

Buzzwords: Copresheaves, Structured cospans, Pullbacks, Operad

differences between syntax of composition and semantics of composition

The syntax of composition refers to the rules and structure defining how subsystems interact within a model, often exemplified by visual representations like undirected wiring diagrams. 1 On the other hand, the semantics of composition involve assigning meaning to models in a composition, interpreting them in terms of real-world phenomena, and analyzing their behavior and implications. 2 The syntax provides a high-level overview of model composition, while the semantics ensure accurate implementation based on defined interactions. 3

→ Petri nets, ODEs, DDEs serve as semantics

Operads : Allows for

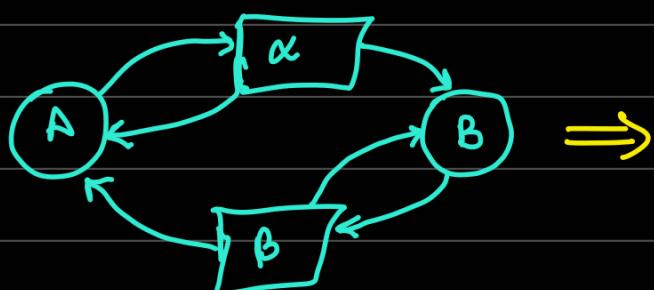


(multiple inputs, single output)

→ Different models can have different semantics but same syntax.

→ Structures may devolve into chaos without proper organizational software. Such software will help in generalizability.

Petri Nets to ODEs

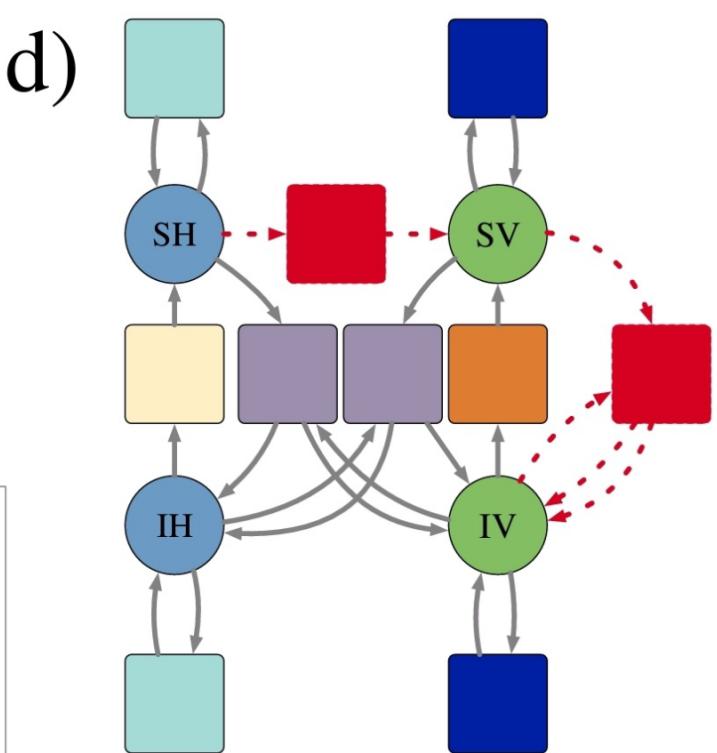
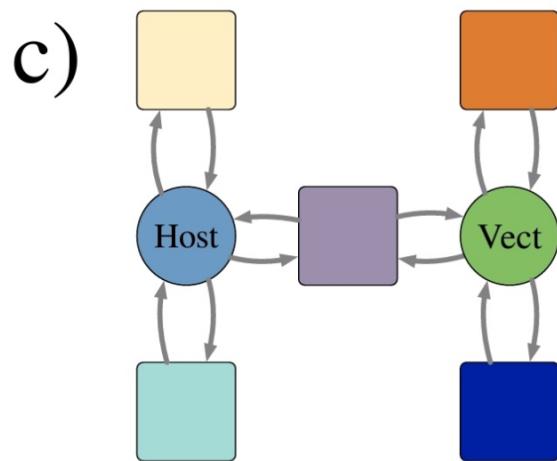
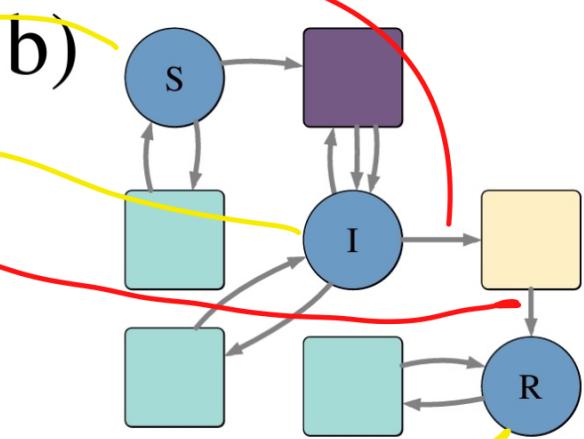
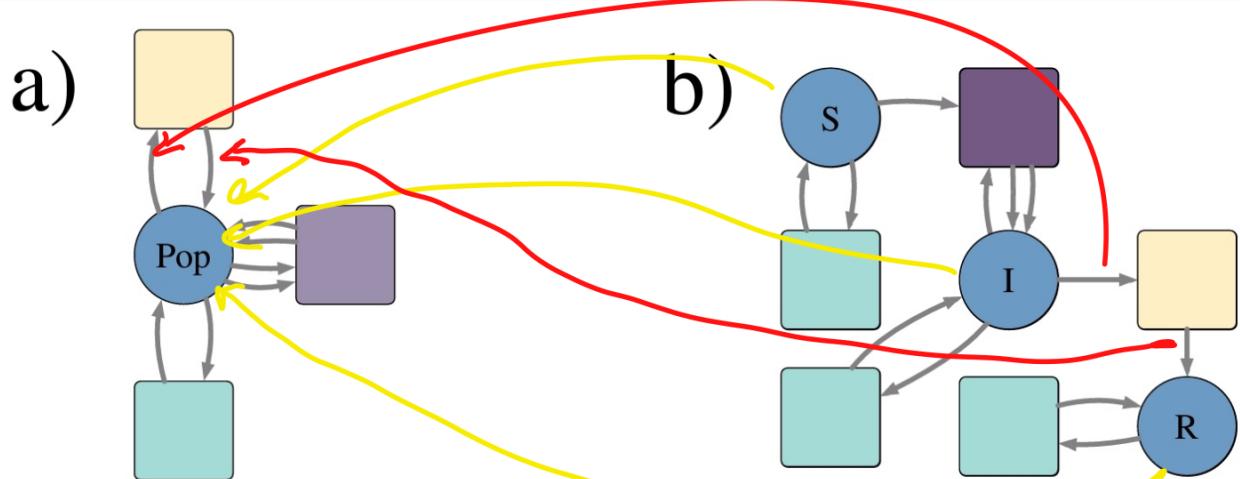


$$\frac{dA}{dt} = -\alpha A + \alpha A + \beta B = \beta B$$

$$\frac{dB}{dt} = -\beta B + \beta B + \alpha A = \alpha A$$

Type Systems

- The idea is that we want to define a general Petri Net that encapsulates the idea of any possible model we can create.
- This general Petri Net is called the type system (P_0)
- A typed Petri Net is a Petri Net P along with a morphism $\phi: P \rightarrow P_0$. P is said to be "typed" by P_0 . The morphism is drawn below for some elements of P .



Legend

- (Blue Circle) Host Population
- (Green Circle) Vector Population
- (Yellow/Orange Square) Change of Disease State Host/Vector
- (Teal/Blue Square) Change of Strata State Host/Vector
- (Purple Square) Interaction Effect
- (Red Square) Forbidden Transition

① → This Petri Net ^{infections} is a type system for ②.
 → 'Pop' represents the total population.

→ So this Petri Net is saying three things:

- ① any one person may spontaneously change their condition of disease (leave Pop, undergo transition, come back to Pop)
- ② any one person may change strata: move to another place, come under quarantine control, etc.
- ③ any two people interact and can perturb the dynamics.
 (e.g.: $S + I \rightarrow I + I$; S and I leave Pop, undergo transition and come back to Pop.)

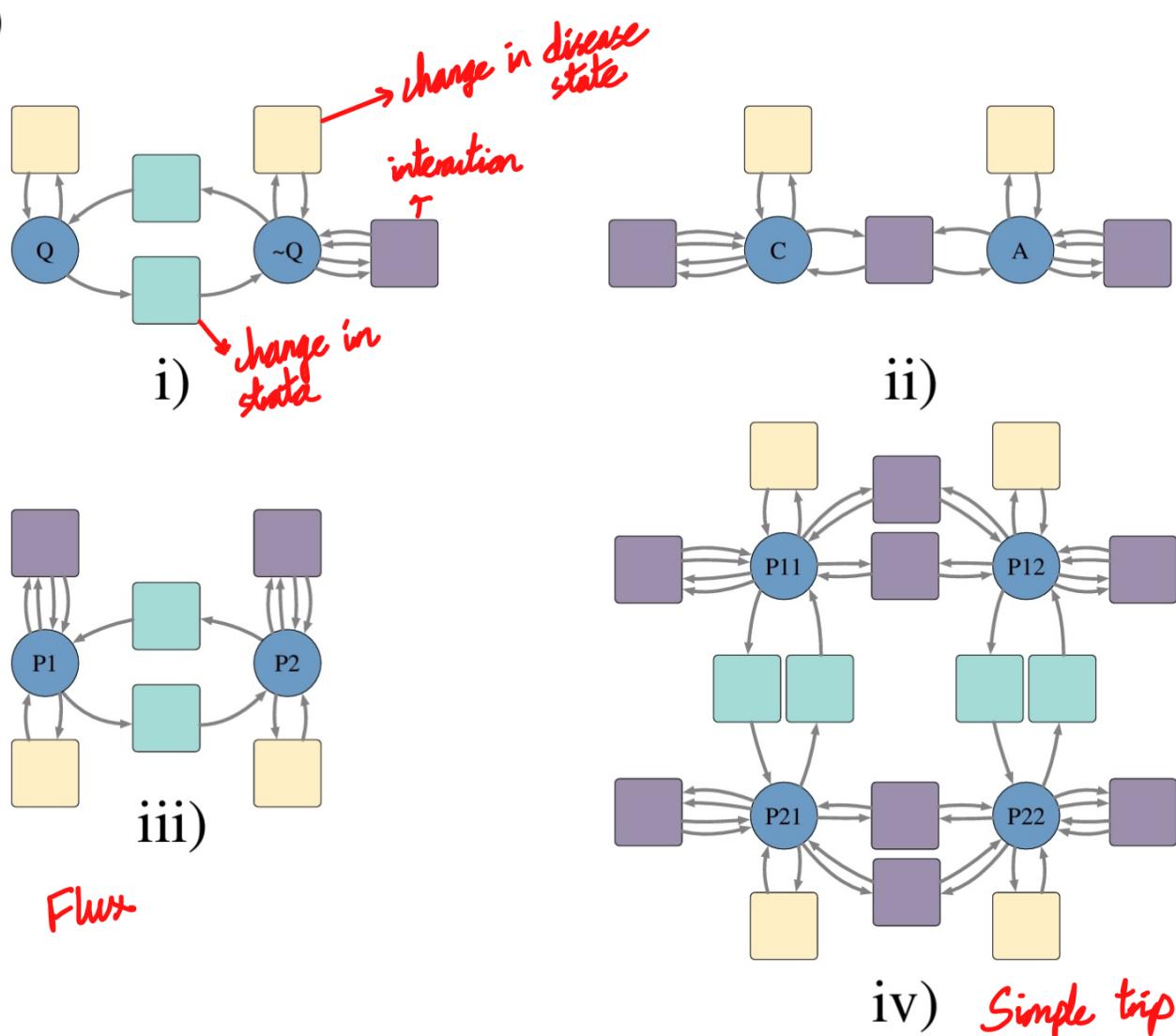
!! Note how $\frac{dPop}{dt} = 0$, each forward arc has a backward arc.

Use similar logic for (c) and (d), noting the forbidden transitions.

Stratification Schemes

- It is important to consider other aspects of an epidemic besides strictly compartmentalizing the population into S, I, R, etc.
- Let's assume the people can 1) quarantine themselves, 2) move around in the world.
- For (2), we use the flux model (migration of populations) or the simple trip model (home assigned, but people free to move around).

b)



($\sim Q$: not quarantined)

(A: adults, C: children)

(P represents a patch of population)

Let's discuss (iv) as it's the most complicated. Note that for P_{ij} , $i \rightarrow$ current location, $j \rightarrow$ residence (as this models the simple trip model).

Note :

- ① Change in state only when j 's are equal, so blue squares only in the vertical columns.
- ② Disease transmission can only happen in the same locations when the i 's are equal, so purple squares only in the horizontal rows.
- ③ The yellow squares take care of spontaneous change in disease state for each patch.

→ All four Petri Nets are typed by $P_{\text{infectious}}$. \checkmark

Stratified Compartmental Models

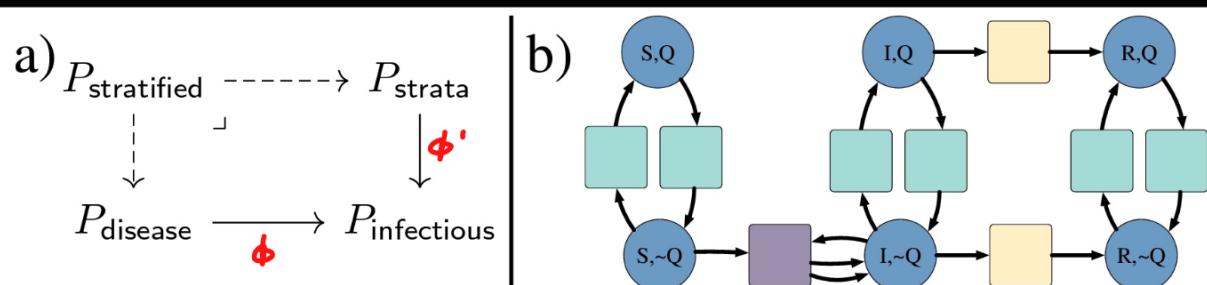


Figure 6: (a) A stratified model over $P_{\text{infectious}}$ is the pullback of a typed epidemiological model $P_{\text{disease}} \rightarrow P_{\text{infectious}}$ and a typed stratification scheme $P_{\text{strata}} \rightarrow P_{\text{infectious}}$. (b) The stratified model of the SIR model depicted in Figure 4(b) and the quarantining model depicted in Figure 5(b.i).

If $\phi: P_{\text{disease}} \rightarrow P_{\text{infectious}}$ is the unstratified disease model and $\phi': P_{\text{strata}} \rightarrow P_{\text{infectious}}$ is the stratification scheme, then $P_{\text{stratified}}$ is said to be the stratification of P_{disease} over P_{strata} .

