Mathematical models of infectious diseases

1) Basic SEIR Model : (Flu)

* We use mathematics to model infectections diseases and compartmental models are one in which the population are assigned to compartments

* The SEIR model is a compartmental model in which the compartments are S, E, I, R → Recovered

susceptible ← S, E ← exposed, I → Infected

(primitive model → SIR)

* It is used to analyse disease dynamics and estimate parameters like total no. of infected or recovered people ; also a epidemiological parameter named 'R_0'

'R_0' → Basic Reproduction Number

for SIR model :

$$\frac{ds}{dt} = -\beta s i$$

$s = \frac{S}{N}$ = fraction of suscep. population

$i = \frac{I}{N}$

β → avg. no. of contacts per person per time

×

probability that a disease transmits b/w a infected person & sus. person.

and, w.k.t rate $\propto$ concentration so we multiplied

$$\frac{dR}{dt} = \gamma I$$

If an infected person is infectious for a average time period 'D'

$$\gamma = \frac{1}{D}$$

we can obtain $\frac{dI}{dt}$ by $\frac{d}{dt}(s + i + r) = \underbrace{\frac{dN}{dt} = 0}_{\text{for no birth \& death rates}}$

⇒ Terms:

* $R_0 = \frac{\beta}{\gamma}$, expected new infections from a single infection in a population where all are susceptible

⇓

understood as typical time contact b/w s & i ⇒ $T_c = \frac{1}{\beta}$ &

Removal ⇒ $T_r = \frac{1}{\gamma}$,

No. of contacts by infected indivial before the infectious has been removed

$$= \frac{T_r}{T_c} = \frac{\beta}{\gamma}$$

⇒

$$\frac{dI}{dt} = \left(R_0 \frac{S}{N} - 1\right)\gamma I$$

for proper epidemic outbreak $\frac{dI(0)}{dt} > 0$

so, $R_0\, S(0) > N$

$$R_0 \frac{S(0)}{N} > 1$$

(Generally $S(0) = 1$) $\boxed{R_0 > 1}$

* Force of infection : $F = \beta i$

If birth rate λ & death rate μ is considered,

$$\frac{dS}{dt} = \lambda - \mu S - \frac{\beta I S}{N}$$

$$\frac{dI}{dt} = \frac{\beta I S}{N} - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R \qquad R_0 = \frac{\beta}{\mu + \gamma}$$

SEIR Model :

* For many diseases we have latency period → time b/w when the person is infections and when the symptoms show up and is known infections.

During this time, we name them as 'Exposed'

If the latency period is a^{-1}, then we use 'a' as parameter for the eq.

Assuming birth and death rates are equal.

$$\frac{dS}{dt} = \overbrace{\mu N}^{\text{Total births}} - \overbrace{\mu S}^{\text{sus. death}} - \frac{\beta I S}{N}$$

$$\frac{dE}{dt} = \frac{\beta I S}{N} - (\mu + a) E$$

$$R_0 = \frac{a}{\mu + a} \times \frac{\beta}{\mu + \gamma}$$

$$\frac{dI}{dt} = aE - (\gamma + \mu) I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

here also, $\sum \frac{d}{dt} = \underline{\underline{0}}$ $\Rightarrow$ total pop. remains same

<u>OBS</u> : General SEIR Model vs SEIR model with vital dynamics

$\lim_{t\to\infty}$ $s, i, e \simeq 0$

$r \simeq 1$

(Ideal)

for $\mu = 0.02$,

$\lim_{t\to\infty}$ $s \approx 0.122$

$e \approx 0.017$

$i = 0.143$

$r = 0.717$

(Near realistic)

* Interacting Subpopulation SEIR Model

$\Downarrow$

Models differ b/w pop. of different age groups

so SEIR model for diff. age groups are modelled & connected through interaction links