

$$\frac{da}{dt} = 1 - a - m - K_1 m^2 a$$

$$\frac{dm}{dt} = K_1 m^2 a + K_2 m^n (a + m - 1)$$

Fixed point : $a=0$ and $m=0$

$$a=0 \Rightarrow K_1 m^2 a = 1 - a - m$$

Putting this in $m=0$

$$(1 - a - m) = K_2 m^n (1 - a - m)$$

$$(1 - a - m) [1 - K_2 m^n] = 0$$

$$\Rightarrow \text{Either } \underline{a+m=1} \text{ or } \underline{m = \frac{1}{K_2^{1/n}}}$$

$$K_1 m^2 a = 0 \quad (\text{From } \star)$$

$$\Rightarrow m=0 \text{ or } a=0$$

Then we have

$$(0, 1)$$

$$\text{or } (1, 0)$$

$$K_1 m^2 a = 1 - a - m$$

$$a(K_1 m^2 + 1) = 1 - m$$

$$\Rightarrow a = \frac{1-m}{1+K_1 m^2}$$

$$a = \frac{1 - \frac{1}{K_2^{1/n}}}{1 + \frac{K_1}{K_2^{2/n}}}$$

$$a = \frac{K_2^{2/n} - K_2^{1/n}}{K_2^{2/n} + K_1}$$

$$\therefore \left(\frac{K_2^{2/n} - K_2^{1/n}}{K_2^{2/n} + K_1}, \frac{1}{K_2^{1/n}} \right)$$

$$f(a, m) = 1 - a - m - K_1 m^2 a$$

$$g(a, m) = K_1 m^2 a + K_2 m^n (a + m - 1)$$

$$\frac{\partial f}{\partial a} = -1 - K_1 m^2$$

$$\frac{\partial f}{\partial m} = -1 - 2K_1 m a$$

$$\frac{\partial g}{\partial a} = K_1 m^2 + K_2 m^n$$

$$\frac{\partial g}{\partial m} = 2K_1 m a + n a K_2 m^{n-1} + (n+1) K_2 m^n - n K_2 m^{n-1}$$

At $(a, m) = (0, 1)$:

$$J = \begin{pmatrix} -1 - K_1 & -1 \\ K_1 + K_2 & K_2 \end{pmatrix}$$

$$\tau = K_2 - K_1 - 1$$

$$\Delta = -K_2(K_1 + 1) + K_1 + K_2 = -K_1 K_2 + K_1 = K_1(1 - K_2)$$

If $K_2 > 1$, Saddle point (usual case)

If $K_2 < 1$, $\Delta > 0$ $\tau < 0$

$$\begin{aligned} \tau^2 - 4\Delta &= (K_2 - K_1 - 1)^2 + 4K_1(K_2 - 1) \\ &= K_2^2 + K_1^2 + 1 + 2K_1 K_2 - 2K_1 - 2K_2 \\ &= (K_1 + K_2)^2 + 1 - 2(K_1 + K_2) \\ &= (K_1 + K_2 - 1)^2 \geq 0 \end{aligned}$$

\therefore if $K_2 < 1$ and $K_1 + K_2 \neq 1$, stable node.
 " and $K_1 + K_2 = 1$, star or degenerate node.

At $(a, m) = (1, 0)$:

$\lambda = 0 : \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
 $\lambda = -1 : \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
 $J = \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}$
 $\tau = -1$
 $\Delta = 0$
 \Rightarrow Non-Isolated Fixed Point

At $(a, m) = \left(\frac{k_2^{2/n} - k_2^{1/n}}{k_2^{2/n} + k_1}, -\frac{1}{k_2^{1/n}} \right)$

$-1 - k_1 k_2^{-2/n}$ $\frac{\partial f}{\partial a}$

$2 k_1 \left(-1 + k_2^{\frac{1}{n}} \right)$ $\frac{\partial f}{\partial m}$

$-1 - \frac{k_1 + k_2^{2/n}}{k_1 k_2^{-2/n} + k_2 (k_2^{-1/n})^n}$ $\frac{\partial g}{\partial a}$

$k_1 \left(-1 + k_2^{\frac{1}{n}} \right) (-2 + n k_2 (k_2^{-1/n})^n)$ $\frac{\partial g}{\partial m}$

$k_2 (k_2^{-1/n})^n - \frac{k_1 \left(-1 + k_2^{\frac{1}{n}} \right) (-2 + n k_2 (k_2^{-1/n})^n)}{k_1 + k_2^{2/n}}$ τ

$\frac{-1 + k_2 (k_2^{-1/n})^n + k_1 \left(-k_2^{-2/n} - \frac{(-1 + k_2^{\frac{1}{n}}) (-2 + n k_2 (k_2^{-1/n})^n)}{k_1 + k_2^{2/n}} \right)}{k_1 k_2^{-2/n} \left(k_1 \left(1 + k_2 (k_2^{-1/n})^n \left(-1 - n + n k_2^{\frac{1}{n}} \right) \right) + k_2^{2/n} \left(3 - (3 + n) k_2 (k_2^{-1/n})^n + k_2^{\frac{1}{n}} (-2 + (2 + n) k_2 (k_2^{-1/n})^n) \right) \right)}$ Δ

Clearly, this expression is too complicated for us to make any general comment.

