

# Week Seven

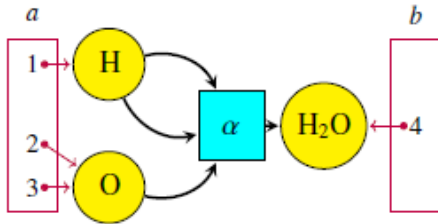
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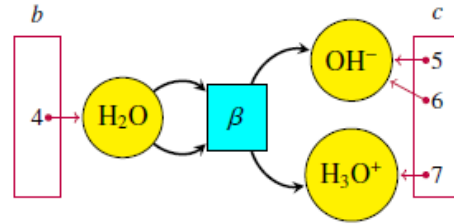
25th August

## Open Petri-net

- These contain **open** places which interact with other petri-nets outside it.

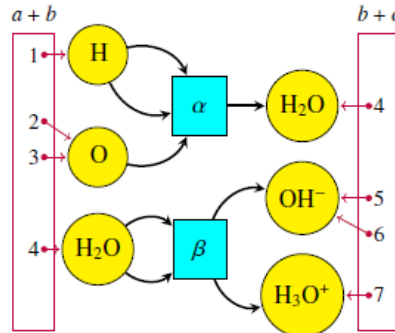


Petri net representing  $2\text{H} + \text{O} \longrightarrow \text{H}_2\text{O}$



Petri net representing self-ionization.

- We use extra objects whose elements get mapped to these open places. This creates a sense of 'port' and ports of the same type (element) can be connected together.



Combined Petri net

- When plugged together, the contents of the open place are shared between the two Petri nets.
- An open petri net is a *structured cospan*.

## Structured Cospan

It is a mathematical object which is of the form:

$$\begin{array}{ccc} & X & \\ i \nearrow & & \nwarrow o \\ L(a) & & L(b). \end{array}$$

- $L$ : A *functor* mapping the object  $A$  to the combined Petri net  $X$ .
- $A$ : The object representing the set of pre-images for both input and output places.

- $X$ : The combined Petri net that results from the structured composition.
- $i$  and  $o$  are called the **legs** of the cospan. They map the input and output places to the closed petri-net  $x$ .
- $a$  and  $b$ : Subsets of  $A$  representing the pre-images of the input and output places, respectively. These subsets are referred to as the **input** and **output** sets.
- $x$ : The specific part of the Petri net, also known as the **apex**, which is determined by the functor  $L$ . The functor  $L$  maps  $a$  to the input places and  $b$  to the output places of this apex.

Given the input set  $a$  and output set  $b$ , the apex  $x$  is the portion of the Petri net where  $a$  and  $b$  correspond to the input and output places, respectively.

In the context of chemical reactions, such as the water formation reaction, the sets  $a$  and  $b$  correspond to the **pre-images** of the input and output places, respectively. Specifically:

- The set  $a$  represents the **inputs** to the water formation reaction.
- The set  $b$  represents the **outputs** of the water formation reaction.

The *functor*  $L$  maps these sets to the corresponding elements in the Petri net:

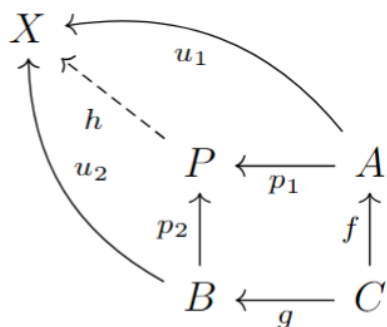
- $L(a)$  maps the elements in  $a$  to the open input places in the Petri net.
- $L(b)$  maps the elements in  $b$  to the open output places in the Petri net.

Using the mappings  $L(a)$  and  $L(b)$ , we can extract the Petri net  $x$  that models the water formation reaction. This Petri net  $x$  incorporates the connections and interactions defined by  $L(a)$  and  $L(b)$ , thereby representing the complete chemical reaction process.

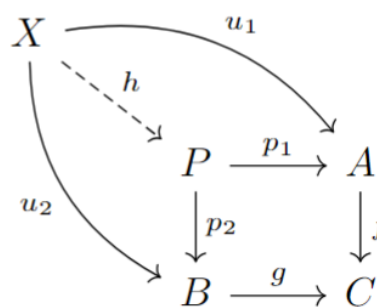
## 26th August

### Pushouts and Pullbacks

Pushouts and Pullbacks are objects which have the universal property that for all objects  $X$ , both the inner and outer squares commute in each figure:



Here, P is **Pushout**



Here, P is **Pullback**

Speaking in terms of sets, we can relate Pushouts to the Union of the individual sets and Pullbacks as Intersection of those sets.

Started learning from Navin's petri net code and tried making my own model (not finished yet). **CLICK HERE** to see *jupyter notebook* which contain the code.