(Yr ...

→ Prop is a kind (special) of monoidal category. Monoidal category's wiring [$(\mathcal{C}, I, \otimes)$] diagrams can interpret parallelism (tensor product $\otimes$, an additional element of the monoidal category tuple) unlike categories [or correspondingly preorders, $\mathcal{C} \cong (P, \leq)$].

→ Free props: [You tell me what to put in there and I'll freely invent this thing]

→ Presentations of props (generators & relations)

→ Graphical proof systems

→ Monoidal poset (simplification in two monoidal oper. orthogonal directions) [partial order & a monotone

The cat.
∴ $\mathcal{C}$ is a poset (meaning there is a unique morphism between any two objects, either one or none)

→ **Props**: symmetric monoidal category
where $ob(\mathcal{C}) = \mathbb{N}$, $I = 0$, $\otimes = +$ $[m \otimes n = m+n]$

Examples of props: 1) Mat - $\mathcal{C}$

$ob(Mat) = \mathbb{N}$ $\quad m \times n \to \mathbb{R}$

Functions $= Mat(m, n) =$ $m \times n$ matrix

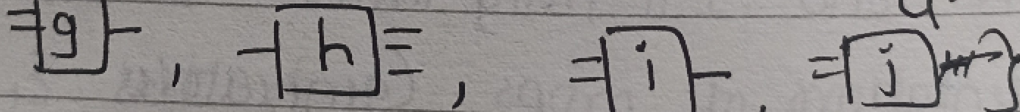
2) Port graphs (we're looking at acyclic port graphs)

→ Free props: Given a set G ("generators") and functions $in: G \to \mathbb{N}$, $out: G \to \mathbb{N}$ the free prop on G is the prop whose morphisms ~~are~~ $m \to n$ are "G-labelled port graphs"

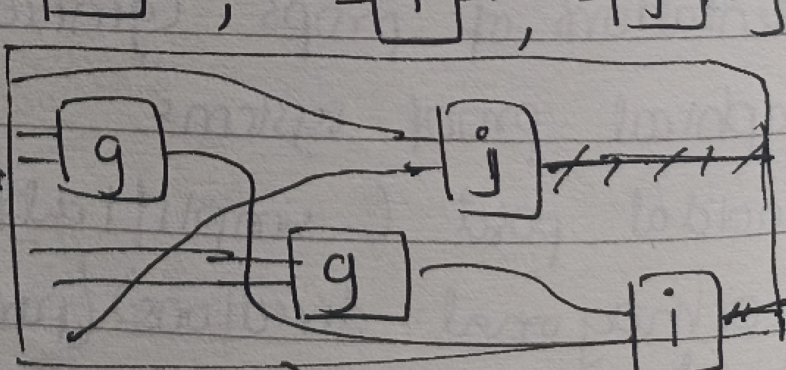
Meaning: $(V, in, out, i, \ell: V \to G)$ preserving

$$\begin{tikzcd} \mathbb{N} & V \arrow[l, "out"'] \arrow[r, "in"] \arrow[d, "\ell"] & \mathbb{N} \\ & G \arrow[ul, "out"] \arrow[ur, "in"'] & \end{tikzcd}$$

Ex:

$G = \{$  $\}$

G-labelled Port graph (a morphism in a morphism between 6 & 2)



(SFG)

signal flow graphs: a free prop

$G = \{\ ,\ ,\ ,\ \sigma\ \} \cup \{\ \mid a \in \mathbb{R}\}$

copy, scale up, add, zero, scale up

$\begin{pmatrix} in = 1 \\ out = 2 \end{pmatrix}$ $\begin{pmatrix} in = 1 \\ out = 0 \end{pmatrix}$ $\begin{pmatrix} in = 2 \\ out = 1 \end{pmatrix}$ $\begin{pmatrix} in = 0 \\ out = 1 \end{pmatrix}$ $\begin{pmatrix} in = 1 \\ out = 1 \end{pmatrix}$

we need generating set, in & out functions to give me free prop

prop morphism/ functor: $SFG \to Mat$ (symmetric monoidal)

In general to give a functor out of a free prop, one just has to say where the generators have to go (on define l?)

(Min rules possible, or more flexible, in free prop) key i.e. the signal is one

$I(\ ) = (1, 1) \in Mat(1, 2)$

$I(\ ) = (\ ) \in Mat(1, 0)$

$I(\ ) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \in Mat(2, 1)$

$I(\ ) = (\ ) \in Mat(0, 1)$

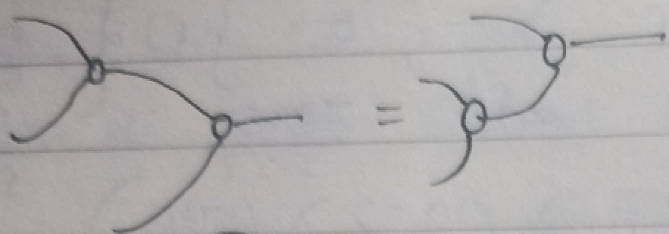
$I(\ ) = (a) \in (1, 1)$

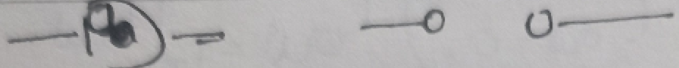
signal flow graphs are syntax & matrices are semantics

so a functor give meaning to semantics

syntax is a way of representing

presentations: free + equations

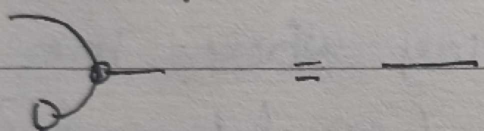
1) 

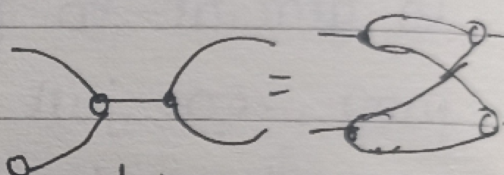
2)  [(0) × ( ) (1,0) matrix (0,1) matrix) = (a) on matrices (1,1) matrix)

kinda like ←

$5^0=1$, $0!=1$

17 known relations (equations) people like people like corlte.

3) 

4) 

→ Isomorphism

SFG / 17 weird pictures $\xrightarrow{\cong}$ Matrix [meaning they (A up) make sense]

~~This is called~~ These are

- sound; (meaning) any time you can get from one set sfg to another using these rules, they give the same matrix
- complete: Any time two matrices are equal two signal flow graphs give same matrix you can get from one to the other (These things will give proof that two matrices are same)

From matrices to behavior:

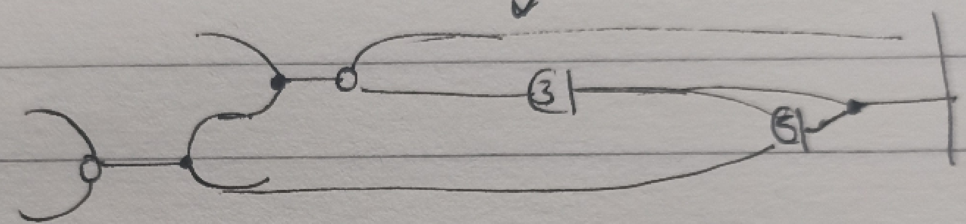
$y = Mx \rightarrow Ny = Mx$

- Behavior: some kind of linear relation between two vector spaces { two $\mathbb{R}^n$s }

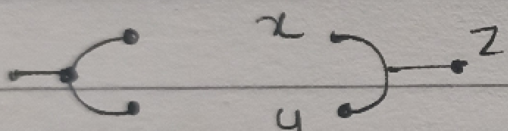
$\{(x,y) \mid Ny = Mx\}$

A (linear) behaviour of type (m,n) is a subspace

All such behaviours ~~are~~ can be represented by signal flow graphs + their reverses ✓



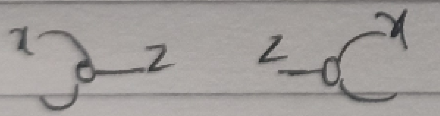
Reverses



$= \{((x,y), z) \in \mathbb{R}^2 \times \mathbb{R} \mid x = y = z\}$

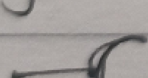
$= \{ x \in \mathbb{R} \mid \}$

$= \{ x \in \mathbb{R} \mid x = 0 \}$

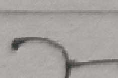


$= \cancel{\{}$ $\cancel{x+y=z}$

$\{((x,y,z) \in \mathbb{R}^2 \times \mathbb{R} \mid x+y=z\}$



$= \{ (x,x) \in \mathbb{R}^2 \}$



$= \{ (x,x) \in \mathbb{R}^2 \}$

we can use these to form cups & caps
[compacts closed category]