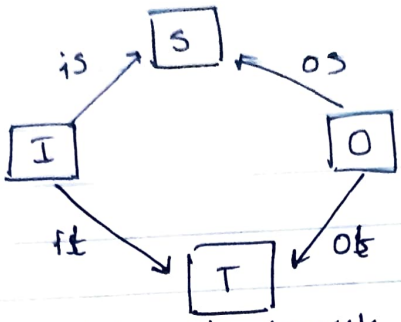
Reflecting on Frame work the code to understand the ~~framework~~.

1) Model specification & Semantics / Defining petri nets (whole-grain petri nets)
A petri net is a combinatorial description of a dynamical Process.



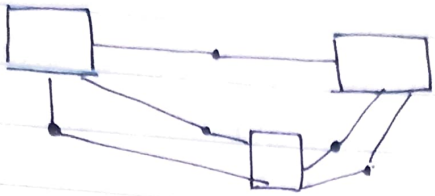
- A whole grain-petri net consists of a finite set of places or species (S), a finite set of transitions (T) and spans $S \xleftarrow{is} I \xrightarrow{it} T$ and $S \xleftarrow{os} O \xrightarrow{ot} T$ defining input & output arcs between S and T

- Petri nets are closed systems (isolated) to ~~open them for composition interaction (composition)~~ formalise the opening of these systems for interaction (composition) we use structured cospans & decorated cospans.
So, An open Petri Net is a whole grain petri net together with a list of finite sets $A_1, A_2, A_3 \ldots A_n$ and functions $A_1 \to S, \ldots A_n \to S$.

2) syntax to compose; Undirected wiring diagrams (UWD)
- UWD are a generic graphical syntax for composing relations, databases tables, structured multi-cospans and other undirected systems.
- Undirected systems: An undirected system is a graph where the edges are bidirectional, meaning that there is no specific direction associated with them.

Date:

- An UWD consists of boxes, a set of ports and a set of junctions. Each port is assigned to a box and wired to a junction

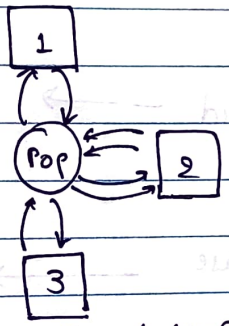


In our context, boxes represent submodels (systems) The systems compose by identifying ~~comp~~ places that are connected in UWD.

3) Stratified Compartmental models:

→ Typed Petri Nets:

- Type system: A general petri Net / a rule the Petri net Pinfectious is a type system for an infectious diseases model.

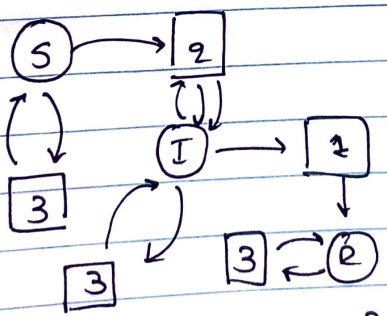


→ Pinfectious = Ptype

i) single species type: Population

ii) Three transistion types: Infection status, non-infection related strata (movement / quarantine), Intraction

- let P:



be a petri-net

then P a typed petri-net is Petri net P together with a morphism $\phi: P \rightarrow P_{type}$

[A morphism between Petri net is a map of places, transitions & input arcs and output arcs that preserve arities (structure) of the arcs & respects the sources & targets of

the arcs.

→ Stratifying models:
We may need to include other factors like migration, quarantine etc to make it more practical. So, the models are stratified.

Consider two typed Petri nets; the unstratified the disease model $\phi: P \to P_{type}$ and a stratification scheme $\phi': P' \to P_{type}$. The stratification of P by P' is defined to be the petri-net with places (respective transitions, input arcs & output arcs) consisting of pairs of places (respective transitions, input arcs & output arcs) in P and P' have the same type.

$$\begin{array}{ccc} P_{stratified} & \longrightarrow & P_{strato}\ (=P') \\ \downarrow & & \downarrow \\ P_{disease} & \longrightarrow & P_{infectious}(P_{type}) \\ (P) & & \end{array}$$

A stratified model over $P_{infectious}$ is the pullback of a typed epidemiological model $P_{disease} \to P_{infectious}$