SEIR model

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Introduction :

- In this report , we are going to analyse about SEIR model and use Julia to model it.
- SEIR model is an extended version of SIR model which is the most common model to model the endemic diseases.
- In this report we will be analysing the SIR model initially and then we will form the block diagrams for SEIR model and use Julia as a tool to model it categorically.

SIR model

- SIR model involves dividing the total population into three categories
  - **Susceptible** : The generic category of people who are susceptible to get infected **S**.
  - **Infected** : The category of people who got infected by the disease **I**.
  - **Recovered** : The category of people who got immunity and recovered from the infection **R**.
- It can be represented by the following block diagram.
- The equations that govern this modelling are :

$$\frac{d\mathbf{S}}{dt} = -\frac{\beta\mathbf{SI}}{N} \tag{1}$$

$$\frac{d\mathbf{I}}{dt} = \frac{\beta\mathbf{SI}}{N} - \gamma\mathbf{I} \tag{2}$$

$$\frac{d\mathbf{R}}{dt} = \gamma\mathbf{I} \tag{3}$$

- The equation (8) represents that the number of susceptible people will decrease once the infected people **I** interact with the susceptible people **S** which is proportional to $\mathbf{S}\times\mathbf{I}$. Therefore the susceptible population will decrease at the rate of $\boxed{\frac{\beta}{N}}$.
- The equation (9) represents that the number of infected people will increase at the rate at which the susceptible decrease and it will also decrease due to a section of people getting recovered at the rate of $\boxed{\gamma}$
- The equation (10) represents the rate of change of recovered people is directly proportional to **I** and is at the rate of $\boxed{\gamma}$

SEIR model

- **SEIR** model is just the extended version of SIR model with a extra group of E which is exposed group. An intermediate group between infected and susceptible.
- The exposed group represents the group of people who are in incubation period of infection.

- The rate of change of number of people in exposed group varies directly as the possibility of infection and the rate will decrease at the rate of latency period σ as they will leave once they get infected and get to infected group.

- So the equations transform into

$$\frac{d\mathbf{S}}{dt} = -\frac{\beta\mathbf{SI}}{N} \tag{4}$$

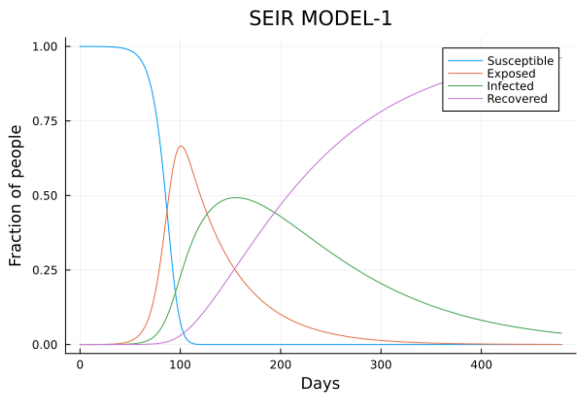
$$\frac{d\mathbf{E}}{dt} = \frac{\beta\mathbf{SI}}{N} - \sigma\mathbf{E} \tag{5}$$

$$\frac{d\mathbf{I}}{dt} = \sigma\mathbf{E} - \gamma\mathbf{I} \tag{6}$$

$$\frac{d\mathbf{R}}{dt} = \gamma\mathbf{I} \tag{7}$$

Observations :

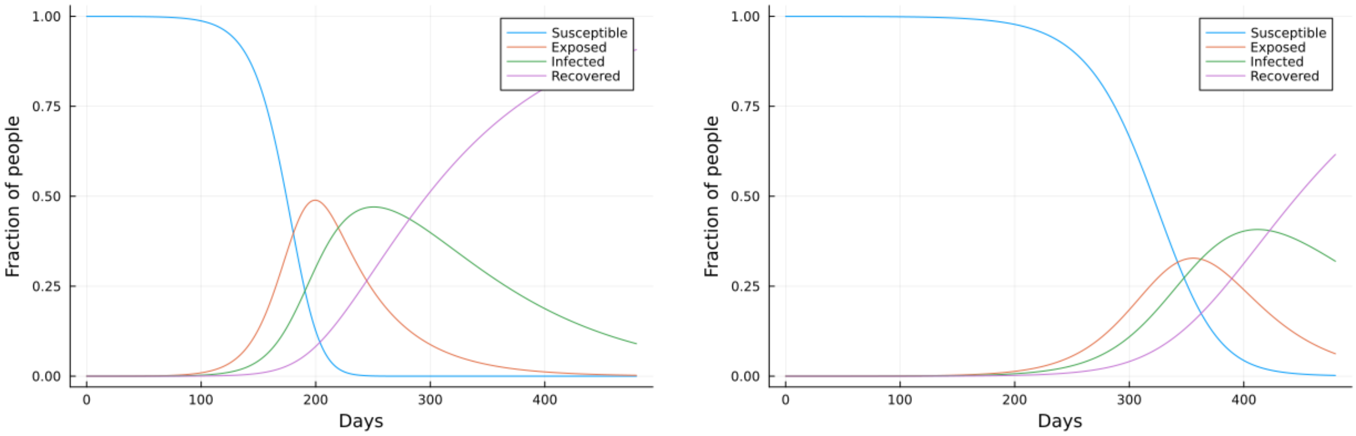
- The initial parameters which are apt for this model would be
  - S_0 = 0.9999234
  - E_0 = 7.66e-5
  - I_0 , R_0 = 0 , 0
- I have coded this model in Julia and the right parameters which I found from internet are ,
  - β = 0.85 . It also makes sense intuitively as infection spreads quickly.
  - σ = 0.02 , This depends on the incubation period for the disease.
  - γ = 0.01 , This depends on the nature of the disease and the immune of people in the locality.
- With all these parameters , the graph obtained after solving the DE is ,



Playing around with Parameters

Beta β

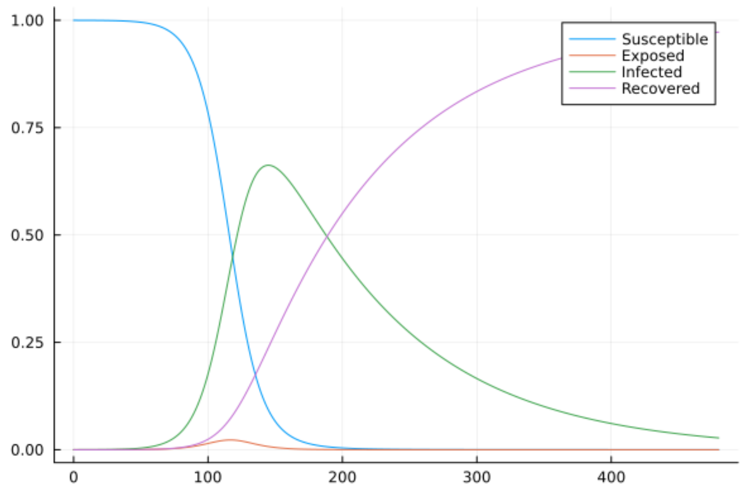
- As said earlier , *beta* determines the rate at which the infection spreads.
- Making β too less (< 0.01) means that the infection is too insignificant and it has no effect on population.
- On increasing the beta , we can infer that the duration of the disease gets affected i.e For less beta , the infection takes longer duration to affect more people and as we increase beta , the duration decreases as shown below.



(a) $\beta = 0.5$ (b) $\beta = 0.2$

Sigma σ

- σ decides the number of exposed group of people and it depends on the incubation period of the disease.
- Increasing the σ reduces the exposed group people as the incubation period will be less which is as good as not having an exposed group of people.

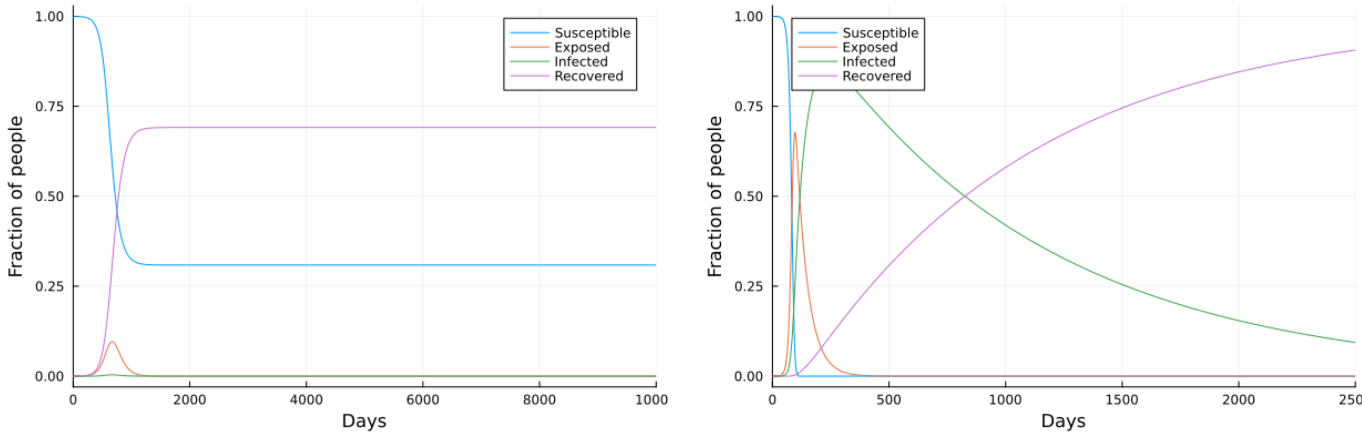


$\sigma = 0.9$

- Making the σ very less means the incubation period of the disease is too high which would imply that the disease wont have much of an impact on the global population.

Gamma γ

- γ is the recover rate of infected people.
- If we put γ 1 ,it would mean that almost everyone who got infected has recovered which would imply that there is no significance of the infection.
- Making γ high makes significant changes only after a long time (in a way like β) and there is very little spike of infected people as shown below.
- Making the γ very less causes a huge maxima in the group of infected people and reduces slowly.



(a) $\beta = 0.5$

(b) $\beta = 0.2$

Introducing new parameters

- Let us now add more complexities to the SEIR model by introducing parameters like Birth rate , Death rate , immunity into the equations.
- Let us include the birth and death rates to be μ i.e the rate at which population gets changed.
- Let us take the death rate due to infection be α i.e the number in infection group will decrease at the rate of α.
- Let us remove a part of people from recovered group who had lost immunity which is represented by the rate ω.
- So the equations become ,

$$\frac{d\mathbf{S}}{dt} = -\frac{\beta \mathbf{SI}}{N} + \omega \mathbf{R} + \mu \mathbf{N} - \mu \mathbf{S} \tag{8}$$

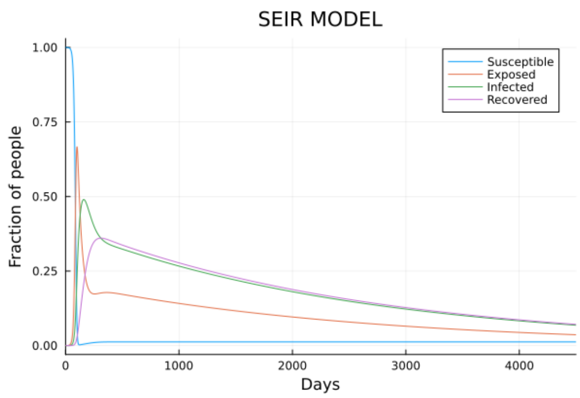
$$\frac{d\mathbf{E}}{dt} = \frac{\beta \mathbf{SI}}{N} - \sigma \mathbf{E} - \mu \mathbf{E} \tag{9}$$

$$\frac{d\mathbf{I}}{dt} = \sigma \mathbf{E} - \gamma \mathbf{I} - (\mu + \alpha)\mathbf{I} \tag{10}$$

$$\frac{d\mathbf{R}}{dt} = \gamma \mathbf{I} - \omega \mathbf{R} - \mu \mathbf{R} \tag{11}$$

Observation :

- Adding these groups into the equations gives the following graph.



- We can infer that in a small amount of time the susceptible population falls near 0.

- The population of Infected group achieves the peak in the mean time and falls rapidly unlike the normal case as we have included the possibility of death here.
- The population of Recovered is limited unlike the normal case as we have included the immune lost case.
- The inclusion of death case hardly affects Exposed group as it is considerable only for a small interval.

Playing around with the parameters :

- We cannot modify the birth/death parameter : μ.
- The parameter α determines the death rate of the infectious disease. Making $\alpha = 0.51$ decreases the population by 0.66 fraction i.e The remaining population is 65508.23.
- Reducing α to 0.21 gives you the remaining population to be 95941.414.
- The immune loss parameter ω is in general very less as the immune system will hold up for a significant amount of time.
- The ideal parameters for a disease would be i.e Obtained from here
  - $\alpha = 0.0001$ i.e the death rate of a disease would be less
  - $\beta = 0.21$ i.e infection rate is around 20
  - $\sigma = 0.1429$ i.e incubation period would be a week
  - $\gamma = 0.07143$ i.e recovery period is generally half the time for the disease to incubate.
  - $\mu =$ 1e-5 i.e Birth,Death rate is in general less 76 years.
  - $\omega = \frac{1}{365}$ i.e On a safer side we take it to be 1 year.
- The graph of all group and **I** vs **S** will look like