

Drawing the Parabolic Trajectory of an Object under Gravity

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Abstract

When you throw an object into the air, its trajectory is a parabola. How do its instantaneous velocity and the vertical and horizontal components of the instantaneous velocity change over the course of its trajectory? What happens when the initial speed is larger? Smaller? What happens if the gravity were larger? Smaller? Zero? Negative? Holding gravity and the initial speed constant, in which direction can you throw to make the object reach the farthest point along the horizon?

1 Introduction

In this article we will show you how to construct a model of a gravity sketch. The reader is assumed to be familiar with the basics of Dr. Geo such as drawing parallel lines and changing the colors/names/styles/... of the points and lines.

This sketch created using Dr. Geo can help a high school physics teacher explore all these questions with her/his students. Figure 1 illustrates the constructed sketch after reading this article.

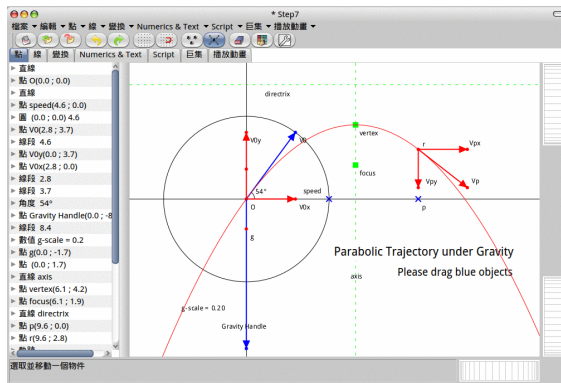


Figure 1. The final gravity sketch.

2 The initial plot

- Enable axes and magnetic grid in edit menu.
- Create points H_a and H_b far out along the x-axis, say (-40,0) and (40,0). Create the straight line passing H_a and H_b as the horizon. Hide and lock H_a and H_b .
- Create a circle centered at the origin O and passing a point *Speed* along the x-axis. Lock O .
- Create a straight line passing O perpendicular to line (H_aH_b) . This is just the y-axis.
- Disable axes and magnetic grid in edit menu.

- f) Create a line segment from O to a free point V_0 on the circle. This is the initial velocity vector.
- g) From V_0 draw lines perpendicular or horizontal to the axes and intersect the x-axis/y-axis at V_{ox}/V_{oy} . Hide the two auxiliary lines. Draw line segments $[OV_{ox}]$ and $[OV_{oy}]$. These are the initial velocities along the two axes.

We also need the gravity vector. But later you would find that it's difficult to drag the gravity vector directly because it's too close to the origin (if we want to see the interesting portion of the parabola). So we create a point *Gravity Handle* along the negative y-axis at a comfortable distance, and create the point g as the end point of the real gravity vector at roughly 0.2 or 0.25 times distance. To do this you need to create a number *g-scale*, set it to 0.2 or 0.25, and use the Homothety transform (i.e. homogeneous dilation) to scale *Gravity Handle* down towards O . The figure 2 bellow illustrates these constructions.

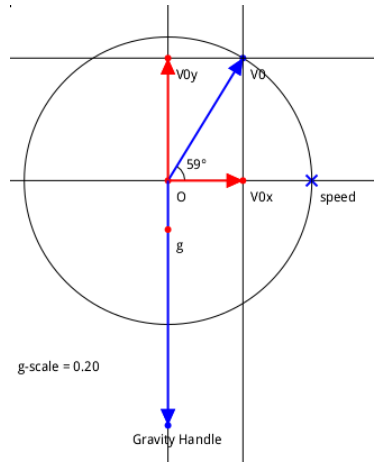


Figure 2. The initial velocity and the gravity vectors.

Download the resulting construction of this step: [step2.fgeo](#).

3 The focus and the directrix of the parabola

To draw the parabola, we need to find its focus and directrix. Obviously the axis of the parabola is the vertical line at $d = V_{ox} \times t_x$ distance away from the y-axis where t_x is the time for the vertical component of the velocity to reach zero. Therefore $\frac{d}{V_{ox}} = \frac{V_{oy}}{-g}$.¹

At time t_x the object also reaches the highest point of its trajectory at the height of:

$$h_x = -\frac{1}{2}g \times t_x^2 = \frac{d \times V_{oy}}{V_{ox} \times 2}, \quad \text{therefore} \quad \frac{2 \times h_x}{V_{oy}} = \frac{d}{V_{ox}}.$$

- a) Use mirror or symmetry to find the point $-g$ on the positive y-axis.
- b) Draw straight line $(-gV_{ox})$. Draw a straight line parallel to $(-gV_{ox})$ and passing V_{oy} . Intersect the latter line with the x-axis at point d . Draw x-axis's perpendicular line at d . This is the *axis* of symmetry of the parabola.
- c) Draw straight line $(V_{ox}V_{oy})$. Draw a straight line parallel to $(V_{ox}V_{oy})$ and passing d . Intersect the latter line with the y-axis at point $2h$. Find the midpoint of $2h$ and O as h .

¹. $V_y(t) = g \times t + V_{oy}$ as $V_y(t_x) = 0$ therefore $t_x = \frac{V_{oy}}{-g}$

Download the resulting construction of this step: [step4.fgeo](#).

4 The parabola itself

Now we can draw the parabola. Recall that it is the locus of all points that are equally distant from the focus as from the directrix.

- Draw a free point p on the x-axis. This point will provide user interaction to control the position of the instantaneous velocity drawing.
- Draw the line vertical to the x-axis and passing p , then intersect the line with the directrix at point q .
- Draw line segment $[Focus\ q]$. Find its midpoint m and draw the straight line (T) perpendicular to the line segment. Intersect (T) with (pq) r .
- Use the locus tool in line menu to draw the parabola, using p as the free (independent) point and r as the relative (dependent) point.
- Hide the auxiliary lines and points except line (T) .

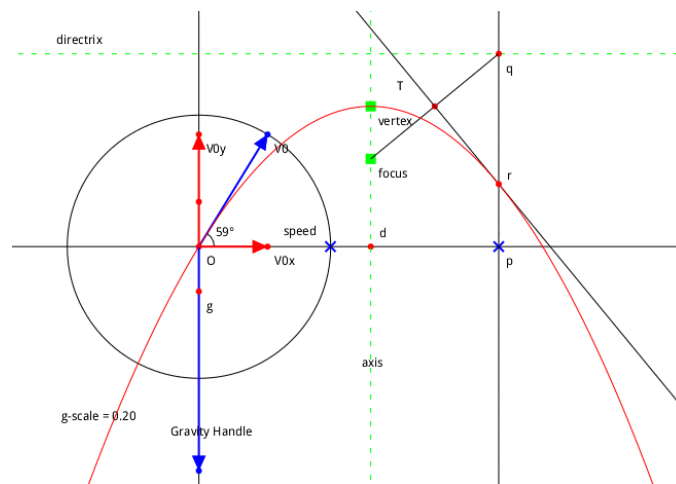


Figure 5. Drawing the parabola using the locus tool.

Download the resulting construction of this step: [step5.fgeo](#).

5 The instantaneous velocity

Finally we will draw the instantaneous velocity at point r .

- Draw vector \vec{Or} . Use it to translate the point V_{ox} to V_{px} . Draw line segment $[rV_{px}]$. This horizontal component of the instantaneous velocity remains constant at all times.
- The instantaneous velocity lies on the tangent line (T). So from V_{px} draw a vertical line to intersect with (T) at V_p . This is the end point of the instantaneous velocity.
- From V_p draw a horizontal line to intersect with straight line (pr) at V_{py} . This is the end point of the vertical component of the instantaneous velocity.

d) Clean up the drawing and add some textual explanations.

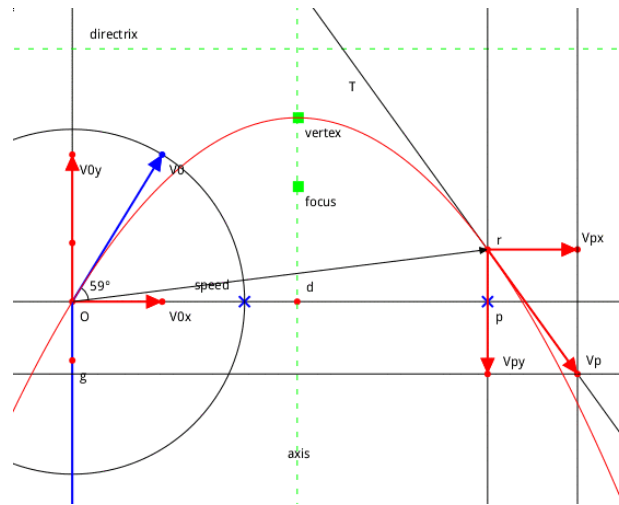


Figure 6. The instantaneous velocity V_p and its components V_{px} and V_{py} .

Download the resulting construction of this step: [step7.fgeo](#).

6 Conclusion

My first attempt created a sketch that became broken when the initial velocity moves out of the first quadrant ($x \geq 0, y \geq 0$). I guess it was because at some step an absolute value (length) was taken. It's probably better to think in terms of make use of parallel lines instead of sign-less lengths when you construct a sketch.

In fact Dr. Geo has very powerful smalltalk scripting capabilities. However, drawing this sketch using only the basic drawing primitives makes it so... classic and so much more fun. I also learned one or two properties about parabola that I never knew as a high school kid (or as an adult).