Lawvere Week-5

Section - 19

* In any category, an object T is a $\boxed{\text{terminal obj.}}$ iff

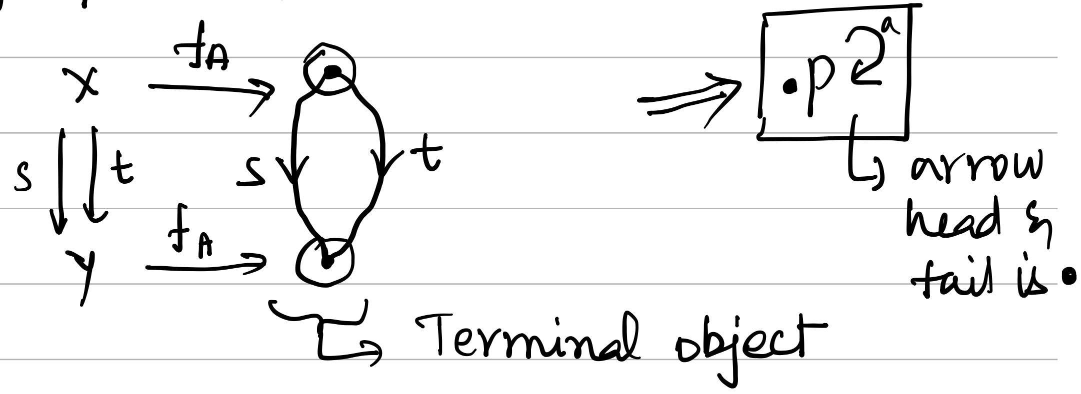
for each X in the category there is exactly <u>one map</u> from $X \to T$

↳ We use only maps to define T not the no. of elements in T

In case of S, a set with one element $\boxed{\bullet}$

" $S^{\circlearrowleft}$, endomap with one element mapped to itself $\boxed{\bullet\circlearrowleft}$

In category of $S^{\downdownarrows}$,



Theorem : In $\mathcal{C}$, if T_1, T_2 are both terminal objects, then T_1 and T_2 are isomorphic

↓

proof : By using fact that there can be only one map to a T, there is only one map from $T_1 \to T_2$ & ofc they're isomorphic

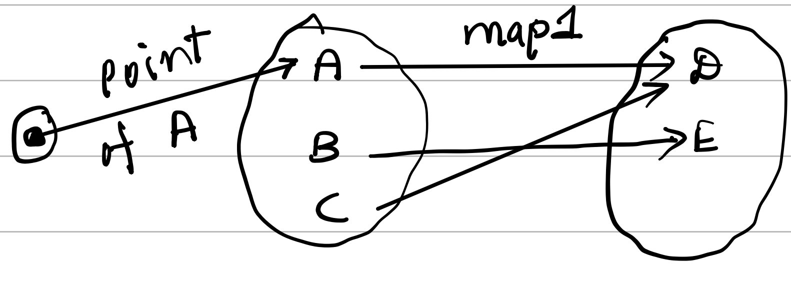
Section - 20

Points of an object :

A point of an object X is a 'map'

$T \to X$ where T is terminal.

In S_1



By doing $(\text{map1}) \circ (\text{point of } A) = \underline{\underline{D}}$

" I can use the elements of the set in terms of a map which is a point " which is needed in a category where we focus on maps

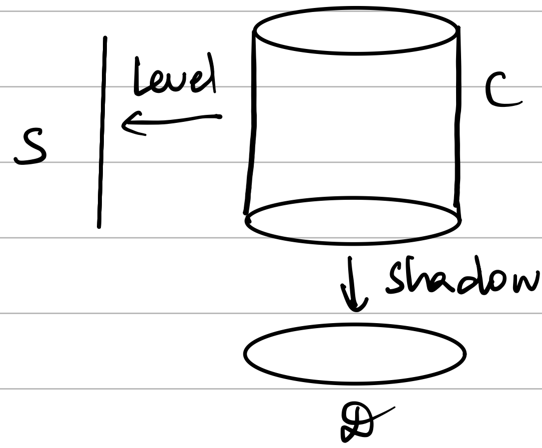
⇒ In $S^{\circlearrowright}$, if $X^{\circlearrowright\alpha}$ is an object, a point from T can map only to "fixed points" in the endomap inorder to obey $f \circ \alpha = \beta \circ f$

⇒ In $N^{\circlearrowright()+1}$, there are no points (no fixed -s)

In case of $S^{\downarrow\downarrow}$, the terminal object can map only to the point(s) where the head & tail of arrow is the same ($\boxed{p \bullet \circlearrowright^{a}}$) – loop

Section– 21

Products in category:



we say,
$C = S \times D$

Like the example above suggest, a product in a category means

* an object P
* & maps from product to factors
$P \xrightarrow{P_1} A \quad P \xrightarrow{P_2} B$

Theorem: Suppose $A \xleftarrow{p_1} P \xrightarrow{p_2} B$ & $A \xleftarrow{q_1} Q \xrightarrow{q_2} B$ are 2 products of A & B, these 2 products are isomorphic