Week Three

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4th July

Started to read the book "**Conceptual Mathematics**" again. Goal is to complete till page 150 (before the start of section 11).

Section 10:

- **Brouwer's theorem:**
  - ⋆ If there is '**no continuous retraction**' of the figure (line segment/disk/ball) to its boundary then every **continuous map** from this figure to itself has a '**fixed point**.
  - ⋆ We prove the above theorem by proving its **contrapositive statement**.
  - ⋆ The "**no continous retract**" in the theorem ensures that "**all points** of the figure are considered".

Personally, I find this theorem really '*beautiful*' and '*concise*'. Although the first part of the theorem might 'seem unnecessary' at first glance, it actually plays a crucial role in the overall argument.

7th July

- Categories of structured sets:
  - ⋆ Let us denote $\mathbf{S}^{\circlearrowleft}$ as the the category of sets and maps, where each element is a category of the type $\mathbf{X}^{\circlearrowleft^{\alpha}}$. A mapping $\mathbf{X}^{\circlearrowleft^{\alpha}} \xrightarrow{f} \mathbf{Y}^{\circlearrowleft^{\beta}}$ is called **structure preserving** if $f \circ \alpha = \beta \circ f$.
  - ⋆ If the mapping f is an *isomorphism*, then both f and its inverse f^{-1} has the **same number of fixed points**.
  - ⋆ The *only* idempotent automorphism is the **Identity map**. Also, the *only* idempotent which has a section or a retraction is the Identity map. (Proof in page 139)
  - ⋆ We can use the involution property to define evenness and oddness of a set (in terms of number of elements), without even counting the elements in it!

    If the set can have an 'involution map' which does not have 'fixed point', then we can surely say there are *even number* of elements (direct half is possible), while for any involution, if there exists a fixed point, then we can say the set must contain *odd number* of elements (direct half isn't possible, one element came extra). (See Exercise 3, page 140)
  - ⋆ These have applications in **Dynamical systems**.

- Irreflexive graphs:
  - ⋆ These categories, denoted by $\mathbf{S}^{\downarrow\downarrow}$, contain elements of the type $X \underset{s}{\overset{t}{\longrightarrow}} P$, where X is the **set of arrows**, P is the **set of points**, s contains info about the arrow's tail (source) and t contains info about the arrow's head (target).
  - ⋆ The category $\mathbf{S}^{\circlearrowleft}$ is a **subcategory** (similar to subset for sets) of Irreflexive graphs.
- Reflexive graphs: These contains elements of the type:

$$\begin{tikzcd} X \arrow[d, shift right=3, "s"'] \arrow[d, shift left=3, "t"] \\ P \arrow[u, "i"'] \end{tikzcd}$$

Here, i is a *common section* for both s and t. ie,

$$\boxed{s \circ i = 1_P \ \ \& \ \ t \circ i = 1_P}$$

I skimmed through pages 146 to 151, reading quickly without paying too much attention to each sentence.