More universal properties

Pullbacks:

For a given objects A & B

$$\begin{tikzcd} V \arrow[dr, "h"] \arrow[drr, bend left, "t"] \arrow[ddr, bend right, "s"'] & & \\ & U \arrow[r, "q"] \arrow[d, "p"'] \arrow[dr, phantom, "\mathrm{I}"] & B \arrow[d, "g"] \\ & A \arrow[r, "f"'] & C \end{tikzcd}$$

II

$U = A \times B$, (p & q are projection morphisms)

If the squares I & II commute for another object C, then there exists a pullback h from $V \to U$ such that it is unique.

For sets:

$$\begin{tikzcd} V \arrow[dr, "h"] \arrow[drr, bend left, "t"] \arrow[ddr, bend right, "s"'] & & \\ & U \arrow[r, "q"] \arrow[d, "p"'] & B \arrow[d, "g"] \\ & A \arrow[r, "f"'] & C \end{tikzcd}$$

$a \in A$ & $b \in B$

$U = A \times B$.

$\longrightarrow$ s.t $\{ (a,b) \in A \times b ; f(a) = g(b) \}$ iff (has to commute)

$\longrightarrow$ $x \in V$

To show: $h(x) = (s(x), t(x))$

$s(x) \in A$ & $t(x) \in B$

<u>To prove:</u> $f(s(x)) = g(t(x))$ (has to commute)

So, $(s(x), t(x)) \in A \times B$ from square II

and as x is an element in V.

this is uniquely mapped to $A \times B$ hence

$h(x) = (s(x), t(x))$

Pushouts:

Duals of pullback