To answer questions like, "Can I make what I want from what I have?" or, "How much will it cost to obtain something?", the following ideas may be employed to build resource theories.

Symmetric Monoidal Preorders

A preorder $(X, \leq)$ may be given extra structure in the following way:

- identify some $I \in X$ — monoidal unit
- define a function $\otimes : X \times X \longrightarrow X$ — monoidal product

such that

a) if $x_1 \leq y_1$ and $x_2 \leq y_2$ then $x_1 \otimes x_2 \leq y_1 \otimes y_2$ — monotonicity

b) $I \otimes x = x = x \otimes I$ — unitality

c) $x \otimes (y \otimes z) = (x \otimes y) \otimes z$ — associativity

d) $x \otimes y = y \otimes x$ — symmetry

Such a structure is called a symmetric monoidal preorder.

→ A monoid is a set M, a function $* : M \times M \rightarrow M$ and some $e \in M$ such that $*$ is unital w.r.t e and associative.

→ e.g: $\mathbf{Bool} = (\mathbb{B}, \leq, \text{true}, \wedge)$

↓ more useful

$\wedge$	F	T
F	F	F
T	F	T

- $(\mathbb{B}, \leq, \text{false}, \vee)$

$\vee$	F	T
F	F	T
T	T	T

- Cost $:= ([0,\infty], \geq, 0, +)$

→ Note: $(X, \leq)^{op} := (X, \geq)$

Monoidal Monotone Maps

- A map $f : (P, \leq_P) \longrightarrow (Q, \leq_Q)$ such that

  - $I_Q \leq_Q f(I_P)$

  - $f(p_1) \otimes_Q f(p_2) \leq_Q f(p_1 \otimes p_2)$

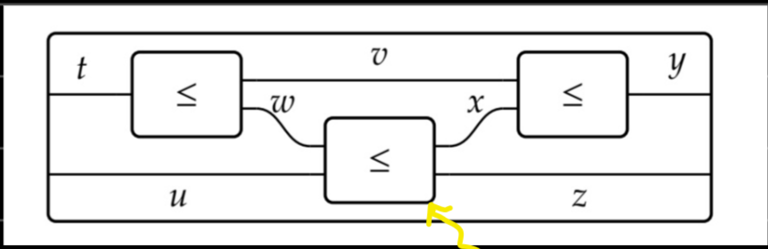
also called lax monoidal monotones

→ e.g.: For Bool $= (\mathbb{B}, \leq, \text{true}, \wedge)$, Cost $= ([0,\infty], \geq, 0, +)$ we have $g : \text{Bool} \longrightarrow \text{Cost}$ with $g(F) := \infty$, $g(T) := 0$.

Enrichment

→

Wiring Diagrams



$u \leq z$, $w \leq x$ and $u \otimes w \leq z \otimes x$