

# The hidden low-dimensional dynamics of large neuronal networks

Frontiers in Neurophotonics 2022

Patrick Desrosiers

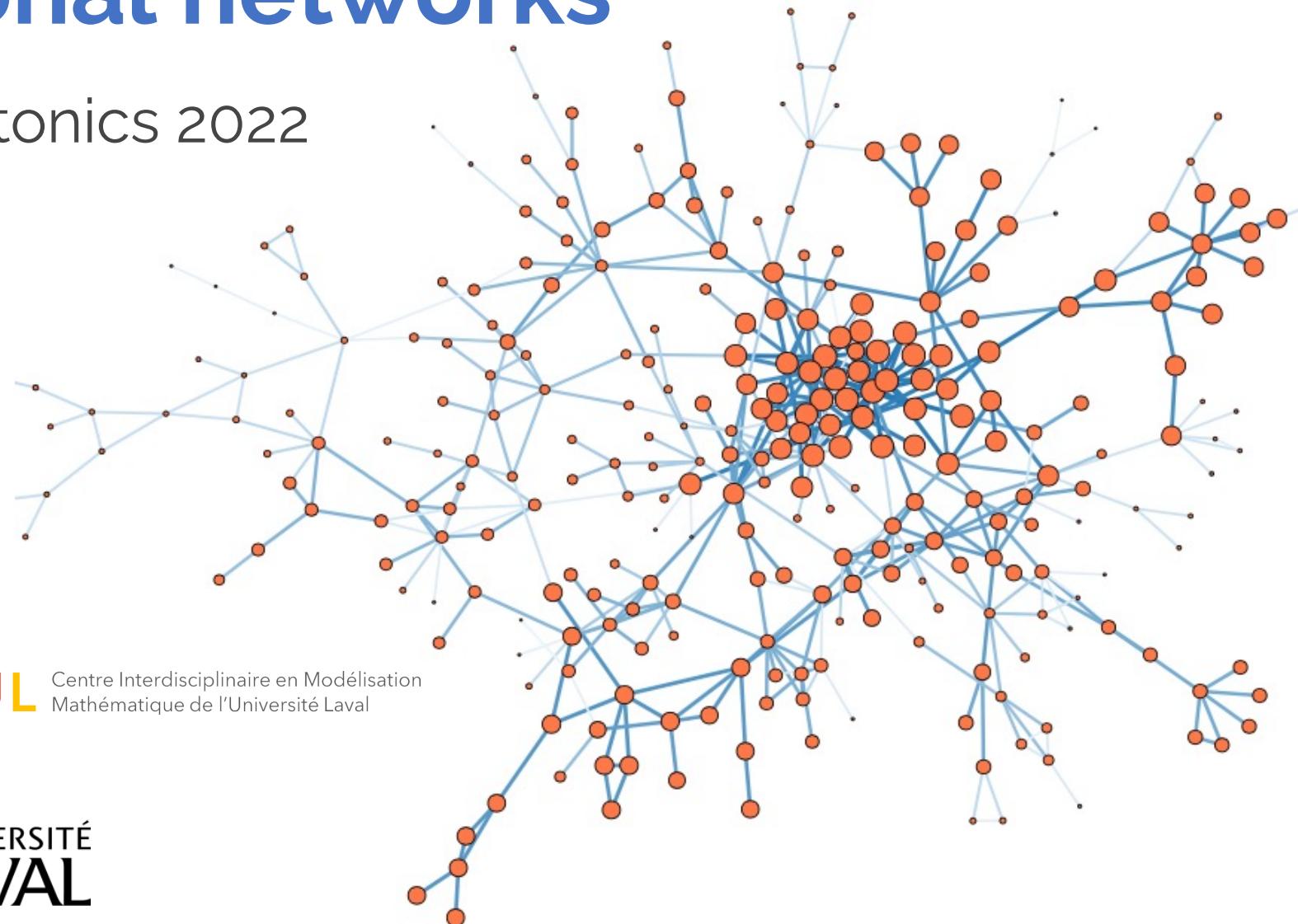


Dynamica



UNIVERSITÉ  
Laval

**CIMMUL** Centre Interdisciplinaire en Modélisation  
Mathématique de l'Université Laval

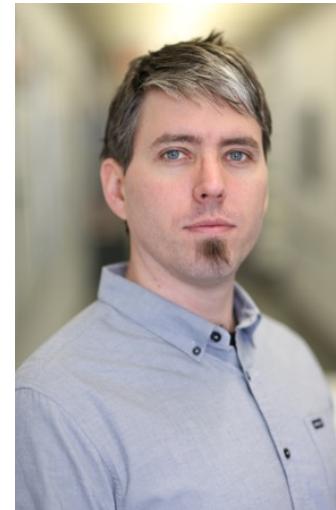




Daniel Côté



Paul De Koninck



Benoit Labonté



Pierre Marquet



Daniel Côté



Paul De Koninck



Benoit Labonté



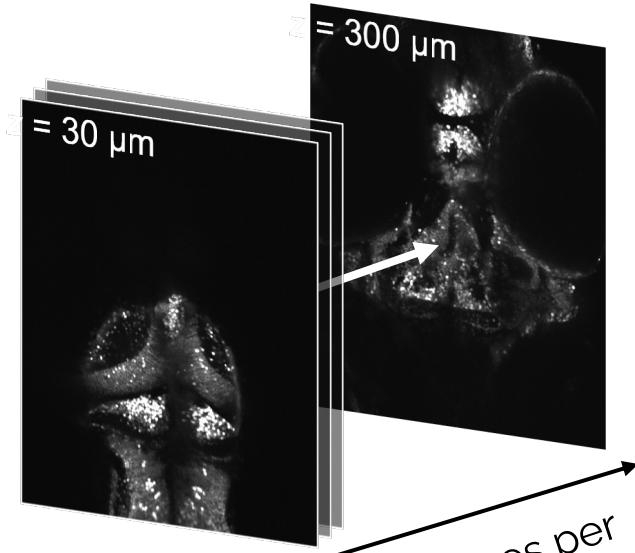
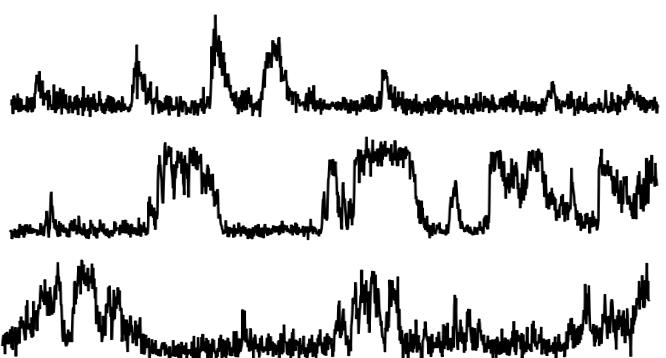
Pierre Marquet



Antoine Légaré

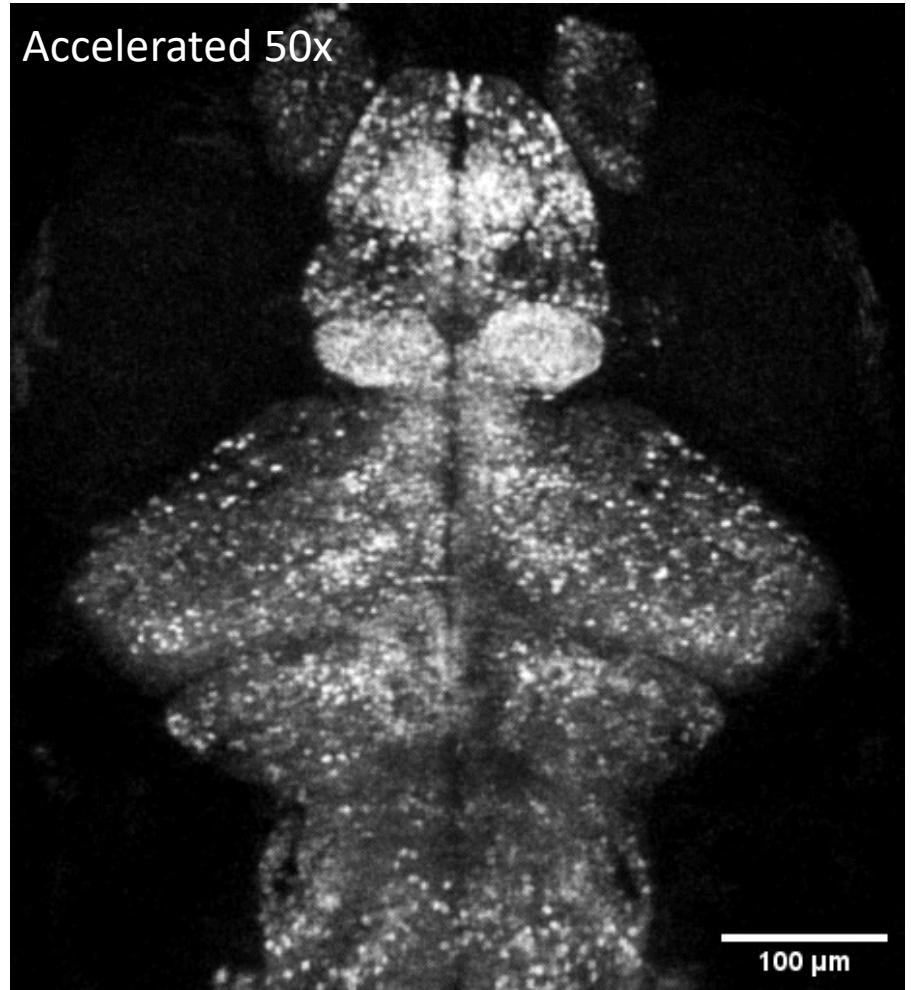


Larval zebrafish



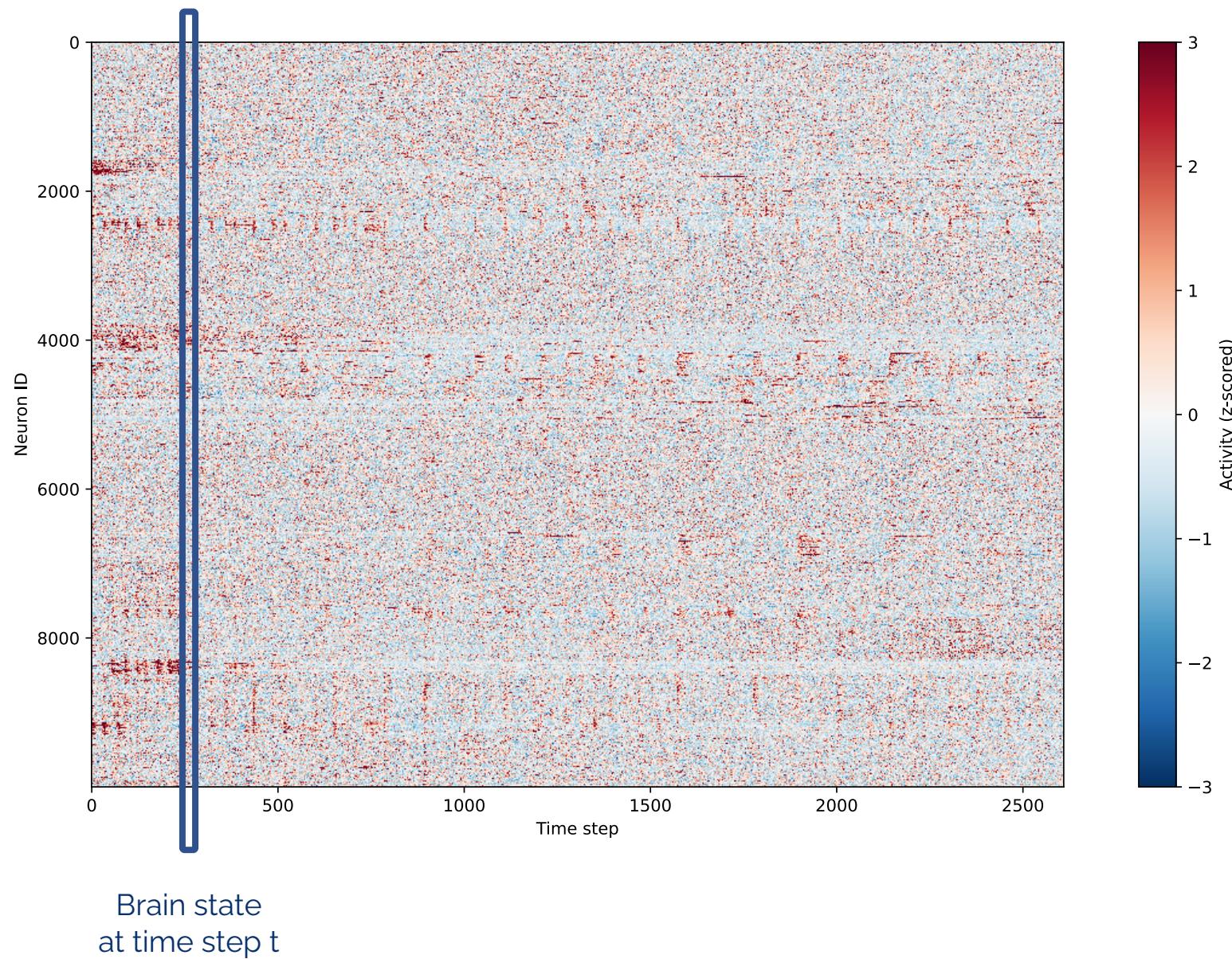
29 planes per second

61500 neurons on average

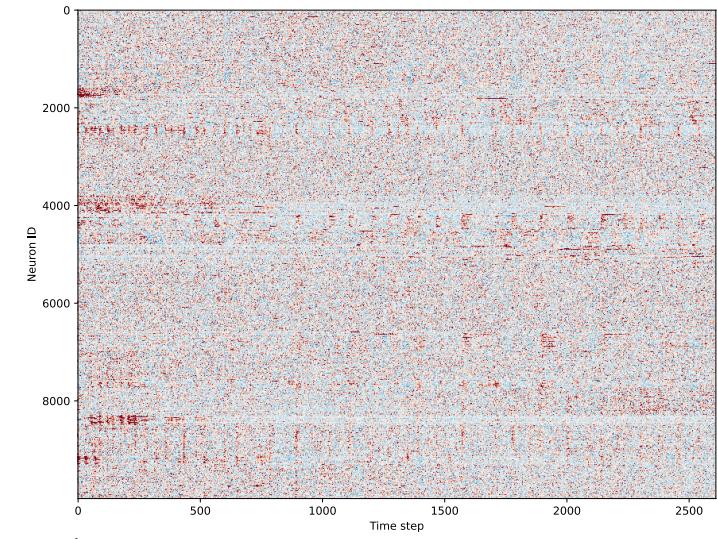


Source: Paul De Koninck' Lab at CERVO

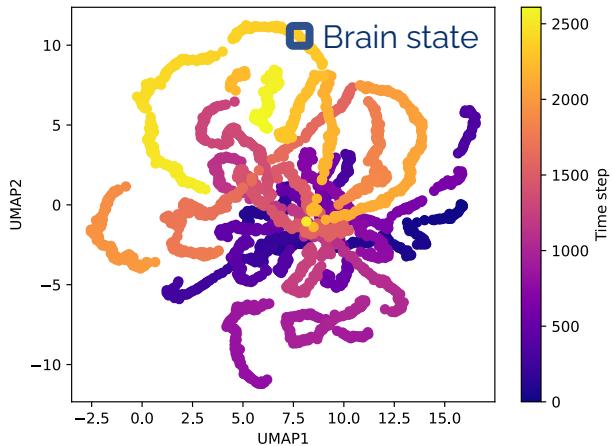
# Neuronal activity in zebrafish brain: data



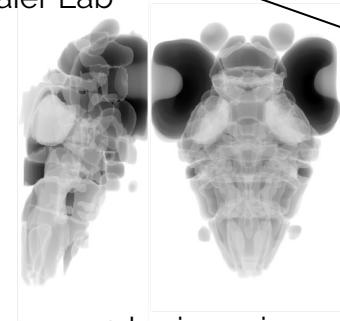
# Neuronal activity in zebrafish brain: possible analyses



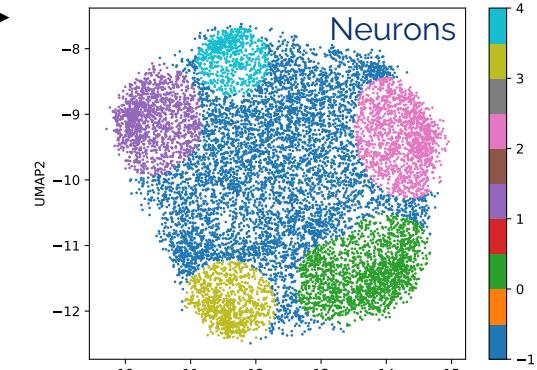
Dimensionality reduction:  
PCA, Isomap, t-SNE, UMAP



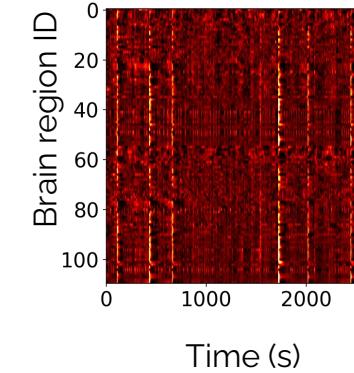
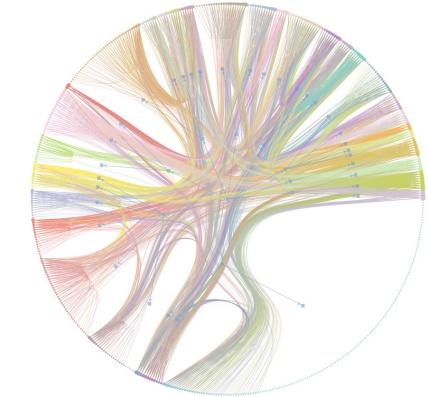
Mapzebrain,  
Herwig Baier Lab



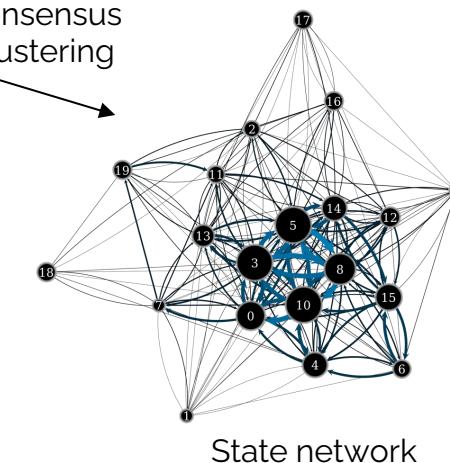
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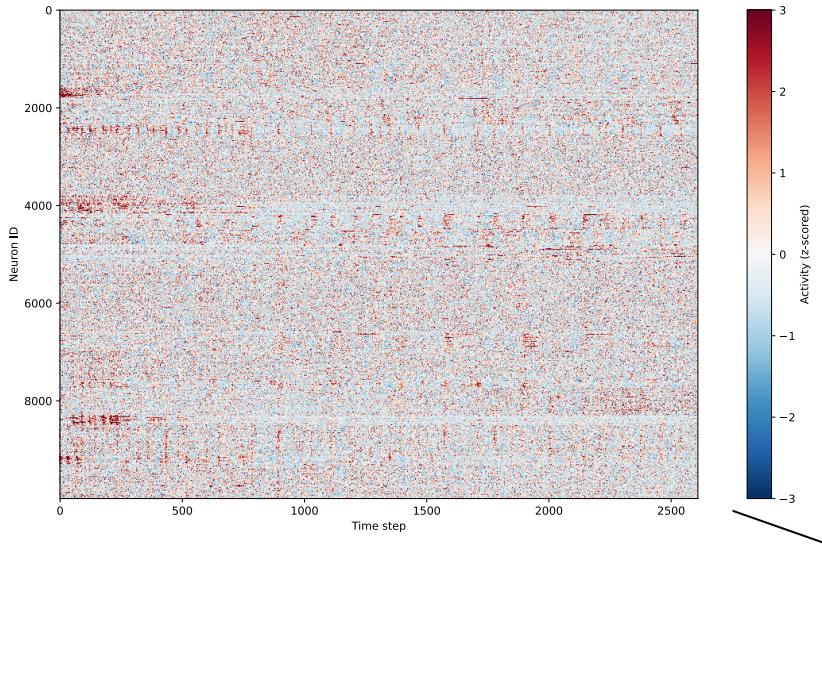
Functional connectivity:  
Pearson correlation, Granger causality, Transfer entropy



Consensus  
clustering



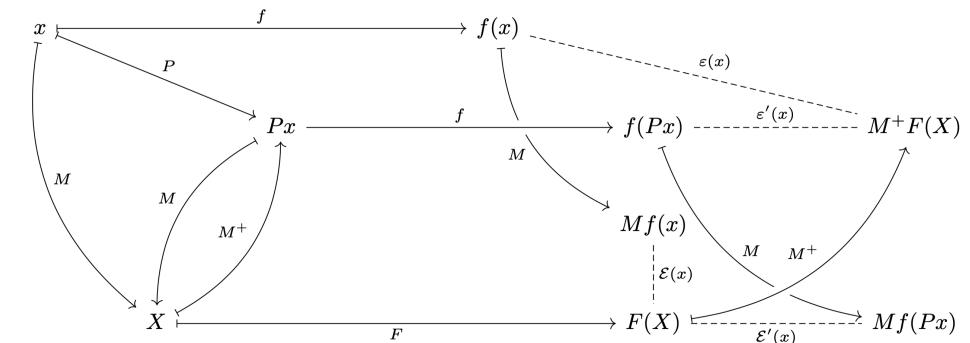
# Neuronal activity in zebrafish brain: possible analyses



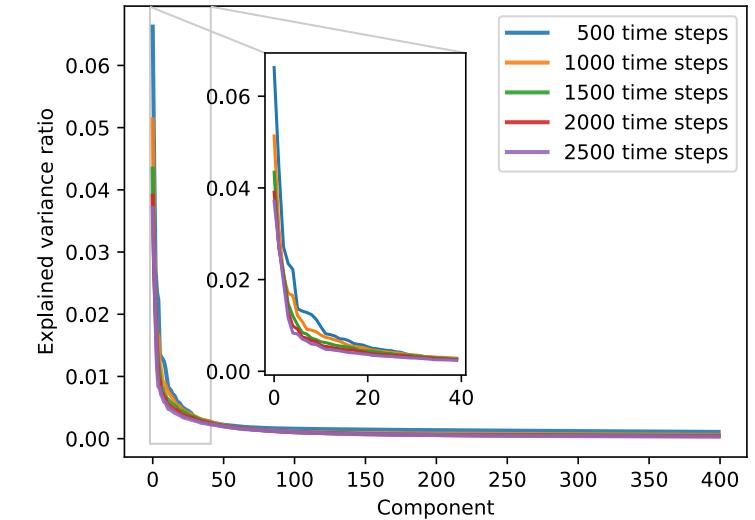
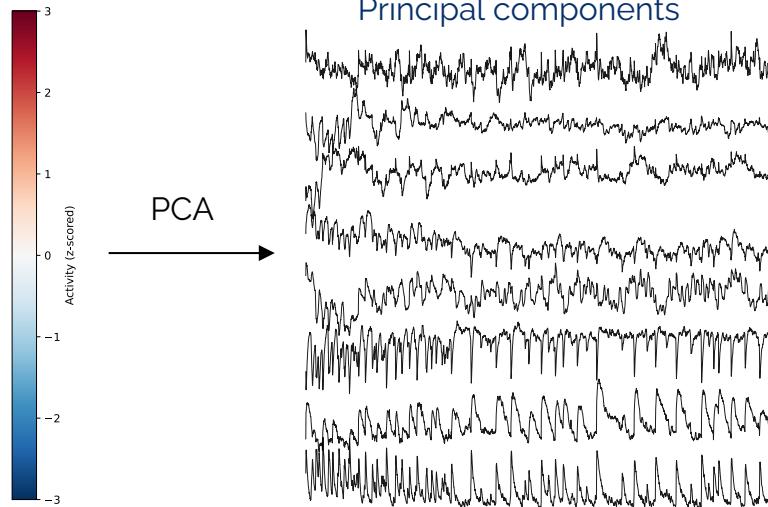
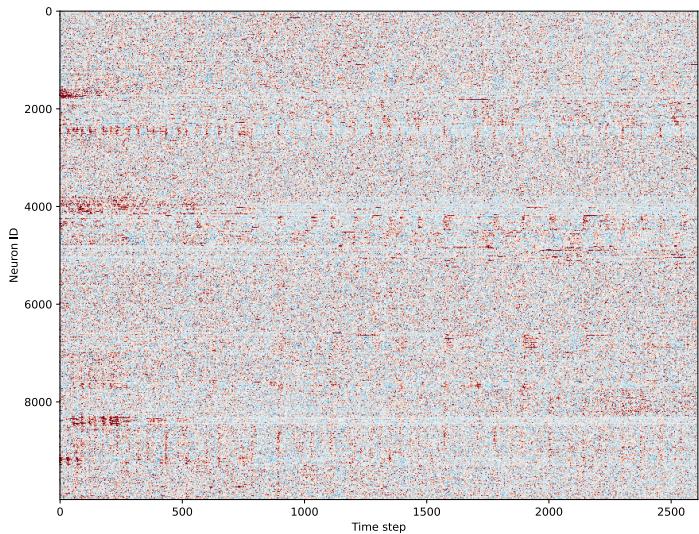
Mathematical modeling of the whole dynamics ?

$$\dot{X} = \frac{d(R \circ x)}{dt} = \mathcal{U}[R] \circ x = J_R \circ f \circ x = F \circ R \circ x = F \circ X.$$

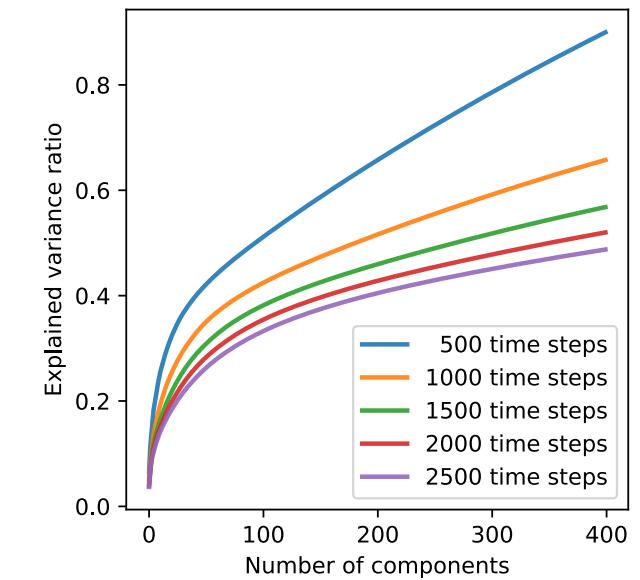
$$\dot{X} = \mathcal{U}[R] \circ x = J_R \circ f \circ x.$$



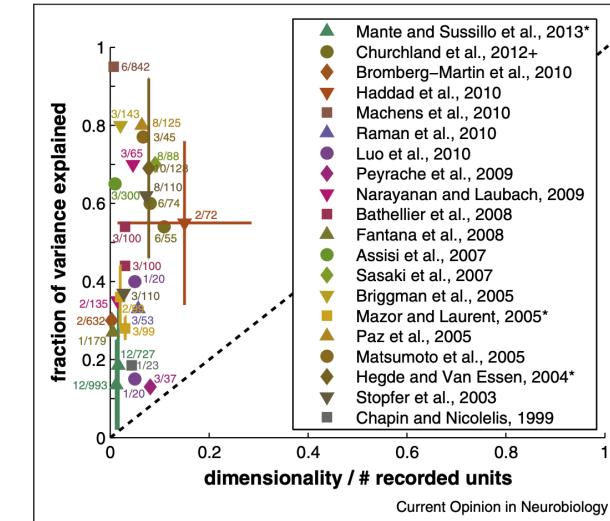
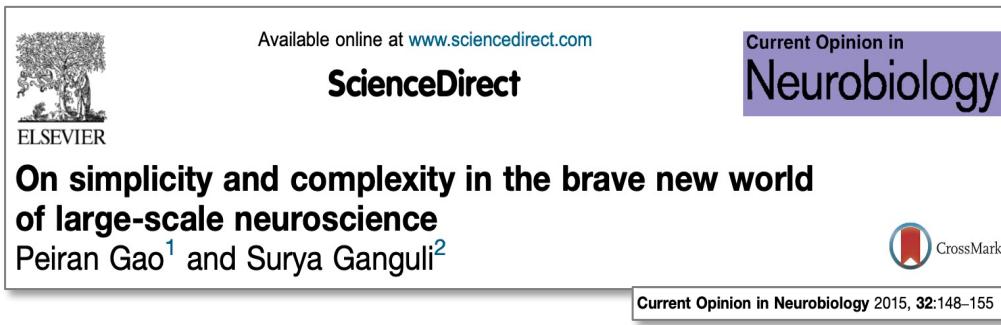
# PCA and low-dimensionality



- Each neuron is approximately equal to a linear combination of a few principal components.
- Suggests that the dimensionality of the whole brain is much smaller than the number of neurons.



# PCA and low-dimensionality



In many experiments (e.g. in insect [20,23–26] olfactory systems, mammalian olfactory [26,27], prefrontal [21,22\*,28–30], motor and premotor [31,32], somatosensory [33], visual [34,35], hippocampal [36], and brain stem [37] systems) a *much* smaller number of dimensions than the number of recorded neurons captures a large amount of variance in neural firing rates.

**Why should we expect low dimensionality for large neuronal networks ?**

# **Neuronal activity: resurgence of the dynamical system approach**

**Cell**

Volume 177, Issue 4, 2 May 2019, Pages 970–985.e20

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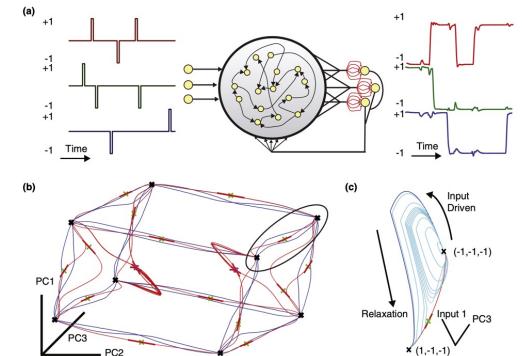
Article

**Neuronal Dynamics Regulating Brain and Behavioral State Transitions**

Aaron S. Andelman <sup>1, 2, 12</sup>, Vanessa M. Burns <sup>3, 12</sup>, Matthew Lovett-Barron <sup>1, 2</sup>, Michael Broxton <sup>4</sup>, Ben Poole <sup>4</sup>, Samuel J. Yang <sup>5</sup>, Logan Grosenick <sup>1, 6</sup>, Talia N. Lerner <sup>1</sup>, Ritchie Chen <sup>1</sup>, Tyler Benster <sup>6</sup>, Philippe Mourrain <sup>7, 8, 9</sup>, Marc Levoy <sup>4</sup>, Kanaka Rajan <sup>10</sup>, Karl Deisseroth <sup>1, 2, 8, 11, 13</sup> , 

 Current Opinion in Neurobiology  
Volume 70, October 2021, Pages 163-170





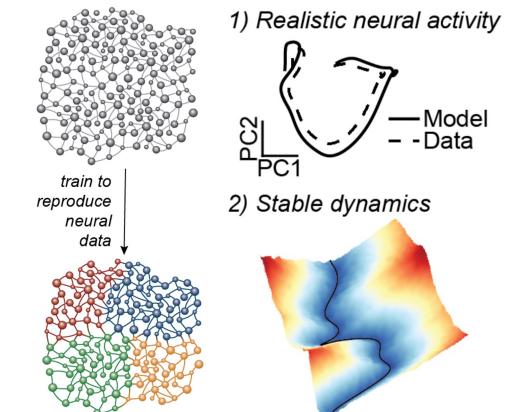
D. Sussillo 2014

# Computation Through Neural Population Dynamics

# The role of population structure in computations through neural dynamics

[Alexis Dubreuil](#) , [Adrian Valente](#) , [Manuel Beiran](#), [Francesca Mastrogiovanni](#) & [Srdjan Ostoic](#) 

Nature Neuroscience 25, 783–794 (2022) | Cite this article



M. G. Perich et al. 2020

# Firing rate model for recurrent neural networks

Grossberg, Amari, Wilson–Cowan, Hopfield, ...

$$\frac{dx_i}{dt} = -x_i + \sigma\left(\sum_j w_{ij}x_j - \mu_i\right)$$

$x_i(t)$  = activity of neuron  $i$  at time  $t$

$\mu_i$  = activation threshold of neuron  $i$

$w_{ij}$  = weight of the connection from neuron  $j$  to neuron  $i$

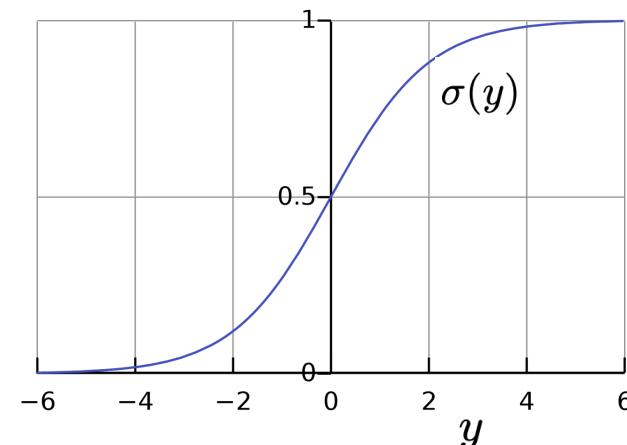
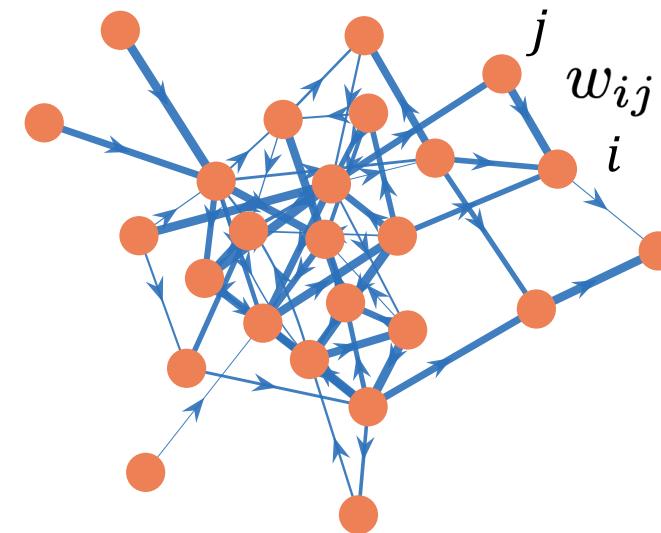
$$i, j \in \{1, 2, \dots, N\}$$

## NEURAL NETWORK DYNAMICS

Annual Review of Neuroscience

Vol. 28:357-376 (Volume publication date 21 July 2005)  
First published online as a Review in Advance on March 22, 2005  
<https://doi.org/10.1146/annurev.neuro.28.061604.135637>

Tim P. Vogels, Kanaka Rajan, and L.F. Abbott



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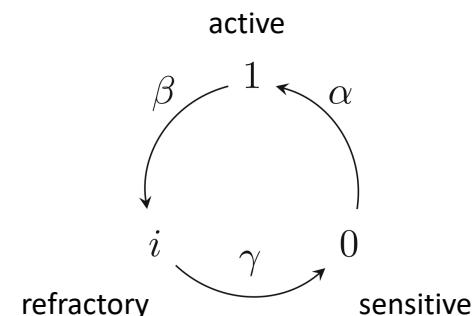


Original Article | Open Access | Published: 05 September 2022

Beyond Wilson–Cowan dynamics: oscillations and chaos without inhibition

Vincent Painchaud, Nicolas Doyon & Patrick Desrosiers

*Biological Cybernetics* (2022) | Cite this article



# Firing rate model for recurrent neural networks

Grossberg, Amari, Wilson–Cowan, Hopfield, ...

$$\frac{dx}{dt} = -x + \sigma(Wx - \mu)$$

$x$  =  $N \times 1$  network state vector at time  $t$

$\mu$  =  $N \times 1$  vector of thresholds

$W$  =  $N \times N$  weight matrix

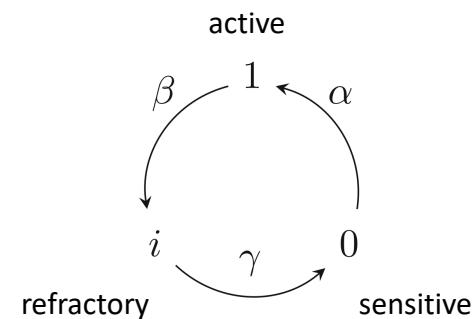


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Beyond Wilson–Cowan dynamics: oscillations and chaos without inhibition

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# Recurrent neural networks can be trained to fit the data

$$\frac{dx}{dt} = -x + \sigma(Wx - \mu)$$

Unknown parameters

PLOS ONE

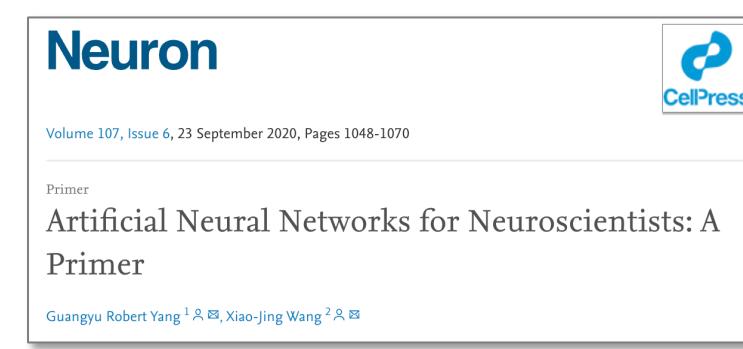
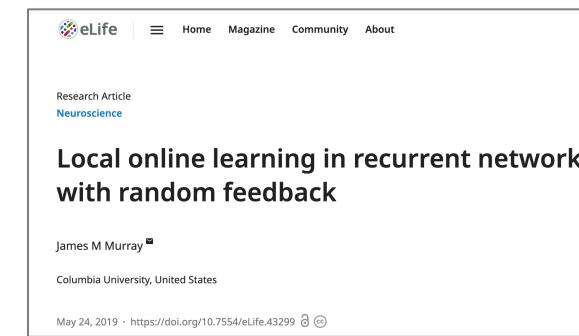
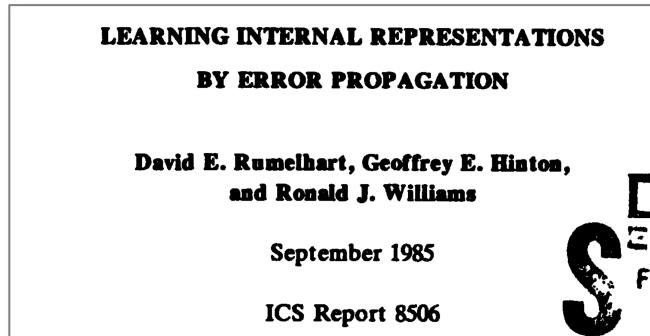
OPEN ACCESS

RESEARCH ARTICLE

full-FORCE: A target-based method for training recurrent networks

Brian DePasquale, Christopher J. Cueva, Kanaka Rajan, G. Sean Escola, L. F. Abbott

Published: February 7, 2018 • <https://doi.org/10.1371/journal.pone.0191527>



# Recurrent neural networks can be trained to fit the data



Jérémie Gince



Anthony Drouin



Simon Hardy



Daniel Côté

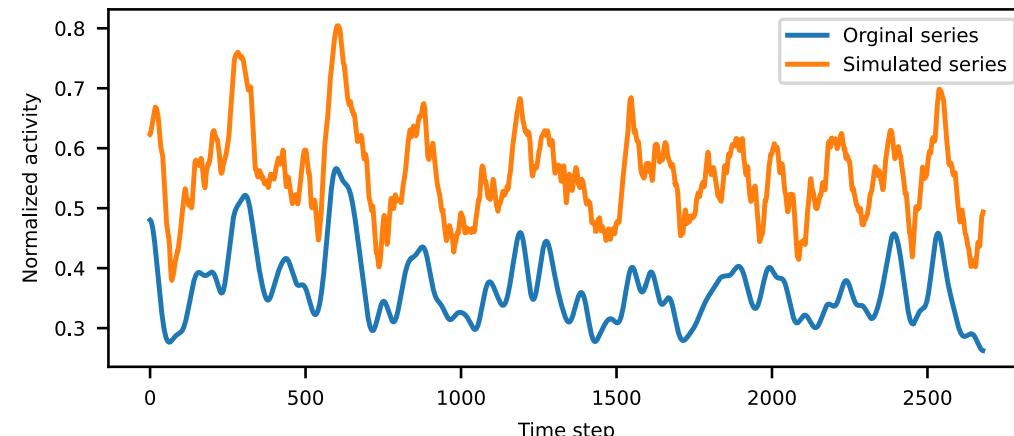


Antoine Légaré

<https://github.com/NeuroTorch/NeuroTorch>

**Current Version (v0.0.1-alpha)**

- Image classification with spiking networks.
- Classification of spiking time series with spiking networks.
- Time series classification with spiking or Wilson-Cowan.
- Reconstruction/Prediction of time series with Wilson-Cowan.
- Reconstruction/Prediction of continuous time series with spiking networks.
- Backpropagation Through Time.



Worst solution  
RMSE=0.19

# Recurrent neural networks can be trained to fit the data



Jérémie Gince



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<https://github.com/NeuroTorch/NeuroTorch>

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- Backpropagation Through Time.

After training:

$$\frac{dx}{dt} = -x + \sigma(Wx - \mu)$$

Learned parameters

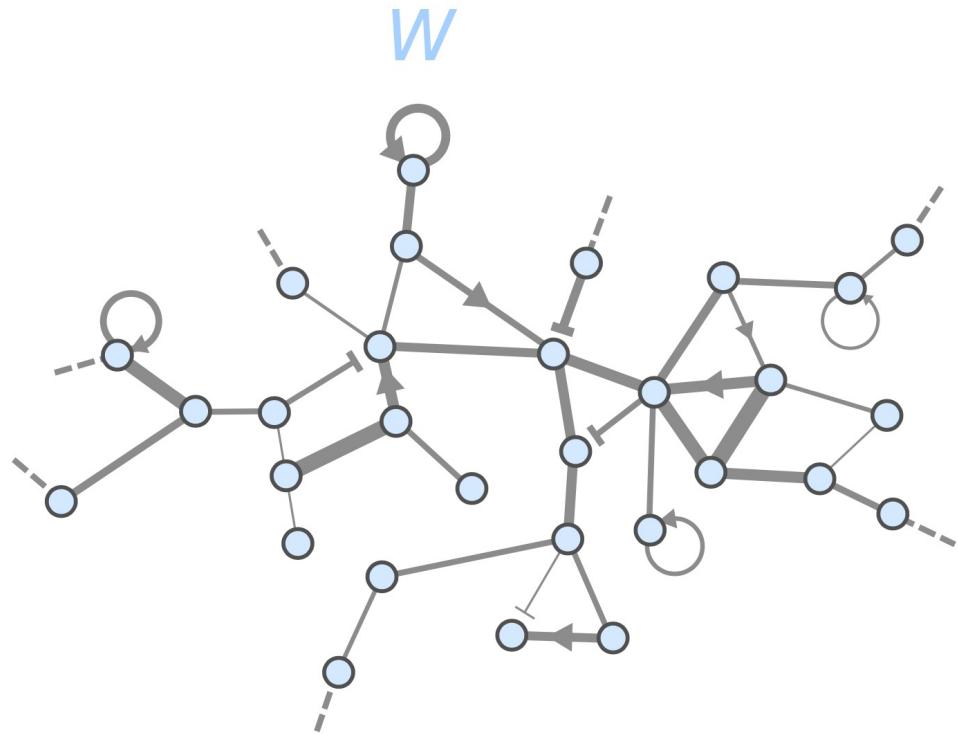
High-dimensional  
dynamical system

**How to reduce the dimensionality of large dynamical systems?**

**Is it justified to reduce the dimensionality?**

# Our point of view: complex systems theory

## *Complex network*

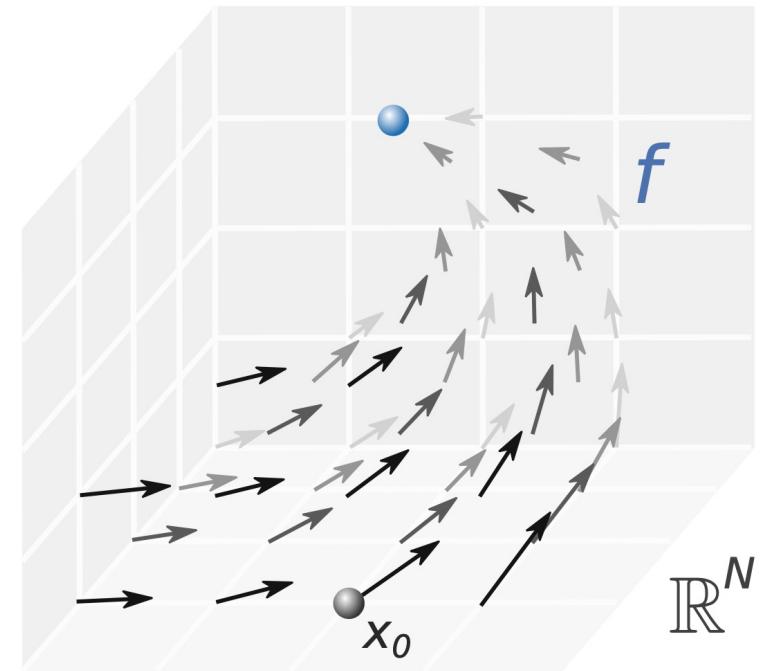


## Vector field



## *High-dimensional dynamics*

$$\dot{x} = f(x; W)$$

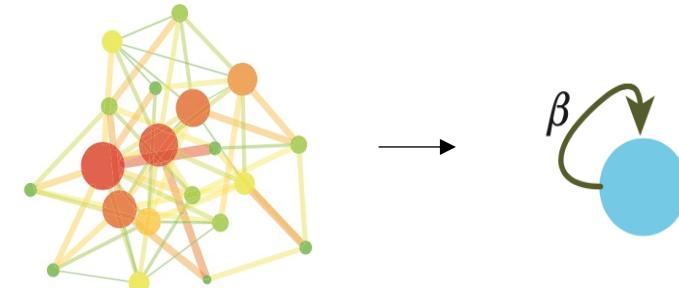


# Our first inspiration: Dimension reduction to study resilience

## Universal resilience patterns in complex networks

Jianxi Gao<sup>1\*</sup>, Baruch Barzel<sup>2\*</sup> & Albert-László Barabási<sup>1,3,4,5</sup>

18 FEBRUARY 2016 | VOL 530 | NATURE | 307



Dimension reduction based on degrees:

$$\frac{dx_i}{dt} = F(x_i) + \sum_j W_{ij}G(x_i, x_j) \quad \rightarrow \quad \frac{dx}{dt} = F(x) + \beta G(x, x)$$

where

$$x = \frac{\sum_{i,j} W_{ij}x_j}{\sum_{i,j} W_{ij}} = \text{degree-weighted average activity}$$

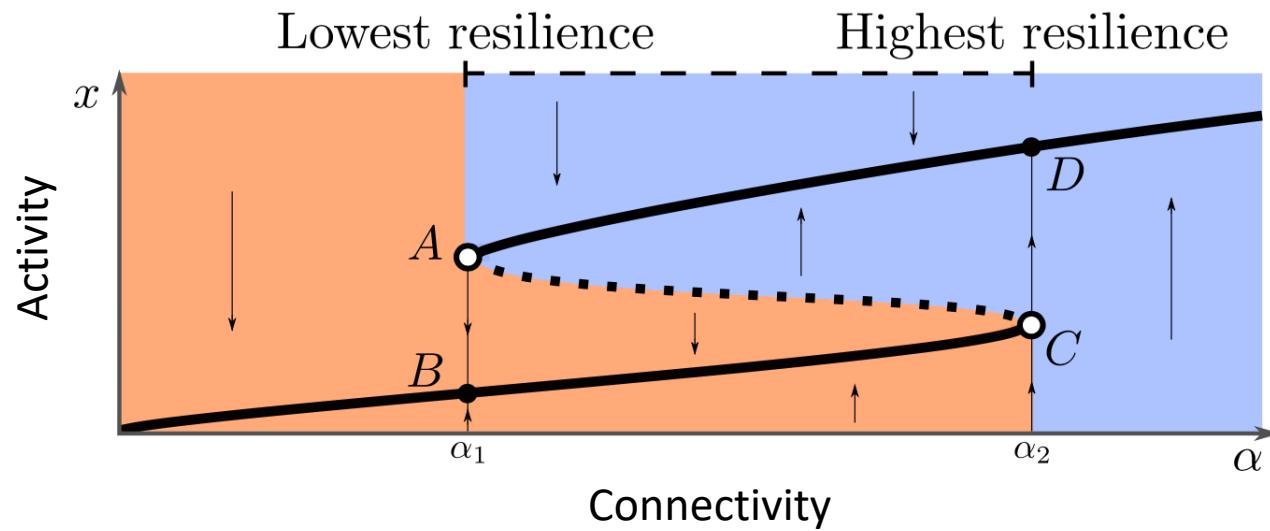
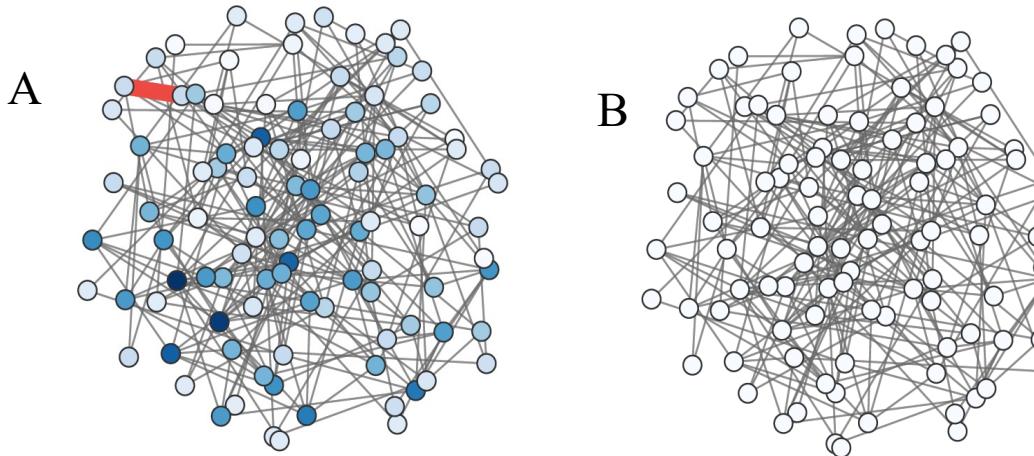
$$\beta = \frac{\sum_{i,j,k} W_{ij}W_{jk}}{\sum_{i,j} W_{ij}} = \text{degree-weighted average degree}$$

$$i, j, k \in \{1, \dots, N\}$$

**Success:** The reduction allows studying resilience.

**Problem:** The reduction does not work well with all networks.

# Resilience in complex systems

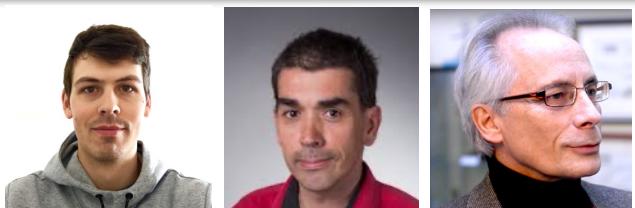


# More complete solutions:

PHYSICAL REVIEW X 9, 011042 (2019)

## Spectral Dimension Reduction of Complex Dynamical Networks

Edward Laurence,<sup>1,2</sup> Nicolas Doyon,<sup>2,3,4</sup> Louis J. Dubé,<sup>1,2</sup> and Patrick Desrosiers<sup>1,2,4</sup>



PHYSICAL REVIEW RESEARCH 2, 043215 (2020)

## Threefold way to the dimension reduction of dynamics on networks: An application to synchronization

Vincent Thibeault<sup>1,2,\*</sup> Guillaume St-Onge<sup>1,2</sup> Louis J. Dubé,<sup>1,2</sup> and Patrick Desrosiers<sup>1,2,3,†</sup>



## DART: Dynamics Approximate Reduction Technique

### Complete dynamics

$N \gg 1$  dimensions

$$\frac{dz_j}{dt} = F(z_j) + \sum_{k=1}^N A_{jk}G(z_j, z_k)$$

$q \ll N$  dimensions

$$\frac{dZ_\mu}{dt} \approx F(Z_\mu) + \sum_{\nu=1}^q \mathcal{A}_{\mu\nu}G(Z_\mu, Z_\nu)$$

### Reduced dynamics



### PAPER

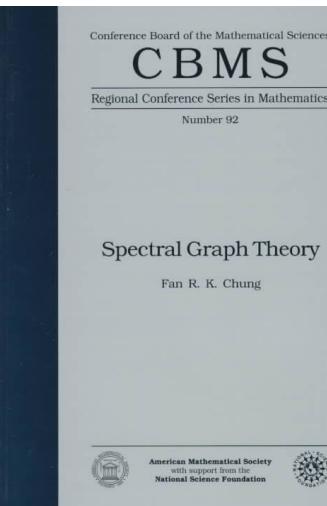
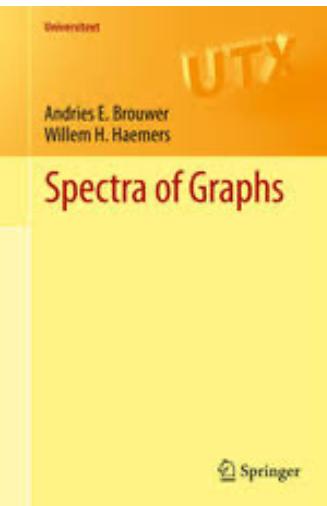
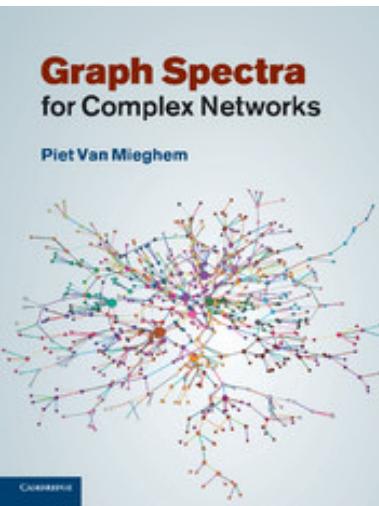
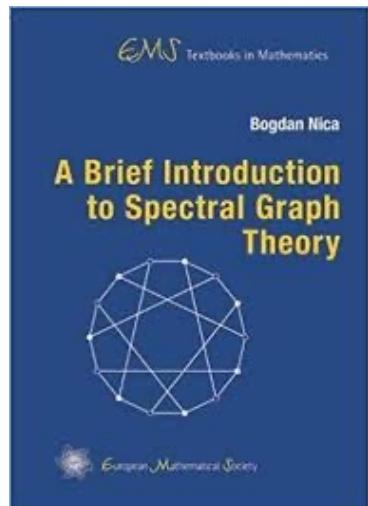
## Dimension reduction of dynamics on modular and heterogeneous directed networks

Marina Vegué,<sup>a,b,\*</sup> Vincent Thibeault,<sup>a,b</sup> Patrick Desrosiers<sup>a,b,c</sup> and Antoine Allard<sup>a,b</sup>



# Theoretical framework: Spectral Graph Theory and Matrix factorization

Good sources of information:



**Learning the parts of objects by non-negative matrix factorization**

Daniel D. Lee\* & H. Sebastian Seung\*†

NATURE | VOL 401 | 21 OCTOBER 1999 | www.nature.com

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 32, NO. 1, JANUARY 2010

**Convex and Semi-Nonnegative Matrix Factorizations**

Chris Ding, Member, IEEE, Tao Li, and Michael I. Jordan, Fellow, IEEE

Old treasure:

A GENERALIZED INVERSE FOR MATRICES  
BY R. PENROSE  
Communicated by J. A. TODD  
Received 26 July 1954

Recent treasure:

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 60, NO. 8, AUGUST 2014  
**The Optimal Hard Threshold for Singular Values is  $4/\sqrt{3}$**

Matan Gavish, Student Member, IEEE, and David L. Donoho, Member, IEEE

# Our latest approach: Singular Value Decomposition

arXiv:2208.04848

The low-rank hypothesis of complex systems:  
From empirical and theoretical evidence to the emergence of higher-order interactions

Vincent Thibeault,<sup>1, 2, \*</sup> Antoine Allard,<sup>1, 2, †</sup> and Patrick Desrosiers<sup>1, 2, 3, ‡</sup>

$$W = U \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_r \\ & & & & 0 \\ & & & & & \ddots \\ & & & & & & 0 \end{pmatrix} V^T$$

*Orthogonal N x N matrix      Diagonal N x N matrix      Orthogonal N x N matrix*

Rank : number of linearly independent rows/columns of a matrix

$$W \underset{\sim}{\sim}$$

$$U_n$$

$N \times n$

$$\begin{matrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \end{matrix}$$

$n \times n$

$$V_n^T$$

$n \times N$

*Optimal low-rank approximation  
( Eckart-Young theorem )  
( Exact for  $n = r$  )*

$$W = \boxed{\sigma_1 X_1} + \sigma_2 X_2 + \dots + \boxed{\sigma_r X_r}$$

rank 1

**largest contribution**                            **smallest contribution**

```
graph TD; A[W] --- B["rank 1"]; B --- C["\u03c3\u2081 X\u2081"]; B --- D["\u03c3\u2082 X\u2082"]; B --- E["\u03c3\u208r X\u208r"]
```

# Flags have low rank



Inspired by a lecture by Alex Townsend on [Rapidly Decreasing Singular Values](#)

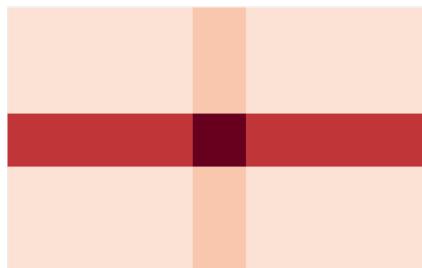
France's flag has rank 1



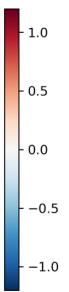
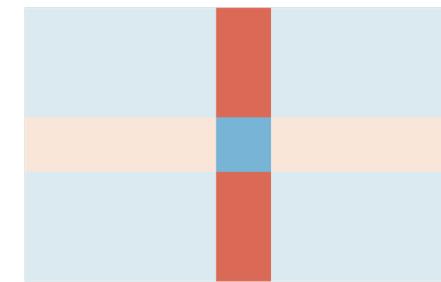
England's flag has rank 2



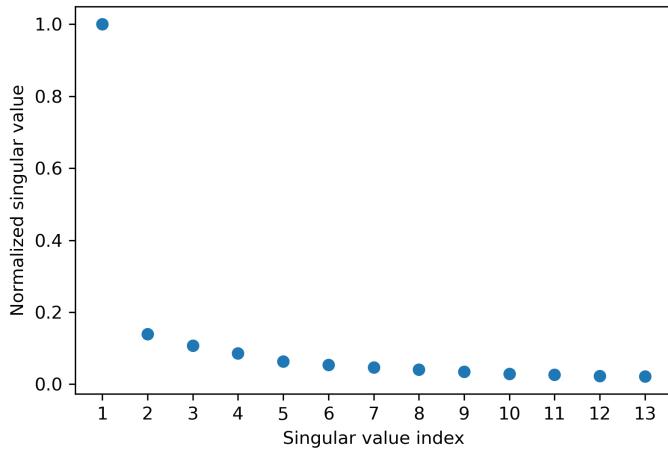
=



+

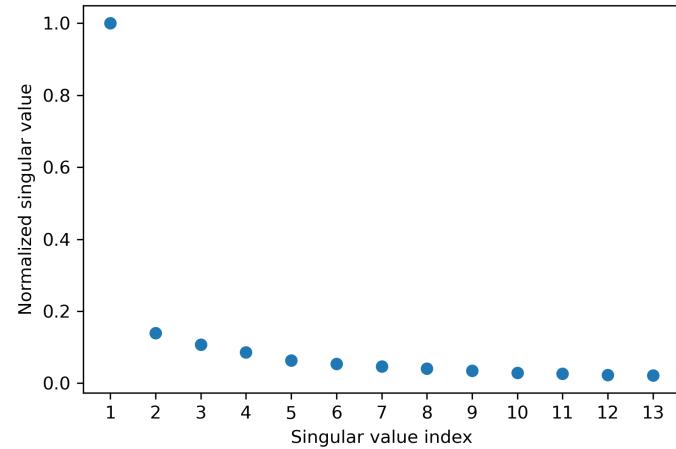


# Nunavik's flag is more complex



Good low-rank  
approximation

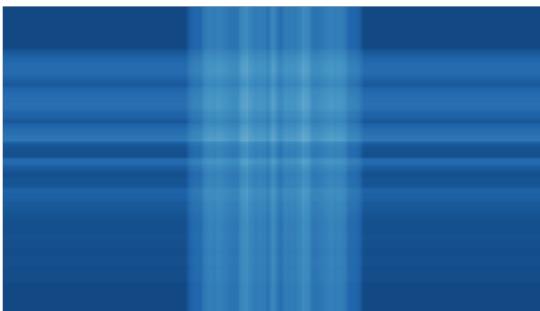
# Nunavik's flag is more complex



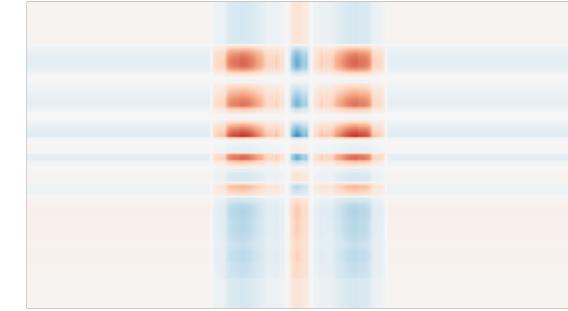
Good low-rank approximation



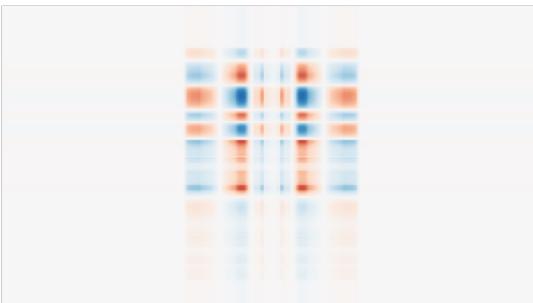
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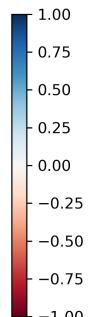


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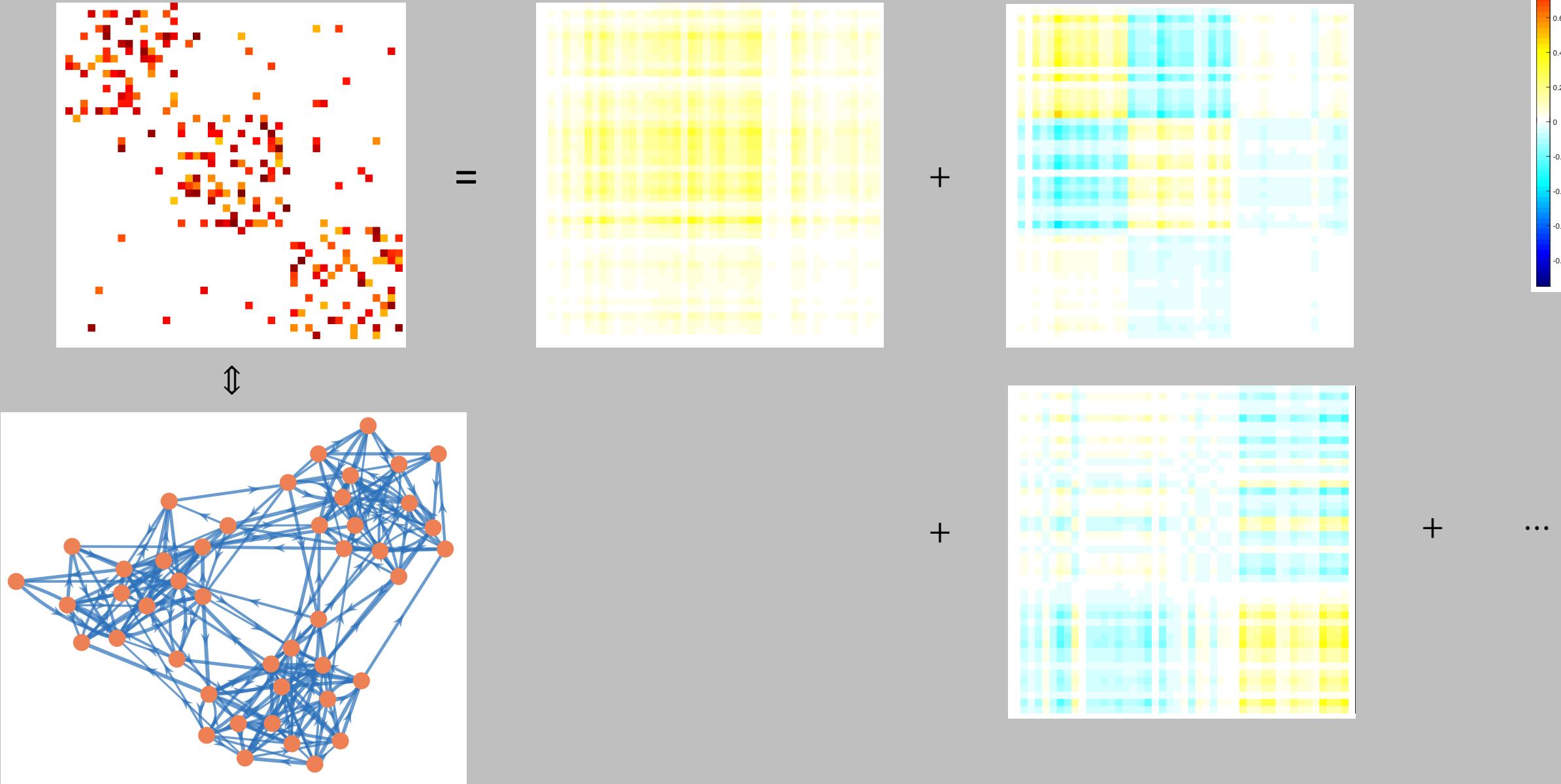


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...



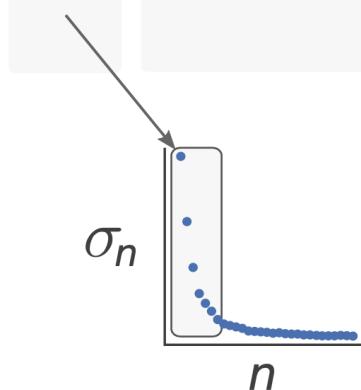
# Low-rank approximations work for some networks



## Fundamental notion: effective rank

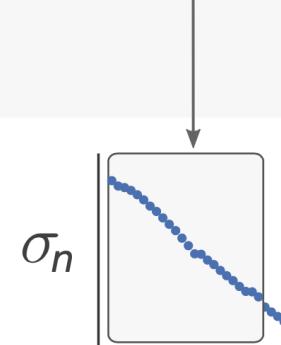
Effective rank of  $W$  = number of significant singular values of  $W$

Example: the stable rank defined as  $\text{srank}(W) = \frac{\sum_{i=1}^N \sigma_i^2}{\sigma_1^2}$



*Low effective rank*

*or*

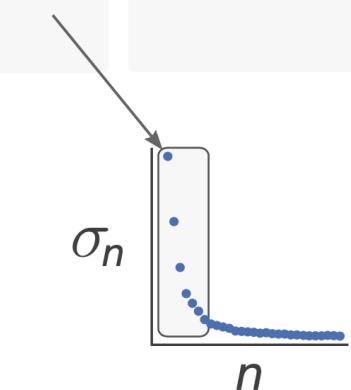


*High effective rank*

?

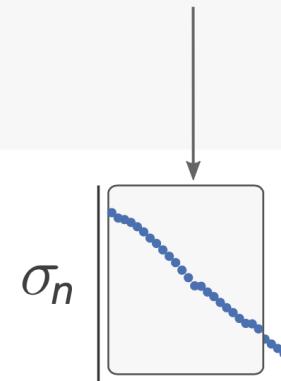
# Fundamental notion: effective rank

Effective rank of  $W$  = approximate dimension of the space generated by all  $Wx$



*Low effective rank*

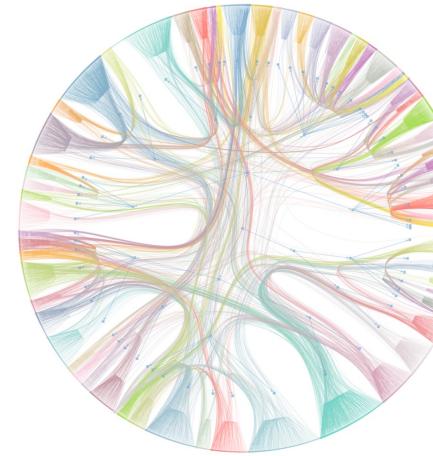
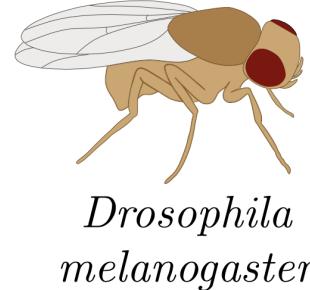
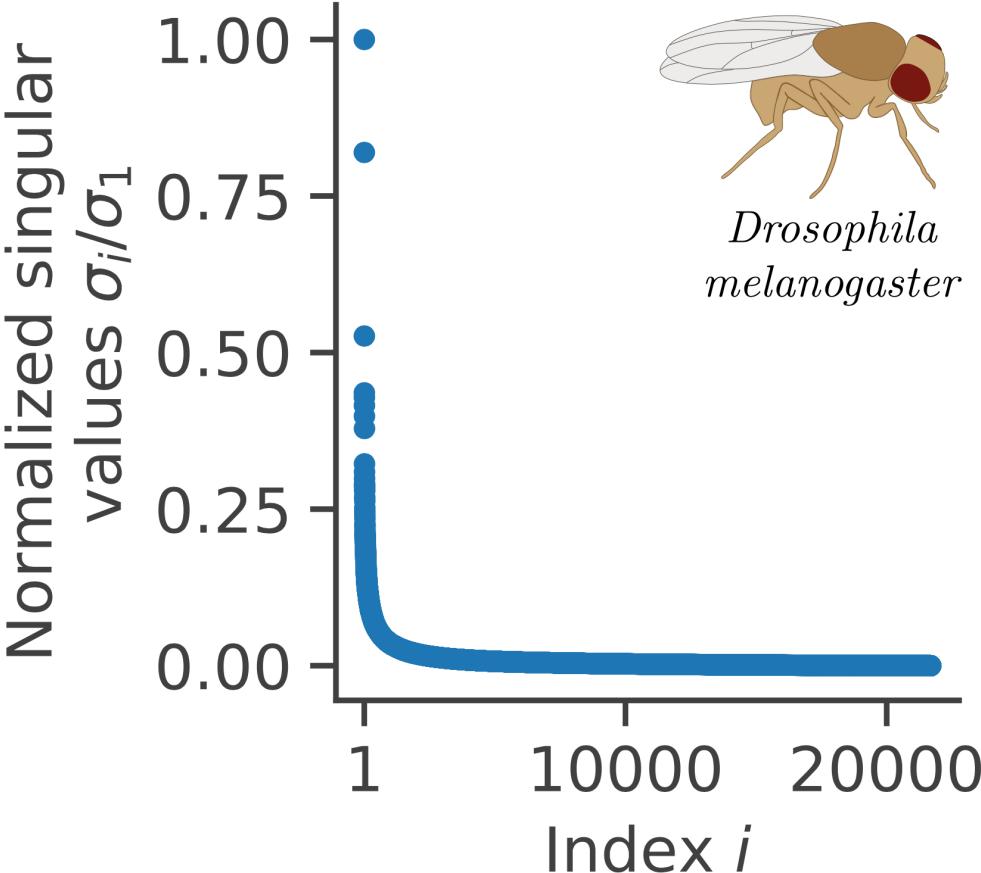
*or*



*High effective rank*

?

# Experimental results: Connectomes have low effective rank



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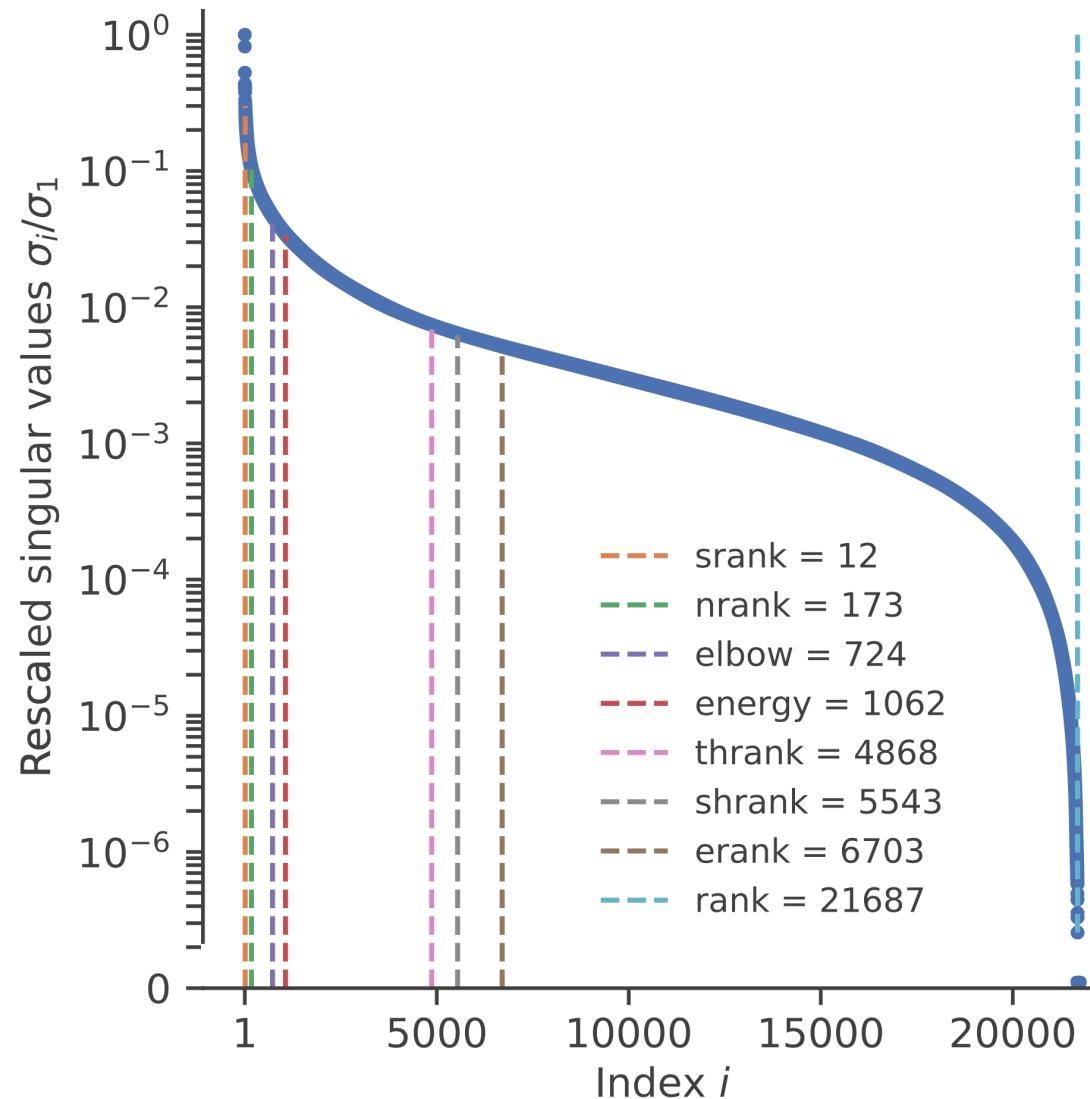
**A connectome and analysis of the adult *Drosophila* central brain**

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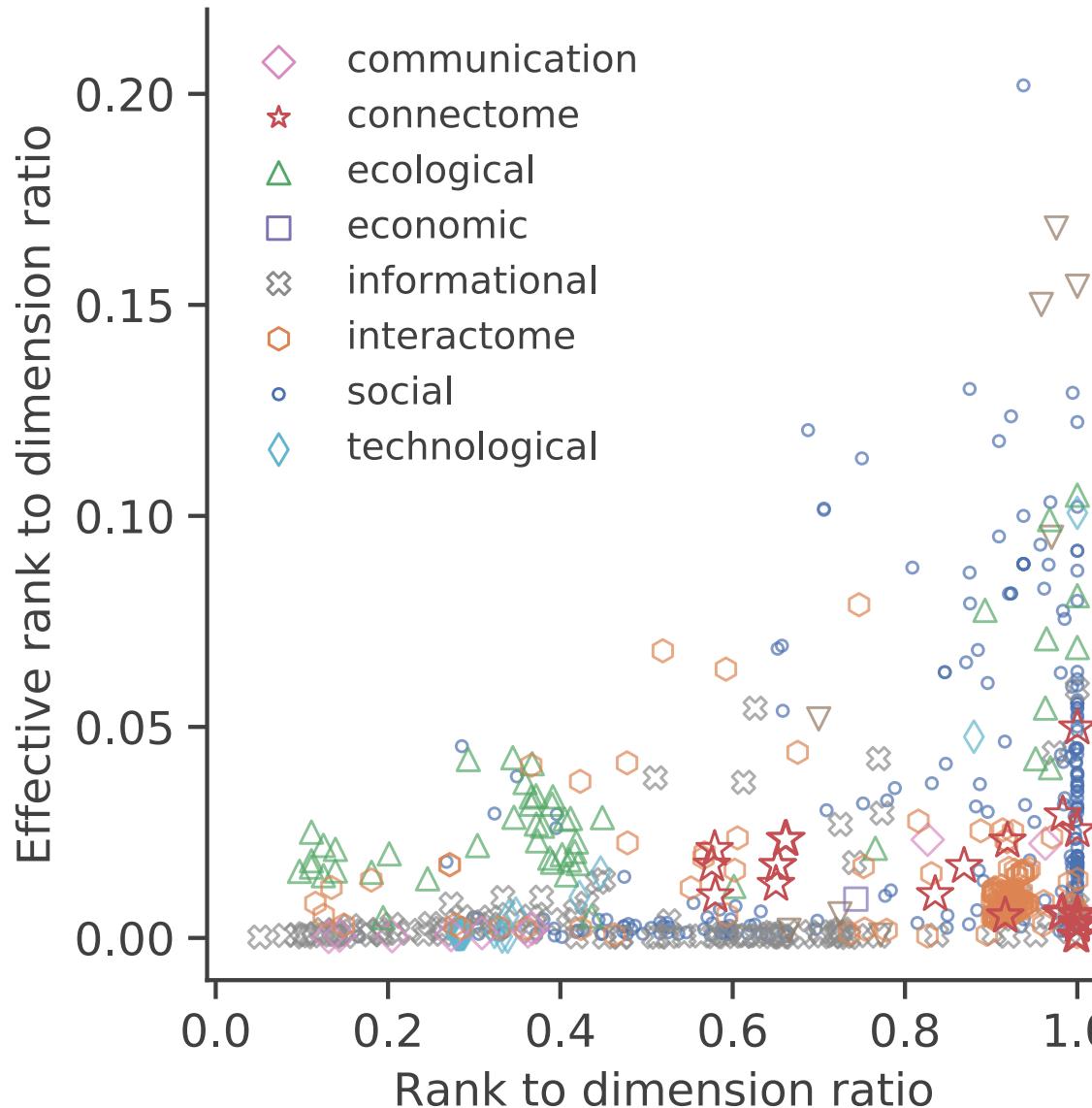
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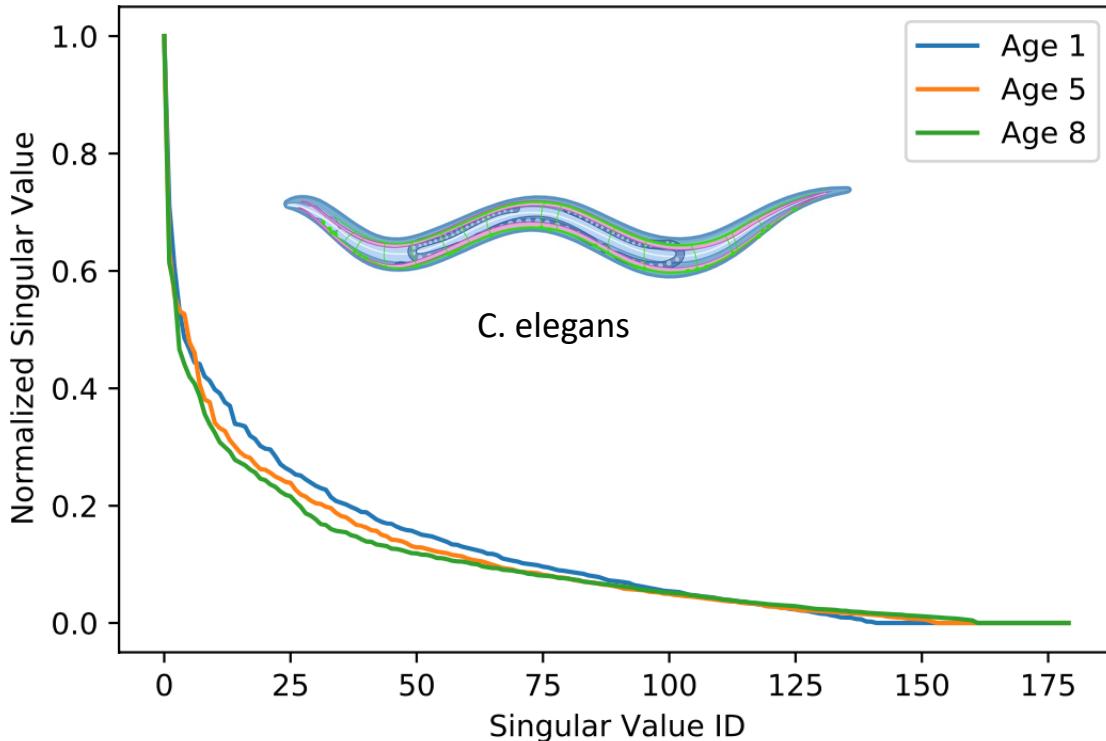
# Experimental results: Connectomes have low effective rank



# Experimental results: real complex networks have low effective rank



# Observation: Maturation seems to reduce effective rank



Singular values of the matrices describing the connectivity of the *C. elegans* brain at different maturation stages. The stable ranks are 21.6 (age 1), 19.7 (age 5), 18.5 (age 8).

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## Connectomes across development reveal principles of brain maturation

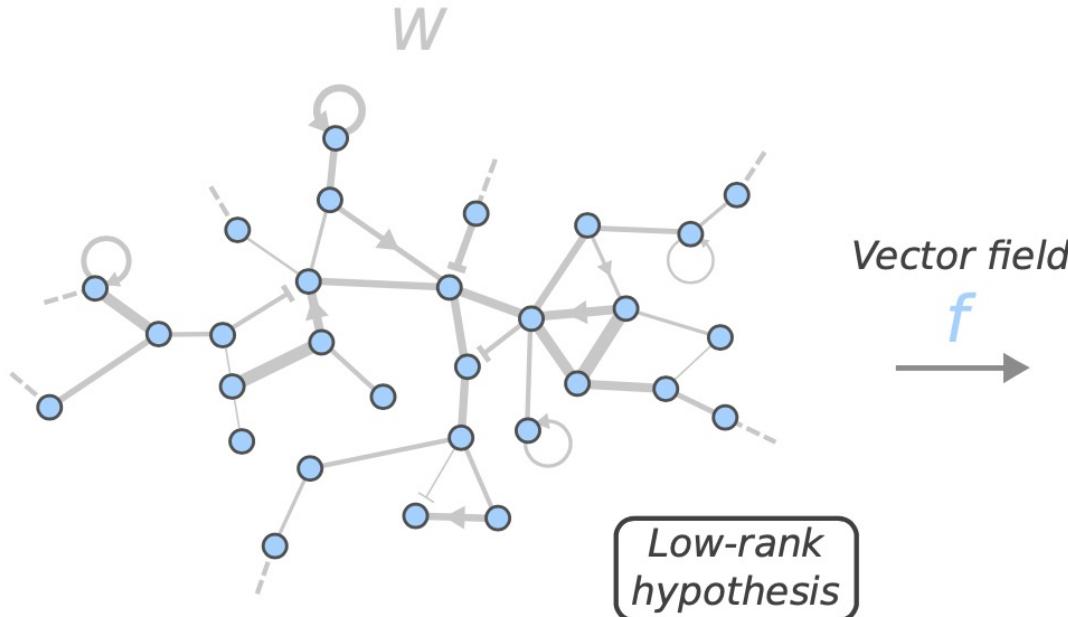
[Daniel Witvliet](#)✉, [Ben Mulcahy](#), [James K. Mitchell](#), [Yaron Meirovitch](#), [Daniel R. Berger](#), [Yuelong Wu](#), [Yufang Liu](#), [Wan Xian Koh](#), [Rajeev Parvathala](#), [Douglas Holmyard](#), [Richard L. Schalek](#), [Nir Shavit](#), [Andrew D. Chisholm](#), [Jeff W. Lichtman](#)✉, [Aravinthan D. T. Samuel](#)✉ & [Mei Zhen](#)✉

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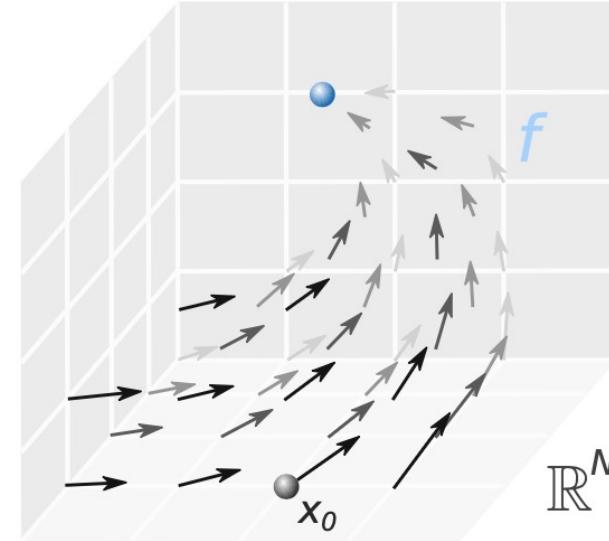
**What are the dynamical consequences of  
low effective rank ?**

### Complex network

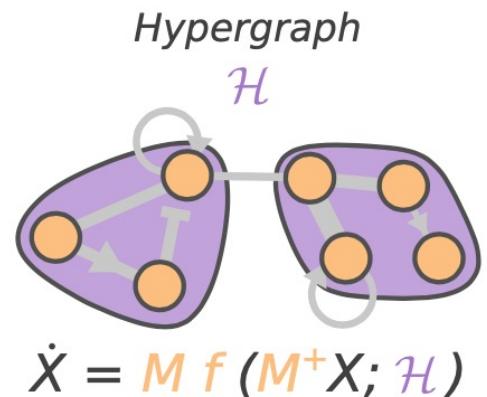


### High-dimensional dynamics

$$\dot{x} = f(x; W)$$



Emergence of  
higher-order  
interactions



Vector field



Low-dimension  
hypothesis

$$Optimal\ vector\ field\\ G = M \circ f \circ M^+$$



$$\dot{X} = G(X; \text{Structure?})\\ Low-dimensional\ dynamics$$

Reduction matrix  
 $M$

Optimal  $M$  determined by the weight matrix  $W$



Complete dynamics :  $\dot{x} = f(x)$

Reduced dynamics :  $\dot{X} = G(X)$  where  $X = Mx$

### THEOREM (SIMPLIFIED)

*The vector field  $G^*$  that minimizes the quadratic error between the projected dynamics  $\dot{p} = f(p)$  with  $p = M^+ Mx$  and the reduced dynamics in  $\mathbb{R}^N$  [ $M^+ G(X)$ ] is*

$$G^*(X) = Mf(M^+ X).$$

*Proof :* Just use least-squares.

## THEOREM (SIMPLIFIED)

The alignment error  $\mathcal{E}_f(x)$  for some  $x \in \mathbb{R}^N$  is upper-bounded by

$$\mathcal{E}_f(x) \leq \frac{1}{\sqrt{n}} \left[ \|V_n^\top J_x(x', y')(I - V_n V_n^\top)x\| + \frac{\sigma_{n+1}}{\sigma_1} \|V_n^\top J_y(x', y')\|_2 \|x\| \right].$$

$\sigma_i$  :  $i$ -th singular values of  $W$

$V_n$  :  $n$ -truncated right singular vector matrix

$J_x, J_y$  : Jacobian matrices evaluated at some point  $x', y'$

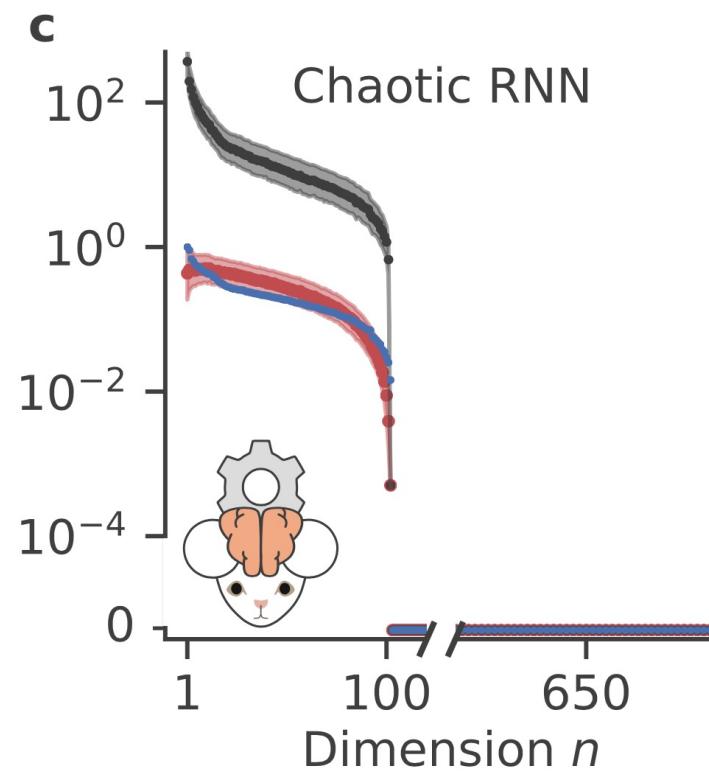
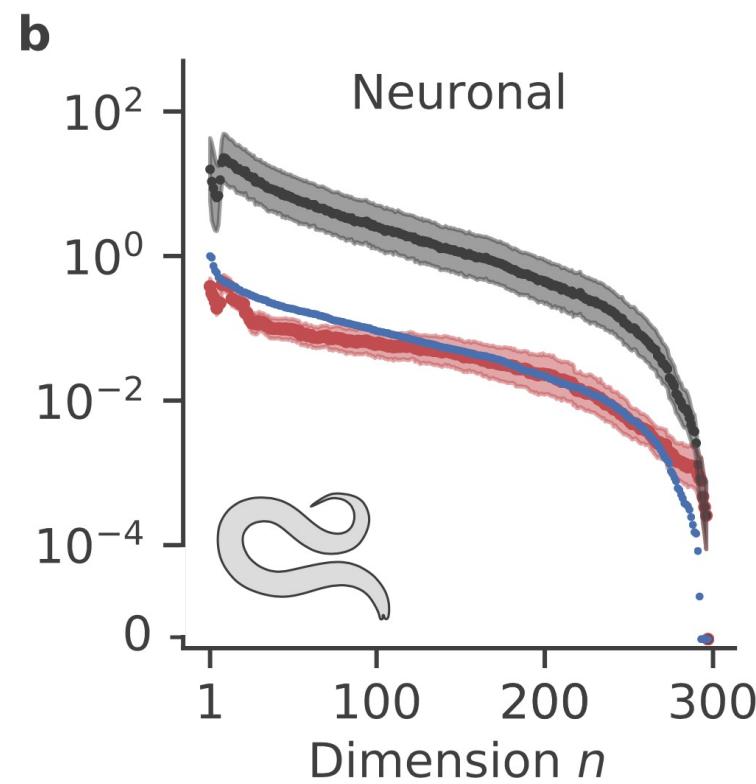
$n$  : dimension of the reduced system

$$J_x(x', y') = aI \text{ and } n \geq \text{rank}(W) \quad \Rightarrow \quad \text{Exact dimension reduction}$$

• • • Average alignment error  $\langle \mathcal{E} \rangle$

• • • Average upper-bound on  $\mathcal{E}(x)$

••• Rescaled singular values  $\frac{\sigma_n}{\sigma_1}$



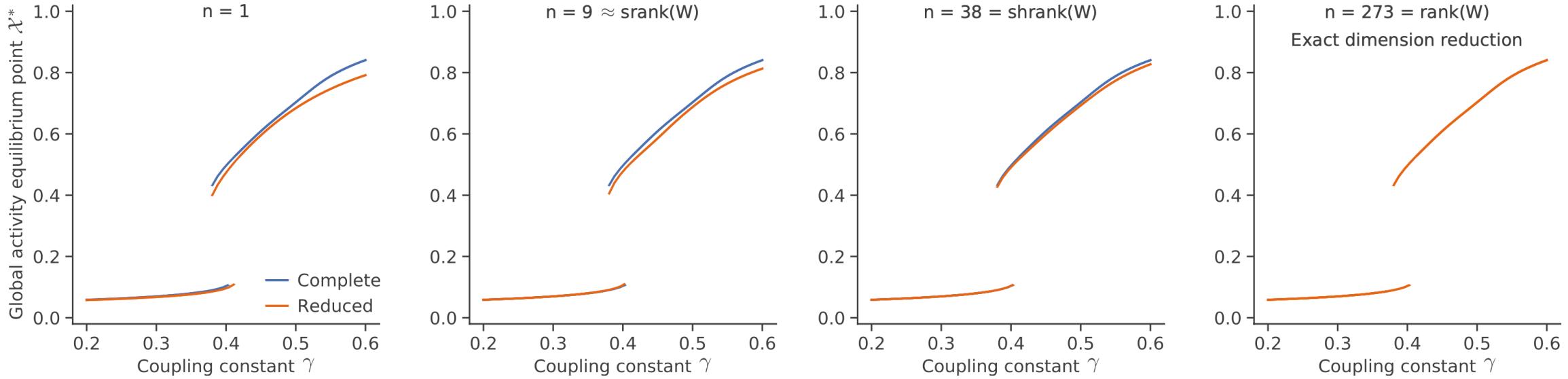


Fig. S8: Comparison between the global observable at equilibrium  $\chi^*$  of the complete (blue) and reduced (orange) Wilson-Cowan dynamics on the (unsigned) *C. elegans* connectomes ( $N = 279$ ,  $\text{rank}(W) = 273$ ) vs. the global coupling  $\gamma$  for  $n \in \{1, 9, 38, 273\}$ . Parameters:  $d = 1$ ,  $a = 0$ ,  $b = 1$ ,  $c = 3$ . For the weight matrix, see the [GitHub repository](#), module `get_real_network.py`, function `get_connectome_weight_matrix(graph_name="celegans")`. The effective ranks of this connectome with weight matrix  $W$  are  $\text{srank}(W) \approx 9$ ,  $\text{thrank}(W) = 27$ ,  $\text{elbow}(W) = 31$ ,  $\text{nrank}(W) \approx 36$ ,  $\text{shrank}(W) = 38$ ,  $\text{energy}(W) = 106$ , and  $\text{erank}(W) \approx 192$ .

## Take-home messages

1. Whole brain neuronal activity can be modeled using firing rate models, which are high dimensional.
2. Real networks, and especially ***connectomes, have low effective rank.***
3. Large neuronal networks with ***firing rate dynamics possess low-dimensional dynamical systems*** that approximately describe the activity at large scale.
4. Alignment errors of reduced vector fields can rapidly decrease following the singular values of complex networks.
5. Our theoretical findings support the use of Principal Component Analysis when analyzing neuronal activity.
6. Dimension reduction can lead to dynamics with *higher-order interactions.*

Still so much work to do ...

# Thank you! Questions?



<https://dynamicalab.github.io/>

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