Product in Categories:

Def: Suppose that A & B are objects in a category C. A product of A & B (in C) is

1. An object P in C
2. A pair of maps, $P \xrightarrow{P_1} A$ & $P \xrightarrow{P_2} B$ satisfying

for every object T & every pair of maps $T \xrightarrow{q_1} A$, $T \xrightarrow{q_2} B$, there is exactly one map $T \xrightarrow{q} P$, s.t $q_1 = P_1 \circ q$ & ~~$q_2 \circ p$~~ $q_2 = P_2 \circ q$.

Illustration: (Using motion in space)

let P be the object called space, A be plane and B be pole/height

$$\begin{tikzcd} & A \\ P \arrow[ur, "P_1"] \arrow[dr, "P_2"'] & \\ & B \end{tikzcd}$$

P_1: shadow ; P_2: level

let time interval be replaced by an object T

$$\begin{tikzcd} & A \\ T \arrow[ur, "q_1"] \arrow[dr, "q_2"'] & \\ & B \end{tikzcd}$$

q_1: time interval in ~~for~~ A

q_2: time interval in B

motion in P is uniquely determined by motions in A and B

$T \xrightarrow{q} P$:

$$\begin{tikzcd} & & A \\ T \arrow[r] \arrow[urr, bend left, "q_1"] \arrow[drr, bend right, "q_2"'] & P \arrow[ur, "P_1"] \arrow[dr, "P_2"'] & \\ & & B \end{tikzcd}$$

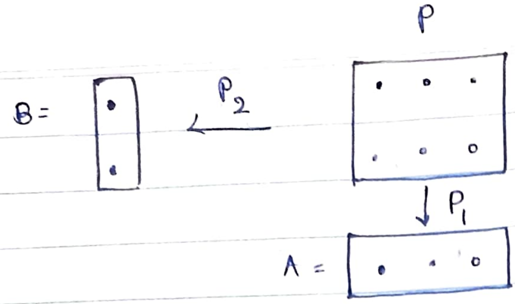
$\longrightarrow$ should commute

$q_1 = P_1 \circ q$ (motion in plane: what was ~~point's show~~ the position of point's shadow at a certain instant of time)

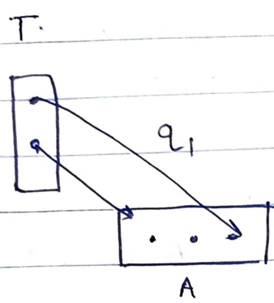
$q_2 = P_2 \circ q$

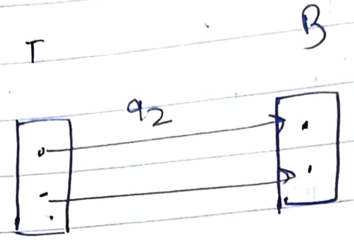
Note: A product is not only an object, but an object $\underbrace{\text{with two maps}}$. (to maintain uniqueness)

Date:

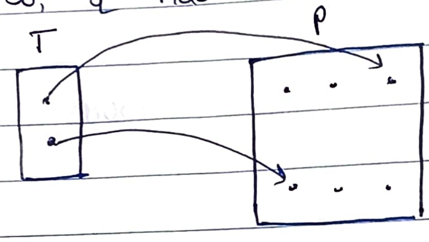


let





so, q has to be



Uniqueness theorem:

Suppose that $A \xleftarrow{p_1} P \xrightarrow{p_2} B$ and $A \xleftarrow{q_1} Q \xrightarrow{q_2} B$ are two products of A and B. Viewing Q as a "test object" gives a map $Q \to P$; because $A \xleftarrow{q_1} Q \xrightarrow{q_2} B$ is also a product we get $P \to Q$ map. These two maps are necessarily inverse to each other. and therefore two objects P, Q are isomorphic.

Two consequences of the theorem:

1) The theorem indicates the uniqueness property as we know, if two objects are isomorphic they are essentially same
2) If there does not exist an isomorphism between P and Q Or then we can't have a product of two objects.