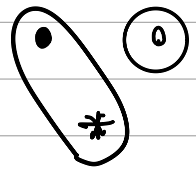
1. Generative Effects

In cat. theory there are these structures that are preserved in a category, and less the structure is preserved more suprises occurs when we observe its operations

Take a system of 3 points which are connected in some 'way' → There are total 5 ways of making a system from these 3 points

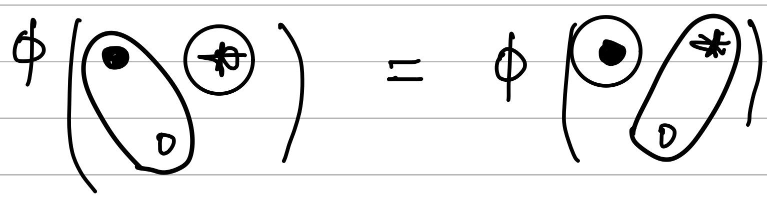


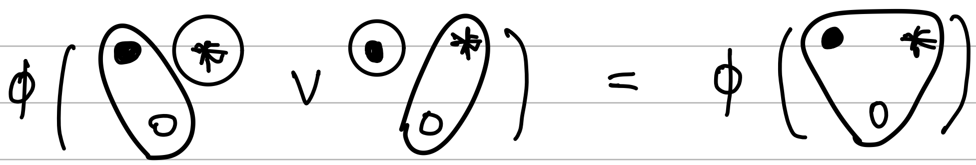
* Suppose ϕ is an observation - 'whether • is connected to * or not' which results in true in 2 cases & false in remaining 3.

Now an operation 'JOIN' is defined. ($\vee$)

↓

* Over lap 2 systems (in mind)
* Ensure TRANSITIVITY

Now  = false

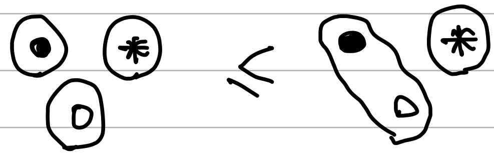
BUT..  = true

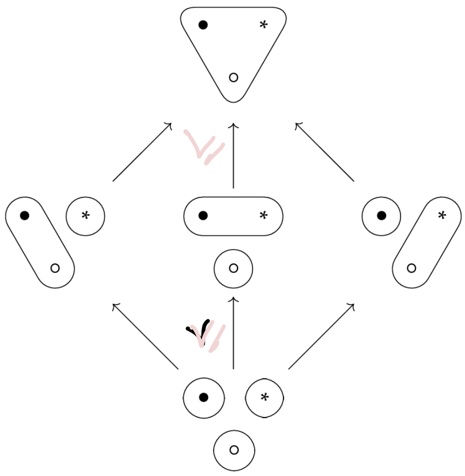
↳ This is an example of generative effect.

'JOIN'ing is not preserved by the observation ϕ

ORDER : Given systems A & B we say that $A \leq B$ if whenever x is connected to y in A, then x is connected to y in B (not necessarily in the other way around)

eg.





→ order hierarchy known as 'Hasse Diagrams'

Note: for any A & B in this diagram ($\vee \to$ join)

$A \leq A \vee B$ or $B \leq A \vee B$

& If $A \leq C$ & $B \leq C$; $A \vee B \leq C$

set $\mathbb{B} = \{\text{true}, \text{false}\}$ has an order false $\leq$ true

use the logic if $A \leq B$ then $A \to B$ here

false 'can' → true, false 'can' → false, true → true but true $\not\to$ false

so, $f \leq t$, $f \leq f$, $t \leq t$ but $t \not\leq f$

Exercise 1.7.

1. $t \vee f$: $t \leq t \vee f$ & $f \leq t \vee f$ so $t \vee f = t$
2. $f \vee t = t$, 3. $t \vee t = t$, 4. $f \vee f = f$

{ If $A \leq B$
if • & * is connected in A then so is in B. }

In our previous ϕ observation,

ϕ preserves the $\leq$ order

but, $\phi(A) \vee \phi(B) \leq \phi(A \cup B)$
(exercise 1.7. captures this)

but this was not preserved by the JOIN operation
($f \vee f = t$)

Disjoint union of 2 sets : $A = \{1, 2\}$

$B = \{1, 6, 7\}$

$A \sqcup B = \{(1,1), (2,1), (1,2), (6,2), (7,2)\}$

contains elements of form $(x,1)$ & $(y,2)$ where $x \in A$, $y \in B$

PARTITION : see book for formal def.

It is basically dividing the elements into groups- no element is common in any 2 groups & together they form the set

* All the diagrams above are partitions of the set { o, •, * }

SYMBOLS :

$\rightarrow$ arbitrary func, $\twoheadrightarrow$ surjective, $\rightarrowtail$ injective

$\cong$ bijective

$F ; G$ means $G(F(x))$