

III

Meets and Joins

certain elements of a pre-ordered set stand out due to some special characterization. This can be either relative or absolute.

We discuss two such elements: join and meet

join: least upper bound (supremum)

meet: greatest lower bound (infimum)

Def: let (P, \leq) be pre order and let $A \subseteq P$ be a subset, we say that an element $p \in P$ is a meet of A if:

- for all $a \in A$, we have $p \leq a$
- for all $a \in A$, q such that $q \leq a$, $(a \in A)$, we have $q \leq p$

meet of two element a and $b \in P$ is denoted by $a \wedge b$ and meet of subset A is denoted p by

$$p = \bigwedge A$$

join of A p is a join of A if:

- for $\forall a \in A$ we have $a \leq p$
- $\exists \forall q$ such that $a \leq q$, $\forall a \in A$ we have $p \leq q$

$p = \bigvee A$ for any subset A , p is (join) is denoted like this: $p = \bigvee A$

- two joins or meets p and q mean

$p \equiv q$, i.e., $p \leq q$ and $q \leq p$ (a and q may not be equal but in this context one can be used instead of other, i.e., equivalent in this context)

- Meets and joins may not exist
- Multiple meets or joins may exist.
- Suppose (P, \leq) is a preorder and $A \subseteq B \subseteq P$ are subsets that have meets. Then:

$$A \wedge B \leq A \wedge A$$

$$\forall A \leq \forall B$$

What are joins and meet?

i) set: join and meet

join \rightarrow union

$(A) \cup (B)$ both elements

meet \rightarrow intersection

ii) Boolean:

join \rightarrow OR

meet \rightarrow AND