to capture all the potentially relevant processes and scales of the climate system. But second, we can learn, understand, and make predictions with *some* models.

In this review, we have identified a number of deliberately simplified models that have proven useful for understanding and predicting the large scale circulation of the atmosphere. We have not identified all possible benchmark models, but have sought to provide a balanced view of the dynamics of the tropical, extra-tropical, and middle atmosphere, highlighting processes on scales large, e.g., planetary waves in the Holton–Mass model, to small, e.g., convection and clouds within radiative-convective equilibrium integrations.

In Section 2, we proposed three principles to help organize models into hierarchies: dynamics, process, and scale. These are motivated, in part, by decisions we make in order to create a numerical model of the atmosphere that captures the large-scale circulation. These decisions include the appropriate governing equations, the relevant processes that drive the circulation, and the domain and resolution (which determine the allowable scales). These principles are not independent of one another. Dynamical hierarchies are designed to isolate particular scales and processes, e.g., the quasi-geostrophic equations focus on Rossby wave processes by filtering out the faster and smaller scale gravity waves. Likewise, the process hierarchy influence the choice of dynamics; if we wish to look at non-hydrostatic effects, we must resolve scales with an order-one aspect ratio, and thus the kilometer scale. The models featured in Sections 3-7 provide several examples of each of these hierarchies that have organically emerged in the literature, as highlighted in Figure 5.

Dynamical hierarchies have played a key role in understanding the mid-latitude circulation, where fast rotation and stratification organize the flow. We define the **equation hierarchy**, see Figure 5, that forms a natural progression of the equation set. The equation hierarchy includes the (i) barotropic vorticity dynamics that capture the evolution of Rossby waves (Section 3.1), (ii) quasi-geostrophic flow on 2 or more layers that generally captures baroclinic instability (Section 3.2), (iii) the dry primitive equation dynamics, e.g., as represented in the the Held-Suarez model, (iv) the moist primitive equation dynamics (as in the Frierson model in Section 5.2 or a standard AGCM), and ending with (v) the non-hydrostatic equations that includes the vertical momentum equation and are accurate at higher horizontal resolutions, e.g. used in CRMs or weather forecast models. In the tropics, rotation is weak and moist processes are of first order importance. As such, the dynamical hierarchies generally only include the more complex end of the equation hierarchy. None-the-less, the primitive equations or non-hydrostatic dynamics can be used with either vertical truncation (Matsuno-Gill model in Section 5.1) or horizontal truncation (weak-temperature gradient approximation in Section 6.1) to simplify the equation set.

863 The focus on processes is most essential for organizing model hierarchies. The purpose of the
 864 dynamics and scale hierarchies are then used to isolate and resolve the processes of interest. An
 865 example of a process hierarchy is the **diabatic hierarchy**, a term we use to describe a series of
 866 GCMs that integrate the primitive equation dynamics on the sphere, with advancing representations
 867 of the processes driving the temperature equation and generally with a resolution on the order of 100
 868 km. At the base of the diabatic hierarchy is (i) the *Held and Suarez* [1994] model (often referred
 869 to as a dry dynamical core) where all diabatic processes are replaced by Newtonian temperature
 870 relaxation. The Held-Suarez model has been used to understand jet stream variability (Section 3.4),
 871 tropical overturning circulation (Section 5.2), stratosphere-troposphere coupling (Section 4.1), and
 872 tracer transport (Section 4.3).

873 The next step in the diabatic hierarchy is to add moisture using a (ii) "gray" radiation schemes.
 874 This scheme decouples the convective latent heating from the radiation scheme so that water vapour
 875 is transparent to the radiation scheme [e.g., *Frierson et al.*, 2006; *O'Gorman and Schneider*, 2008].
 876 The next step in the diabatic hierarchy is (iii) to include a full radiation in the absence of clouds (or
 877 with a prescribed cloud climatology) that allows water vapor to interact with radiation in a simplified
 878 context [e.g., *Merlis et al.*, 2013; *Jucker and Gerber*, 2017]. These models have elucidated the
 879 circulation of the tropics and coupling between high and low latitudes (Section 5.2). At the next
 880 step in the diabatic hierarchy are (iv) atmospheric General Circulation Models that account for the
 881 importance of cloud and aerosol processes in the diabatic forcing of the circulation (Section 6).
 882 The most complex end of the diabatic hierarchy is to include the (v) carbon cycle and interactive
 883 chemistry which enables a more realistic representation of the processes governing radiative gases,
 884 clouds, and aerosols. The complex end of the diabatic hierarchy continues to evolve with time, as
 885 more processes are included in Earth System models and computational resources continue to grow.

886 Another process hierarchy that helps to organized the model hierarchies focusses the lower
 887 boundary conditions. Atmospheric models can be created with oceans that have (i) constant or fixed
 888 SST, (ii) aquaplanets with a so-called "slab" ocean which only captures the local thermodynamics
 889 of the atmosphere-ocean coupling, and (iii) a slab ocean with q-fluxes which include idealized
 890 horizontal transport. These models can also be configured to have idealized land and topography
 891 by changing the heat capacity and boundary layer roughness. The representation of the land surface
 892 conditions can be idealized or more realistic e.g., bucket hydrology vs. water runoff or a full
 893 representation of vegetation, aerosols, and carbon chemistry. The atmospheric models can also be
 894 forced with (iv) observed SSTs or (v) with an interactive ocean (vertical and horizontal) to form a
 895 coupled atmosphere-ocean model.

896 Figure 6 illustrates a hierarchy available within the CESM framework, incorporating elements
 897 of both the diabatic hierarchy and varying configurations of the land surface. The SimpleER project
 898 [*Polvani et al.*, 2017] makes many of these models an integral part of the CESM structure. The
 899 Isca framework [*Vallis et al.*, 2018], based on the GFDL modeling system, includes many of the
 900 lower steps of the hierarchy, but also includes hooks to add complexity and to build models of other
 901 planetary atmospheres as needed. One aspect of the process hierarchy that moves beyond these
 902 GCMs is to include more comprehensive treatments of microphysical processes that determine the
 903 distribution of clouds. An example is the Weather Research and Forecasting (WRF) model, which
 904 offers different options for representation of atmospheric processes, such as microphysics, and the
 905 treatment of boundary conditions.

906 Our final principal for organizing the models is scale. One example in which a hierarchy has
 907 naturally developed is for studying convective organization (Section 6.1). The model domain can vary
 908 from very high resolution in a small domain (to understand in-cloud properties) to low resolution
 909 on a global scale to understand planetary scale organization such as the MJO. Scale hierarchies
 910 are also implicit in dynamical hierarchies. Simplified models have proven useful in problems that
 911 intrinsically involve a great spread in scales, such as the QBO, where the evolution of planetary scale
 912 jets is driven by small scale gravity waves, and could only recently be captured in AGCMs.

913 Looking forward, we believe that model hierarchies will continue to help us improve climate
 914 and weather models. In particular, the gap in our understanding of the coupling processes between

915 clouds, convection, and circulation is mirrored in part with a gap in simple models that isolate the key
 916 processes regulating these interactions. There is a large jump between idealized moist models that
 917 effectively neglect cloud-aerosol processes [e.g., *Frierson et al.*, 2006; *Merlis et al.*, 2013; *Jucker and Gerber*,
 918 2017] and comprehensive GCMs that seek to parametrize all the unresolved scales
 919 which are critical to clouds and aerosols. This gulf partially reflects the difference between what
 920 can be done by an individual research group and a full modeling center. Further development of
 921 simpler GCMs that capture the essential elements of cloud and aerosol interactions are needed. It
 922 requires identifying sufficiently elegant model configurations, in the language of *Held* [2014], that
 923 would merit investment by a modeling center, or consortium of research groups, to bring in sufficient
 924 expertise.

925 Radiative-convective equilibrium integrations are in part aimed at this gulf between large scale
 926 dynamics and clouds processes. There is still a fundamental separation between them and the
 927 real atmosphere, however, where wind shear plays a vital role in organizing convection. This
 928 gap is visually emphasized in Figure 6 by the profound changes in circulation in CESM between
 929 simulations in RCE and an aquaplanet model (where the large-scale flow is determined by rotation
 930 and the temperature). Adding the building blocks of rotation and shear into RCE integrations may
 931 help establish these links.

932 In this review we have highlighted various ‘benchmark’ models for understanding and modelling
 933 the large-scale circulation of the atmosphere. We emphasize that their *connectedness* is essential;
 934 indeed it is what defines a hierarchy. A simple model must be connected in some way to a com-
 935 prehensive model and/or to reality for it to have value, else it becomes irrelevant. Connectedness does
 936 not always need to occur in a sequence of small steps; in some cases a simple model may connect
 937 almost directly to observations or experiment (the Lindzen–Holton–Plumb model of the QBO may
 938 be an example). However, such a leap is the exception, and in most cases a simple model connects
 939 to reality via a sequence of other models.

940 Model hierarchies will continue to play a role in our understanding of climate projections; in
 941 fact, we argue they should play an increasingly important role. We do not believe in global warming
 942 because a GCM tells us it is so; rather, we believe in it because of very basic physical laws. However,
 943 in their simplest manifestation those laws have little quantitative predictive capability for Earth’s
 944 climate. At the other extreme, when comprehensive models are forced into the warmer regimes
 945 that may lie in our planet’s future, we do not have the ability to compare parametrizations with
 946 observations. A purpose of the model hierarchies is then to provide a pathway connecting robust
 947 physical laws to our complex reality, via models of varying levels of complication. Ideally, this
 948 enables us to both understand the processes involved and to make useful and trustworthy predictions.

949 Acknowledgments

950 We wish to thank Dee Burke who created Figure 5. We would like to sincerely thank Isaac Held
 951 and two anonymous reviewers whose comments led us to greatly improve earlier versions of this
 952 manuscript. We also wish to thank the Editor Alan Robock and the editorial staff, especially Julie
 953 Dickson.

954 We also wish to acknowledge various funding sources who support the research of the authors.
 955 PM and GKV acknowledge the Natural Environment Research Council and Met Office ParaCon
 956 project NE/N013123/1. EPG acknowledges support from the US National Science Foundation
 957 through grant AGS-1546585. BM acknowledges support by the Regional and Global Model Analysis
 958 component of the Earth and Environmental System Modeling Program of the U.S. Department of
 959 Energy’s Office of Biological & Environmental Research Cooperative Agreement No. DE-FC02-
 960 97ER62402, and the National Center for Atmospheric Research which is a major facility sponsored by
 961 the National Science Foundation under Cooperative Agreement No. 1852977. TMM acknowledges
 962 Natural Science and Engineering Research Council of Canada grant RGPIN-2014-05416 and a
 963 Canada Research Chair. SS acknowledges the Australian Research Council grant number ARC
 964 FL150100035. AS acknowledges the Simons Foundation award number 354584. AHS acknowledges
 965 the National Science Foundation grant AGS-1758603. AV is supported by the German Ministry of

966 Education and Research (BMBF) and FONA: Research for Sustainable Development (www.fona.de)
 967 under grant agreement 01LK1509A. PZG acknowledges the State Research Agency of Spain, Grant
 968 number CGL2015-72259-EXP.

969 References

- 970 Adames, A. F., and D. Kim (2016), The MJO as a dispersive, convectively coupled moisture wave:
 971 Theory and observations., *Journal of the Atmospheric Sciences*, 73, 913–941.
- 972 Ahn, M.-S., D. Kim, K. R. Sperber, I.-S. Kang, E. Maloney, D. Waliser, and H. Hendon (2017), MJO
 973 simulation in CMIP5 climate models: MJO skill metrics and process-oriented diagnosis, *Climate
 974 Dynamics*, 49(11), 4023–4045.
- 975 Albern, N., A. Voigt, S. A. Buehler, and V. Grützun (2018), Robust and Nonrobust Impacts of
 976 Atmospheric Cloud-Radiative Interactions on the Tropical Circulation and Its Response to Surface
 977 Warming, *Geophysical Research Letters*, 45(16), 8577–8585.
- 978 Albers, J. R., and T. Birner (2014), Vortex preconditioning due to planetary and gravity waves prior
 979 to sudden stratospheric warmings, *Journal of the Atmospheric Sciences*, 71(11), 4028–4054.
- 980 Alexander, M. J., M. Geller, C. McLandress, S. Polavarapu, P. Preusse, F. Sassi, K. Sato, S. Ecker-
 981 mann, M. Ern, A. Hertzog, Y. Kawatani, M. Pulido, T. A. Shaw, M. Sigmond, R. Vincent, and
 982 S. Watanabe (2010), Recent developments in gravity-wave effects in climate models and the global
 983 distribution of gravity-wave momentum flux from observations and models, *Quarterly Journal of
 984 the Royal Meteorological Society*, 136(650), 1103–1124.
- 985 Andersen, J. A., and Z. Kuang (2012), Moist Static Energy Budget of MJO-like Disturbances in the
 986 Atmosphere of a Zonally Symmetric Aquaplanet, *Journal of Climate*, 25(8), 2782–2804.
- 987 Andrews, D., and M. McIntyre (1978), Generalized Eliassen-Palm and Charney-Drazin theorems
 988 for waves on axisymmetric mean flows in compressible atmospheres, *Journal of the Atmospheric
 989 Sciences*, 35(2), 175–185.
- 990 Arnold, N. P., and D. A. Randall (2015), Global-scale convective aggregation: Implications for the
 991 Madden-Julian Oscillation, *Journal of Advances in Modeling Earth Systems*, 7(4), 1499–1518.
- 992 Balasubramanian, G., and S. T. Garner (1997), The role of momentum fluxes in shaping the life cycle
 993 of a baroclinic wave, *Journal of the Atmospheric Sciences*, 54(4), 510–533.
- 994 Baldwin, M. P., and T. J. Dunkerton (2001), Stratospheric harbingers of anomalous weather regimes,
 995 *Science*, 294(5542), 581–584.
- 996 Baldwin, M. P., L. J. Gray, T. J. Dunkerton, K. Hamilton, P. H. Haynes, W. J. Randel, J. R. Holton,
 997 M. J. Alexander, I. Hirota, T. Horinouchi, D. B. A. Jones, J. S. Kinnison, C. Marquardt, K. Sato,
 998 and M. Takahashi (2001), The quasi-biennial oscillation, *Reviews of Geophysics*, 39(2), 179–229.
- 999 Barnes, E. A., and D. L. Hartmann (2011), Rossby wave scales, propagation, and the variability of
 1000 eddy-driven jets, *Journal of the Atmospheric Sciences*, 68(12), 2893–2908.
- 1001 Barnes, E. A., D. L. Hartmann, D. M. Frierson, and J. Kidston (2010), Effect of latitude on the
 1002 persistence of eddy-driven jets, *Geophysical Research Letters*, 37(11).
- 1003 Becker, T., B. Stevens, and C. Hohenegger (2017), Imprint of the convective parameterization
 1004 and sea-surface temperature on large-scale convective self-aggregation, *Journal of Advances in
 1005 Modeling Earth Systems*, 9(2), 1488–1505.
- 1006 Bellon, G., and A. H. Sobel (2010), Multiple Equilibria of the Hadley Circulation in an Intermediate-
 1007 Complexity Axisymmetric Model, *Journal of Climate*, 23(7), 1760–1778.
- 1008 Blanco-Fuentes, J., and P. Zurita-Gotor (2011), The driving of baroclinic anomalies at different
 1009 timescales, *Geophysical Research Letters*, 38(23).
- 1010 Blossey, P. N., C. S. Bretherton, and M. C. Wyant (2009), Subtropical Low Cloud Response to a
 1011 Warmer Climate in a Superparameterized Climate Model. Part II: Column Modeling with a Cloud
 1012 Resolving Model, *Journal of Advances in Modeling Earth Systems*, 1(3).
- 1013 Bony, S., and K. A. Emanuel (2005), On the Role of Moist Processes in Tropical Intraseasonal
 1014 Variability: Cloud–Radiation and Moisture–Convection Feedbacks, *Journal of the Atmospheric
 1015 Sciences*, 62(8), 2770–2789.
- 1016 Bony, S., B. Stevens, I. H. Held, J. F. Mitchell, J.-L. Dufresne, K. A. Emanuel, P. Friedlingstein,
 1017 S. Griffies, and C. Senior (2013), Carbon Dioxide and Climate: Perspectives on a Scientific

- 1018 Assessment, in *Climate Science for Serving Society: Research, Modeling and Prediction Priorities*,
 1019 pp. 391–413, Springer Netherlands.
- 1020 Bony, S., B. Stevens, D. M. W. Frierson, C. Jakob, M. Kageyama, R. Pincus, T. G. Shepherd, S. C.
 1021 Sherwood, A. P. Siebesma, A. H. Sobel, M. Watanabe, and M. J. Webb (2015), Clouds, circulation
 1022 and climate sensitivity, *Nature Geoscience*, 8, 261–268.
- 1023 Bony, S., B. Stevens, D. Coppin, T. Becker, K. A. Reed, A. Voigt, and B. Medeiros (2016),
 1024 Thermodynamic control of anvil cloud amount, *Proceedings of the National Academy of Sciences*,
 1025 113(32), 8927–8932.
- 1026 Boos, W. R., and Z. Kuang (2010), Dominant control of the South Asian monsoon by orographic
 1027 insulation versus plateau heating, *Nature*, 463, 218.
- 1028 Bordoni, S., and T. Schneider (2008), Monsoons as eddy-mediated regime transitions of the tropical
 1029 overturning circulation, *Nature Geoscience*, 1, 515–519.
- 1030 Boucher, O., D. Randall, P. Artaxo, C. Bretherton, G. Feingold, P. Forster, V.-M. Kerminen, Y. Kondo,
 1031 H. Liao, U. Lohmann, P. Rasch, S. K. Satheesh, S. Sherwood, B. Stevens, and X. Y. Zhang (2013),
 1032 *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth
 1033 Assessment Report of the Intergovernmental Panel on Climate Change*, chap. Clouds and Aerosols,
 1034 Cambridge University Press.
- 1035 Box, G. E. P. (1978), Robustnesss in the strategy of scientific model building, *Army Research Office
 1036 Workshop on Robustness in Statistics, April 11-12*.
- 1037 Bretherton, C. S., and M. F. Khairoutdinov (2015), Convective self-aggregation feedbacks in near-
 1038 global cloud-resolving simulations of an aquaplanet, *Journal of Advances in Modeling Earth
 1039 Systems*, 7, 1765–1787.
- 1040 Bretherton, C. S., and A. H. Sobel (2002), A Simple Model of a Convectively Coupled Walker
 1041 Circulation Using the Weak Temperature Gradient Approximation, *Journal of Climate*, 15(20),
 1042 2907–2920.
- 1043 Bretherton, C. S., P. N. Blossey, and M. Khairoutdinov (2005), An energy-balance analysis of
 1044 deep convective self-aggregation above uniform SST, *Journal of the Atmospheric Sciences*, 62,
 1045 4273–4292.
- 1046 Bretherton, C. S., P. N. Blossey, and M. E. Peters (2006), Interpretation of simple and cloud-resolving
 1047 simulations of moist convection–radiation interaction with a mock-Walker circulation, *Theoretical
 1048 and Computational Fluid Dynamics*, 20(5), 421–442.
- 1049 Brewer, A. W. (1949), Evidence for a world circulation provided by the measurements of helium and
 1050 water vapour distribution in the stratosphere, *Quarterly Journal Royal Meteorology Society*, 75,
 1051 351–363.
- 1052 Brient, F., and S. Bony (2013), Interpretation of the positive low-cloud feedback predicted by a
 1053 climate model under global warming, *Climate Dynamics*, 40(9-10), 2415–2431.
- 1054 Burns, S. P., A. H. Sobel, and L. M. Polvani (2006), Asymptotic solutions of the axisymmetric moist
 1055 Hadley circulation in a model with two vertical modes, *Theoretical and Computational Fluid
 1056 Dynamics*, 20(5), 443–467.
- 1057 Burrows, D. A., G. Chen, and L. Sun (2017), Barotropic and Baroclinic Eddy Feedbacks in the
 1058 Midlatitude Jet Variability and Responses to Climate Change-Like Thermal Forcings, *Journal of
 1059 the Atmospheric Sciences*, 74(1), 111–132.
- 1060 Butler, A. H., D. W. Thompson, and R. Heikes (2010), The steady-state atmospheric circulation
 1061 response to climate change-like thermal forcings in a simple general circulation model, *Journal
 1062 of Climate*, 23(13), 3474–3496.
- 1063 Byrne, M. P., A. G. Pendergrass, A. D. Rapp, and K. R. Wodzicki (2018), Response of the Intertropical
 1064 Convergence Zone to Climate Change: Location, Width, and Strength, *Current Climate Change
 1065 Reports*, 4(4), 355–370.
- 1066 Byrne, N. J., T. G. Shepherd, T. Woollings, and R. A. Plumb (2016), Annular modes and apparent
 1067 eddy feedbacks in the Southern Hemisphere, *Geophysical Research Letters*, 43(8), 3897–3902.
- 1068 Caldwell, P. M., M. D. Zelinka, K. E. Taylor, and K. Marvel (2016), Quantifying the Sources of
 1069 Intermodel Spread in Equilibrium Climate Sensitivity, *Journal of Climate*, 29(2), 513–524.
- 1070 Cane, M. A., S. E. Zebiak, and S. C. Dolan (1986), Experimental forecasts of El Niño, *Nature*, 321,
 1071 827–832.

- 1072 Ceppi, P., and D. L. Hartmann (2016), Clouds and the atmospheric circulation response to warming,
 1073 *Journal of Climate*, 29, 783–799.
- 1074 Ceppi, P., and T. G. Shepherd (2017), Contributions of climate feedbacks to changes in atmospheric
 1075 circulation, *Journal of Climate*, 30(22), 9097–9118.
- 1076 Ceppi, P., Y.-T. Hwang, D. M. W. Frierson, and D. L. Hartmann (2012), Southern Hemisphere jet
 1077 latitude biases in CMIP5 models linked to shortwave cloud forcing, *Geophysical Research Letters*,
 1078 39, L19,708.
- 1079 Ceppi, P., Y.-T. Hwang, X. Liu, D. M. Frierson, and D. L. Hartmann (2013), The relationship between
 1080 the ITCZ and the Southern Hemispheric eddy-driven jet, *Journal of Geophysical Research: Atmospheres*, 118(11), 5136–5146.
- 1081 Ceppi, P., F. Brient, M. D. Zelinka, and D. L. Hartmann (2017), Cloud feedback mechanisms and
 1082 their representation in global climate models, *Wiley Interdisciplinary Reviews: Climate Change*,
 1083 (4).
- 1085 Charney, J. G. (1963), A Note on Large-Scale Motions in the Tropics, *Journal of the Atmospheric
 1086 Sciences*, 20, 607–609.
- 1087 Charney, J. G., and P. G. Drazin (1961), Propagation of planetary-scale disturbances from the lower
 1088 into the upper atmosphere, *Journal of Geophysical Research*, 66(1), 83–109.
- 1089 Charney, J. G., R. Fjörtoft, and J. von Neuman (1950), Numerical Integration of the Barotropic
 1090 Vorticity Equation, *Tellus*, 2, 237–254.
- 1091 Chen, G., and I. M. Held (2007), Phase speed spectra and the recent poleward shift of Southern
 1092 Hemisphere surface westerlies, *Geophysical Research Letters*, 34(21).
- 1093 Chiang, J. C. H., and A. H. Sobel (2002), Tropical tropospheric temperature variations caused by
 1094 ENSO and their influence on the remote tropical climate, *Journal of Climate*, 15, 2616–2631.
- 1095 Chikira, M. (2013), Eastward-Propagating Intraseasonal Oscillation Represented by Chikira -
 1096 Sugiyama Cumulus Parameterization. Part II: Understanding Moisture Variation under Weak
 1097 Temperature Gradient Balance, *Journal of the Atmospheric Sciences*, 71, 615–639.
- 1098 Chou, C., J. D. Neelin, and H. Su (2001), Ocean-atmosphere-land feedbacks in an idealized monsoon,
 1099 *Quarterly Journal of the Royal Meteorological Society*, 127(576), 1869–1891.
- 1100 Chung, E.-S., and B. J. Soden (2018), On the compensation between cloud feedback and cloud
 1101 adjustment in climate models, *Climate Dynamics*, 50(3), 1267–1276.
- 1102 Codron, F. (2012), Ekman heat transport for slab oceans, *Climate Dynamics*, 38(1), 379–389.
- 1103 Coppin, D., and S. Bony (2015), Physical mechanisms controlling the initiation of convective self-
 1104 aggregation in a General Circulation Model, *Journal of Advances in Modeling Earth Systems*,
 1105 7(4), 2060–2078.
- 1106 Crueger, T., and B. Stevens (2015), The effect of atmospheric radiative heating by clouds on the
 1107 Madden-Julian Oscillation, *Journal of Advances in Modeling Earth Systems*, 7(2), 854–864.
- 1108 Crueger, T., B. Stevens, and R. Brokopf (2013), The Madden-Julian Oscillation in ECHAM6 and
 1109 the Introduction of an Objective MJO Metric, *Journal of Climate*, 26(10), 3241–3257.
- 1110 Dai, A. (2006), Precipitation Characteristics in Eighteen Coupled Climate Models, *Journal of
 1111 Climate*, 19(18), 4605–4630.
- 1112 Dal Gesso, S., A. P. Siebesma, and S. R. de Roode (2015), Evaluation of low-cloud climate feedback
 1113 through single-column model equilibrium states, *Quarterly Journal of the Royal Meteorological
 1114 Society*, 141(688), 819–832.
- 1115 Daleu, C. L., R. S. Plant, S. J. Woolnough, S. Sessions, M. J. Herman, A. Sobel, S. Wang, D. Kim,
 1116 A. Cheng, G. Bellon, et al. (2015), Intercomparison of methods of coupling between convection
 1117 and large-scale circulation: 1. Comparison over uniform surface conditions, *Journal of Advances
 1118 in Modeling Earth Systems*, 7, 1576–1601.
- 1119 Daleu, C. L., R. S. Plant, S. J. Woolnough, S. Sessions, M. J. Herman, A. Sobel, S. Wang, D. Kim,
 1120 A. Cheng, G. Bellon, et al. (2016), Intercomparison of methods of coupling between convection and
 1121 large-scale circulation: 2. Comparison over nonuniform surface conditions, *Journal of Advances
 1122 in Modeling Earth Systems*, 8, 387–405.
- 1123 Del Genio, A. (2012), Representing the Sensitivity of Convective Cloud Systems to Tropospheric
 1124 Humidity in General Circulation Models, *Surveys in Geophysics*, Volume 33, 1–20.

- 1125 DelSole, T. (2001), A simple model for transient eddy momentum fluxes in the upper troposphere,
 1126 *Journal of the Atmospheric Sciences*, 58(20), 3019–3035.
- 1127 Derbyshire, S. H., I. Beau, P. Bechtold, J.-Y. Grandpeix, J.-M. Piriou, J.-L. Redelsperger, and P. M.
 1128 Soares (2004), Sensitivity of moist convection to environmental humidity, *Quarterly Journal
 1129 Royal Meteorology Society*, 130, 3055–3080.
- 1130 Dobson, G. M. B. (1956), Origin and distribution of polyatomic molecules in the atmosphere,
 1131 *Proceedings of the Royal Society of London*, 236, 187–193.
- 1132 Eady, E. T. (1949), Long waves and cyclone waves, *Tellus*, 1, 33–52.
- 1133 Edelmann, W. (1963), *On the behaviour of disturbances in a baroclinic channel. Summary Rept. No.
 1134 2, Research in Objective Weather Forecasting, Part F, Contract AF61(052)-373*, 35 pp., Deutscher
 1135 Wetterdienst, Offenbach.
- 1136 Edmon, H., B. Hoskins, and M. McIntyre (1980), Eliassen-palm cross sections for the troposphere,
 1137 *Journal of the Atmospheric Sciences*, 37(12), 2600–2616.
- 1138 Emanuel, K. A. (1987), An air-sea interaction model of intraseasonal oscillations in the tropics.,
 1139 *Journal of the Atmospheric Sciences*, 44, 2324–2340.
- 1140 Feldl, N., and S. Bordoni (2016), Characterizing the Hadley circulation response through regional
 1141 climate feedbacks, *Journal of Climate*, 29, 613–622.
- 1142 Feldl, N., S. Bordoni, and T. M. Merlis (2017), Coupled high-latitude climate feedbacks and their
 1143 impact on atmospheric heat transport, *Journal of Climate*, 30(1), 189–201.
- 1144 Feldstein, S. B., and I. M. Held (1989), Barotropic decay of baroclinic waves in a two-layer beta-plane
 1145 model, *Journal of the Atmospheric Sciences*, 46(22), 3416–3430.
- 1146 Frierson, D. M. W. (2007), The Dynamics of Idealized Convection Schemes and Their Effect on the
 1147 Zonally Averaged Tropical Circulation, *Journal of the Atmospheric Sciences*, 64(6), 1959–1976.
- 1148 Frierson, D. M. W., I. M. Held, and P. Zurita-Gotor (2006), A Gray-Radiation Aquaplanet Moist
 1149 GCM. Part I: Static Stability and Eddy Scale, *Journal of the Atmospheric Sciences*, 63(10),
 1150 2548–2566.
- 1151 Fuchs, Z., and D. J. Raymond (2002), Large-scale modes of a nonrotating atmosphere with water
 1152 vapor and cloud-radiation feedbacks, *Journal of the Atmospheric Sciences*, 59(10), 1669–1679.
- 1153 Fuchs, Z., and D. J. Raymond (2007), A simple, vertically resolved model of tropical disturbances
 1154 with a humidity closure, *Tellus A*, 59(3), 344–354.
- 1155 Garny, H., T. Birner, H. Bönisch, and F. Bunzel (2014), The effects of mixing on age of air, *Journal
 1156 of Geophysical Research: Atmospheres*, 119(12), 7015–7034.
- 1157 Geen, R., F. H. Lambert, and G. K. Vallis (2018), Regime Change Behavior during Asian Monsoon
 1158 Onset, *Journal of Climate*, 31(8), 3327–3348.
- 1159 Gerber, E. P., and L. M. Polvani (2009), Stratosphere–troposphere coupling in a relatively simple
 1160 AGCM: The importance of stratospheric variability, *Journal of Climate*, 22(8), 1920–1933.
- 1161 Gerber, E. P., and G. K. Vallis (2007), Eddy–zonal flow interactions and the persistence of the zonal
 1162 index, *Journal of the Atmospheric Sciences*, 64(9), 3296–3311.
- 1163 Gerber, E. P., L. M. Polvani, and D. Anchukiewicz (2008), Annular mode time scales in the inter-
 1164 governmental panel on climate change fourth assessment report models, *Geophysical Research
 1165 Letters*, 35(22).
- 1166 Gerber, E. P., A. Butler, N. Calvo, A. Charlton-Perez, M. Giorgetta, E. Manzini, J. Perlitz, L. M.
 1167 Polvani, F. Sassi, A. A. Scaife, T. A. Shaw, S.-W. Son, and S. Watanabe (2012), Assessing and
 1168 understanding the impact of stratospheric dynamics and variability on the Earth system, *Bulletin
 1169 of the American Meteorological Society*, 93, 845–859.
- 1170 Ghil, M., and A. W. Robertson (2000), Solving problems with GCMs: General circulation models
 1171 and their role in the climate modeling hierarchy, *International Geophysics Series*, 70, 285–326.
- 1172 Gill, A. E. (1980), Some Simple Solutions for Heat-Induced Tropical Circulation, *Quarterly Journal
 1173 of the Royal Meteorological Society*, 106, 447–462.
- 1174 Grose, W. L., and B. J. Hoskins (1979), On the influence of orography on large-scale atmospheric
 1175 flow, *Journal of the Atmospheric Sciences*, 36(2), 223–234.
- 1176 Harrop, B. E., and D. L. Hartmann (2016), The Role of Cloud Radiative Heating in Determining the
 1177 Location of the ITCZ in Aquaplanet Simulations, *Journal of Climate*, 29(8), 2741–2763.

- 1178 Hartmann, D. L., and K. Larson (2002), An important constraint on tropical cloud - climate feedback,
 1179 *Geophysical Research Letters*, 29(20), 12–1–12–4, 1951.
- 1180 Hartmann, D. L., and P. Zuercher (1998), Response of baroclinic life cycles to barotropic shear,
 1181 *Journal of the Atmospheric Sciences*, 55(3), 297–313.
- 1182 Hartmann, D. L., J. R. Holton, and Q. Fu (2001), The heat balance of the tropical tropopause, cirrus,
 1183 and stratospheric dehydration, *Geophysical Research Letters*, 28(10), 1969–1972.
- 1184 Held, I. (2000), The General Circulation of the Atmosphere, Paper presented at 2000 Woods Hole
 1185 Oceanographic Institute Geophysical Fluid Dynamics Program.
- 1186 Held, I. (2005), The Gap between Simulation and Understanding in Climate Modeling., *Bulletin of*
 1187 *the American Meteorological Society*, 86, 1609–1614.
- 1188 Held, I. (2014), Simplicity amid Complexity, *Science*, 343(6176), 1206–1207.
- 1189 Held, I. M. (2001), The Partitioning of the Poleward Energy Transport between the Tropical Ocean
 1190 and Atmosphere, *Journal of the Atmospheric Sciences*, 58, 943–948.
- 1191 Held, I. M., and A. Y. Hou (1980), Nonlinear Axially Symmetric Circulations in a Nearly Inviscid
 1192 Atmosphere, *Journal of the Atmospheric Sciences*, 37, 515–533.
- 1193 Held, I. M., and M. J. Suarez (1994), A Proposal for the Intercomparison of the Dynamical Cores
 1194 of Atmospheric General Circulation Models, *Bulletin of the American Meteorological Society*,
 1195 75(10), 1825–1830.
- 1196 Held, I. M., and M. Zhao (2008), Horizontally Homogeneous Rotating Radiative-Convective Equi-
 1197 libria at GCM Resolution, *Journal of the Atmospheric Sciences*, 65(6), 2003–2013.
- 1198 Herman, M. J., and D. J. Raymond (2014), WTG cloud modeling with spectral decomposition of
 1199 heating, *Journal of Advances in Modeling Earth Systems*, 6(4), 1121–1140.
- 1200 Holloway, C. E. (2017), Convective aggregation in realistic convective-scale simulations, *Journal of*
 1201 *Advances in Modeling Earth Systems*, 9(2), 1450–1472.
- 1202 Holton, J. R., and R. S. Lindzen (1972), An updated theory for the quasi-biennial cycle of the tropical
 1203 stratosphere, *Journal of the Atmospheric Sciences*, 29, 1076–1080.
- 1204 Holton, J. R., and C. Mass (1976), Stratospheric vacillation cycles, *Journal of the Atmospheric*
 1205 *Sciences*, 33(11), 2218–2225.
- 1206 Hoskins, B. J. (1983), Dynamical processes in the atmosphere and the use of models, *Quarterly*
 1207 *Journal of the Royal Meteorological Society*, 109(459), 1–21.
- 1208 Hoskins, B. J., and D. J. Karoly (1981), The steady linear response of a spherical atmosphere to
 1209 thermal and orographic forcing, *Journal of the Atmospheric Sciences*, 38(6), 1179–1196.
- 1210 Hu, Q., and D. A. Randall (1994), Low-Frequency Oscillations in Radiative-Convective Systems,
 1211 *Journal of the Atmospheric Sciences*, 51(8), 1089–1099.
- 1212 Hunt, G. E., V. Ramanathan, and R. M. Chervin (1980), On the role of clouds in the general
 1213 circulation of the atmosphere, *Quarterly Journal of the Royal Meteorological Society*, 106(447),
 1214 213–215.
- 1215 Inoue, K., and L. E. Back (2015a), Column-integrated Moist Static Energy Analysis on Various Time
 1216 Scales during TOGA COARE, *Journal of the Atmospheric Sciences*, 72, 4148–4166.
- 1217 Jansen, M., and R. Ferrari (2013), Equilibration of an atmosphere by adiabatic eddy fluxes, *Journal*
 1218 *of the Atmospheric Sciences*, 70(9), 2948–2962.
- 1219 Jeevanjee, N., P. Hassanzadeh, S. Hill, and A. Sheshadri (2017), A perspective on climate model
 1220 hierarchies, *Journal of Advances in Modeling Earth Systems*, 9(4), 1760–1771.
- 1221 Jucker, M., and E. P. Gerber (2017), Untangling the Annual Cycle of the Tropical Tropopause Layer
 1222 with an Idealized Moist Model, *Journal of Climate*, 30(18), 7339–7358.
- 1223 Judt, F. (2018), Insights into Atmospheric Predictability through Global Convection-Permitting
 1224 Model Simulations, *Journal of the Atmospheric Sciences*, 75(5), 1477–1497.
- 1225 Julian, P. R., and R. M. Chervin (1978), A Study of the Southern Oscillation and Walker Circulation
 1226 Phenomenon, *Monthly Weather Review*, 106(10), 1433–1451.
- 1227 Kang, S. M., D. M. W. Frierson, and I. M. Held (2009), The Tropical Response to Extratropical
 1228 Thermal Forcing in an Idealized GCM: The Importance of Radiative Feedbacks and Convective
 1229 Parameterizatie, *Journal of the Atmospheric Sciences*, 66, 2812–2827.

- 1230 Khairoutdinov, M., and K. Emanuel (2013), Rotating radiative-convective equilibrium simulated by
1231 a cloud-resolving model, *Journal of Advances in Modeling Earth Systems*, 5, 816–825.
- 1232 Kidston, J., and E. Gerber (2010), Intermodel variability of the poleward shift of the austral jet stream
1233 in the CMIP3 integrations linked to biases in 20th century climatology, *Geophysical Research
1234 Letters*, 37(9).
- 1235 Kim, D., A. Sobel, E. D. Maloney, D. M. W. Frierson, and I.-S. Kang (2011), A Systematic Rela-
1236 tionship between Intraseasonal Variability and Mean State Bias in AGCM Simulations, *Journal
1237 of Climate*, 24(21), 0894–8755.
- 1238 Kim, D., A. H. Sobel, A. D. D. Genio, Y. Chen, S. J. Camargo, M.-S. Yao, M. Kelley, and
1239 L. Nazarenko (2012), The tropical subseasonal variability simulated in the NASA GISS general
1240 circulation model., *Journal of Climate*, 25, 4641–4659.
- 1241 Kim, D., P. Xavier, E. Maloney, M. Wheeler, D. Waliser, K. Sperber, H. Hendon, C. Zhang,
1242 R. Neale, Y.-T. Hwang, and H. Liu (2014a), Process-oriented MJO simulation diagnostic: Moisture
1243 sensitivity of simulated convection, *Journal of Climate*, 27, 5379–5395.
- 1244 Kim, D., J.-S. Kug, and A. H. Sobel (2014b), Propagating vs. non-propagating Madden-Julian
1245 oscillation events., *Journal of Climate*, 27, 111–125.
- 1246 Kim, D., M.-S. Ahn, I.-S. Kang, and A. D. D. Genio (2015), Role of Longwave Cloud-Radiation
1247 Feedback in the Simulation of the Madden-Julian Oscillation, *Journal of Climate*, 28(17), 6979–
1248 6994.
- 1249 Kim, H. K., and S. Y. Lee (2001), Hadley Cell Dynamics in a Primitive Equation Model. Part II:
1250 Nonaxisymmetric Flow, *Journal of the Atmospheric Sciences*, 58, 2859–2871.
- 1251 Knutti, R., and M. A. A. Rugenstein (2015), Feedbacks, climate sensitivity and the limits of linear
1252 models, *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical
1253 and Engineering Sciences*, 373(2054).
- 1254 Kuang, Z. (2008), Modeling the interaction between cumulus convection and linear waves using a
1255 limited domain cloud system resolving model, *J. Atmos. Sci.*, 65, 576–591.
- 1256 Kuang, Z. (2012), Weakly Forced Mock Walker Cells, *Journal of the Atmospheric Sciences*, 69(9),
1257 2759–2786.
- 1258 Kuang, Z., and C. S. Bretherton (2006), A mass flux scheme view of a high-resolution simulation of
1259 a transition from shallow to deep cumulus convection, *Journal of the Atmospheric Sciences*, 63,
1260 1895–1909.,
- 1261 Lachmy, O., and N. Harnik (2014), The transition to a subtropical jet regime and its maintenance,
1262 *Journal of the Atmospheric Sciences*, 71(4), 1389–1409.
- 1263 Lee, S., and I. M. Held (1993), Baroclinic wave packets in models and observations, *Journal of the
1264 Atmospheric Sciences*, 50(10), 1413–1428.
- 1265 Lee, S., S.-W. Son, K. Grise, and S. B. Feldstein (2007), A mechanism for the poleward propagation
1266 of zonal mean flow anomalies, *Journal of the Atmospheric Sciences*, 64(3), 849–868.
- 1267 Levine, X. J., and T. Schneider (2011), Response of the Hadley Circulation to Climate Change in an
1268 Aquaplanet GCM Coupled to a Simple Representation of Ocean Heat Transport, *Journal of the
1269 Atmospheric Sciences*, 68, 769–783.
- 1270 Levine, X. J., and T. Schneider (2015), Baroclinic Eddies and the Extent of the Hadley Circulation:
1271 An Idealized GCM Study, *Journal of the Atmospheric Sciences*, 72(7), 2744–2761.
- 1272 Li, Y., D. W. J. Thompson, Y. Huang, and M. Zhang (2014), Observed linkages between the
1273 northern annular mode/North Atlantic Oscillation, cloud incidence, and cloud radiative forcing,
1274 *Geophysical Research Letters*, 41(5), 1681–1688.
- 1275 Li, Y., D. W. J. Thompson, and S. Bony (2015), The Influence of Atmospheric Cloud Radiative
1276 Effects on the Large-Scale Atmospheric Circulation, *Journal of Climate*, 8, 7263–7278.
- 1277 Lindzen, R. S., and J. R. Holton (1968), A theory of the quasi-biennial oscillation, *Journal of the
1278 Atmospheric Sciences*, 25, 1095–1107.
- 1279 Lindzen, R. S., and A. V. Hou (1988), Hadley Circulations for Zonally Averaged Heating Centered
1280 off the Equator, *Journal of the Atmospheric Sciences*, 45(17), 2416–2427.
- 1281 Linz, M., R. A. Plumb, E. P. Gerber, and A. Sheshadri (2016), The relationship between age of
1282 air and the diabatic circulation of the stratosphere, *Journal of the Atmospheric Sciences*, 73(11),
1283 4507–4518.

- 1284 Linz, M., R. A. Plumb, E. P. Gerber, F. J. Haenel, G. Stiller, D. E. Kinnison, A. Ming, and J. L.
 1285 Neu (2017), The strength of the meridional overturning circulation of the stratosphere, *Nature
 1286 Geoscience*.
- 1287 Lorenz, D. J. (2014), Understanding midlatitude jet variability and change using Rossby wave
 1288 chromatography: Poleward-shifted jets in response to external forcing, *Journal of the Atmospheric
 1289 Sciences*, 71(7), 2370–2389.
- 1290 Lorenz, D. J., and E. T. DeWeaver (2007), Tropopause height and zonal wind response to global
 1291 warming in the IPCC scenario integrations, *Journal of Geophysical Research: Atmospheres*,
 1292 112(D10).
- 1293 Lorenz, D. J., and D. L. Hartmann (2001), Eddy–zonal flow feedback in the Southern Hemisphere,
 1294 *Journal of the Atmospheric Sciences*, 58(21), 3312–3327.
- 1295 Lutsko, N. J., I. M. Held, P. Zurita-Gotor, and A. K. O’Rourke (2017), Lower-Tropospheric Eddy
 1296 Momentum Fluxes in Idealized Models and Reanalysis Data, *Journal of the Atmospheric Sciences*,
 1297 74(11), 3787–3797.
- 1298 Ma, D., and Z. Kuang (2016), A mechanism-denial study on the Madden-Julian Oscillation with
 1299 reduced interference from mean state changes, *Geophysical Research Letters*, 43(6), 2989–2997.
- 1300 Maher, P., G. Vallis, S. Sherwood, M. Webb, and P. Sansom (2018), The Impact of Parameterized
 1301 Convection on Climatological Precipitation in Atmospheric Global Climate Models, *Geophysical
 1302 Research Letters*, 45, 3728–3736.
- 1303 Majda, A. J., and S. N. Stechmann (2009), The skeleton of tropical intraseasonal oscillations, *PNAS*,
 1304 106, 8417–8422.
- 1305 Maloney, E. D. (2009), The moist static energy budget of a composite tropical intraseasonal oscillation
 1306 in a climate model, *Journal of Climate*, 22, 711–729.
- 1307 Maloney, E. D., and A. H. Sobel (2004), Surface fluxes and ocean coupling in the tropical intrasea-
 1308 sonal oscillation, *Journal of Climate*, 17, 4368–4386.
- 1309 Manabe, S., and R. F. Strickler (1964), Thermal Equilibrium of the Atmosphere with a Convective
 1310 Adjustment, *Journal of the Atmospheric Sciences*, 21(4), 361–385.
- 1311 Manzini, E., A. Y. Karpechko, J. Anstey, M. P. Baldwin, T. Birner, R. X. Black, C. Cagnazzo,
 1312 N. Calvo, A. J. Charlton-Perez, B. Christiansen, P. Davini, E. P. Gerber, M. Giorgetta, L. Gray,
 1313 S. Hardiman, Y.-Y. Lee, D. Marsh, B. A. McDonald, L. M. Polvani, A. Purich, A. A. Scaife,
 1314 D. Shindell, S.-W. Son, E. Volodin, S. Watanabe, J. Wilson, S. Yukimoto, and G. Zappa (2014),
 1315 Northern winter climate change: Assessment of uncertainty in CMIP5 projections related to
 1316 stratosphere-troposphere coupling, *Journal of Geophysical Research: Atmospheres*, 119.
- 1317 Mapes, B., and R. Neale (2011), Parameterizing convective organization to Escape the Entrainment
 1318 Dilemma, *Journal of Advances in Modeling Earth Systems*, 3 (M06004), 20.
- 1319 Mapes, B. E. (2016), Gregarious convection and radiative feedbacks in idealized worlds, *Journal of
 1320 Advances in Modeling Earth Systems*, 8(2), 1029–1033.
- 1321 Matsuno, T. (1966), Quasi-geostrophic motions in the equatorial area, *Journal of the Meteorological
 1322 Society of Japan*, 44, 25–42.
- 1323 Matsuno, T. (1971), A dynamical model of the stratospheric sudden warming, *Journal of the
 1324 Atmospheric Sciences*, 28(8), 1479–1494.
- 1325 McIntyre, M. E., and T. Palmer (1983), Breaking planetary waves in the stratosphere, *Nature*,
 1326 305(5935), 593–600.
- 1327 Medeiros, B., B. Stevens, I. M. Held, M. Zhao, D. L. Williamson, J. G. Olson, and C. S. Bretherton
 1328 (2008), Aquaplanets, Climate Sensitivity, and Low Clouds, *Journal of Climate*, 21(19), 4974–
 1329 4991.
- 1330 Medeiros, B., B. Stevens, and S. Bony (2015), Using aquaplanets to understand the robust responses
 1331 of comprehensive climate models to forcing, *Climate Dynamics*, 44(7–8), 1957–1977.
- 1332 Merlis, T. M. (2015), Direct weakening of tropical circulations from masked CO₂ radiative forcing,
 1333 *PNAS*, 112, 13,167–13,171.
- 1334 Merlis, T. M., T. Schneider, S. Bordoni, and I. Eisenman (2013), Hadley circulation response to
 1335 orbital precession. Part I: Aquaplanets, *Journal of Climate*, 26, 740–753.
- 1336 Merlis, T. M., W. Zhou, I. M. Held, and M. Zhao (2016), Surface temperature dependence of tropical
 1337 cyclone-permitting simulations in a spherical model with uniform thermal forcing, *Geophysical*

- 1338 *Research Letters*, 43(6), 2859–2865.
- 1339 Miyamoto, Y., Y. Kajikawa, R. Yoshida, T. Yamaura, H. Yashiro, and H. Tomita (2013), Deep
1340 moist atmospheric convection in a subkilometer global simulation, *Geophysical Research Letters*,
1341 40(18), 4922–4926.
- 1342 Muller, C., and S. Bony (2015), What favors convective aggregation and why?, *Geophysical Research
1343 Letters*, 42(13), 5626–5634.
- 1344 Neelin, J. D., and N. Zeng (2000), A Quasi-Equilibrium Tropical Circulation Model-Formulation,
1345 *Journal of the Atmospheric Sciences*, 57(11), 1741–1766.
- 1346 Neelin, J. D., I. M. Held, and K. H. Cook (1987), Evaporation-wind feedback and low-frequency
1347 variability in the tropical atmosphere, *Journal of the Atmospheric Sciences*, 44, 2341–2348.
- 1348 Neu, J. L., and R. A. Plumb (1999), Age of air in a “leaky pipe” model of stratospheric transport,
1349 *Journal of Geophysical Research: Atmospheres*, 104(D16), 19,243–19,255.
- 1350 Nof, D. (2008), Simple Versus Complex Climate Modeling, *Eos, Transactions American Geophysical
1351 Union*, 89(52), 544–545.
- 1352 O’Gorman, P. A. (2011), The effective static stability experienced by eddies in a moist atmosphere,
1353 *Journal of the Atmospheric Sciences*, 68(1), 75–90.
- 1354 O’Gorman, P. A., and T. Schneider (2008), The Hydrological Cycle over a Wide Range of Climates
1355 Simulated with an Idealized GCM, *Journal of Climate*, 21(15), 3815–3832.
- 1356 O’Rourke, A. K., and G. K. Vallis (2013), Jet interaction and the influence of a minimum phase speed
1357 bound on the propagation of eddies, *Journal of the Atmospheric Sciences*, 70(8), 2614–2628.
- 1358 Otto, A., F. E. L. Otto, O. Boucher, J. Church, G. Hegerl, P. M. Forster, N. P. Gillett, J. Gregory,
1359 G. C. Johnson, R. Knutti, N. Lewis, U. Lohmann, J. Marotzke, G. Myhre, D. Shindell, B. Stevens,
1360 and M. R. Allen (2013), Energy budget constraints on climate response, *Nature Geoscience*, 6,
1361 415–416.
- 1362 Oueslati, B., and G. Bellon (2015), The double ITCZ bias in CMIP5 models: interaction between
1363 SST, large-scale circulation and precipitation, *Climate Dynamics*, 44(3), 585–607.
- 1364 Phillips, N. A. (1956), The general circulation of the atmosphere: A numerical experiment, *Quarterly
1365 Journal of the Royal Meteorological Society*, 82(352), 123–164.
- 1366 Plumb, R. A. (1996), A “tropical pipe” model of stratospheric transport, *Journal of Geophysical
1367 Research*, 101, 3957–3972.
- 1368 Plumb, R. A., and M. K. Ko (1992), Interrelationships between mixing ratios of long-lived strato-
1369 spheric constituents, *Journal of Geophysical Research: Atmospheres*, 97(D9), 10,145–10,156.
- 1370 Plumb, R. A., and A. D. McEwan (1978), The instability of a forced standing wave in a viscous
1371 stratified fluid: A laboratory analogue of the quasi-biennial oscillation, *Journal of the Atmospheric
1372 Sciences*, 35, 1827–1839.
- 1373 Polvani, L. M., and P. J. Kushner (2002), Tropospheric response to stratospheric perturbations in a
1374 relatively simple general circulation model, *Geophysical Research Letters*, 29(7).
- 1375 Polvani, L. M., and A. H. Sobel (2002), The Hadley Circulation and the Weak Temperature Gradient
1376 Approximation, *Journal of the Atmospheric Sciences*, 59(10), 1744–1752.
- 1377 Polvani, L. M., A. C. Clement, B. Medeiros, J. J. Benedict, and I. R. Simpson (2017), When Less Is
1378 More: Opening the Door to Simpler Climate Models, *EOS*.
- 1379 Popke, D., B. Stevens, and A. Voigt (2013), Climate and climate change in a radiative-convective
1380 equilibrium version of ECHAM6, *Journal of Advances in Modeling Earth Systems*, 5(1), 1–14.
- 1381 Popp, M., and L. G. Silvers (2017), Double and Single ITCZs with and without Clouds, *Journal of
1382 Climate*, 30(22), 9147–9166.
- 1383 Pritchard, M. S., and C. S. Bretherton (2014), Causal evidence that rotational moisture advection is
1384 critical to the superparameterized Madden-Julian Oscillation, *Journal of the Atmospheric Sciences*,
1385 71, 800–815.
- 1386 Privé, N. C., and R. A. Plumb (2007), Monsoon Dynamics with Interactive Forcing. Part II: Impact
1387 of Eddies and Asymmetric Geometries, *Journal of the Atmospheric Sciences*, 64(5), 1431–1442.
- 1388 Randall, D., C. DeMott, C. Stan, M. Khairoutdinov, J. Benedict, R. McCrary, K. Thayer-Calder,
1389 and M. Branson (2016), Simulations of the Tropical General Circulation with a Multiscale Global
1390 Model, *Meteorological Monographs*, 56, 15.1–15.15.

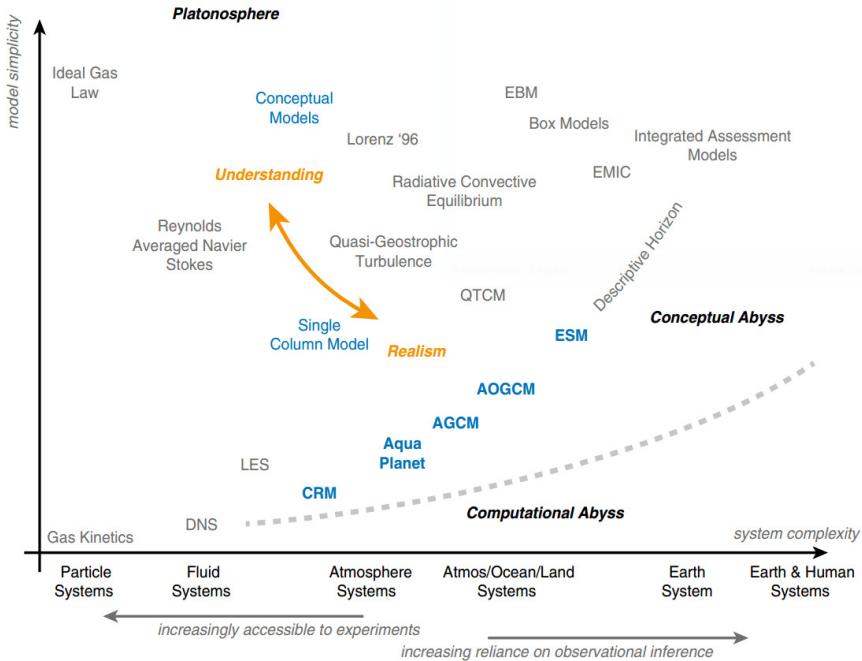
- 1391 Randall, D. A., Harshvardhan, D. A. Dazlich, and T. G. Corsetti (1989), Interactions among Ra-
 1392 diation, Convection, and Large-Scale Dynamics in a General Circulation Model., *Journal of the*
 1393 *Atmospheric Sciences*, 46, 1943–1970.
- 1394 Randel, W. J., and I. M. Held (1991), Phase speed spectra of transient eddy fluxes and critical layer
 1395 absorption, *Journal of the Atmospheric Sciences*, 48(5), 688–697.
- 1396 Ray, E. A., F. L. Moore, K. H. Rosenlof, S. M. Davis, H. Boenisch, O. Morgenstern, D. Smale,
 1397 E. Rozanov, M. Hegglin, G. Pitari, et al. (2010), Evidence for changes in stratospheric transport
 1398 and mixing over the past three decades based on multiple data sets and tropical leaky pipe analysis,
 1399 *Journal of Geophysical Research: Atmospheres*, 115(D21).
- 1400 Raymond, D. J. (2001), A New Model of the Madden–Julian Oscillation, *Journal of the Atmospheric*
 1401 *Sciences*, 58(18), 2807–2819.
- 1402 Raymond, D. J. (2007), Testing a cumulus parameterization with a cumulus ensemble model in
 1403 weak temperature gradient mode, *Quarterly Journal of the Royal Meteorological Society*, 133,
 1404 1073–1085.
- 1405 Raymond, D. J., and Z. Fuchs (2007), Convectively coupled gravity and moisture modes in a simple
 1406 atmospheric model, *Tellus A*, 59(5), 627–640.
- 1407 Raymond, D. J., and Z. Fuchs (2009), Moisture Modes and the Madden-Julian Oscillation, *Journal*
 1408 *of Climate*, 22(11), 3031–3046.
- 1409 Raymond, D. J., and X. Zeng (2005), Modelling tropical atmospheric convection in the context of the
 1410 weak temperature gradient approximation, *Quarterly Journal of the Royal Meteorological Society*,
 1411 131(608), 1301–1320.
- 1412 Reed, K. A., and D. R. Chavas (2015), Uniformly rotating global radiative-convective equilibrium in
 1413 the Community Atmosphere Model, version 5, *Journal of Advances in Modeling Earth Systems*,
 1414 7(4), 1938–1955.
- 1415 Reed, K. A., and B. Medeiros (2016), A reduced complexity framework to bridge the gap between
 1416 AGCMs and cloud-resolving models, *Geophysical Research Letters*, 43(2), 860–866.
- 1417 Ringer, M. A., T. Andrews, and M. J. Webb (2014), Global-mean radiative feedbacks and forcing
 1418 in atmosphere-only and coupled atmosphere-ocean climate change experiments, *Geophysical*
 1419 *Research Letters*, 41(11), 4035–4042, 2014GL060347.
- 1420 Rivière, G. (2009), Effect of latitudinal variations in low-level baroclinicity on eddy life cycles
 1421 and upper-tropospheric wave-breaking processes, *Journal of the Atmospheric Sciences*, 66(6),
 1422 1569–1592.
- 1423 Rivière, G., and I. Orlanski (2007), Characteristics of the Atlantic storm-track eddy activity and its
 1424 relation with the North Atlantic Oscillation, *Journal of the Atmospheric Sciences*, 64(2), 241–266.
- 1425 Robinson, W. A. (2000), A baroclinic mechanism for the eddy feedback on the zonal index, *Journal*
 1426 *of the Atmospheric Sciences*, 57(3), 415–422.
- 1427 Rodwell, M. J., and B. J. Hoskins (1996), Monsoons and the dynamics of deserts, *Quarterly Journal*
 1428 *of the Royal Meteorological Society*, 122(534), 1385–1404.
- 1429 Romps, D. M. (2012), Weak pressure gradient approximation and its analytical solutions, *Journal of*
 1430 *the Atmospheric Sciences*, 69, 2835–2845.
- 1431 Salmon, R. (1980), Baroclinic instability and geostrophic turbulence, *Geophysical & Astrophysical*
 1432 *Fluid Dynamics*, 15(1), 167–211.
- 1433 Satoh, M., K. Aramaki, and M. Sawada (2016), Structure of Tropical Convective Systems in Aqu-
 1434 Planet Experiments: Radiative-Convective Equilibrium Versus the Earth-Like Experiment, *SOLA*,
 1435 12, 220–224.
- 1436 Scherhag, R. (1952), Die explosionsartigen Stratosphenerwärmungen des Spätwinters, 1951–1952.,
 1437 *Berlin Deutscher Wetterdienst (U.S. Zone)*, 38, 51–63.
- 1438 Schneider, S. H., and R. E. Dickinson (1974), Climate modeling, *Reviews of Geophysics*, 12(3),
 1439 447–493.
- 1440 Schneider, T., and C. C. Walker (2006), Self-Organization of Atmospheric Macroturbulence into
 1441 Critical States of Weak Nonlinear Eddy–Eddy Interactions, *Journal of the Atmospheric Sciences*,
 1442 63, 1569–1586.
- 1443 Scott, R., and P. Haynes (2000), Internal vacillations in stratosphere-only models, *Journal of the*
 1444 *Atmospheric Sciences*, 57(19), 3233–3250.

- 1445 Scott, R., and L. M. Polvani (2006), Internal variability of the winter stratosphere. Part I: Time-
 1446 independent forcing, *Journal of the Atmospheric Sciences*, 63(11), 2758–2776.
- 1447 Sessions, S., S. Sugaya, D. J. Raymond, and A. H. Sobel (2010), Multiple equilibria in a cloud-
 1448 resolving model, *J. Geophys. Res.*, 115, D12,110, doi:10.1029/2009JD013,376.
- 1449 Sessions, S., S. Sentic, and M. J. Herman (2016), The role of radiation in organizing convection
 1450 in weak temperature gradient simulations, *Journal of Advances in Modeling Earth Systems*, 8,
 1451 244–271.
- 1452 Shaw, T. A. (2014), On the Role of Planetary-Scale Waves in the Abrupt Seasonal Transition of the
 1453 Northern Hemisphere General Circulation, *Journal of the Atmospheric Sciences*, 71(5), 1724–
 1454 1746.
- 1455 Sheshadri, A., R. A. Plumb, and E. P. Gerber (2015), Seasonal variability of the polar stratospheric
 1456 vortex in an idealized AGCM with varying tropospheric wave forcing, *Journal of the Atmospheric
 1457 Sciences*, 72(6), 2248–2266.
- 1458 Shi, X., and C. S. Bretherton (2014), Large-scale character of an atmosphere in rotating radiative-
 1459 convective equilibrium, *Journal of Advances in Modeling Earth Systems*, 6, 616–629.
- 1460 Sigmond, M., J. F. Scinocca, V. V. Kharin, and T. G. Shepherd (2013), Enhanced seasonal forecast
 1461 skill following stratospheric sudden warmings, *Nature Geoscience*.
- 1462 Silvers, L. G., B. Stevens, T. Mauritsen, and M. Giorgetta (2016), Radiative convective equilibrium
 1463 as a framework for studying the interaction between convection and its large-scale environment,
 1464 *Journal of Advances in Modeling Earth Systems*, 8(3), 1330–1344.
- 1465 Simmons, A. J., and B. J. Hoskins (1978), The life cycles of some nonlinear baroclinic waves,
 1466 *Journal of the Atmospheric Sciences*, 35(3), 414–432.
- 1467 Simmons, A. J., and B. J. Hoskins (1980), Barotropic influences on the growth and decay of nonlinear
 1468 baroclinic waves, *Journal of the Atmospheric Sciences*, 37(8), 1679–1684.
- 1469 Simpson, I. R., P. Hitchcock, T. G. Shepherd, and J. F. Scinocca (2013), Southern annular mode
 1470 dynamics in observations and models. Part I: The influence of climatological zonal wind biases
 1471 in a comprehensive GCM, *Journal of Climate*, 26(11), 3953–3967.
- 1472 Sjoberg, J. P., and T. Birner (2014), Stratospheric wave–mean flow feedbacks and sudden stratospheric
 1473 warmings in a simple model forced by upward wave activity flux, *Journal of the Atmospheric
 1474 Sciences*, 71(11), 4055–4071.
- 1475 Slingo, A., and J. M. Slingo (1988), The response of a general-circulation model to cloud longwave
 1476 radiative forcing. Part I: Introduction and initial experiments, *Quarterly Journal of the Royal
 1477 Meteorological Society*, 114, 1027–1062.
- 1478 Sobel, A. H., and C. S. Bretherton (2000), Modeling Tropical Precipitation in a Single Column,
 1479 *Journal of Climate*, 13(24), 4378–4392.
- 1480 Sobel, A. H., and E. D. Maloney (2012), An idealized semi-empirical framework for modeling the
 1481 Madden-Julian oscillation, *Journal of the Atmospheric Sciences*, 69, 1691–1705.
- 1482 Sobel, A. H., and E. D. Maloney (2013), Moisture modes and the eastward propagation of the MJO,
 1483 *Journal of the Atmospheric Sciences*, 70, 187–192.
- 1484 Sobel, A. H., J. Nilsson, and L. M. Polvani (2001), The weak temperature gradient approximation
 1485 and balanced tropical moisture waves, *Journal of the Atmospheric Sciences*, 58, 3650–3665.
- 1486 Sobel, A. H., G. Bellon, and J. Bacmeister (2007), Multiple equilibria in a single-column model of
 1487 the tropical atmosphere, *Geophysical Research Letters*, 34.
- 1488 Stephens, G. L., T. L'Ecuyer, R. Forbes, A. Gettelman, J.-C. Golaz, A. Bodas-Salcedo, K. Suzuki,
 1489 P. Gabriel, and J. Haynes (2010), Dreary state of precipitation in global models, *Journal of
 1490 Geophysical Research: Atmospheres*, 115(D24).
- 1491 Stevens, B., S. Bony, and M. Webb (2012), Clouds On-off Climate Intercomparison Experiment
 1492 (Cookie), <http://www.euclipse.eu/downloads/Cookie.pdf>.
- 1493 Stevens, B., S. C. Sherwood, S. Bony, and M. J. Webb (2016), Prospects for narrowing bounds on
 1494 earth's equilibrium climate sensitivity, *Earth's Future*, 4(11), 512–522.
- 1495 Sun, Y., S. Solomon, A. Dai, and R. Portmann (2006), How Often Does It Rain?, *Journal of Climate*,
 1496 19, 916–934.
- 1497 Taguchi, M., and S. Yoden (2002), Internal interannual variability of the troposphere–stratosphere
 1498 coupled system in a simple global circulation model. Part I: Parameter sweep experiment, *Journal*

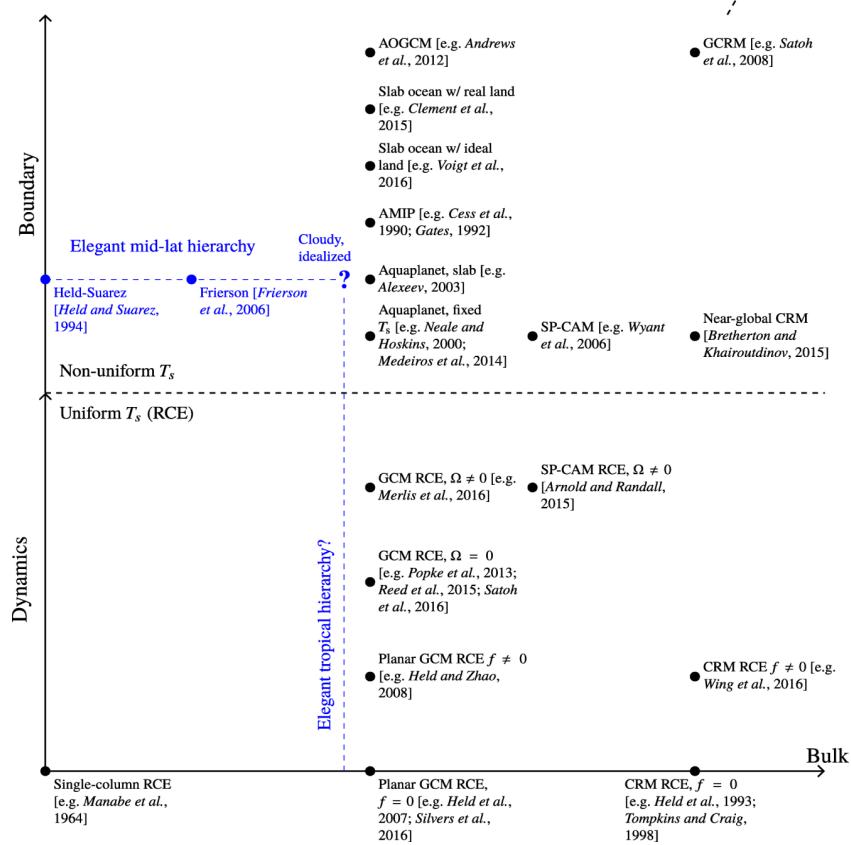
- 1499 *of the Atmospheric Sciences*, 59(21), 3021–3036.
- 1500 Taguchi, M., T. Yamaga, and S. Yoden (2001), Internal variability of the troposphere–stratosphere
1501 coupled system simulated in a simple global circulation model, *Journal of the Atmospheric
1502 Sciences*, 58(21), 3184–3203.
- 1503 Thompson, D. W., and J. M. Wallace (2000), Annular modes in the extratropical circulation. Part I:
1504 Month-to-month variability, *Journal of Climate*, 13(5), 1000–1016.
- 1505 Thorncroft, C., B. Hoskins, and M. McIntyre (1993), Two paradigms of baroclinic-wave life-cycle
1506 behaviour, *Quarterly Journal of the Royal Meteorological Society*, 119(509), 17–55.
- 1507 Tokioka, T., K. Yamazaki, A. Kitoh, and T. Ose (1988), The Equatorial 30–60 day Oscillation and the
1508 Arakawa-Schubert Penetrative Cumulus Parameterization, *Journal of the Meteorological Society
1509 of Japan. Series II*, 66(6), 883–901.
- 1510 Tomita, H., H. Miura, S. Iga, T. Nasuno, and M. Satoh (2005), A global cloud-resolving simulation:
1511 Preliminary results from an aqua planet experiment, *Geophysical Research Letters*, 32(8).
- 1512 Vallis, G. K. (2016), Geophysical fluid dynamics: whence, whither and why?, *Proceedings of
1513 the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 472(2192),
1514 doi:10.1098/rspa.2016.0140.
- 1515 Vallis, G. K. (2017), *Atmospheric and oceanic fluid dynamics*, Cambridge University Press.
- 1516 Vallis, G. K., E. P. Gerber, P. J. Kushner, and B. A. Cash (2004), A mechanism and simple dynamical
1517 model of the North Atlantic Oscillation and annular modes, *Journal of the Atmospheric Sciences*,
1518 61(3), 264–280.
- 1519 Vallis, G. K., G. Colyer, R. Geen, E. Gerber, M. Jucker, P. Maher, A. Paterson, M. Pietschnig, J. Penn,
1520 and S. I. Thomson (2018), Isca, v1.0: a framework for the global modelling of the atmospheres of
1521 Earth and other planets at varying levels of complexity, *Geoscientific Model Development*, 11(3),
1522 843–859.
- 1523 Voigt, A., and T. A. Shaw (2015), Circulation response to warming shaped by radiative changes of
1524 clouds and water vapour, *Nature Geoscience*, 8, 102–.
- 1525 Voigt, A., and T. A. Shaw (2016), Impact of regional atmospheric cloud radiative changes on shifts of
1526 the extratropical jet stream in response to global warming, *Journal of Climate*, 29(23), 8399–8421.
- 1527 Walker, C. C., and T. Schneider (2006), Eddy influences on Hadley circulations: Simulations with
1528 an idealized GCM, *Journal of the Atmospheric Sciences*, 63, 3333–3350.
- 1529 Wang, B. (2005), Chapter 10: Theories, in *Intraseasonal Variability in the Atmosphere-Ocean
1530 Climate System*, edited by W. K. M. Lau and D. E. Waliser, pp. 307–360, Praxis Publishing.
- 1531 Wang, S., A. H. Sobel, and Z. Kuang (2013), Cloud-resolving simulation of TOGA-COARE using
1532 parameterized large-scale dynamics, *Journal of Geophysical Research: Atmospheres*, 118(12),
1533 6290–6301.
- 1534 Wang, S., A. H. Sobel, and J. Nie (2016), Modeling the MJO in a cloud-resolving model with
1535 parameterized large-scale dynamics: vertical structure, radiation, and horizontal advection of dry
1536 air, *Journal of Advances in Modeling Earth Systems*, 8, 121–139.
- 1537 Wing, A. A., K. Emanuel, C. E. Holloway, and C. Muller (2017a), Convective Self-Aggregation in
1538 Numerical Simulations: A Review, *Surveys in Geophysics*, pp. 1–25.
- 1539 Wing, A. A., K. A. Reed, M. Satoh, B. Stevens, S. Bony, and T. Ohno (2017b), Radiative-Convective
1540 Equilibrium Model Intercomparison Project, *Geoscientific Model Development Discussions*, 2017,
1541 1–34.
- 1542 Xia, X., and E. K. Chang (2014), Diabatic damping of zonal index variations, *Journal of the
1543 Atmospheric Sciences*, 71(8), 3090–3105.
- 1544 Yang, D., and A. P. Ingersoll (2013), Triggered Convection, Gravity Waves, and the MJO: A Shallow-
1545 Water Model, *Journal of the Atmospheric Sciences*, 79, 2476–2486.
- 1546 Yoo, C., and S.-W. Son (2016), Modulation of the boreal wintertime Madden-Julian oscillation by
1547 the stratospheric quasi-biennial oscillation, *Geophysical Research Letters*, 43(3), 1392–1398.
- 1548 Yuval, J., and Y. Kaspi (2016), Eddy activity sensitivity to changes in the vertical structure of
1549 baroclinicity, *Journal of the Atmospheric Sciences*, 73(4), 1709–1726.
- 1550 Zelinka, M. D., and D. L. Hartmann (2010), Why is longwave cloud feedback positive?, *Journal of
1551 Geophysical Research: Atmospheres*, 115(D16), D16,117.

- 1552 Zhang, M., C. S. Bretherton, P. N. Blossey, P. H. Austin, J. T. Bacmeister, S. Bony, F. Bréint, S. K.
1553 Cheedela, A. Cheng, A. D. Del Genio, S. R. De Roode, S. Endo, C. N. Franklin, J.-C. Golaz,
1554 C. Hannay, T. Heus, F. A. Isotta, J.-L. Dufresne, I.-S. Kang, H. Kawai, M. Köhler, V. E. Larson,
1555 Y. Liu, A. P. Lock, U. Lohmann, M. F. Khairoutdinov, A. M. Molod, R. A. J. Neggers, P. Rasch,
1556 I. Sandu, R. Senkbeil, A. P. Siebesma, C. Siegenthaler-Le Drian, B. Stevens, M. J. Suarez, K.-M.
1557 Xu, K. von Salzen, M. J. Webb, A. Wolf, and M. Zhao (2013), CGILS: Results from the first
1558 phase of an international project to understand the physical mechanisms of low cloud feedbacks
1559 in single column models, *Journal of Advances in Modeling Earth Systems*, 5(4), 826–842.
- 1560 Zhang, R., S. M. Kang, and I. M. Held (2010), Sensitivity of climate change induced by the weakening
1561 of the Atlantic meridional overturning circulation to cloud feedback, *Journal of Climate*, 23, 378–
1562 389.
- 1563 Zhu, H., and A. H. Sobel (2012), Comparison of a single-column model in weak temperature
1564 gradient mode to its parent AGCM., *Quarterly Journal of the Royal Meteorological Society*, 138,
1565 1025–1034.
- 1566 Zurita-Gotor, P. (2007), The relation between baroclinic adjustment and turbulent diffusion in the
1567 two-layer model, *Journal of the Atmospheric Sciences*, 64(4), 1284–1300.
- 1568 Zurita-Gotor, P., and G. K. Vallis (2009), Equilibration of baroclinic turbulence in primitive equations
1569 and quasigeostrophic models, *Journal of the Atmospheric Sciences*, 66(4), 837–863.
- 1570 Zurita-Gotor, P., J. Blanco-Fuentes, and E. P. Gerber (2014), The impact of baroclinic eddy feedback
1571 on the persistence of jet variability in the two-layer model, *Journal of the Atmospheric Sciences*,
1572 71(1), 410–429.

a) Bony et al. 2013



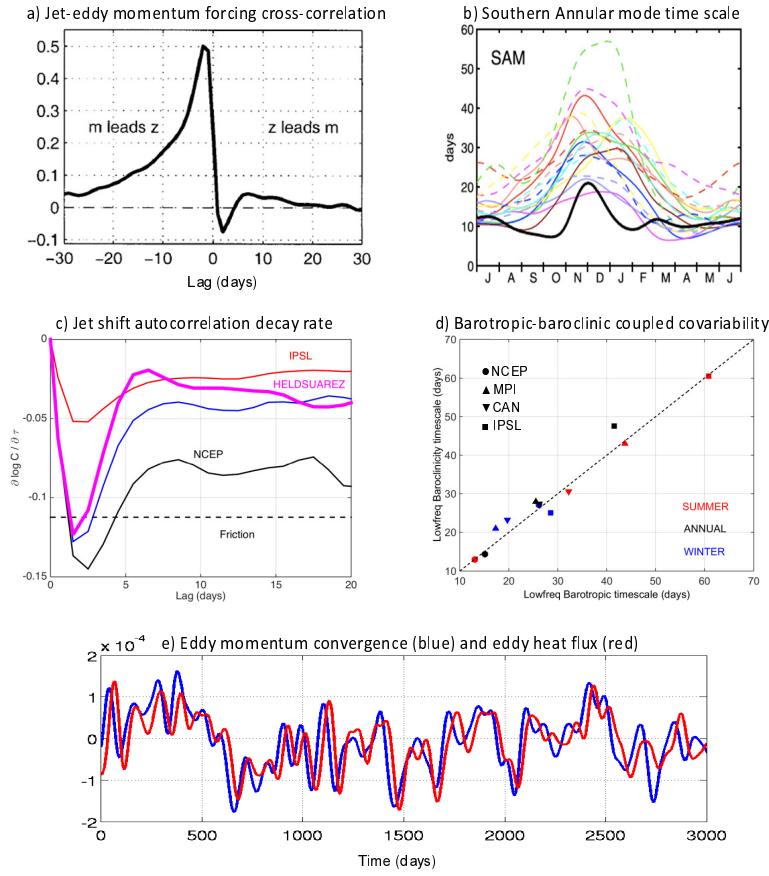
b) Jeevanjee et al. 2017



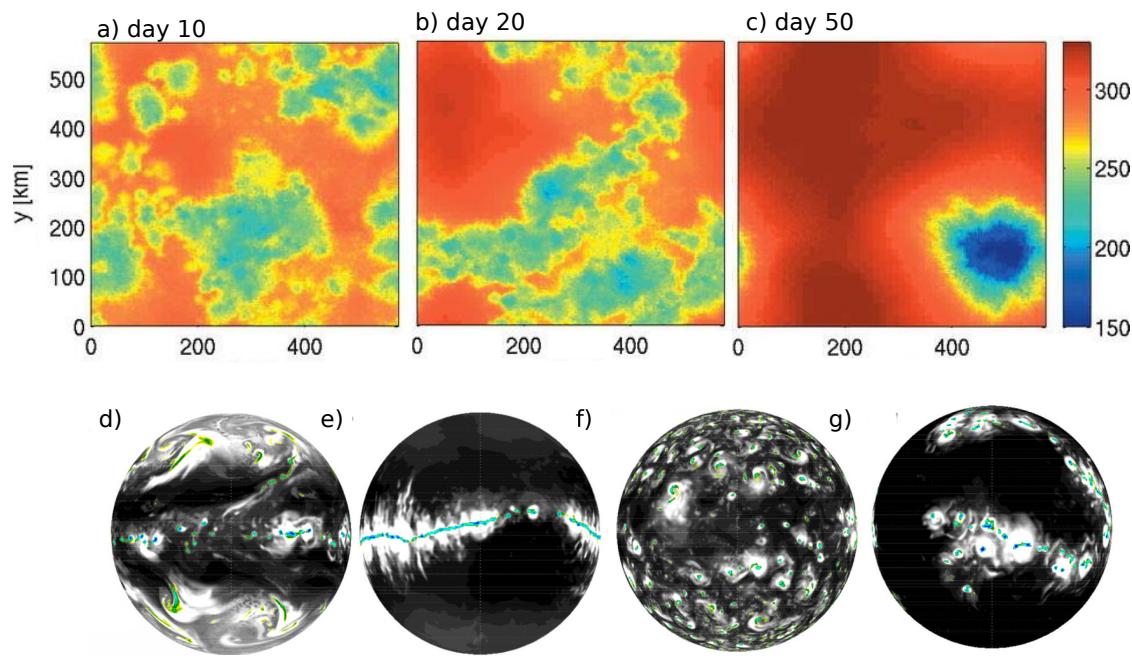
1573

Figure 1. Categorizing atmospheric climate models in terms of complexity a) from Bony et al. [2013] and b) grouped in terms of model configurations for Earth's climate as in Jeevanjee et al. [2017].

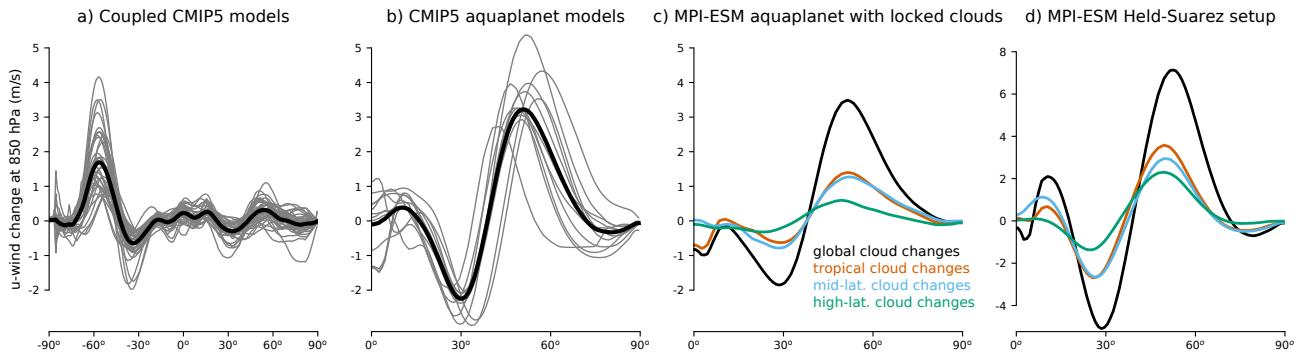
1574



1575 **Figure 2.** (a) Lagged correlation between zonal mean wind (z) and eddy momentum forcing (m) from
 1576 *Lorenz and Hartmann [2001]*. (b) Autocorrelation timescale of the Southern Annular Mode for observations
 1577 (black thick solid) and CMIP3 models (colors), adapted from *Gerber et al. [2008]*. (c) Logarithmic decay rate
 1578 of autocorrelation for zonal wind anomalies in observations (black), two CMIP5 climate models (IPSL: red,
 1579 CAN:blue) and the Held and Suarez model (magenta). (d) Scatterplot between low-frequency logarithmic decay
 1580 rates of baroclinicity and barotropic wind anomalies (average from 5-20 day lags) for the models and seasons
 1581 indicated. (e) Sample timeseries of the low-frequency eddy momentum (blue) and heat (red) flux contributions
 1582 to the upper-layer Eliassen-Palm divergence in the QG simulations of *Zurita-Gotor et al. [2014]*



1583 **Figure 3.** Radiative-Convective Equilibrium simulations in a CRM: top panel is daily OLR for a fixed SST
 1584 (301K) run after a) 10, b) 20 and c) 50 days of the simulation, adapted from Bretherton *et al.* [2005]. The
 1585 bottom panel is OLR for high resolution GCM aquaplanet stimulations using zonally symmetric SSTs similar to
 1586 observation d) with rotation (Earth like), e) without rotation, and uniform SSTs f) with and g) without rotation
 1587 (RCE case), adapted from Satoh *et al.* [2016].



1588 **Figure 4.** Extratropical cloud-circulation coupling. The impact of clouds on the eddy driven jet stream
1589 response to global warming in a hierarchy of GCMs. The zonal-mean time-mean change in 850 hPa zonal
1590 wind (ms^{-1}) for each latitude ($^{\circ}$) for the ensemble mean (bold line) and individual models (gray) for a) CMIP5
1591 coupled Earth system models with $4 \times \text{CO}_2$ and b) aquaplanet CMIP5 models with prescribed-SST and 4 K SST
1592 warming. For the MPI-ESM model in aquaplanet prescribed-SST setup, simulations with the cloud-locking
1593 method and imposed global (black) and regional (colors) cloud changes show the cloud-radiative contribution
1594 to the eddy driven jet response to warming (panel c). The global and regional cloud impacts are reproduced in
1595 panel d) using a dry Held-Suarez setup of the MPI-ESM model perturbed with the radiative forcing from cloud
1596 changes of panel c. Because panels b-d are for aquaplanet simulations, only the Northern hemisphere is shown.
1597 Note the different y-scale in panel d, which reflects the increased jet sensitivity of the Held-Suarez setup. Figure
1598 adapted from Voigt and Shaw [2016].

dynamical (equation)

barotropic
quasi-geostrophic
dry primitive equations
moist primitive equations
non-hydrostatic



process (diabatic)

dry dynamical core
moist model gray radiation
full radiation without clouds
prescribed clouds and aerosols
interactive chemistry



process (boundary condition)

constant SST
slab ocean
slab ocean with q-fluxes
observed SST with land (AMIP)
coupled ocean-atmosphere with land

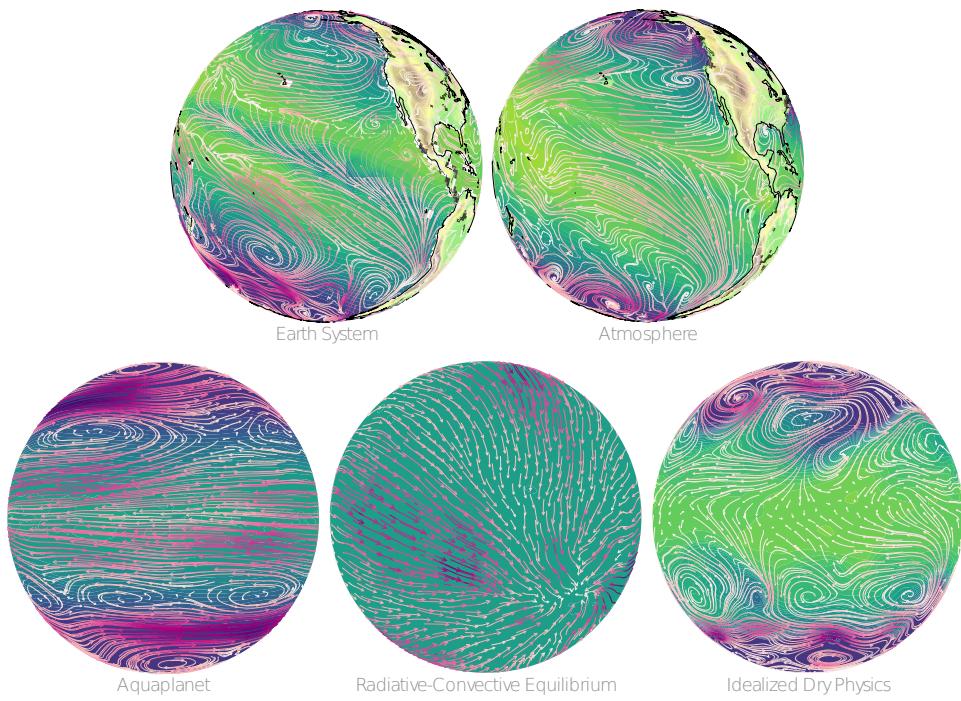
Hierarchies for atmospheric circulation

scale (convective organization)

turbulence
cloud scale
cloud system
mesoscale convective systems
synoptic to planetary scale

Maher et al. 2019

1599 **Figure 5.** The three principles view of the model hierarchies used for understanding the large-scale circulation.
1600 The dynamical hierarchy is shown in terms of the equation hierarchy. The process hierarchy is described in
1601 terms of a diabatic hierarchy and the boundary conditions. Convective organization is used as an example to
1602 illustrate the scale hierarchy. For each list, the first element is the simplest or smallest-scale and builds down to
1603 the most complex or largest-scale. This is an example to illustrate the concept of the three principles and does
1604 not capture all the available models.



1605 **Figure 6.** Models available within the hierarchy in the CESM system. (Top) The Earth system model and
1606 atmosphere only models (with prescribed SST). (bottom) Aquaplanet, RCE and idealized dry physics. The
1607 colour contours over the ocean are SST and over land topography. Streamlines are the near-surface wind
1608 (thicker lines are stronger winds). Each globe is a monthly mean except for the idealized dry model which is a
1609 snapshot.