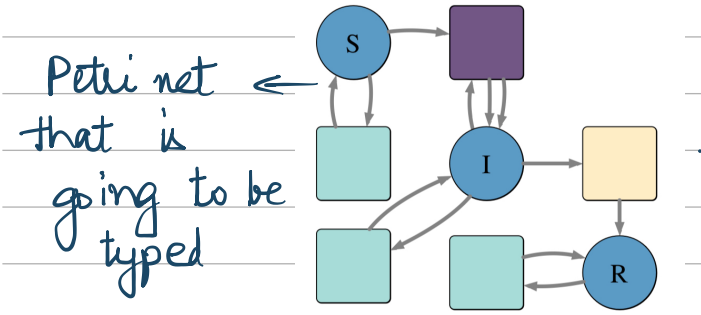
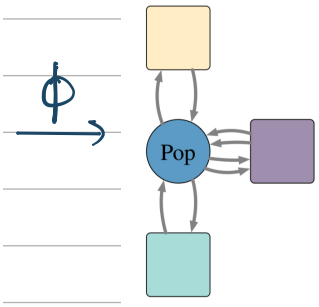
Stratification of Models

Typed Petri net : $P \xrightarrow{\phi} P_t$

P_t ↓ type system





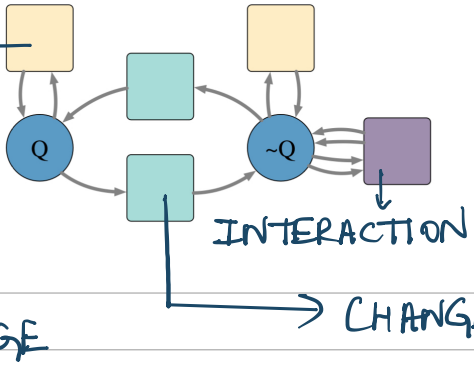
Here :

* S, I, R are brought to single place 'pop'
* Doing so all transitions have input & output arcs to pop only

WHY DOING THIS ?

* The type system is more generalised and acts like an apex of a co-span.
* In this model what we do is treat $P_{infections}$ as apex of the cospan.
* One of its leg is an unstratified model (a petri net), in the above fig, the SIR petrinet (P) is the unstratified model.
* We then use a 'stratification scheme' to stratify a model over the type system. This str. scheme is also a typed petri net, typed by the same - $P_{infections}$

Here's the example of 'Quarantining' as a stratification scheme :

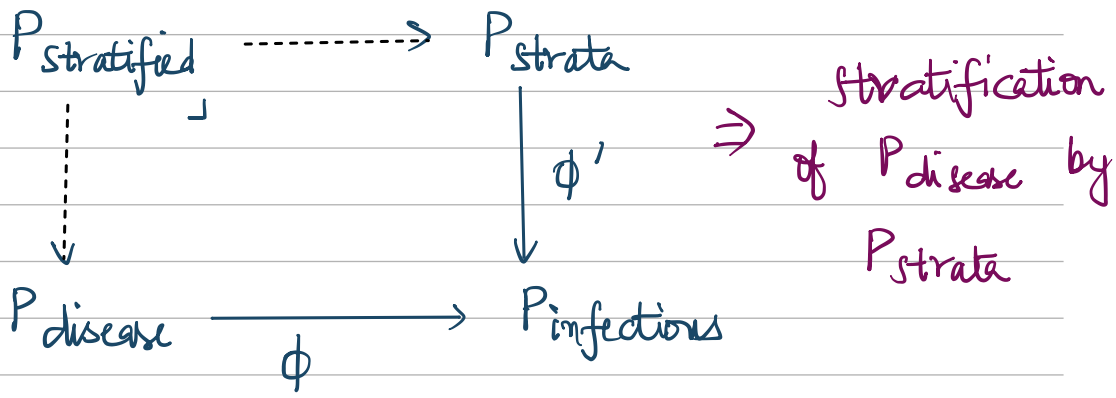


Q → Pop. in quarantine
~Q → Pop. not in quarantine

Here Q, ~Q are 'strata' (like S, I, R)

* This will be the other leg of the cospan.
* One thing to notice is that $P_{infec.}$ has transitions that are spontaneous, and overall population is conserved.

The stratified model will be a 'pullback' of a disease model & stratification scheme over the type system



$$P_{stratified} = P_{dis.} \times_{P_{infectious}} P_{strata}$$

⇒ $P_{stratified}$ is also a petrinet with places as pairs of places each in $P_{disease}$ & P_{strata}, like in a normal pullback

* By knowing pullbacks are associative & commutative we can first stratify a model with one scheme then another scheme or other way around and both are same

* Also we can first find pullback of 2 stratification schemes create a super scheme and pullback this with $P_{disease}$ and it still results the same as above.

Here's the mappings: