

# Resource Theories: Monoidal Preorders and Enrichment

## Resource theories: (Takeaway)

The ideas of how to go from what we have to what we want.

We will study two components (majorly) to understand how questioning: given what I have,

- i) Is it possible to get what I want?
- ii) What is the minimum cost to get what I want?
- iii) Set of ways to get what I want?  
will help in transformation.

[Getting from A to B]

## I Symmetric Monoidal preorders [additional structure]

→ Def: (symmetric Monoidal structure on a preorder) consists of two constituents:  $(X, \leq)$

- (i) Monoidal unit : an element  $\top \in X$
- (ii) a function  $\otimes : X \times X \rightarrow X$  called monoidal product

$$[\otimes(x_1, x_2) = x_1 \otimes x_2]$$

For which the following conditions must satisfy.

monoidality  $x_1, x_2, y_1, y_2 \in X$  if  $x_1 \leq y_1$  and  $x_2 \leq y_2$

then  $x_1 \otimes x_2 \leq y_1 \otimes y_2$   $\otimes(x_1, x_2) \leq \otimes(y_1, y_2)$

unitality  $x, y \in X$  equations  $\otimes(\top, x) = x$  and  $\otimes(y, \top) = y$

Associativity  $x, y, z \in X$   $(x \otimes y) \otimes z = x \otimes (y \otimes z)$

symmetry  $x, y \in X$   $x \otimes y = y \otimes x$

then this monoidal preorder =  $(X, \leq, \top, \otimes)$  monoidal preordered set

order  $\rightarrow$   $\leq$  monoidal relation

- If we replace  $\leq$  with  $\geq$  it is called weak monoidal structure

- Main Monoidal (symmetric) preorders :

- 1) The Booleans:

$$(B, \leq, \text{true}, \wedge)$$

$$B = \{\text{True}, \text{False}\}$$

True  $\geq$  false

$$\top = \text{true}$$

$$\wedge = \text{AND}$$

- 2) The last:

$$([0, \infty], \geq, 0, +)$$

usual meaning

- The opposite of Monoidal pre-orders:

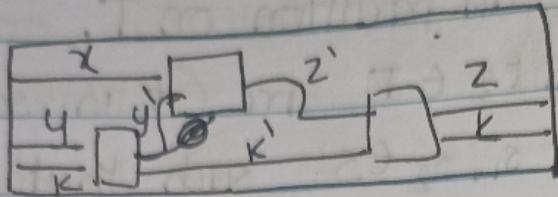
Suppose  $X = (X, \leq)$  is a preorder  $X^{\text{op}} = (X, \geq)$  is its opposite. If  $(X, \leq, \top, \otimes)$  is a symmetrised monoidal pre-order means  $(X, \geq, \top, \otimes)$  is also a pre-

Symmetric Monoidal preorder.

## II Wiring Diagram:

Wiring diagrams are visual representations for building new relationships from old.

consider:

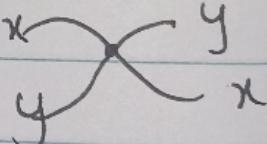


components

How does a wiring diagram represent 'symmetric Monoidal preorder'

- The wires are the elements of the preordered set.
- The boxes are relation order relations.
- The parallel lines (like  $x \& y'$  or  $y \& k'$ ) represent monoidal product.
- The nothing wire/line represents monoidal unit so,

Symmetry can be shown like this.



$$\text{Associativity : } \frac{x}{y} = \frac{x}{z}$$

$$= \frac{y}{z}$$

~~① some other components of a working diagram;~~

ribution than we can use ~~the~~ →

e) Discard axiom of choice.

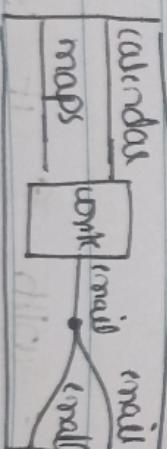
$a+b \rightarrow c+d$   
(and we don't mind this!)

The case of information (only) as element

can be copied like an email.

t) copy axiom

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### III Monoidal Monotone maps:

Def: Let  $P = (P, \leq_P, I_P, \otimes_P)$  and  $Q = (Q, \leq_Q, I_Q, \otimes_Q)$  be monoidal preorders.

A monoidal monotone map from  $P$  to  $Q$  is a monotone map  $f: (P, \leq_P) \rightarrow (Q, \leq_Q)$  such that:

- a)  $I_Q \leq_Q f(I_P)$
- b)  $f(p_1) \otimes_Q f(p_2) \leq_Q f(p_1 \otimes_P p_2)$   
for  $p_1, p_2 \in P$

- strong monotone maps:

- a')  $I_Q \cong f(I_P)$
- b')  $f(p_1) \otimes_Q f(p_2) \cong f(p_1 \otimes_P p_2)$

- Strict monotone maps:

- a'')  $I_Q = f(I_P)$
- b'')  $f(p_1) \otimes_Q f(p_2) = f(p_1 \otimes_P p_2)$