Electrical Circuits

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Notes on Baez's paper

Sect. 2: Circuits of Linear Resistors

- Modeled circuits as open/closed graphs and introduced L-graphs (graphs with resistor-labeled edges).
- Defined nodes N and boundary nodes ∂N.
- Derived KVL and KCL from the extended power functional via the principle of least power.
- Introduced the **Laplacian operator** Δ, showing that the least power principle reduces to $\Delta\phi = 0$.
- Defined the power functional Q, which gives the minimized power for boundary voltages ψ.
- Showed that dQ maps to boundary currents i, fully characterizing circuit behavior at the boundary.
- Concluded that circuits with identical external behavior share the same **Dirichlet form**.

Sect. 3: Passive Linear Circuits

- Expressed KCL, KVL and power functional in Laplace domain.
- We defined passive linear circuit which is an **L-graph** but morphism r is Z and L is $\mathbb{F}^+$ (field)
- Thm 3.3 talks about existence of potential function for least power and its properties.
- I didn't understand Lemma 3.4.
- We now define a **Semi category** (cat without identity morphism) of circuits where morphisms from finite sets S to T are dirchlet form P.
- Composition can be interpreted as power by whole circuit = power used by parts of circuit.

Sect. 4: Category of Circuits

- Defined Cospans and the category Cospan($\mathcal{C}$), whose morphisms are not individual cospans, but rather equivalence classes of cospans under the notion of isomorphism. They are dagger.

- Hypergraph, Hypergraph functors and Closed Compact cats (compactness in the sense, input wire can be rotated by 180° to get output wire).
- A **decorated cospan** consists of:
  - ⋆ A **base category** C.
  - ⋆ A **cospan** in C: $A \to N \leftarrow B$
  - ⋆ A **decoration functor** $F : C \to D$, where $F(N)$ represents additional structure on N in another category D.

  The **decoration** could be:
  - ⋆ Extra algebraic structure (e.g., a monoid or algebra).
  - ⋆ Labels, weights, or attributes attached to N.
  - ⋆ A resource or cost function.
- Didn't understood lemma 4.3 and 4.4.