apsara

Date: ____

Generalizing (Notion of Category Theory)

VI level-shifting

Given

- Any set S, there is a set $Rel(S)$ of binary relations on S. An element $R \in Rel(S)$ is subset of $S \times S$. The set $Rel(S)$ can be given an order (via subset relation / coarse & fine)

  $R \subseteq R'$ if whenever $R(s_1, s_2)$ holds then so does $R'(s_1, s_2)$

- ~~Every preorder relation~~: For any set S, there is also a set $Pos(S)$, consisting of all the preorder relations on S. Infact there is a preorder structure $\underline{\sqsubseteq}$ on $Pos(S)$ we say

  if $a \leq b$ ~~a~~ $\overset{\text{implies}}{\quad}$ $a \leq' b$ for every $a, b \in S$

preorder of preorder

- We have an inclusion for every preorder relation

$I: Pos(S) \rightarrow Rel(S)$, is a right adjoint of a GC

Its left adjoint $Cl: Rel(S) \rightarrow Pos(S)$

Pullbacks, Partitions, Galois connection.

suppose we are given any function $g: S \to T$, it ~~will~~ induces a Galois connection:

$g_!: Prt(S) \longrightarrow Prt(T)$ (left adjoint)

$g^*: Prt(T) \longrightarrow Prt(S)$ (right adjoint)

For ~~a~~ category theorist:

$$
\begin{tikzcd}
S \arrow[r, "g"] \arrow[d, two heads, "c"'] & T \arrow[d, two heads, "r"] \\
P \arrow[r] & P \sqcup_S T
\end{tikzcd}
\qquad
\begin{tikzcd}
S \arrow[r, "g"] \arrow[d, two heads] \arrow[dr, "g;c"] & T \arrow[d, two heads, "c"] \\
Im(g;c) \arrow[r] & P
\end{tikzcd}
$$

consider

$g_!: Prt(S) \to Prt(T)$

take an element in $Prt(S)$ i.e a partition $\sim_S$ of S

$g_!$ gives us $\sim_T$ an partition on T.

we say that $t_1, t_2 \in T$ are in same part $t_1 \sim_T t_2$

if there exists $s_1, s_2 \in S$ such that $s_1 \sim_S s_2$ and

$g(s_1) = t_1$ and $g(s_2) = t_2$. So, $g_!(\sim_S) = \sim_T$

$g^*: Prt(T) \to Prt(S)$

We get a partition $\sim_S$ on S ~~from~~ it by saying

that $s_1 \sim s_2$ iff $g(s_1) \sim_T g(s_2)$.

So, $g^*(\sim_T) = \sim_S$