Monoids:

- Takeaway: The generic category of monoids is used to describe the idea of a “functor”.
- Monoid: A category with exactly one element is called a monoid. Hence, all the maps are endomaps.

  Let M be a monoid with the object #, the maps are (endomaps) represented by n (where n is a natural number) n: # → # and in order to assess we will define the composition for this specific monoid category as multiplication of the two maps and hence the identity map would be 1 [as, 1(n)=1*n=n*1=n]. But this object seems featureless, so to understand this in terms of an object with more structure we can interpret the monoids in following way:

  M → S (the idea is to understand what this represents).

  Where S is sets category with only object as set of natural numbers N. The maps are such that they preserve the structure of composition of M (only those maps are present in this category). Let, they be represented as f_n and defined as $f_n=n*x$ where n is the corresponding map number in monoid category and x is an element in N: f_n: N → N. The identity map would be $f_1 = x$

  Here, the $f_nf_m = n*(m*x) = nmx = f_{nm}$ is the structure preserving rule.

  This interpretation is comprehensible (due to more features and structure) and preserves the structure of the original category.

  This transformation from one category to another category is called a functor (It should also preserve the notion of domain and codomain).

  Such a functor also sheds light on the sense in which we can use the symbol of raising to minus one as a vast generalization of 'inverse.' If we change the example slightly, taking rational numbers instead of natural numbers as the maps in M, we'll find that $(f_3)^{-1} = f_{(3\wedge-1)}$.

  The inverse map of a map in the list of interpretations is also an example of the maps in the list, so if a 'named' map is invertible, the inverse can also be named. In the example above, A f_3 is invertible and its inverse is named by the inverse of 3. But if the maps in M consist only of the natural numbers, and # is interpreted as the set of rational numbers, then f_3 has an inverse, but now it is not named since there is no natural number inverse of 3.