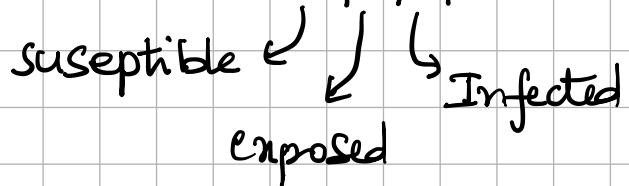


# M<sub>a</sub>thematical models of infectious diseases

## I) Basic SEIR Model : ( Flu )

- \* We use mathematics to model infectious diseases and compartmental models are one in which the population are assigned to compartments
- \* The SEIR model is a compartmental model in which the compartments are S, E, I, R → Recovered

( primitive model → SIR )



- \* It is used to analyse disease dynamics and estimate parameters like total no. of infected or recovered people; also a epidemiological parameter named 'R'

'R' → Basic Reproduction Number

for SIR model :

$$\frac{dS}{dt} = -\beta Si$$

$S = \frac{S}{N}$  = fraction  
of suscep.  
population

$\beta$  → avg. no. of contacts per person per time

$$i = \frac{I}{N}$$

×

probability that a disease transmits b/w a infected person & sus. person.

and, w.r.t rate  $\propto$  concentration so we multiplied

$$\frac{dR}{dt} = \gamma I$$

If an infected person is infectious for a average time period ' $D$ '

$$\gamma = \frac{1}{D}$$

We can obtain  $\frac{dI}{dt}$  by  $\frac{d(S + I + R)}{dt} = \frac{dN}{dt} = 0$

for no birth & death rates

$\Rightarrow$  Terms:

\*  $R_0 = \frac{\beta}{\gamma}$   $\rightarrow$  expected new infections from a single infection in a population where all are susceptible



understood as

typical time contact b/w

$$S \& I \Rightarrow T_c = \frac{1}{\beta} \text{ s}$$

$$\text{Removal} \Rightarrow T_r = \frac{1}{\gamma},$$

No. of contacts by infected individual before the infection has been removed

$$= \frac{T_r}{T_c} = \frac{\beta}{\gamma}$$

$$\frac{dI}{dt} = (R_0 \frac{S}{N} - 1) \gamma I$$

for proper epidemic

outbreak  $\frac{dI(0)}{dt} > 0$

$$\therefore R_0 S(0) > N$$

$$R_0 \frac{S(0)}{N} > 1$$

(Generally  $R_0 > 1$ )

\* Force of infection :  $F = \beta i$

If birth rate  $\lambda$  & death rate  $\mu$  is considered,

$$\frac{dS}{dt} = \lambda - \mu S - \frac{\beta i S}{N}$$

$$\frac{di}{dt} = \frac{\beta i S}{N} - \gamma i - \mu i$$

$$\frac{dR}{dt} = \gamma i - \mu R$$

$$R_0 = \frac{\beta}{\mu + \gamma}$$

SEIR Model :

\* For many diseases we have latency period  $\rightarrow$  time b/w when the person is infectious and when the symptoms show up and is known infectious.

During this time, we name them as 'Exposed'

If the latency period is  $a^+$ , then we use 'a' as parameter for the eq.

Assuming birth and death rates are equal.

$$\frac{dS}{dt} = \underbrace{\mu N}_{\text{Total birth}} - \underbrace{\mu S}_{\text{sus. death}} - \frac{\beta i S}{N}$$

$$\frac{dE}{dt} = \frac{\beta i S}{N} - (\mu + a) E$$

$$\frac{dI}{dt} = a E - (\gamma + \mu) I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

$$R_0 = \frac{a}{\mu + a} \times \frac{\beta}{\mu + \gamma}$$

here also,  $\sum \frac{d}{dt} = 0 \Rightarrow$  total pop. remains same

DBS : General SEIR Model vs SEIR model with vital dynamics

$$\lim_{t \rightarrow \infty} S, I, E \approx 0$$

$$\tau \approx 1 \\ (\text{Ideal})$$

$$\text{for } \mu = 0.02,$$

$$\lim_{t \rightarrow \infty} S \approx 0.122$$

$$E \approx 0.017$$

$$I = 0.143$$

$$\tau = 0.717$$

(Near realistic)

## \* Interacting Subpopulation SEIR Model



Models differ b/w pop. of different age groups

so SEIR model for diff. age groups are modelled & connected through interaction links