Libraries used are:

1. **Catlab**: Provides tools for category theory, allowing manipulation of mathematical structures and creation of wiring diagrams.
2. **AlgebraicPetri**: Facilitates modeling and simulating Petri nets, useful for representing distributed or concurrent systems.
3. **AlgebraicDynamics.UWDDynam**: Supports modeling open dynamical systems with external interactions and evolving states.
4. **DifferentialEquations**: Solves differential equations, enabling simulation of time-dependent dynamical systems.
5. **LabelledArrays**: Creates arrays with labeled elements, simplifying tracking of variables in systems with named components.
6. **Plots**: Offers comprehensive plotting tools for visualizing mathematical models and simulation results.

```
using Catlab, Catlab.CategoricalAlgebra, Catlab.Programs, Catlab.WiringDiagrams, Catlab.Graphics
```

```
using AlgebraicPetri
```

```
using AlgebraicDynamics.UWDDynam
```

```
using DifferentialEquations
```

```
using LabelledArrays
```

```
using Plots
```

Functions which takes in *"place names"* and *"transition names"* and creates an **open petri-net** using Catlab's *"OpenLabelledPetriNet"* method.

- `HE_addition` creates petri-nets which represent **He-4 addition reactions**.
- `decay_reaction` creates petri-nets which represent **decay chains**.

```
HE_addition (generic function with 1 method)
```

```
# Function to generate binary fusion reactions (He_ + Element => Product)
function HE_addition(elements::Tuple, transition_name::Symbol)
    return Open(LabelledPetriNet(
        elements,   # Places: Input and Output elements
        transition_name => ((elements[1], elements[2]) => (elements[3]))  # Transition rule
    ))
end

# Function to generate decay reactions (Element => Intermediate => Product)
```

```
decay_reaction (generic function with 1 method)
```

```
function decay_reaction(elements::Tuple, transition_names::Tuple)
    return Open(LabelledPetriNet(
        elements,   # Places: Input and Output elements
        transition_names[1] => ((elements[1]) => (elements[2])),  # First Decay
        transition_names[2] => ((elements[2]) => (elements[3]))   # Second Decay
    ))
end
```

Creating open petri-nets for each reaction in the **"alpha process"**.

```
StructuredMulticospan(Multicospan(
                                AlgebraicPetri.LabelledPetriNet {T:2, S:3, I:2, O:2, Name:0}
```

T	tname
1	Ni_Co
2	Co_Fe

S	sname
1	Ni
2	Co
3	Fe_stable

I	it	is
1	1	1
2	2	2

O	ot	os
1	1	2
2	2	3

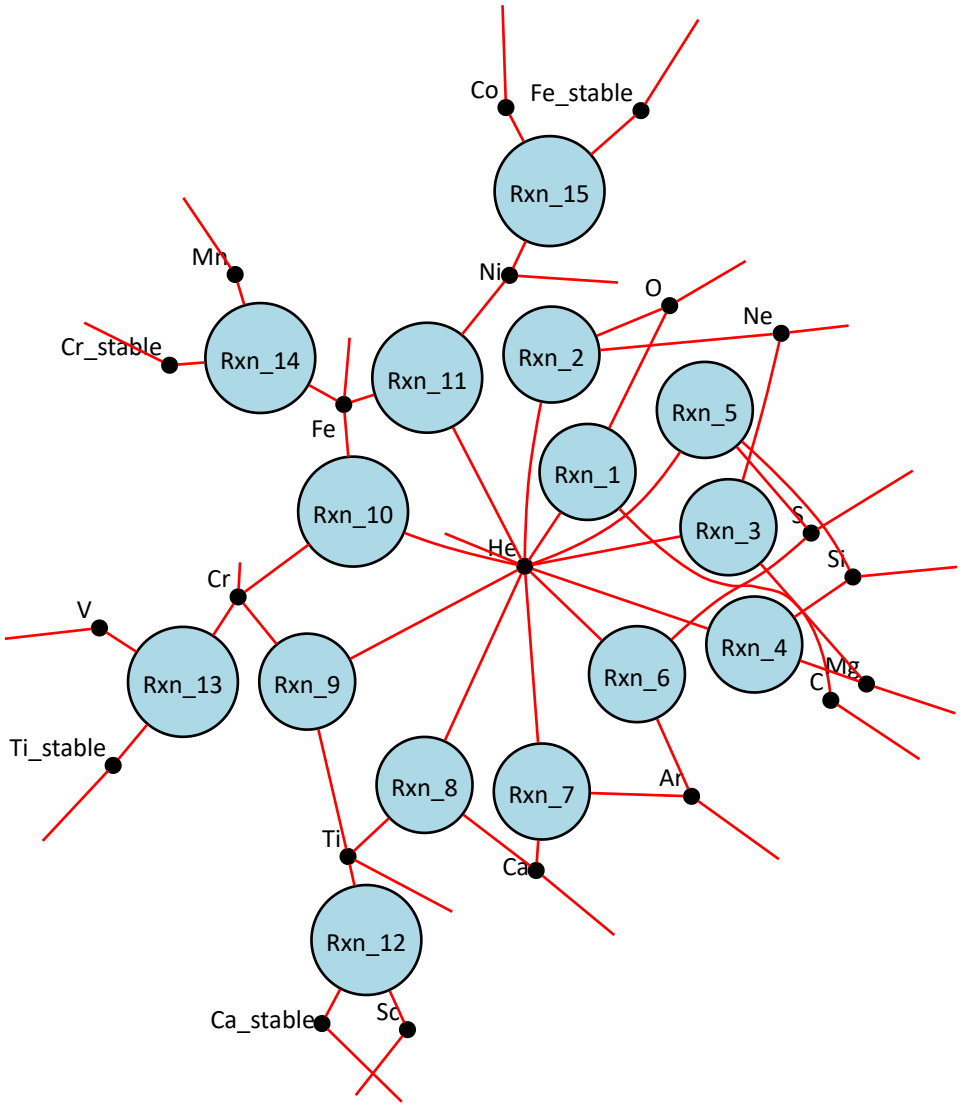
```
begin
    # Generate all the fusion reactions
    Rxn_1 = HE_addition((:He, :C, :O), :He_C)
    Rxn_2 = HE_addition((:He, :O, :Ne), :He_O)
    Rxn_3 = HE_addition((:He, :Ne, :Mg), :He_Ne)
    Rxn_4 = HE_addition((:He, :Mg, :Si), :He_Mg)
    Rxn_5 = HE_addition((:He, :Si, :S), :He_Si)
    Rxn_6 = HE_addition((:He, :S, :Ar), :He_S)
    Rxn_7 = HE_addition((:He, :Ar, :Ca), :He_Ar)
    Rxn_8 = HE_addition((:He, :Ca, :Ti), :He_Ca)
    Rxn_9 = HE_addition((:He, :Ti, :Cr), :He_Ti)
    Rxn_10 = HE_addition((:He, :Cr, :Fe), :He_Cr)
    Rxn_11 = HE_addition((:He, :Fe, :Ni), :He_Fe)

    # Generate all the decay reactions
    Rxn_12 = decay_reaction((:Ti, :Sc, :Ca_stable), (:Ti_Sc, :Sc_Ca))
    Rxn_13 = decay_reaction((:Cr, :V, :Ti_stable), (:Cr_V, :V_Ti))
    Rxn_14 = decay_reaction((:Fe, :Mn, :Cr_stable), (:Fe_Mn, :Mn_Cr))
    Rxn_15 = decay_reaction((:Ni, :Co, :Fe_stable), (:Ni_Co, :Co_Fe))

end
```

Thanks to Hirithik, the below code snippet helps to visualise the composition more clearly.

```
display_uwd (generic function with 1 method)
```

```
display_uwd(ex) = to_graphviz(ex, box_labels=:name, junction_labels=:variable,
graph_attrs=Dict(
        "layout" => "neato",
        "bgcolor" => "white",
        "overlap" => "false",
        "splines" => "true"
    ),
    node_attrs=Dict(
        "shape" => "circle",
        "style" => "filled",
        "fillcolor" => "lightblue",
        "fontname" => "Calibri",
        "fontsize" => "10"
    ),
    edge_attrs=Dict(
        "color" => "red",
        "penwidth" => "1"
    ))
```

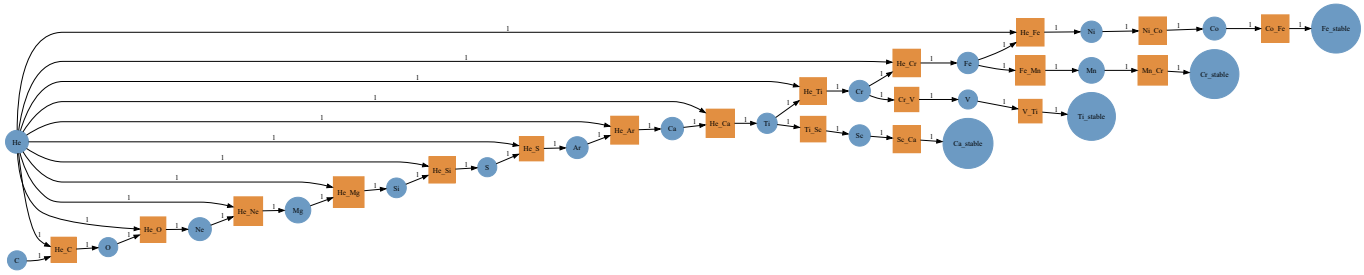
Using `@relation` to create a composition pattern, in which the relation between each individual reaction (open petri-net) is described.




```
begin
    alpha_chain_composition_pattern = @relation (He, C, O, Ne, Mg, Si, S, Ar, Ca,
    Ti, Cr, Fe, Ni, Sc, Ca_stable, V, Ti_stable, Mn, Cr_stable, Co, Fe_stable) where
    (He, C, O, Ne, Mg, Si, S, Ar, Ca, Ti, Cr, Fe, Ni, Sc, Ca_stable, V, Ti_stable,
    Mn, Cr_stable, Co, Fe_stable) begin
        # Rxn_1 through Rxn_15 mapped to the respective elements in the alpha chain
        Rxn_1(He, C, O)
        Rxn_2(He, O, Ne)
        Rxn_3(He, Ne, Mg)
        Rxn_4(He, Mg, Si)
        Rxn_5(He, Si, S)
        Rxn_6(He, S, Ar)
        Rxn_7(He, Ar, Ca)
        Rxn_8(He, Ca, Ti)
        Rxn_9(He, Ti, Cr)
        Rxn_10(He, Cr, Fe)
        Rxn_11(He, Fe, Ni)
        Rxn_12(Ti, Sc, Ca_stable)
        Rxn_13(Cr, V, Ti_stable)
        Rxn_14(Fe, Mn, Cr_stable)
        Rxn_15(Ni, Co, Fe_stable)
    end
```

```
    # Visualizing the forest composition pattern with to_graphviz
    #to_graphviz(alpha_chain_composition_pattern, box_labels = :name,
    junction_labels = :variable)
    display_uwd(alpha_chain_composition_pattern)
```

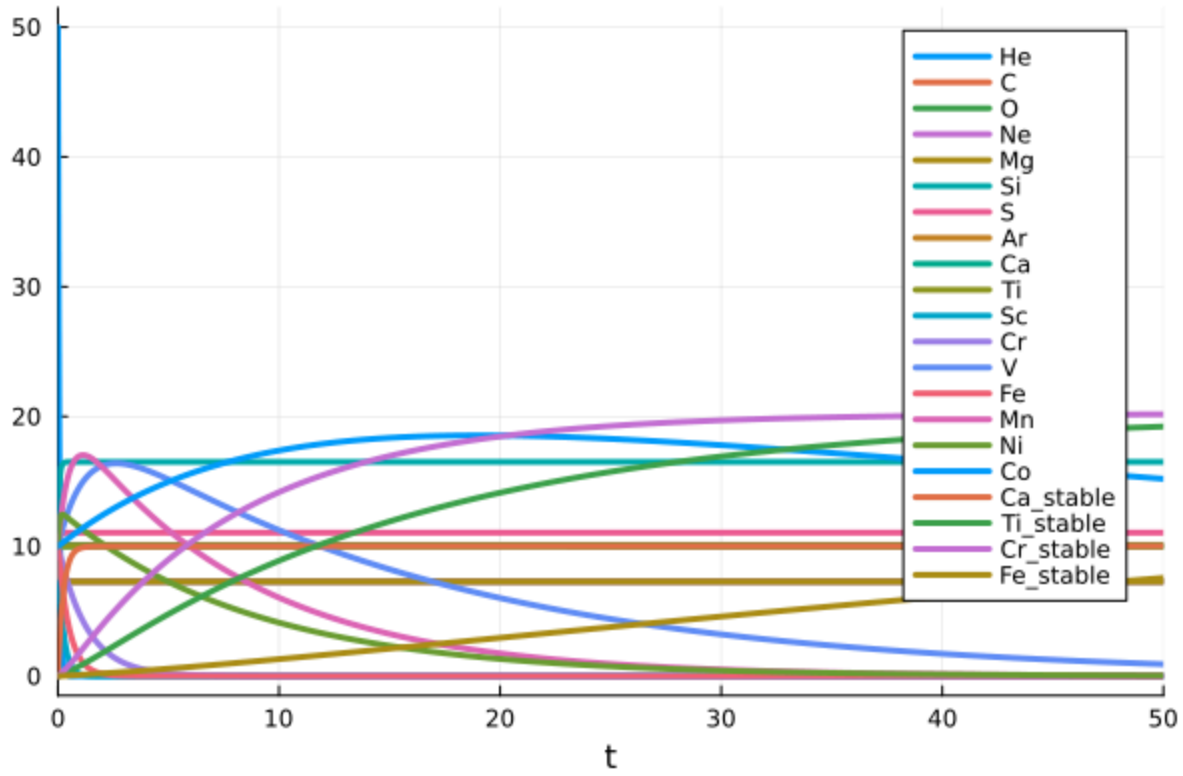
`oapply` is used to apply the composition pattern on the open petri-nets.




```
begin
    rxn_composite = oapply(alpha_chain_composition_pattern,
        Dict(
            :Rxn_1 => Rxn_1,
            :Rxn_2 => Rxn_2,
            :Rxn_3 => Rxn_3,
            :Rxn_4 => Rxn_4,
            :Rxn_5 => Rxn_5,
            :Rxn_6 => Rxn_6,
            :Rxn_7 => Rxn_7,
            :Rxn_8 => Rxn_8,
            :Rxn_9 => Rxn_9,
            :Rxn_10 => Rxn_10,
            :Rxn_11 => Rxn_11,
            :Rxn_12 => Rxn_12,
            :Rxn_13 => Rxn_13,
            :Rxn_14 => Rxn_14,
            :Rxn_15 => Rxn_15
        )
    )

    to_graphviz(rxn_composite, program="osage")
end
```

Now using appropriate rate constants and initial conditions, we can solve this dynamical system by taking the ODEs using `ODEProblem()` and using `solve()` to find the solution to the differential equations. The solution is then plotted using `plot()`.

Transition	Half-Life	Rate Constant (/day)
Cr-48 → V-48	21.6 hours	0.77
Ti-44 → Sc-44	60 years	3.208e-5
Sc-44 → Ca-44	3.97 hours	4.189
V-48 → Ti-48	15.9735 days	0.0626
Fe-52 → Mn-52	8.275 hours	2.01
Mn-52 → Cr-52	5.59 days	0.124
Ni-56 → Co-56	6.1 days	0.1136
Co-56 → Fe-56	77.1 days	0.009

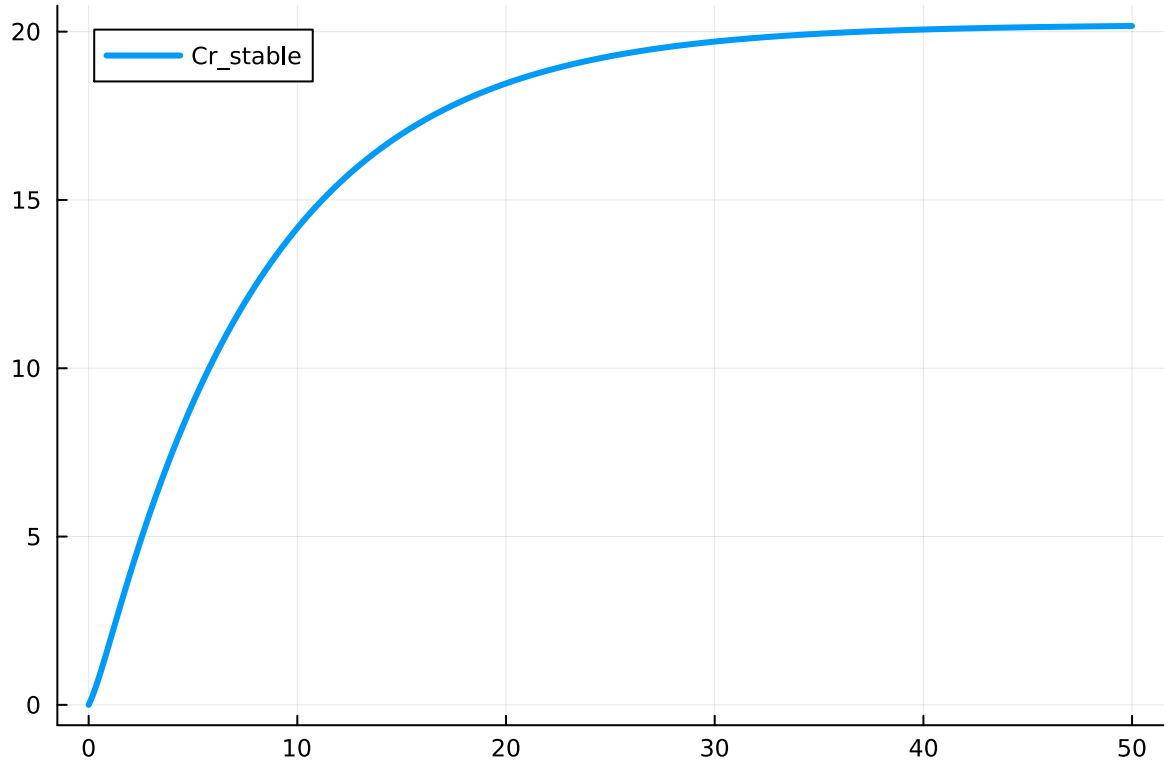



```
begin
    # Assigning parameter values and initial conditions and finding the solution for
    this Dynamical System
    p = LVector(
        He_C = 0.1,
        He_O = 0.2,
        He_Ne = 0.3,
        He_Mg = 0.4,
        He_Si = 0.1,
        He_S = 0.1,
        He_Ar = 0.1,
        He_Ca = 0.1,
        He_Ti = 0.1,
        He_Cr = 0.1,
        He_Fe = 0.1,
        Ti_Sc = 3.208e-5,
        Sc_Ca = 4.189,
        Cr_V = 0.77,
        V_Ti = 0.0626,
        Fe_Mn = 2.01,
        Mn_Cr = 0.124,
        Ni_Co = 0.1136,
        Co_Fe = 0.009
    )
    u0 = LVector(
        He = 50,
        C = 10,
        O = 10,
        Ne = 10,
        Mg = 10,
        Si = 10,
        S = 10,
```

```
        Ar = 10,
        Ca = 10,
        Ti = 10,
        Sc = 10,
        Cr = 10,
        V = 10,
        Fe = 10,
        Mn = 10,
        Ni = 10,
        Co = 10,
        Ca_stable = 0,
        Ti_stable = 0,
        Cr_stable = 0,
        Fe_stable = 0
    )
    soln = solve(ODEProblem(vectorfield(apex(rxn_composite)), u0, (0.0, 50.0), p))
    plot(soln, linewidth=3)
end
```

You can also view the solution curve for each element using the command:

`plot(soln.t, soln[20, :], label="Cr_stable", linewidth=3)` (Cr_stable is the 20th component of soln vector)


Cr_stable
20
15
10
5
0
0
10
20
30
40
50