Takeaway: understanding categorical frameworks like category, functors, natural transformations & universal properties/construction using ~~the~~ one of its applications, databases.

→ Database:
- It is a system of interlocking tables
- Data becomes information when it is stored in a given formation

Elements of a table (one of the many) in a database

1) Primary key (ID- column): a unique entry for each row, so it is called row label.

2) Foreign key (keys): columns that can be used to interlock the tables, or establish relationship between tables.

row: an instant of ~~ta~~ the entity the table represents
(records)

column (Field): represents a property / ~~f~~
(attribute)

- reference structure for a given a database: how table interlocks via ~~f~~ foreign keys.

database → (how info is being stored)

Explain the schema:

- Tables name as nodes in Hasse diagram (graph)
- Each ~~arrow~~ Foreign key ~~(for~~ refers one table to another: (mapping/ morphism)

and these ~~mo~~ are internal references.

- ~~B~~ Arrows from each table to universal ~~nodes~~ data (like strings, ~~nodes~~ numbers) etc is called external reference

– the schema diagrams/graphs depict categories C
– the tables (arrangement of ~~dat~~ in certain columns) is a functor $F: C \to set$ [set of columns

———— x ———— x ———— x ————

Categories: (Graph is a presentation of free category)

→ Def$^n$ (Not so detailed)

→ Defining free categories with a graph G:

The set of natural numbers $N$ can be represented using free category where no. of object is one but ~~and~~ the paths are infinite.

Check the group theory thing

→ Finite presentation of categories.

———— x ———— x ———— x ————

Examples (Beautiful ones) → free categories.
categories

1) Categories for representing set of natural numbers (length of path corresponding to a natural number)
Mon

2) Monoids.

Finitely presented category [additional conditions on free categories]

→ Preorder as category & category as preorder

(Because both are directed graphs)

Preorder is a finitely presented category.

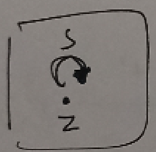
(Rather ~~Hasse diagram~~)

- Preorder $(P, \leq)$ has
- specifies a category $\mathbb{P}$.

where $ob(\mathbb{P}) = P$ exists.
~~reflexive condi~~ morphism $p \to q$ iff $p \leq q$

(reflexivity ensures identity
transitivity ensures composition)

Ex: 3.21:
For $e_1$: $f = g$
$e_2$: $f;f = f$
$e_3$: $f;h = g;i$
$e_4$: No equations.



consider this graph with one vertex & one arrow.

→ Free category perspective:

one object & infinite morphisms as there are infinite paths from z to z like z, s, s;s, s;s;s,

→ Preorder perspective:

$s;s = s = z$

all the paths from a ~~targ~~ source to target are same

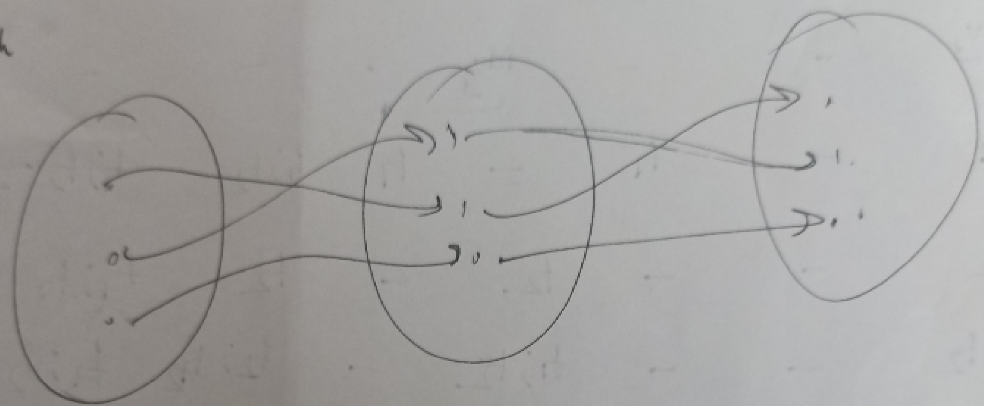
→ ~~Free~~ category presentation

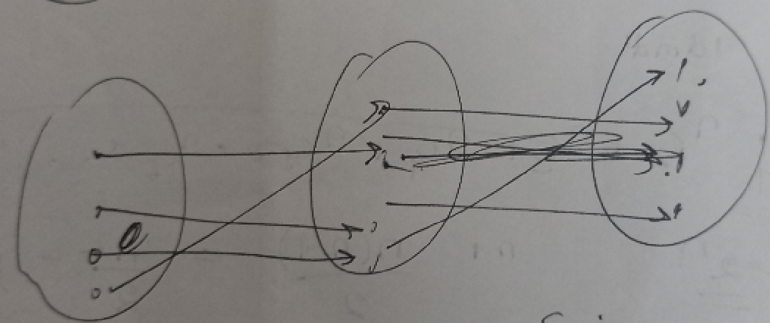
$s;s = s \neq z$

$M :=$ set of morphisms in the category

Ex 1.3.16

Graph





$z \circ s = s$.

$D = \{ z, s, s;s, s;s;s$

$s;s \quad s;s;s$.

$s;s;s;s = s$

| | $idv_1$ | $idv_2$ | $idv_3$ | $f_1$ | $f_2$ | $f_1;f_2$ |
|---|---|---|---|---|---|---|
| $idv_1$ | $idv_1$ | – | – | $f_1$ | – | $f_1;f_2$ |
| $idv_2$ | – | $idv_2$ | – | – | $f_2$ | – |
| $idv_3$ | – | – | $idv_3$ | – | – | – |
| $f_1$ | – | $f_1$ | – | $f_1$ | $f_1;f_2$ | $f_1;f_2$ |
| $f_2$ | – | – | $f_2$ | – | $f_2$ | $f_1;f_2$ |
| $f_1;f_2$ | – | – | $f_1;f_2$ | – | $f_1;f_2$ | $f_1;f_2$ |

n idindl

$$\overbrace{{}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + {}^nC_5 + {}^nC_6 + \dots {}^nC_n}$$

$$= \underline{\underline{2^n}} \qquad n + \frac{n(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2} + n$$

$$= \frac{n^2}{2} + \frac{n}{2}$$

$$= \frac{n}{2}(n+1)$$

$$\begin{tikzcd} a \arrow[r] & b \arrow[d] \\ d \arrow[u] & c \arrow[l] \end{tikzcd}$$