

## Generalizing (Notion of Category Theory)

### VI Level-shifting

Given

- Any set  $S$ , there is a set  $\text{Rel}(S)$  of binary relations on  $S$ . An element  $r \in \text{Rel}(S)$  is subset of  $S \times S$ .

The set  $\text{Rel}(S)$  can be given an order (via subset relation / compare func)

$r \subseteq r'$  if whenever  $R((s_1, s_2))$  holds then so does  $R'(s_1, s_2)$

- Every ~~preorder~~ relation: For any set  $S$ , there is also a set  $\text{Pos}(S)$ , consisting of all the ~~preordering~~ relations on  $S$ . In fact there is a preorder structure on  $\text{Pos}(S)$  we say

if  $a \leq b$  ~~implies~~  $a \leq b$  for every  $a, b \in S$

preorder  
& preorder

- We have an inclusion for every preorder relation:

$\Gamma: \text{Pos}(S) \rightarrow \text{Rel}(S)$ , is a right adjoint of  $\alpha: \text{Rel}(S) \rightarrow \text{Pos}(S)$

Its left adjoint  $\alpha: \text{Rel}(S) \rightarrow \text{Pos}(S)$

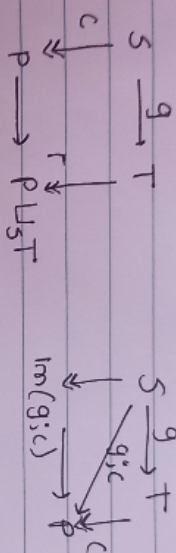
pullbacks, partitions, Galois connection.

Suppose we are given any function  $g: S \rightarrow T$ , it will induce a Galois connection:

$$g! : \text{Pt}(S) \rightarrow \text{Pt}(T) \quad (\text{left adjoint})$$

$$g^* : \text{Pt}(T) \rightarrow \text{Pt}(S) \quad (\text{right adjoint})$$

For a category  $\mathbf{Thomst}$ :



consider

$$g! : \text{Pt}(S) \rightarrow \text{Pt}(T)$$

take an element in  $\text{Pt}(S)$  is a partition  $u$  of  $S$   
 $g!$  gives us  $u$  a partition on  $T$ .

we say that  $h, t_2 \in T$  are  $t$  in same part  $h \sim t_2$   
 if there exists  $s_1, s_2 \in S$  such that  $s_1 \sim s_2$  and

$$g(s_1) = h \text{ and } g(s_2) = t_2. \text{ So, } g!(u) = u$$

$$g^* : \text{Pt}(T) \rightarrow \text{Pt}(S)$$

We get a partition  $u$  on  $S$  from  $u$  by saying  
 that  $s_1, s_2$  iff  $g(s_1) \sim g(s_2)$ .

$$\text{So, } g^*(u) = u$$