David Spivak and Brendan Fong's

Applied Category Theory

Generative Effects ; Orders and Galois connections.

Generative effects:
Consider a system (Domain object), an observation (map), observed ~~object~~ (codomain object) system and a system level operation (like a structure). If the observation doesn't preserve the structure of the system, it leads to something called generative effect. (surprises)
We see later that this is an ~~additional structure~~ ~~in~~ a phenomenon where an additional structured arises in observed system.

Takeaway:
To remove the surprises or remove generative effect (in order to preserve certain structures, here meets and joins)

<u>Some Basic concepts</u>:
~~Def: If A is a set~~, a P

I Partition:
<u>Def</u>: If A is a set, a partition of A consists of a set P, for each $p \in P$, a non empty subset $A_p \subseteq A$ such that

i) $A = \bigcup_{p \in P} A_p$ (union of $\forall A_p$ is A)

ii) $A_p \cap A_q = \emptyset$ if $p \neq q$ (intersection of two parts is zero)

(there can be many partitions)
P is called set of part labels
A_p as the p^{th} part

- one-to-one correspondance between ways to partition A and the equivalence relations on A:

Def Given a set A and an equivalence relation $\sim$ (infix) on A, we say that the quotient is $A/\sim$ of A under $\sim$ is the set of parts of the corresponding partition.

Intuition: It says that for an equivalence relation $\sim$ on A gives you a partition, so depending on how many (and what those) ~~partitions are we~~ get relations ($\sim$: *, +, ...} are we get ~~partition~~ ~~&~~ corresponding partitions.

- A partition on a set A can also be understood in terms of surjective functions out of A.

Def Given a surjective function $f: A \twoheadrightarrow P$, where ~~p~~ P is any other set, the preimage $f^{-1}(p) \subseteq A$, one for each element $p \in P$ form a partition of A

Intuition:
To build a partition of A we say that all the preimages ~~of~~, $f^{-1}(p)$ belongs to the same subset of A and become a part, for this to be surjective (hence, partition) we know $n(A) \geq n(P)$
If $n(P) = k$ and $n(A) = l$
~~I $k \neq l$ the number of partitions of A = $k!\,k^{l-k+1}$ ($\cancel{k-1}$) ${}^{l}C_k$~~
~~II $k = l$ — the number of partitions of A = $l!\,l = k!\,k$~~

I ~~or~~ the number of partitions on A:

$$\sum_{i=1}^{k} {}^{l}C_i \; i! \; i^{l-i}$$

~~II~~

~~It says~~

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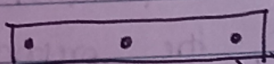
Date:

II. Pre-order:
It is essentially a set P X endowed with a binary relation $\leq$, i.e. eg. $(X, \leq)$

Def: A pre-order relation on a set X is a binary relation on X, such that (notation $X : \leq$)
a) $x \leq x$ (reflexivity)
b) if $x \leq y$ and $y \leq z$ then $x \leq z$ (transitivity)

special pre-orders:

1) Discrete pre-orders: Every set X can be considered as a discrete pre order $(X, =)$ if the only order relations on X are of the form $x \leq x$ & if $x \neq y$ then neither $x \leq y$ nor $y \leq x$ hold.
depicted simply as:



(Hasse Diagram)

2) codiscrete pre-order:
~~Here~~ It is a preorder $(X, \leq)$ where <u>all</u> x & $y \in X$ we have $x \leq y$ (and hence also $y \leq x$)

3) Partial order: (skeletal pre-orders), Posets)
If a pre-order $(X, \leq)$ ~~hets~~ additionally has:
$x \in X$ and $y \in Y$
for $x \cong y$ (i.e., $x \leq y$ and $y \geq x$) $\leftrightarrow$ $x = y$

4) Opposite pre-order: Given a pre order $(P, \leq)$, we define the opposite pre-order $(P, \leq^{op})$ to have same set of elements, but with $p \leq^{op} q$ iff $q \leq p$

5) Total order:
A pre-order $(X, \leq)$ which has additional property: * if $x \leq y$ and $y \leq x$ $\rightarrow$ $x = y$
* all x & $y \in X$ have either $x \leq y$ or $y \leq x$

- power set: A set ~~of~~ consisting of all the set of X

Graph:

def: A graph $G = (V, A, s, t)$ consists of a set V whose elements are called vertices, a set A whose elements are called arrows, and two functions $s, t : A \to V$ known as the source and targe functions respectively. Given $a \in A$ with $s(a) = v$ and $t(a) ~~w~~ = w$ we mean,

$$v \xrightarrow{a} w$$

By a path in G, we mean any sequence of arrows such that the target of one is the source of the next.
(Ex 1.37, page no. 14)

- Hasse Diagram:
A graph (directed) to represent pre-orders. In Hasse diagram an arrow is drawn from ~~a~~ x_1 to ~~b~~ $x_2 \in X$ $[(X, \le)]$ if ~~s(a)~~ ~~s(x_1)~~ $s(a) = x_1$ and $t(a) = x_2]$
iff $x_1 \le x_2$.
We will also see that we can extract some other features like join and meet.

$$\begin{tikzcd} b & & c \\ & a \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$a \le c$ and $a \le b$

- Upper Sets: A pre-order $(P, \le)$, an upper set in P is a subset U of P satisfying the condition that ~~p~~ if $p \in U$ and $p \le q$, then $q \in U$
[Ex: Booleans $(B, \le)$]
and U is called set of upper sets and pre-order of U of Booleans $(B, \le)$

$$\{True, False\}$$
$$\uparrow$$
$$U(B) = \{true\}$$
$$\uparrow$$
$$\varnothing$$

$$B = \begin{matrix} True \\ \uparrow \\ False \end{matrix}$$

- Product pre-order:

Def: Given a pre-order $(P, \leq)$ and $(Q, \leq)$ we may define a preorder structure on the product set $P \times Q$ by setting $(p,q) \leq (p', q')$ iff $p \leq p'$ and $q \leq q'$

Ex:

$$P = \begin{tikzcd} c & & b \\ & a \arrow[ul] \arrow[ur] & \end{tikzcd} \qquad Q = \begin{tikzcd} 2 \\ 1 \arrow[u] \end{tikzcd}$$

$$P \times Q = \begin{tikzcd} (c,2) & & (b,2) \\ & (a,1) \arrow[ul] \arrow[ur] & \end{tikzcd}$$