

# Network analysis of collective motion



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Complex Networks Winter Workshop

# Collective behavior in complex systems



Calcium imaging (BJK)

Caters Clips (youtube)

COLLMOT (youtube)

# Collective behavior in network science?



Calcium imaging (BJK)



Caters Clips (youtube)



COLLMOT (youtube)

# Question

Collective behavior and collective motion is ubiquitous in the study of complex systems.

Is it a *network science* problem?

How would we know?

# Question

Networks! But still more to do with the analysis...

J Stat Phys (2013) 153:270–288  
DOI 10.1007/s10955-013-0827-4

## Complex Network Structure of Flocks in the Standard Vicsek Model

Gabriel Baglietto · Ezequiel V. Albano · Julián Candia



Revealing the hidden networks of interaction in mobile animal groups allows prediction of complex behavioral contagion

Sara Brin Rosenthal<sup>a,1</sup>, Colin R. Twomey<sup>b,1</sup>, Andrew T. Hartnett<sup>a</sup>, Hai Shan Wu<sup>b</sup>, and Iain D. Couzin<sup>b,c,d,2</sup>

Departments of <sup>a</sup>Physics and <sup>b</sup>Ecology and Evolutionary Biology, Princeton University, Princeton, NJ 08544; <sup>c</sup>Department of Collective Behaviour, Max Planck Institute for Ornithology, D-78547 Konstanz, Germany; and <sup>d</sup>Chair of Biodiversity and Collective Behavior, Department of Biology, University of Konstanz, D-78547 Konstanz, Germany

Notworks! ?!??



## Statistical mechanics for natural flocks of birds

William Bialek<sup>a</sup>, Andrea Cavagna<sup>b,c</sup>, Irene Giardina<sup>b,c,1</sup>, Thierry Mora<sup>d</sup>, Edmondo Silvestri<sup>b,c</sup>, Massimiliano Viale<sup>b,c</sup>, and Aleksandra M. Walczak<sup>e</sup>

<sup>a</sup>Joseph Henry Laboratories of Physics and Lewis—Sigler Institute for Integrative Genomics, Princeton University, Princeton, NJ 08544; <sup>b</sup>Istituto Complessi, Consiglio Nazionale delle Ricerche, Rome, Italy; <sup>c</sup>Dipartimento di Fisica, Università Sapienza, Rome, Italy; <sup>d</sup>Laboratoire de Physique de l’École Normale Supérieure, Centre National de la Recherche Scientifique and University Paris VI, Paris, France; and <sup>e</sup>Laboratoire de Physique Théorique de l’École Normale Supérieure, Centre National de la Recherche Scientifique and University Paris VI, Paris, France

Edited by Boris I. Shraiman, University of California, Santa Barbara, CA, and approved January 28, 2012 (received for review November 18, 2011)

Flocking is a typical example of emergent collective behavior, where interactions between individuals produce collective patterns on the large scale. Here we show how a quantitative microscopic theory for directional ordering in a flock can be derived directly from field data. We construct the minimally structured

experimental data. This distribution is the one with maximum entropy (10).

It should be emphasized that the maximum entropy distribution is not a “modeling assumption;” rather it is the most general assumption. Any other model that accounts for the

In contrast to most networks, the connectivity in a flock of birds is intrinsically dynamic—birds move and change their neighbors. Thus, it may not make sense to talk about a matrix of correlations  $C_{ij}$  or interactions  $J_{ij}$  between labeled individuals. On the other hand, the continuous and rapid change of neighbors induced by motion implies that the interaction  $J_{ij}$  between bird  $i$  and bird  $j$  cannot depend directly on their specific identities but only on some function of their relative positions.

# Further motivation

- Is there a physical (i.e. Hamiltonian) formalism that can capture canonical models of collective behavior?
  - (similar to Topaz et al. (2015), but with TDA)

# Approach

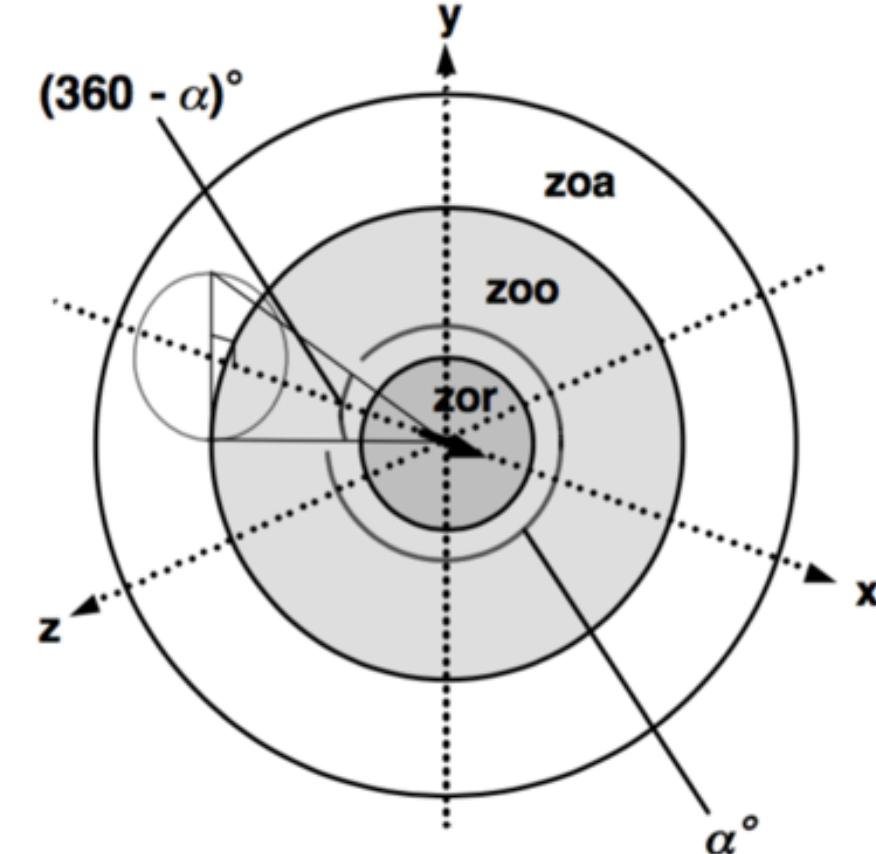
Study networks induced from simulated collective behavior

- Different parameterizations of the generative model
- Different ways of comparing networks over time
- Different ways of generating the adjacency matrix

# The "Couzin model" //

Collective Memory and Spatial Sorting in Animal Groups

IAIN D. COUZIN<sup>\*†</sup>, JENS KRAUSE<sup>†</sup>, RICHARD JAMES<sup>‡</sup>, GRAEME D. RUXTON<sup>§</sup>  
AND NIGEL R. FRANKS<sup>¶</sup>



<sup>†</sup>Centre for Biodiversity and Conservation, School of Biology, University of Leeds, Leeds LS2 9JT, U.K., <sup>‡</sup>Department of Physics, University of Bath, Bath BA2 7AY, U.K., <sup>§</sup>Division of Environmental and Evolutionary Biology, Graham Kerr Building, University of Glasgow, Glasgow G12 8QQ, U.K. and

<sup>¶</sup>Centre for Behavioural Biology, School of Biological Sciences, University of Bristol, Bristol BS8 1UG, U.K.

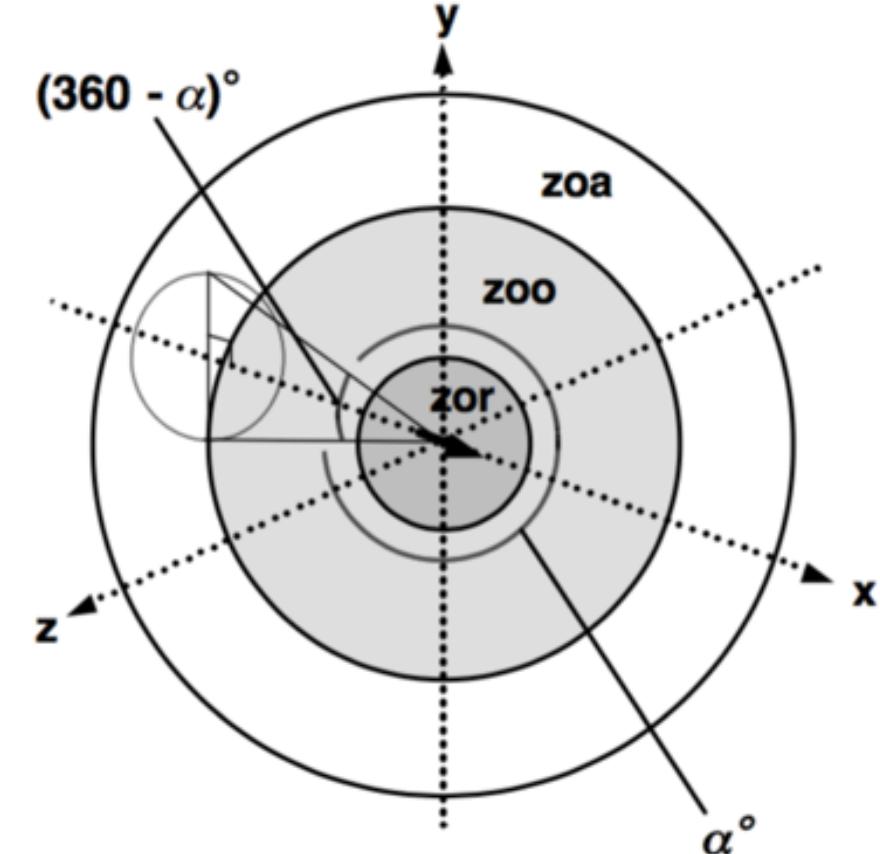
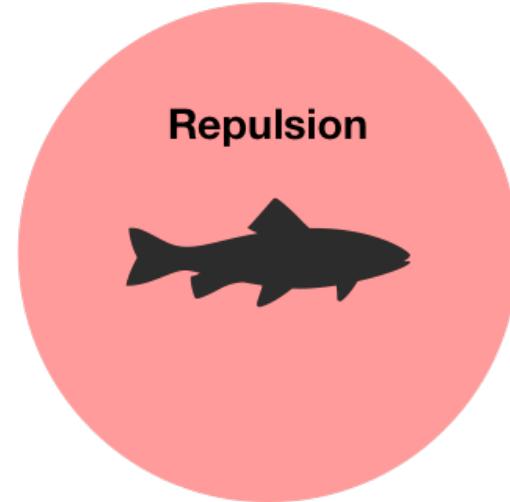
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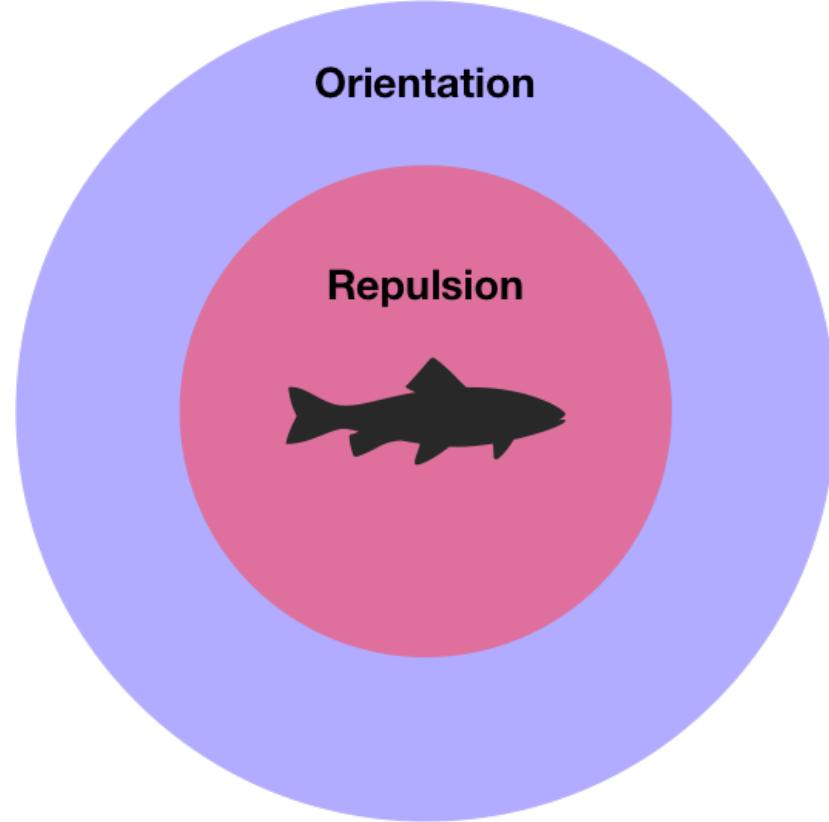
<sup>¶</sup>Centre for Behavioural Biology, School of Biological Sciences, University of Bristol, Bristol BS8 1UG, U.K.



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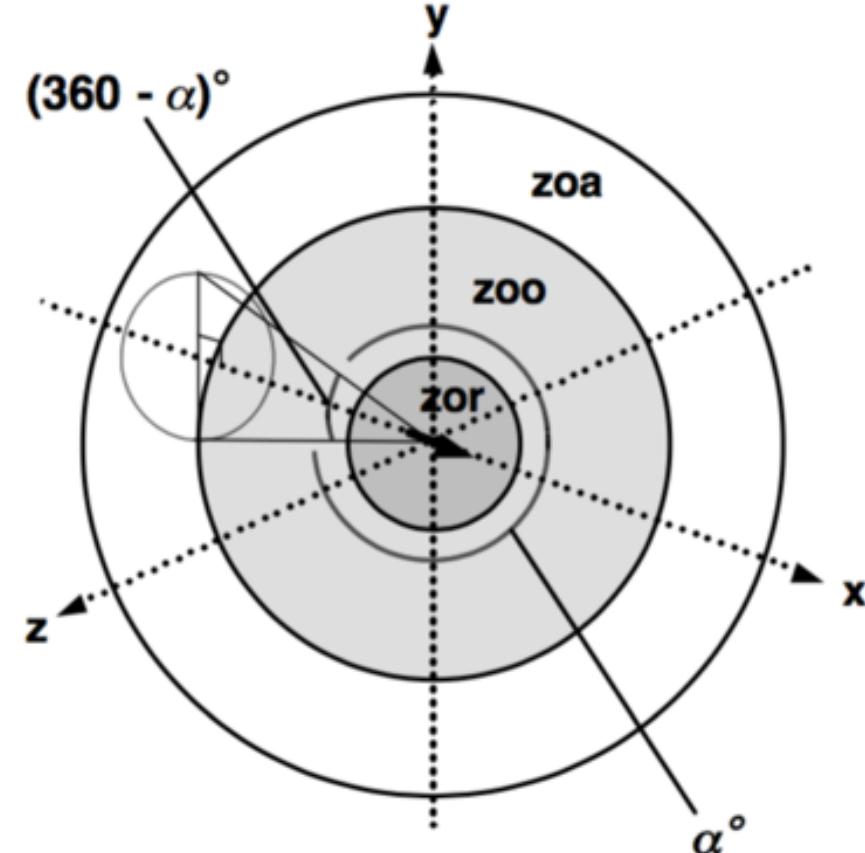
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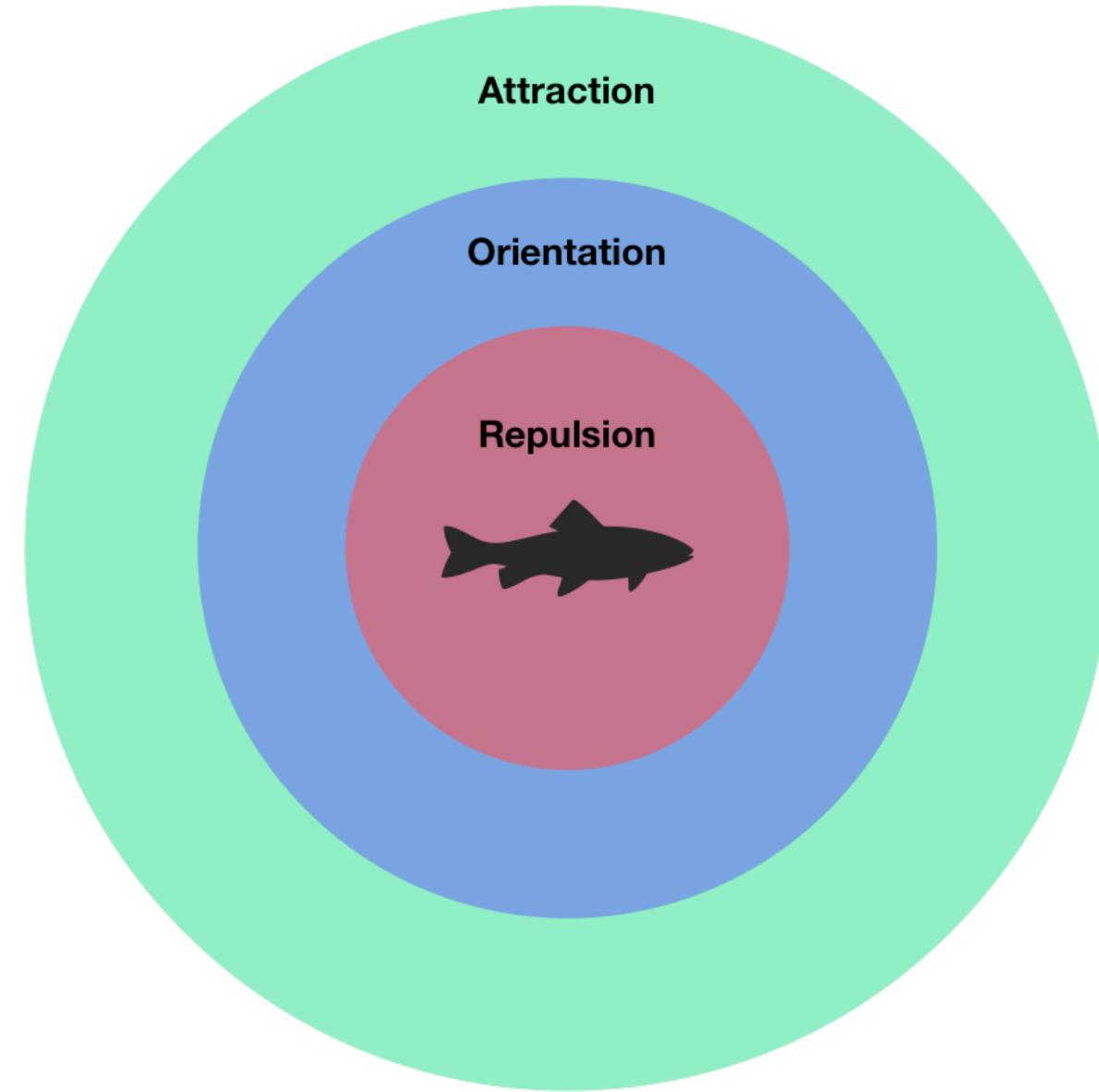


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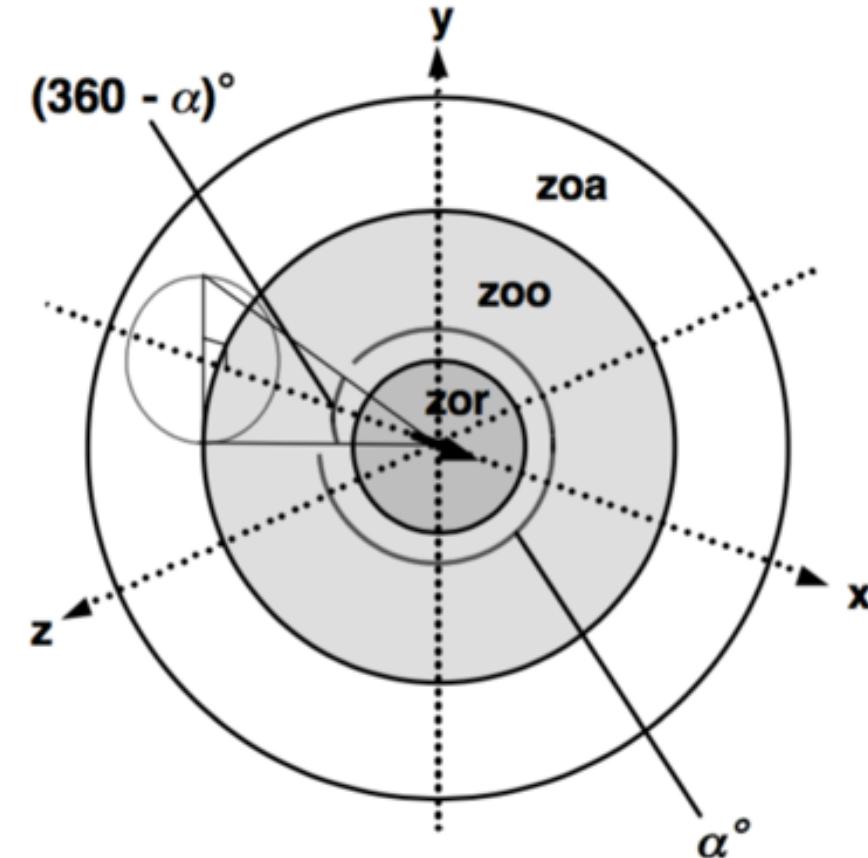


## Collective Memory and Spatial Sorting in Animal Groups

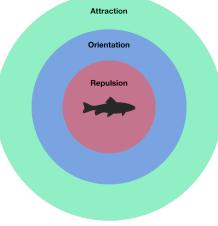
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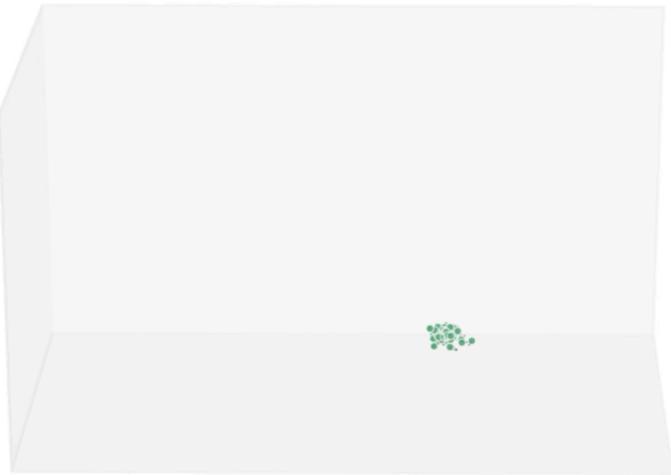
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# Four regimes of collective behavior



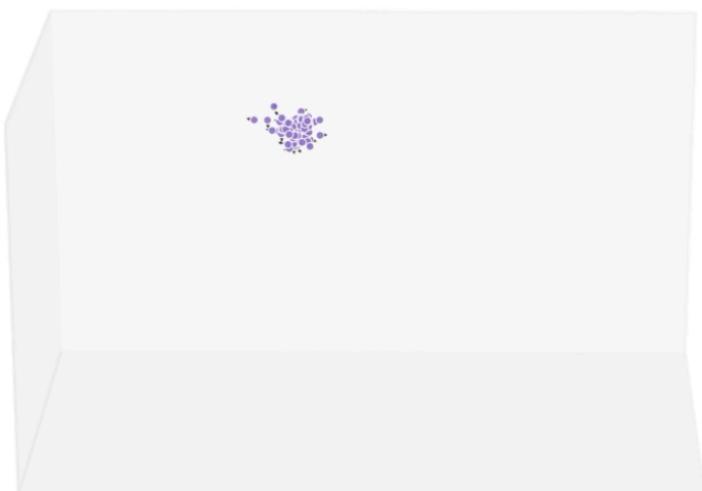
Highly parallel group



Milling / torus



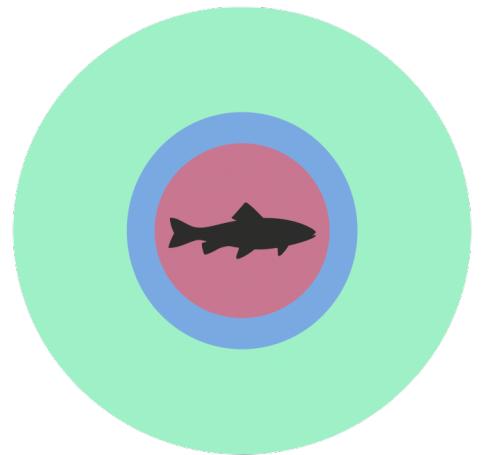
Dynamic parallel group



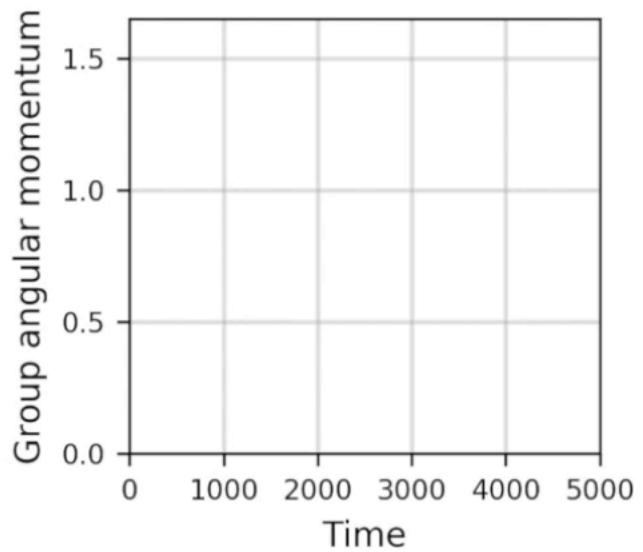
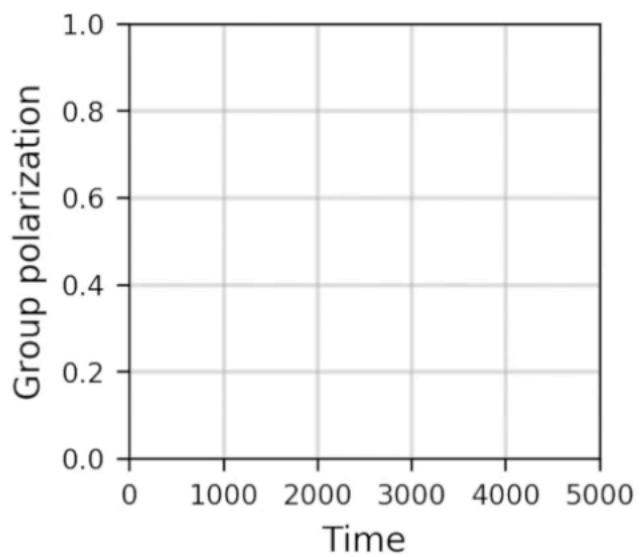
Swarm



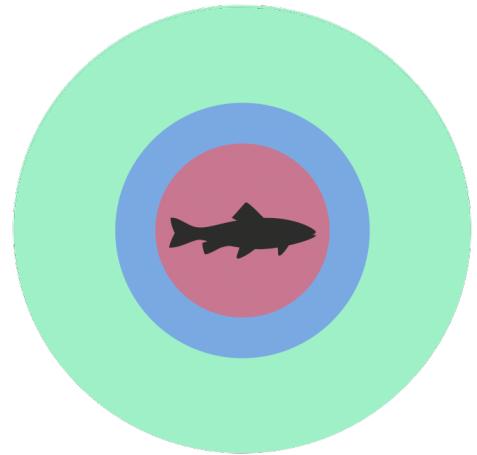
# Swarm-like behavior



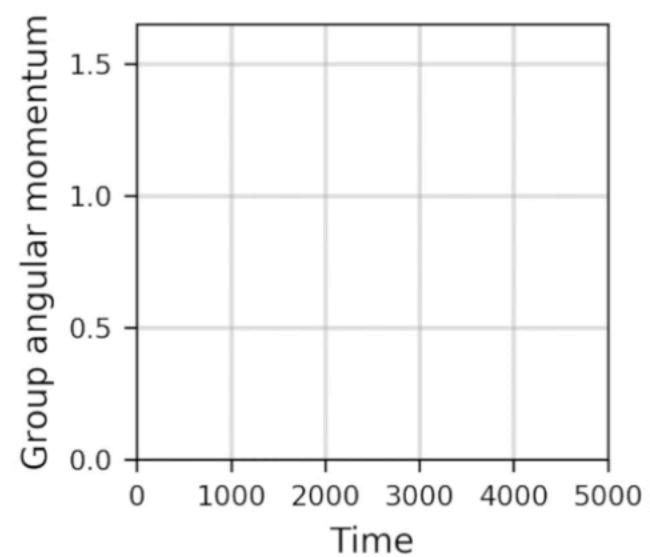
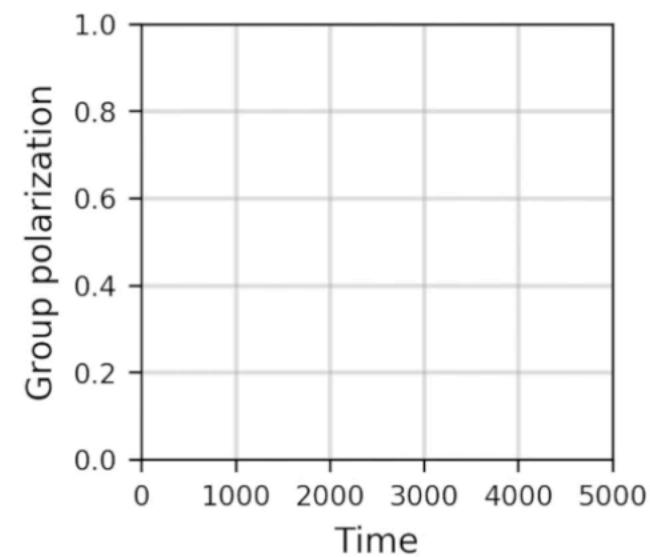
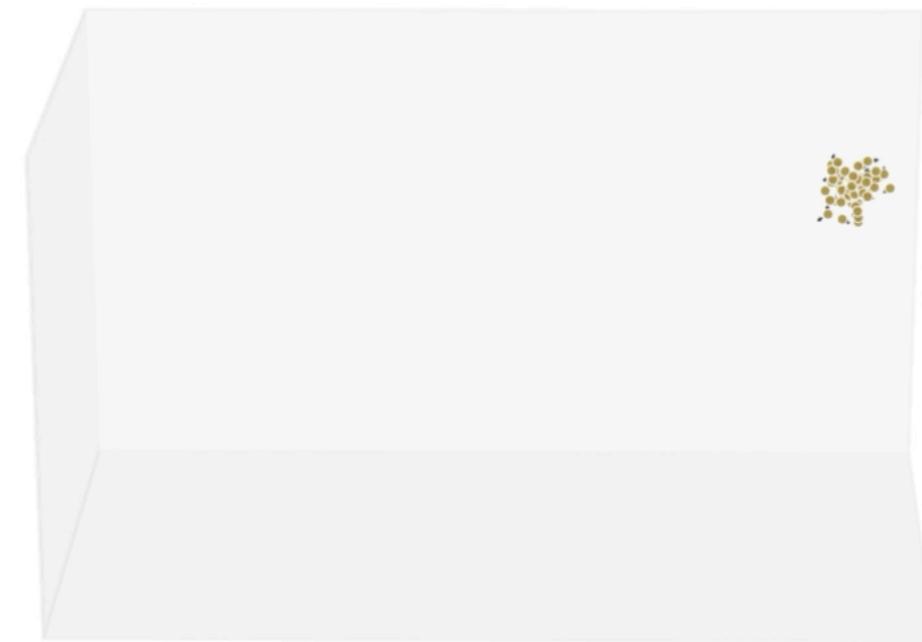
Swarm



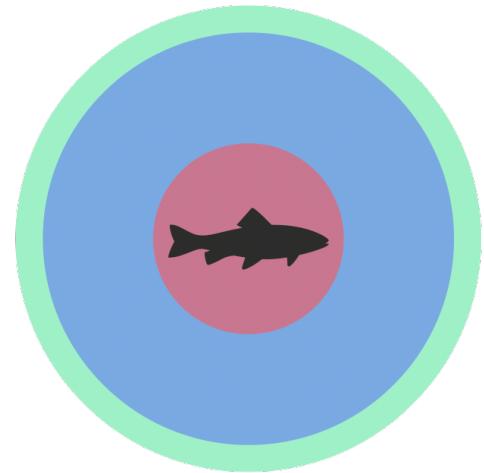
# Milling / torus behavior



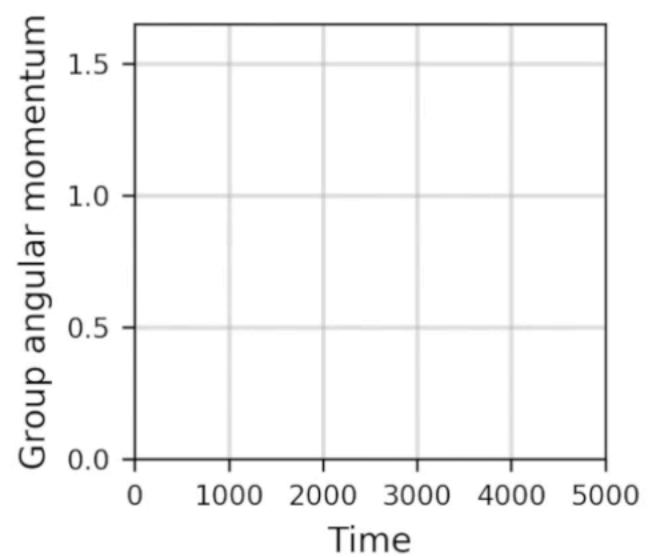
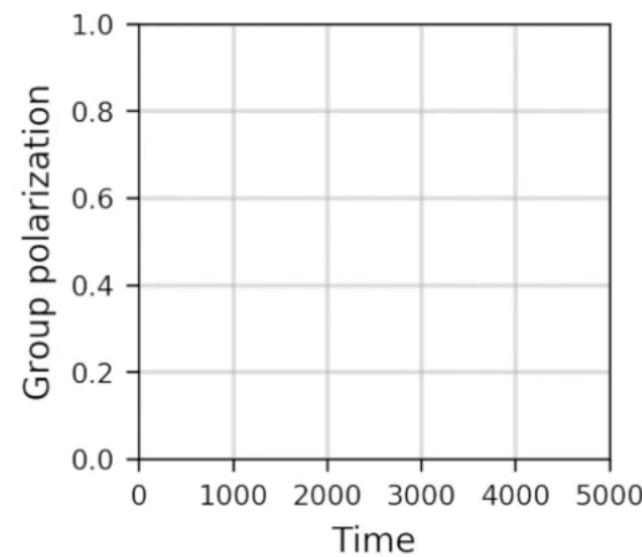
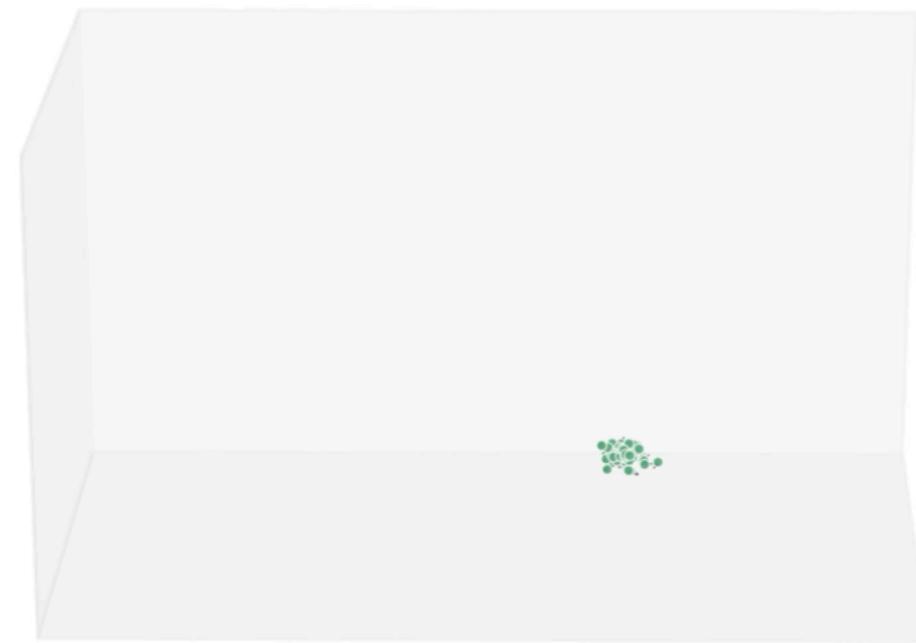
Milling / torus



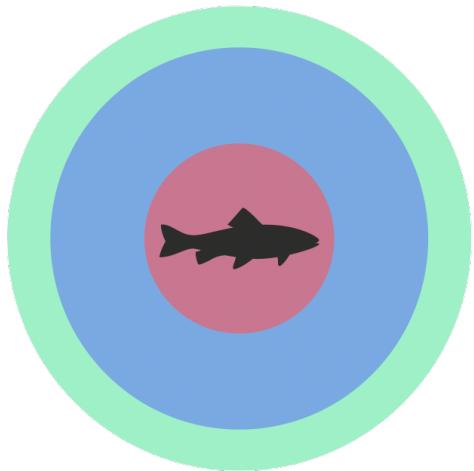
# Highly parallel group



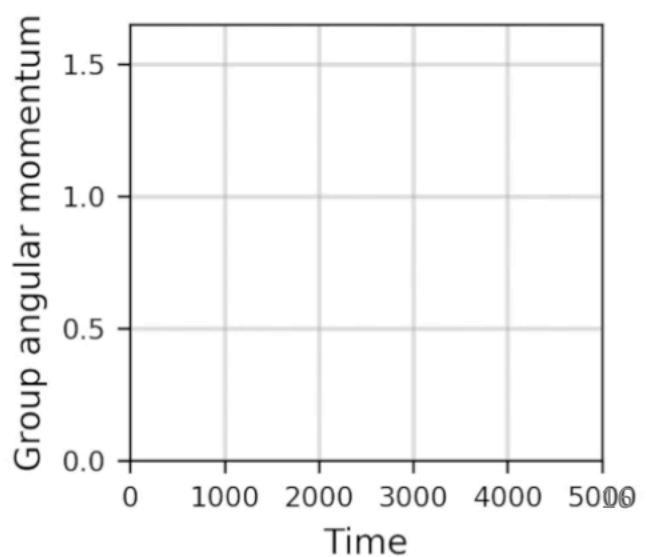
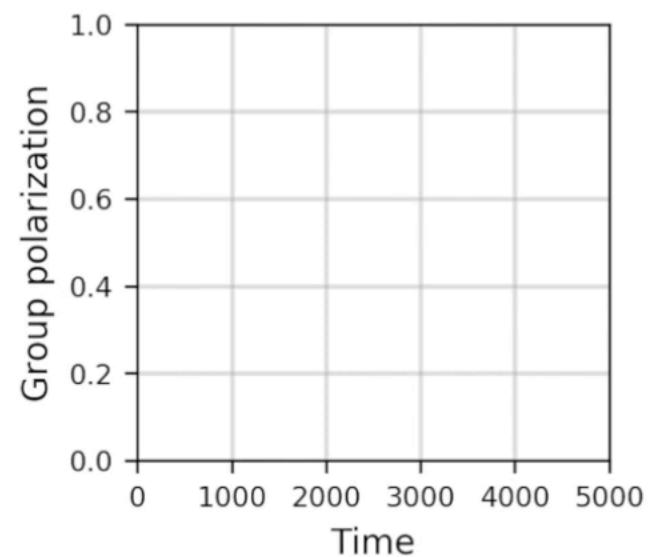
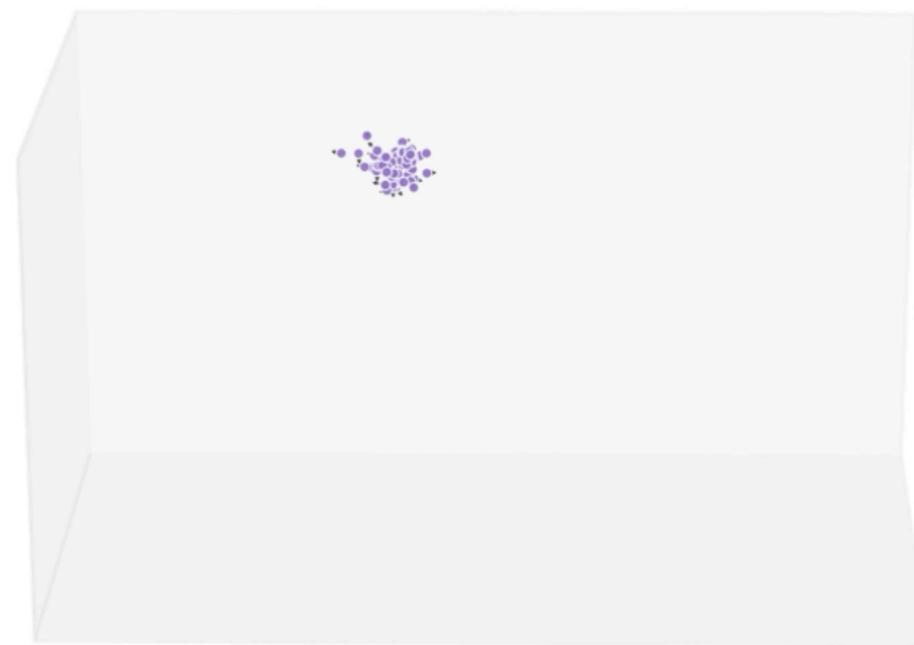
Highly parallel group



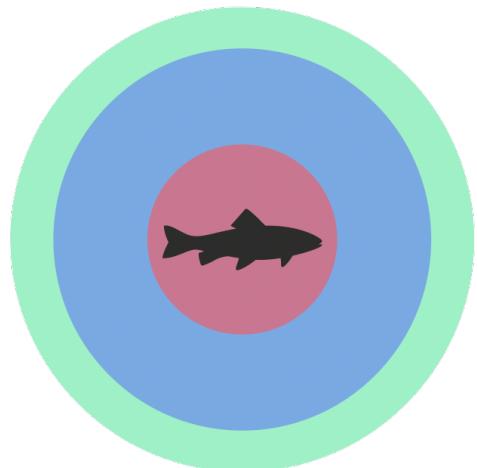
# Dynamic parallel group



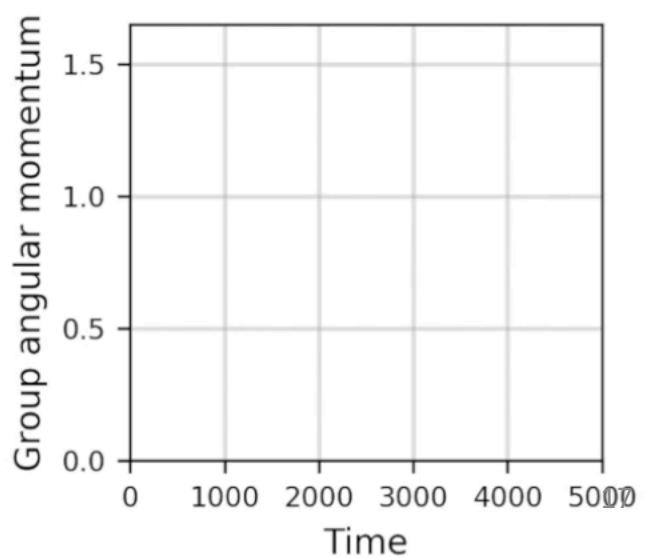
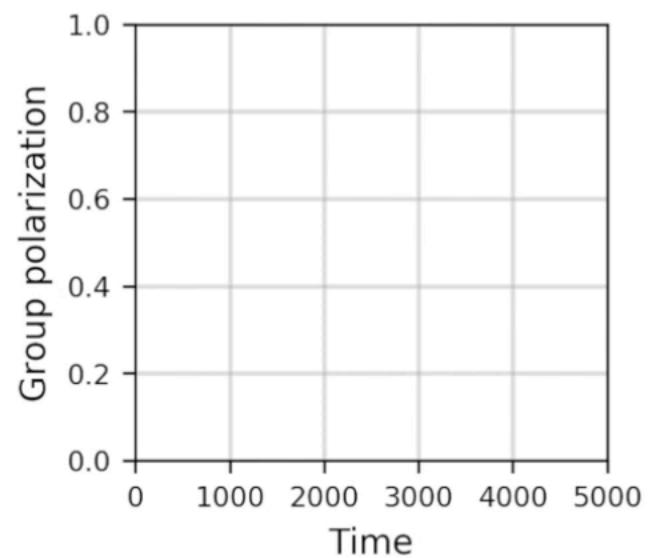
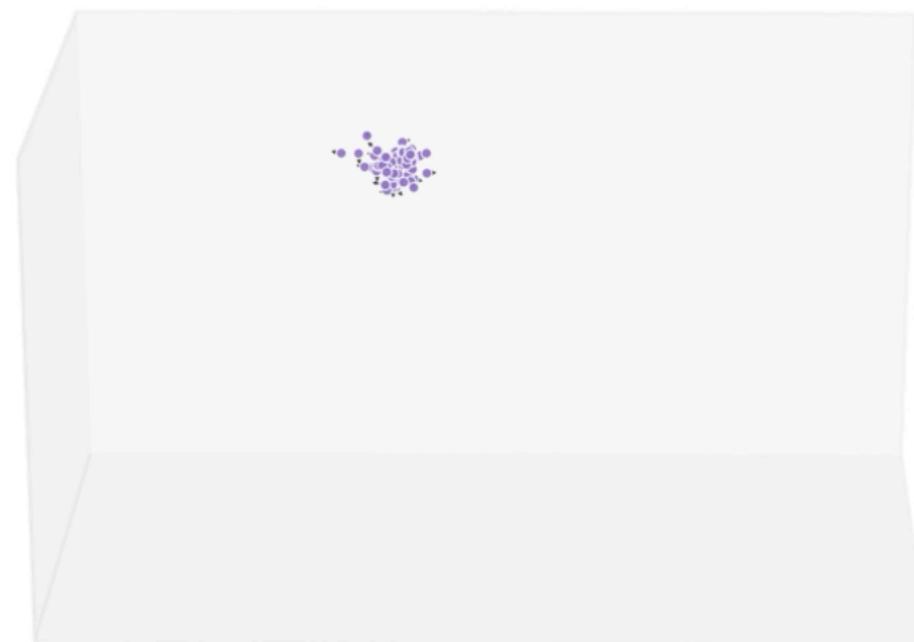
Dynamic parallel group



# What is the network here?



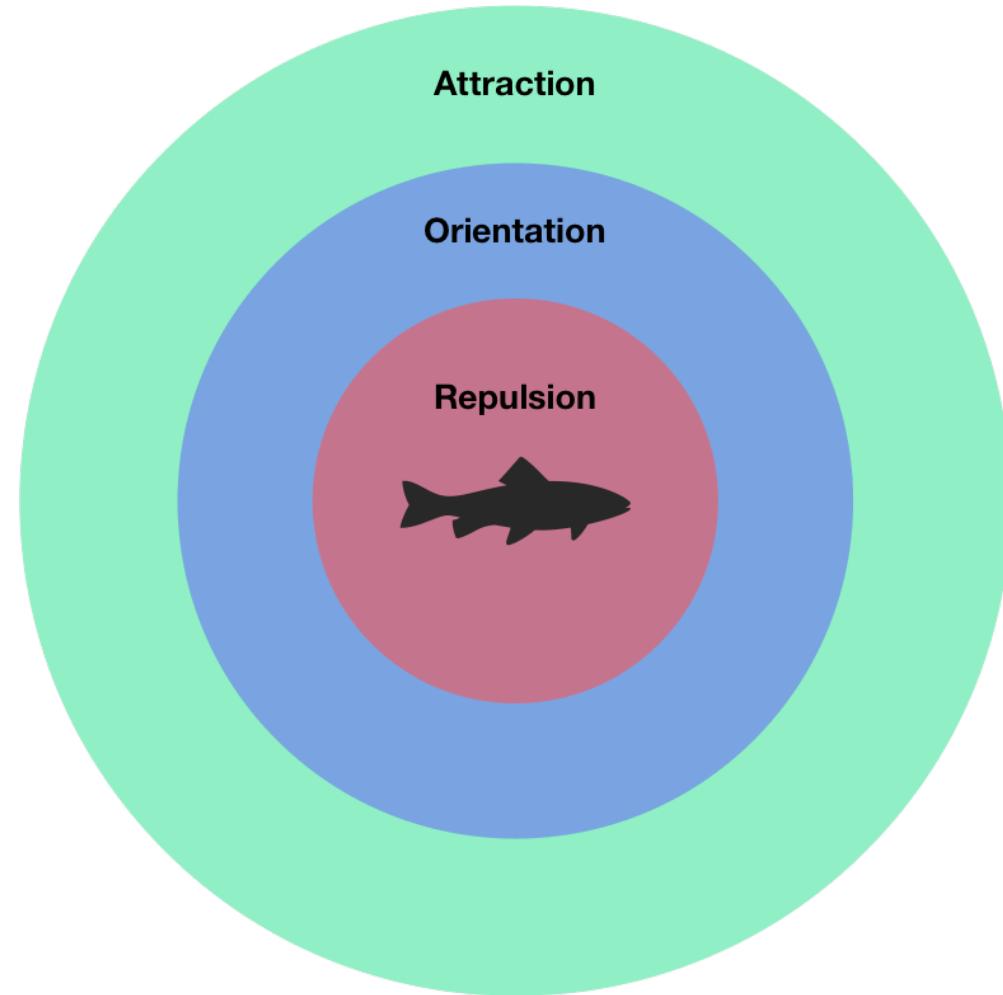
Dynamic parallel group



# What is the network here?

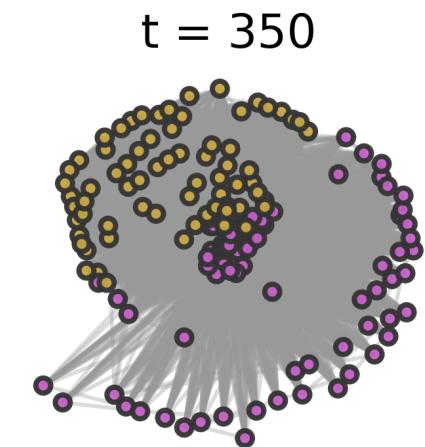
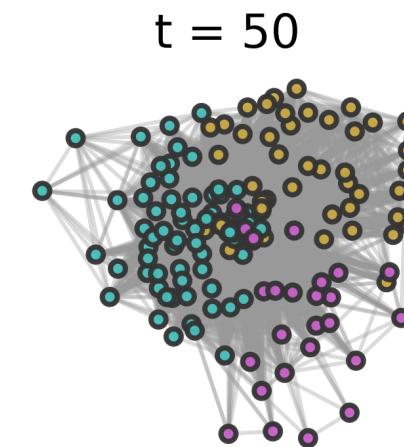
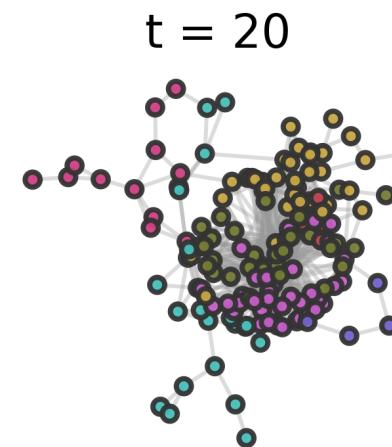
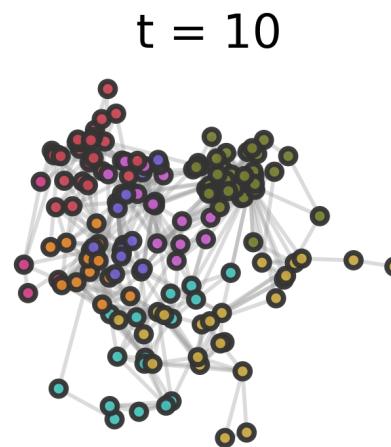
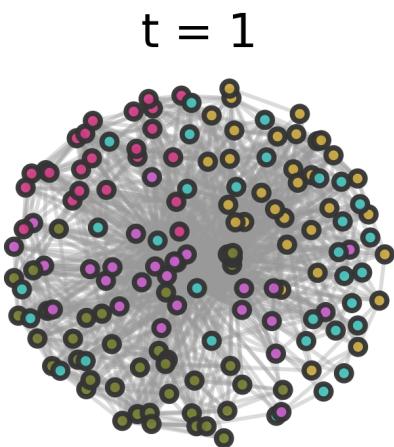
$$A_a = \boxed{\text{green}}$$
$$A_o = \boxed{\text{blue}}$$
$$A_r = \boxed{\text{red}}$$

$A$

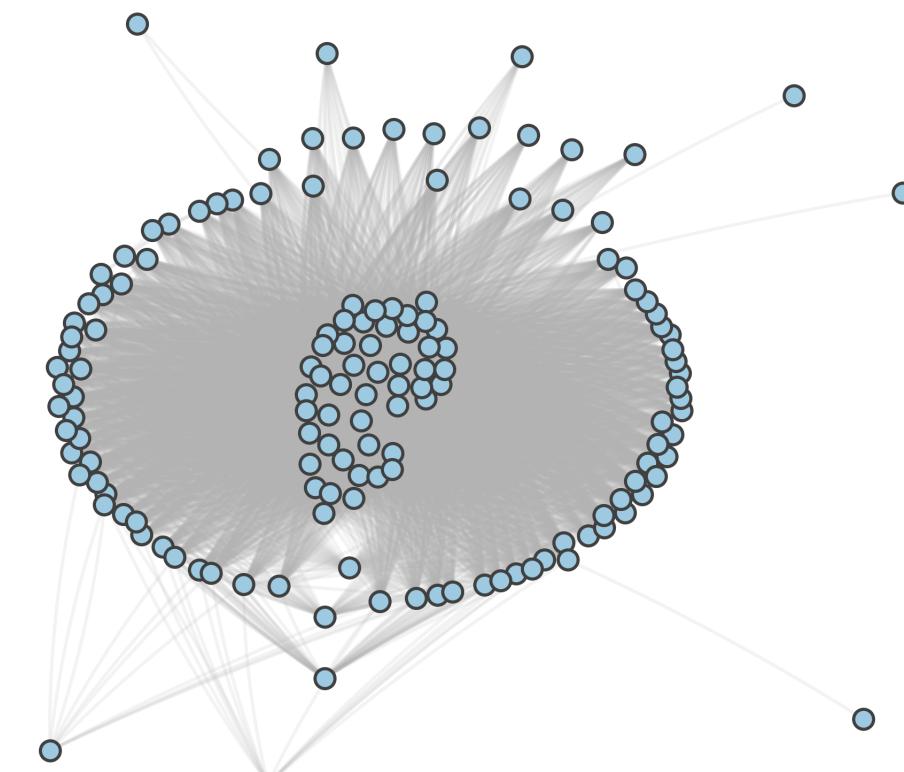
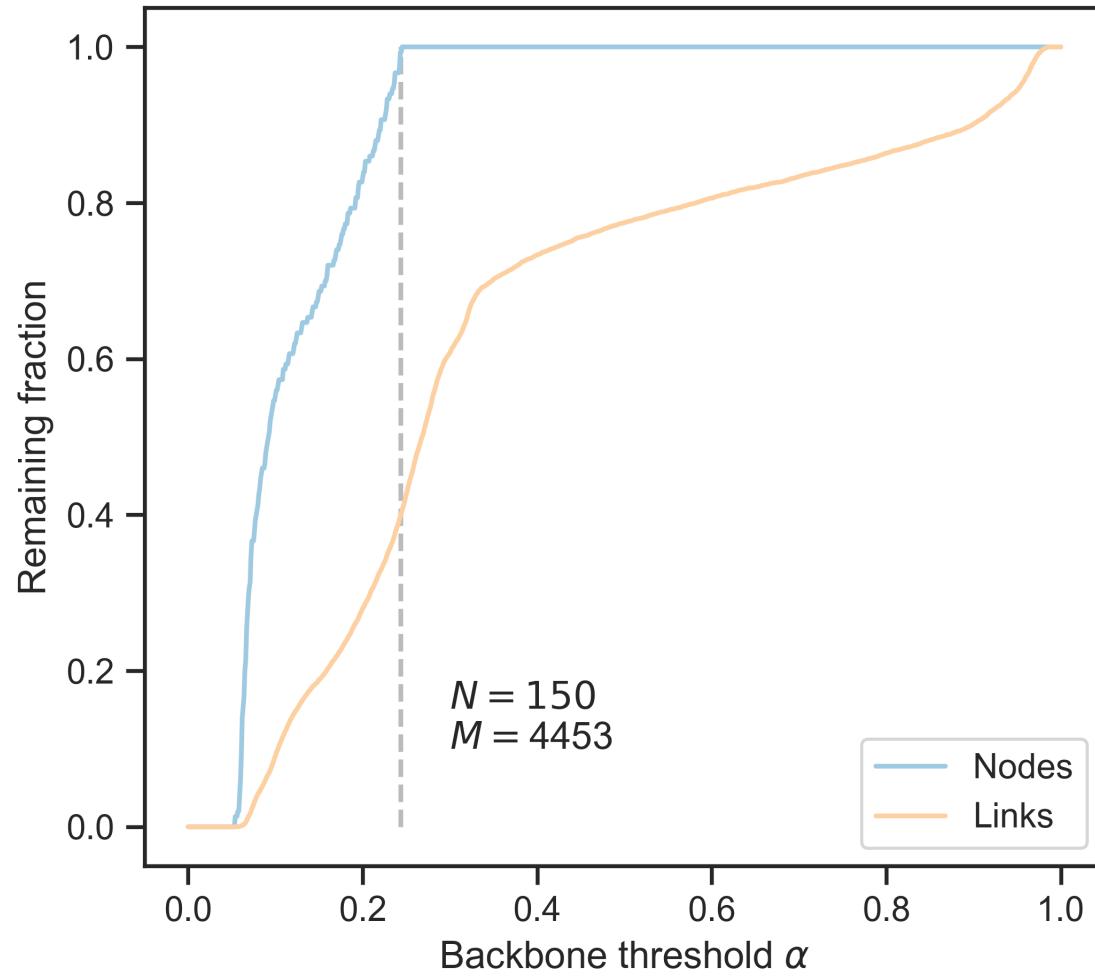


Extract graphs at every timestep  
to create a *temporal network*

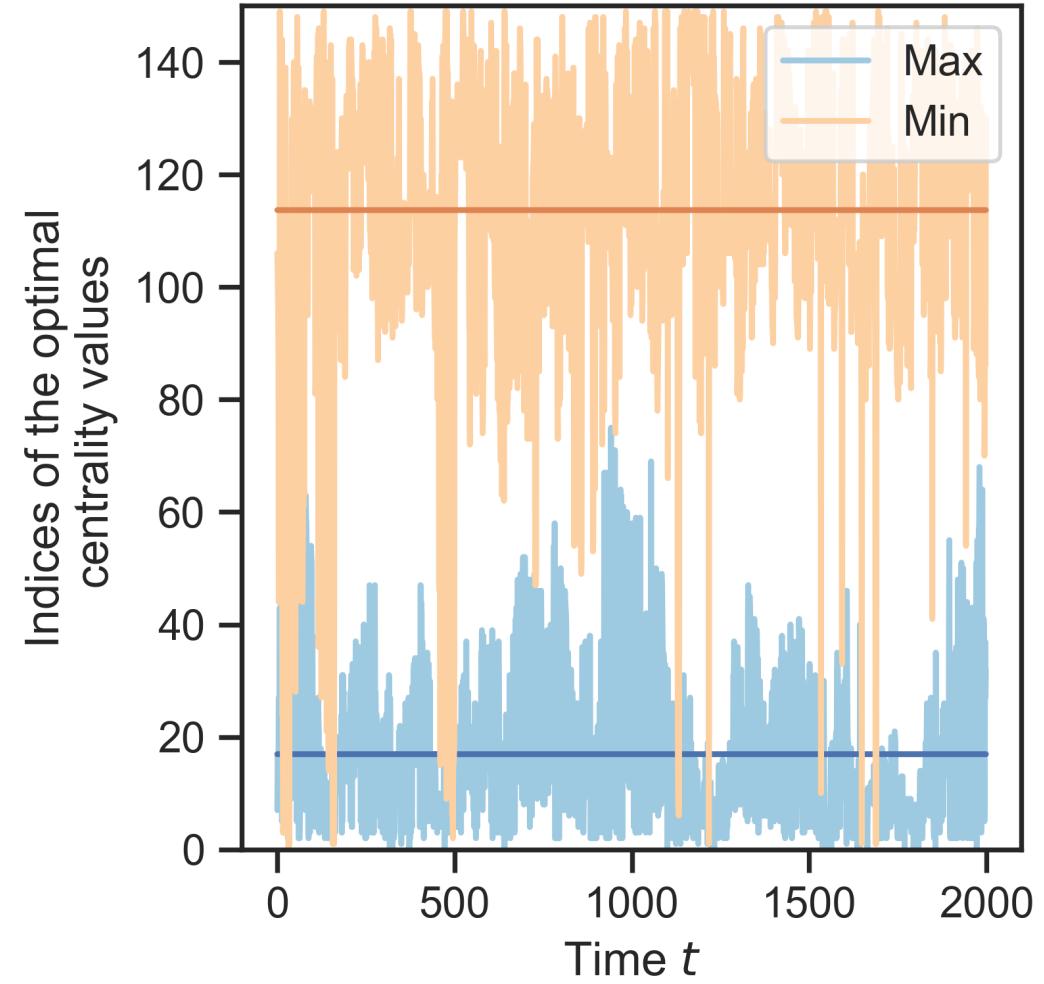
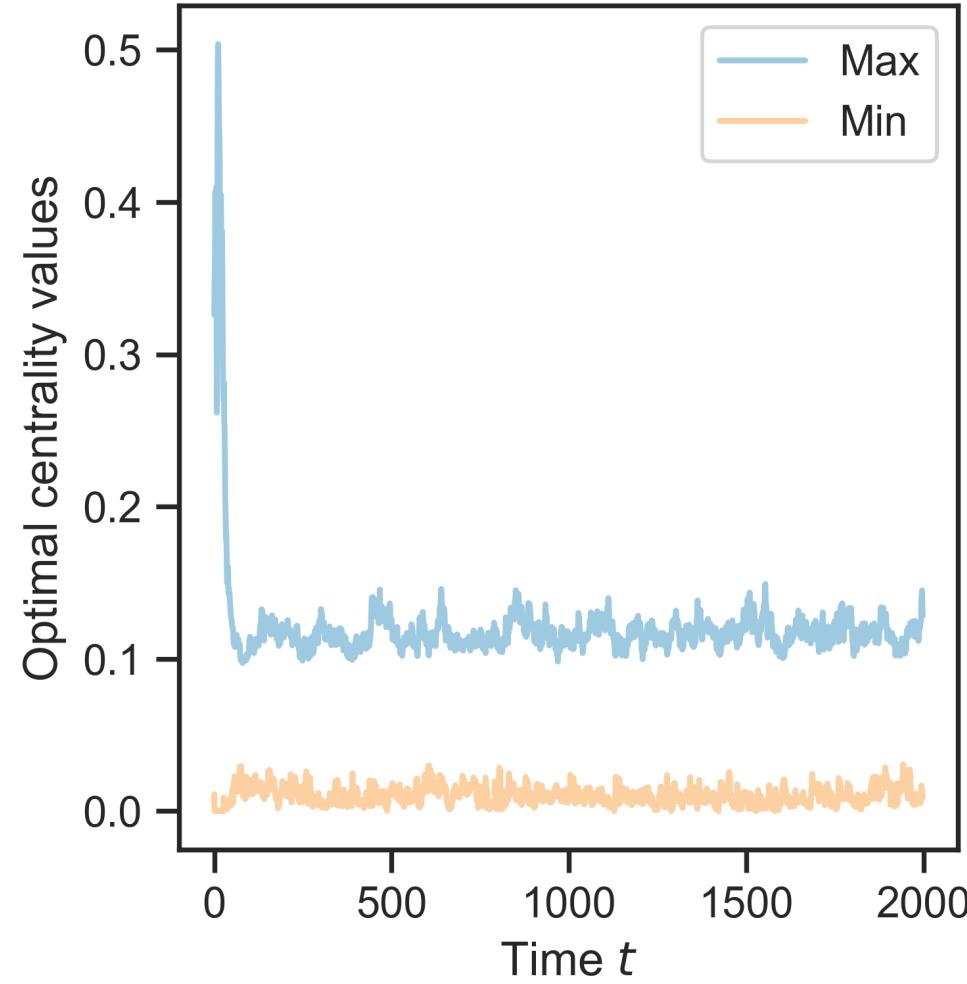
For example



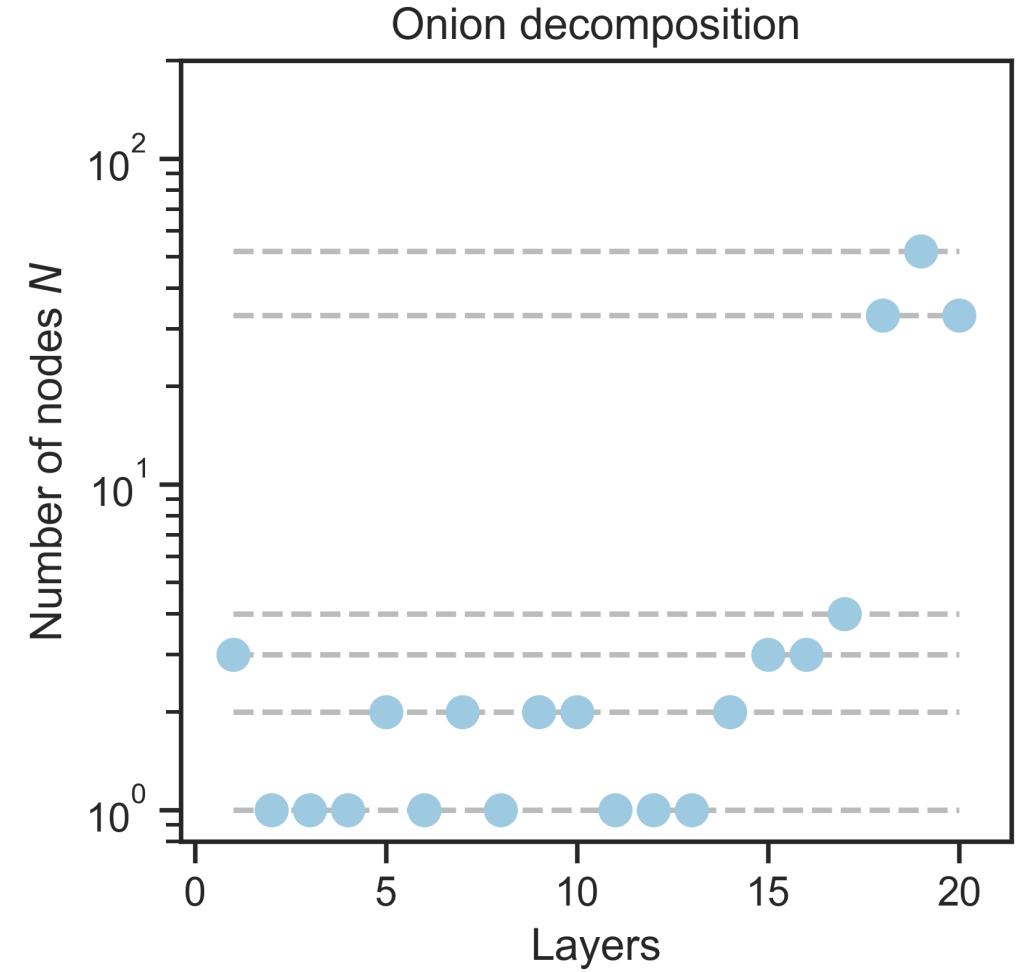
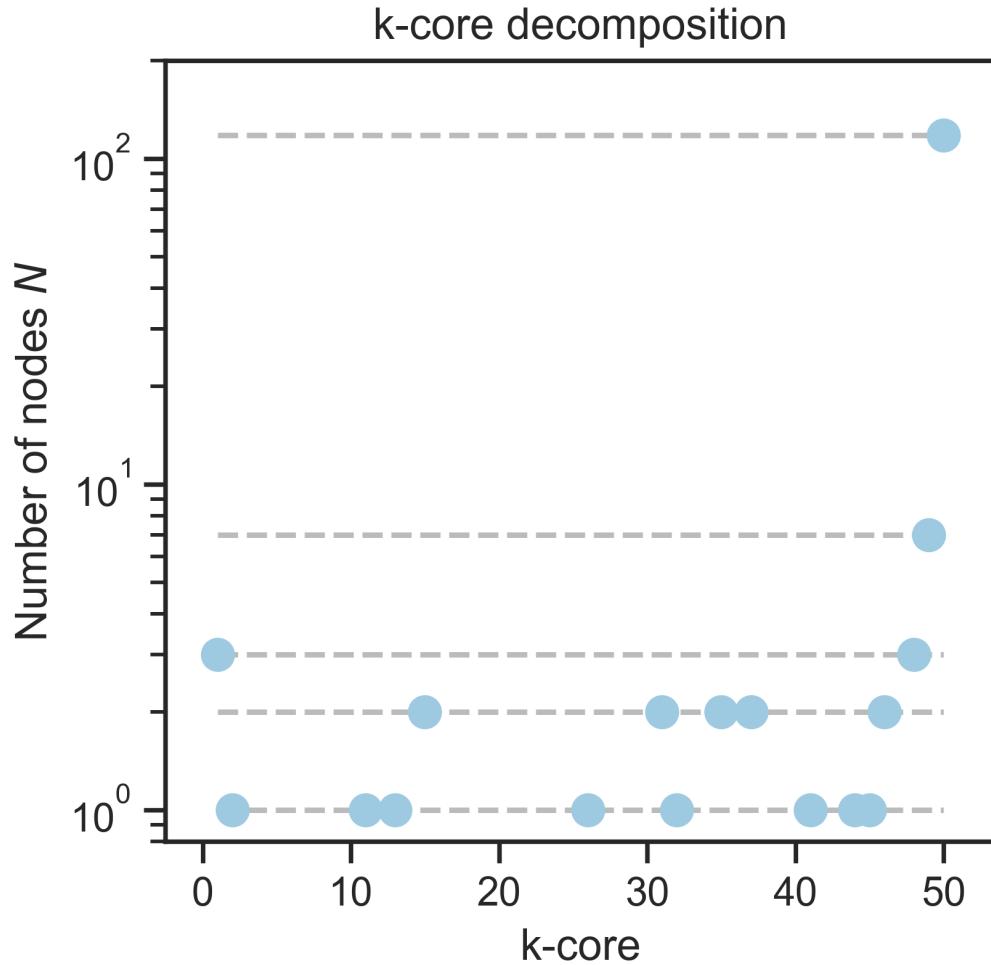
# Results: Temporal Network Backbone



# Results: Change in “Spectral Leadership”



# Results: Decomposition and Coreness



# Approach

Study networks induced from simulated collective behavior

- **Different parameterizations of the generative model**
- Different ways of comparing networks over time
- Different ways of generating the adjacency matrix

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Comparing real networks  
of collective behavior (coming soon)

# netrd 0.2.0

```
pip install netrd
```





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A library for network {reconstruction, distances, dynamics} <https://netrd.readthedocs.io/en/latest/>

753 commits

1 branch

3 releases

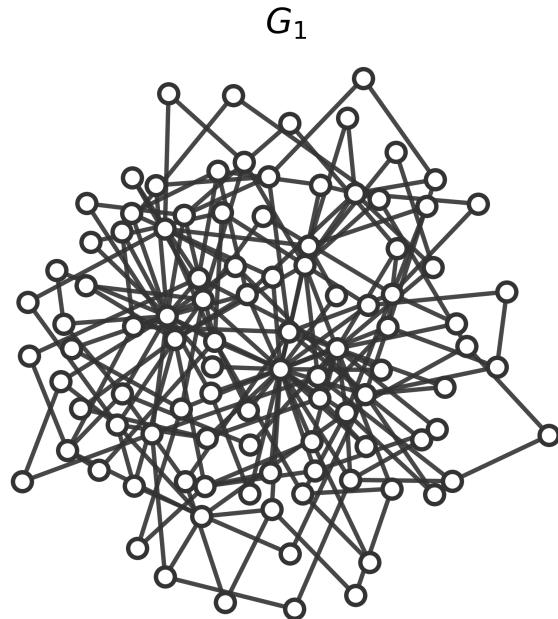
16 contributors

MIT

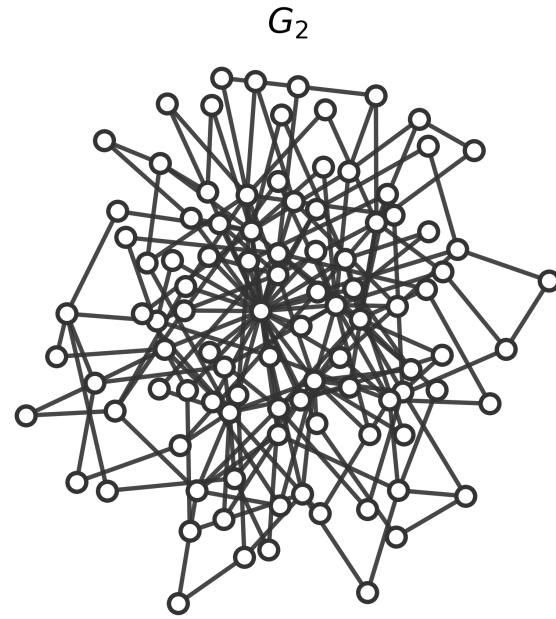


 sdmccabe	Rename entropy to entropy_from_seq (#264) <a href="#">...</a>	Latest commit cb40a35 2 days ago
..		
 <a href="#">distance</a>	Correct documentation of dk-series (#263)	2 days ago
 <a href="#">dynamics</a>	Voter noise (#229)	4 months ago
 <a href="#">reconstruction</a>	Adapt the naive transfer entropy reconstructor to our entropy utility...	3 days ago
 <a href="#">utilities</a>	Rename entropy to entropy_from_seq (#264)	2 days ago
 <a href="#">__init__.py</a>	Updated outer __init__ to not use wildcards.	5 months ago

# netrd: graph distance



How close (similar) are these two graphs?



# netrd: graph distance

$G_1$



How close (similar) are these two mugs?

$G_2$



# netrd: graph distance

$G_1$



$G_2$



How *close* (similar) are these two mugs?

How do their *positions* differ?

How do their *volumes* differ?

How do their *temperatures* differ?

How do their *functions* differ?

# netrd: graph distance

$G_1$



$G_2$



How *close* (similar) are these two mugs?

How do their *positions* differ?

**meters**

How do their *volumes* differ?

**liters**

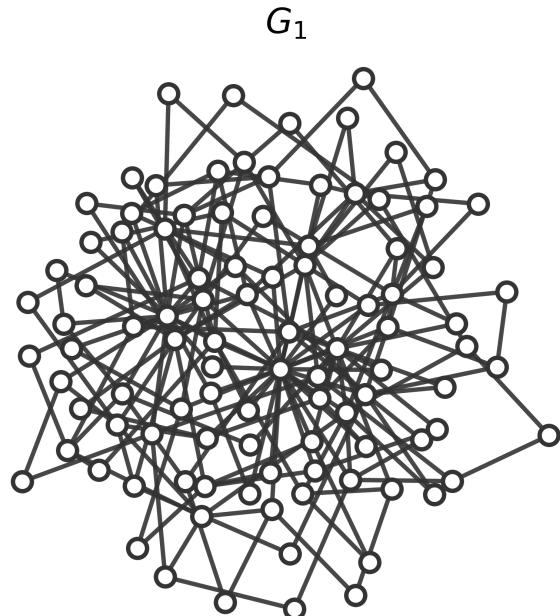
How do their *temperatures* differ?

**degrees**

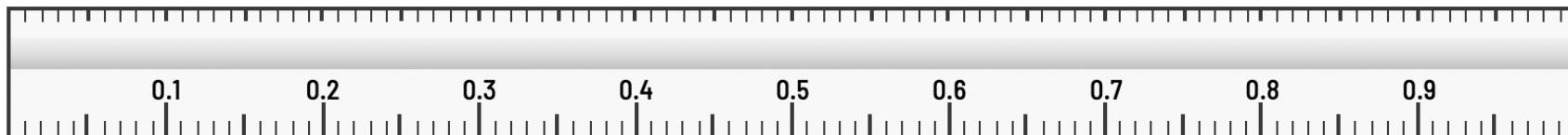
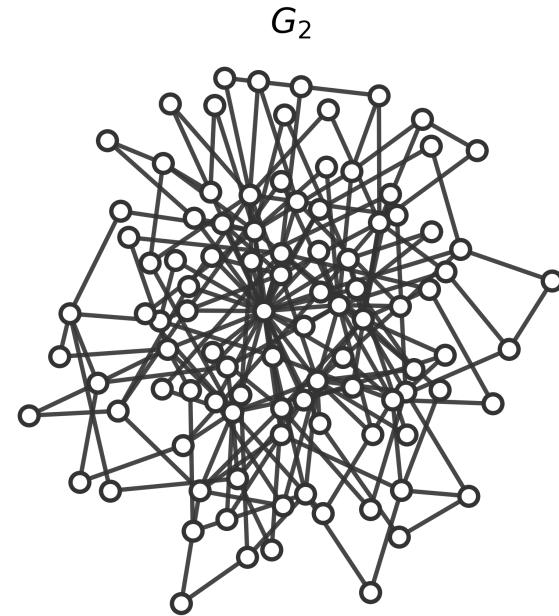
How do their *functions* differ?

**...ask someone at a café?**

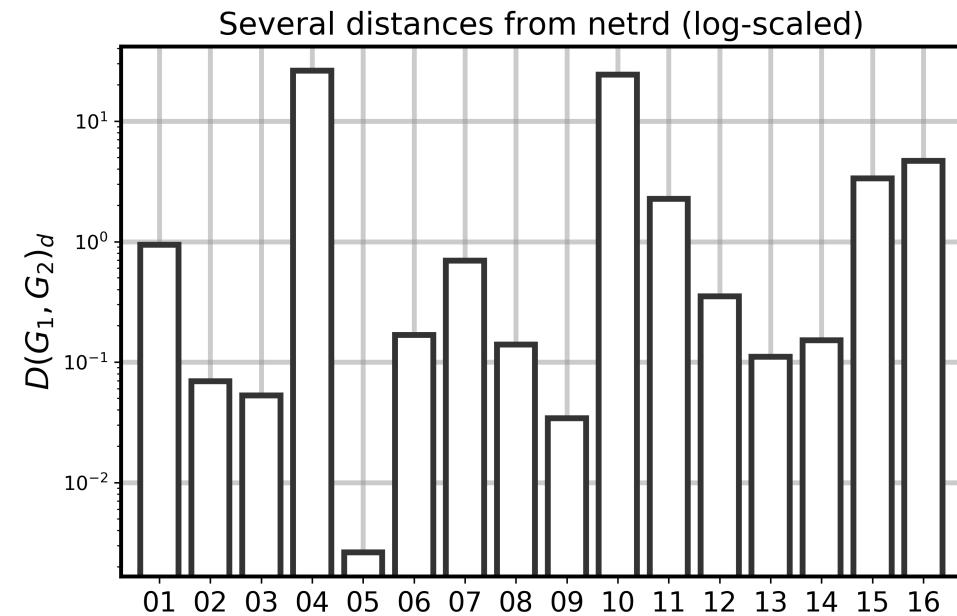
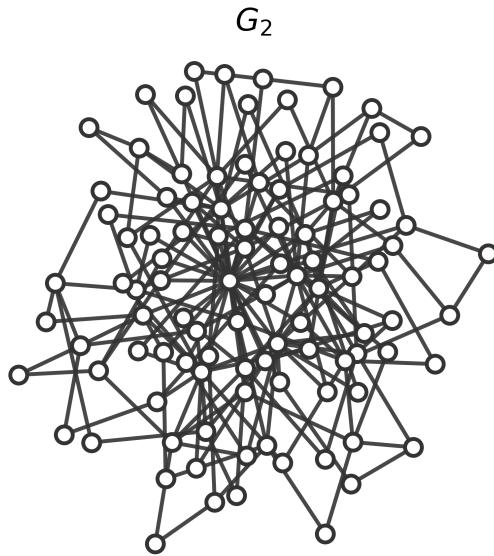
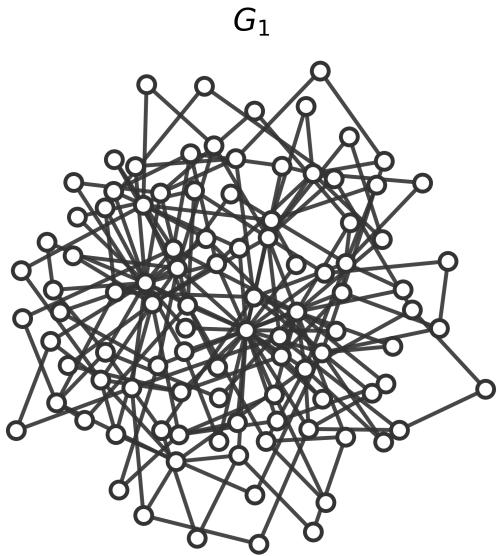
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How close (similar) are these two graphs?



# netrd: graph distance



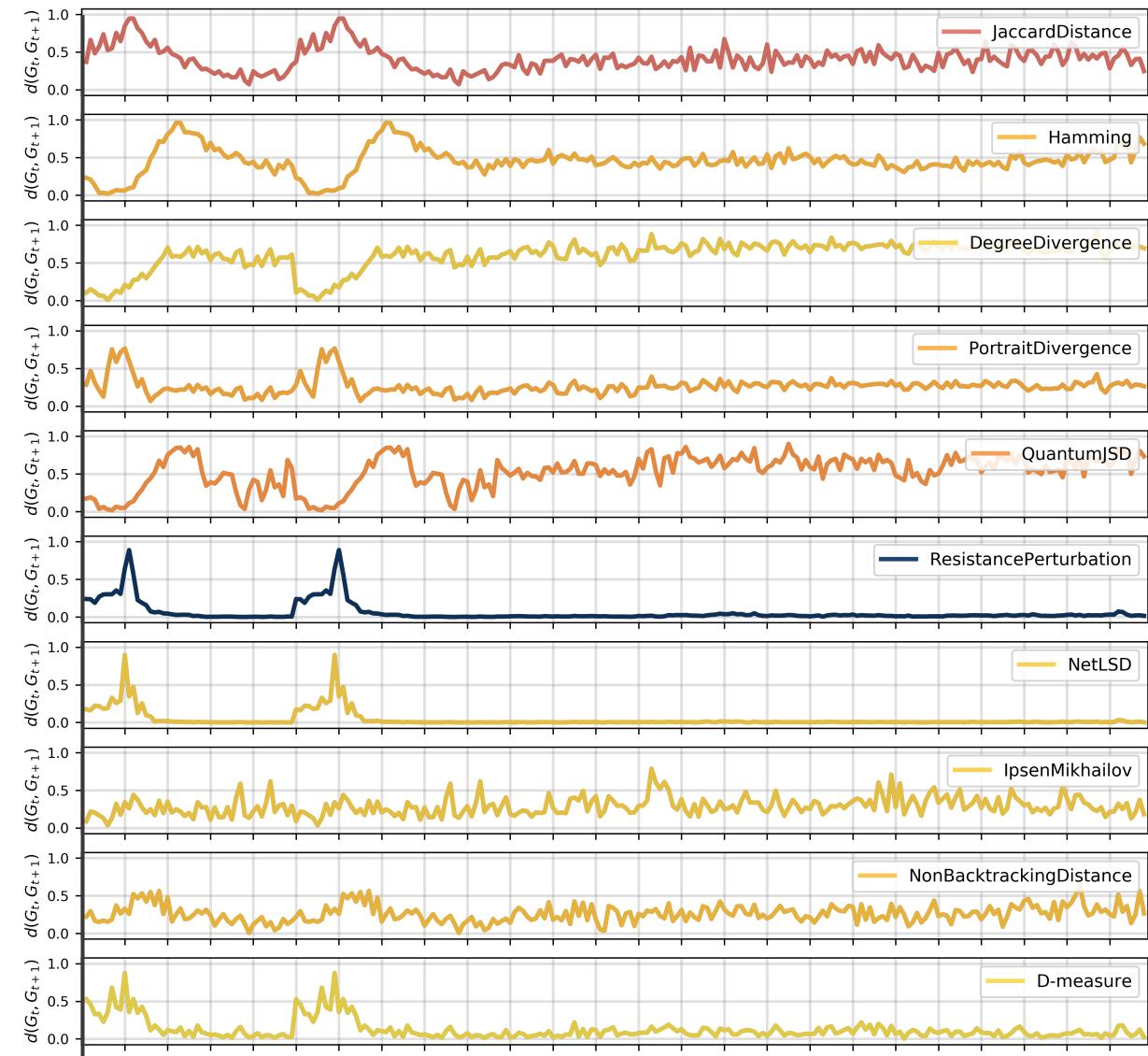
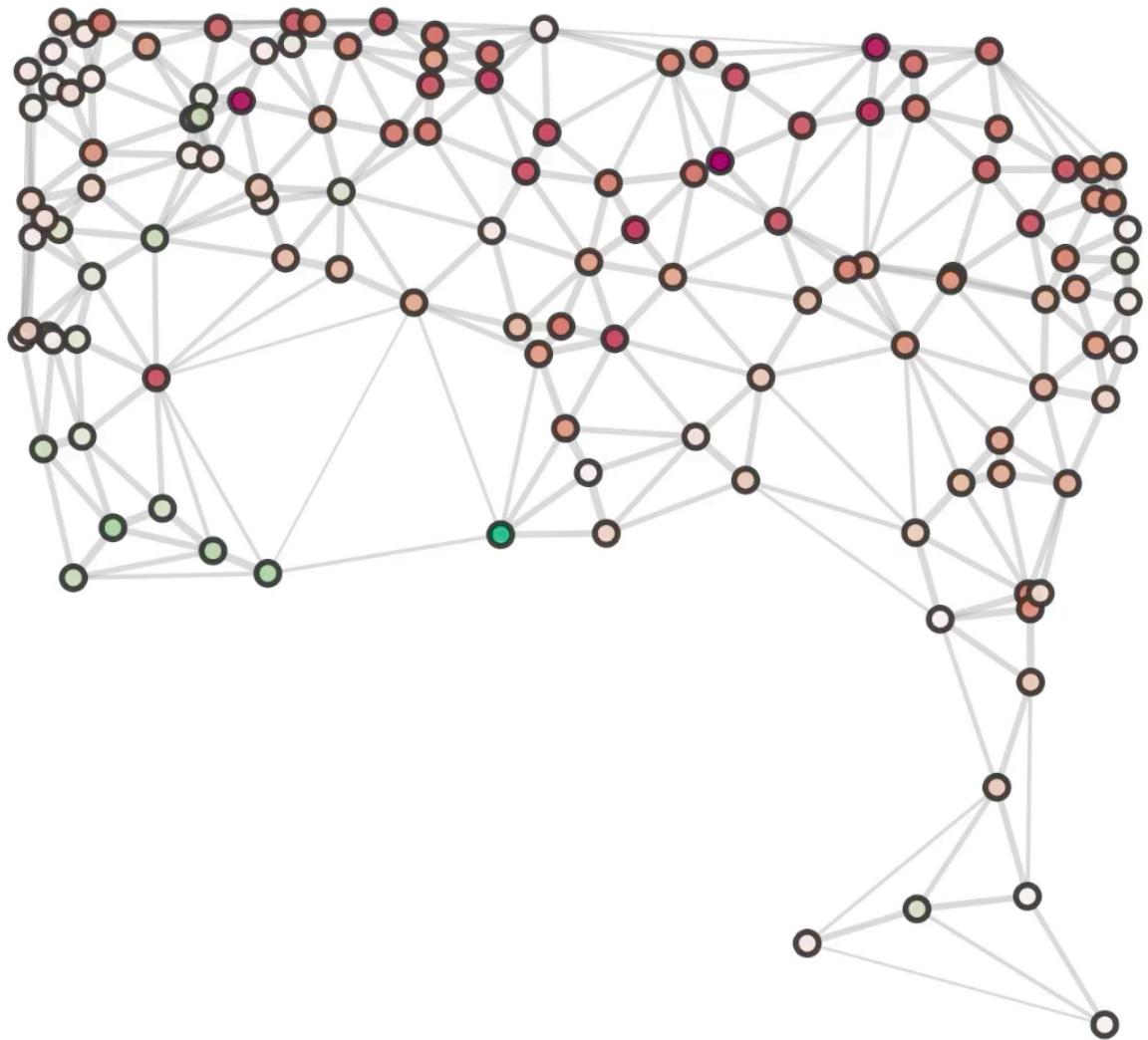
- 01: JaccardDistance
- 02: Hamming
- 03: HammingIpsenMikhailov
- 04: Frobenius
- 05: PolynomialDissimilarity
- 06: PortraitDivergence
- 07: OnionDivergence
- 08: QuantumSpectralJSD
- 09: DegreeDivergence
- 10: ResistancePerturbation
- 11: NetLSD
- 12: CommunicabilitySequence
- 13: IpsenMikhailov
- 14: NonBacktrackingSpectral
- 15: NetSimile
- 16: DeltaCon

# Graph distances between $G(t)$ and $G(t+1)$

Network, changing  
over time

Different graph distances  
between networks at  
 $t$  and  $t+1$

# Graph distances between $G(t)$ and $G(t+1)$



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# Approach

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# Tons more cool stuff to do

- If we can **extract these networks**, we could apply tools from **information theory** to unravel the **role of communication patterns** in maintaining the collective behavior (message passing, information storage/transfer, emergence, etc...).
- If not... maybe there isn't a "network science" for these systems (which would be useful to know!)

# Network analysis of collective motion

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Francis Normand

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Thanks to: Conor Heins, Iain  
Couzin, Dan Bath, **all of you!**

