Compartmental Models

- Divide population into different compartments, behavior entirely dependent on what set they belong in
- Conserve persons across compartments along transitions (law of mass action)
- Most basic model - SIR (susceptible, infected, recovered) (assumes people are born susceptible)
  - S = poor oblivious souls
  - I = infected with the disease
  - R = recovered from the disease, have partial/full immunity
- Modelled as ODEs

Basic Reproduction Number ($R_0 = \beta\tau$, β = new infections per unit time, τ = mean infection period)
"expected of cases directly generated by one case in a population where all individuals are susceptible"

Effective Reproduction Number (R_t)
"average number of new infections caused by a single infected individual at time *t* in the partially susceptible population", often a more useful metric

More than 1 - one infected person transmits to more than one
Less than 1 - one infected person transmits to less than one

SIER Model

Susceptible, Exposed (cannot spread), Infected (can spread), Recovered

$$\frac{dS}{dt} = \mu N - \mu S - \frac{\beta I S}{N}$$

$$\frac{dE}{dt} = \frac{\beta I S}{N} - (\mu + a)E$$

$$\frac{dI}{dt} = aE - (\gamma + \mu)I$$

$$\frac{dR}{dt} = \gamma I - \mu R.$$

$1/\alpha$ = mean latent period, time taken for disease to show symptoms and become potential source for infecting others
$1/\gamma$ = mean infectious period, how long you are infected for
β = mean contact rate
μ = birth rate = death rate, keeping $N = S + I + E + R$ constant

Effects on changing parameters is what you would expect as is obvious from the name of the parameters. However nature of the plots change at times, notably the "I-hill" doesn't form

Making Models Complex

Add more compartments: Vaccinated, Deceased, Fully immune, Recovered, Immunocompromised
Incorporate vital dynamics
Incorporate time-dependent parameters

Adding age-structured models
Incorporating isolation after diagnosis

Diagrammatically it involves adding more circles in the petri-net diagrams (compartments), and possibly more squares (transitions)

Non-constant Population + Mood

Incorporate death rates, birth rates to be unequal
vital dynamics: γ = birth rate, κ = death rate, ω = lose immunity rate; population mood: θ = negative mood, ϕ = positive mood

$$dS/dt = \mu N + \omega R - \kappa S - \beta IS/N - \theta IE/N + \phi dR/dt$$

$$dE/dt = \beta IS/N - \alpha E - \kappa E$$

$$dI/dt = \alpha E - \gamma I - \kappa I$$

$$dR/dt = \gamma I - \kappa R$$

$$dR'/dt = \kappa(I + E)$$

What can be incorporated? Vaccination, time dependent parameters (make birth, death rates a function of the mood), more compartments

Interesting things to note - This model leads to the total population looking like the side-way projection of a mountain range followed by world extinction :(