Week Four

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Section 11

- "*A Set has the potentiality to carry all sorts of structure with the help of maps.*" Sets lack **structure** and can be mapped to any other set. However, introducing a map between two sets creates a notion of structure. In category theory, this "structure" can be **preserved** or **destroyed** by mappings between categories, highlighting a **key difference** between categories (sets+maps) and sets.
- An automorphism of a finite set is also known as a **permutation** of the set.
- Suppose $A^{\circlearrowright^{\alpha}}$ and $B^{\circlearrowright^{\beta}}$ have A isomorphic to B as sets, we **cannot** conclude that $A^{\circlearrowright^{\alpha}}$ is isomorphic to $B^{\circlearrowright^{\beta}}$. (Page 159, Ex.3 and Ex.4)

Section 12

- ⋆ The category $S^{\circlearrowright}$ has **practical uses:** *Dynamical systems/Automata*. We have the set X (in $S^{\circlearrowright}$) of all the different **possible states** of the system, and the endomap α of X which takes each state x to the state in which the system will be one unit of time later.
- In a **finite** dynamical system, every state eventually **settles** into a cycle.
- "**Family Trees**" are categories of sets with **two endomaps**, namely, 'mother' and 'father'.

Section 13: Monoids

- A category with exactly **one object** is a **monoid**.
- **'Structure-preserving'** interpretation of one category into another is a **functor**.
- A **discrete-time dynamical system** is just a **functor** from a 'monoid' (whose mappings are natural numbers) to the 'category of sets'. For **continuous-time**, use real numbers for mappings in the monoid. (Page 168,169)