Legged Robotics, HW 8
Homework due on 03-18-2021, Topic: Trajectory Optimization. Email solutions to pranav@uic.edu.

1. **Trajectory optimization of walker with hip torque and push-off**
Model: Figure 1, left shows one step of a walker and the right shows the schematic. The walker has legs of length ℓ, mass m, a center of mass at distance c from the hinge point, inertia about the center of mass I, a point-mass of M, a ramp of slope γ (taken zero here). Gravity is g and points downwards. The angle between the stance leg and the normal to the ramp is θ_1 and the swing leg makes and angle of θ_2 with the stance leg as shown. There is a linear actuator at the foot (not shown) that applies an impulsive push-off P and a hip actuator that applies a torque T. The slope has been included in the derivation, but will be taken to be zero, $\gamma = 0$. Assume the following: $M = 1$, $m = 0.5$, $I = 0.02$, $\ell = 1$, $c = 0.5$, $g = 1$, and $\gamma = 0$

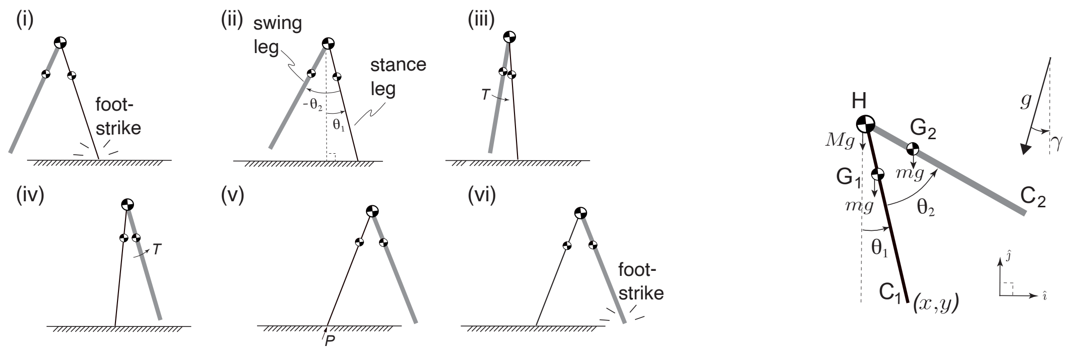

Figure 1: (Left) One step of a powered walker. (Right) A schematic of the passive walker as a floating base.

Optimization problems: For both these problems assume that the state at the Poincaré section which is after foot-strike is θ_1, $\dot{\theta}_1$, θ_2, $\dot{\theta}_2$. You goal is to find periodic solutions such that the following costs are minimized

(a) Assume $P = 0.1$ and minimize $\int T^2 dt$ where T is the hip torque, t is the time, and the integral is over a single step. The solution to this case is: cost is zero and $T(t) = 0$ (Why?). This case can be used to check if you code is performing as intended. No need to submit this code, its only for you to check your code. Aside, a similar solution ($T(t) = 0$) exists if γ is some small value and $P = 0$ (the passive solution).

(b) minimize $+P^2 + \int T^2 dt$ (where P and T are both unknowns) while imposing a step velocity of 1 and step length of 0.5. Make plots of T vs. time and print out the cost and P. Note that all parameters are non-dimensional.

HINTS:

(a) Use either collocation or single shooting. No need to use both.

(b) Consider using step time (t_{step}) as an optimization variable. This will make the optimization converge fast. (Why?)

(c) The step length is $d_{\text{step}} = 2\ell \sin(0.5\theta_2^*)$ where θ_2^* is the hip angle at foot-strike.

(d) The step velocity is given by $v_{\text{step}} = \frac{v_{\text{step}}}{t_{\text{step}}}$.

(e) The equations of motion are provided in the file *poweredderive_lagrange.m*

(f) The solution when $P = 0.1$ and $T = 0$ (without using optimization) is in the file *poweredwalker.m* and is for your reference.