

Universal Mapping Properties:

Takeaway: Abstracting basic ideas like addition and multiplication

Universal Properties:

I. Terminal Objects:

The basic idea of terminal objects is to move away from the idea of elements in an object of category and to describe everything in terms of maps and composition.

Definition: An object S in a category C is called terminal if for each object X of category C there is exactly one map C -map: $X \rightarrow S$.

Terminal objects in succeeding richer categories:

1. Sets: A singleton set. A point object (singleton set) is used to abstract away the idea of element here.
2. Category of endomaps: For an object $a: X \rightarrow X$ let f be the map to the terminal object $b: Y \rightarrow Y$, then according to the structure preservation rule $fa=bf$. This is possible for all the $a: X \rightarrow X$ only when Y has one element.
3. Irreflexive graphs category: The object in this category is a pair of sets with a pair of maps.
 $s, t: X \rightarrow P$ and $s', t': X' \rightarrow P'$ be two objects and f be the map from the first to second, then according to structure preserving rule
 $f_A: X \rightarrow X'$
 $f_D: P \rightarrow P'$ and $f_D s = s' f_A, f_D t = t' f_A$
Now for the f to be unique X' and P' must have a single element.

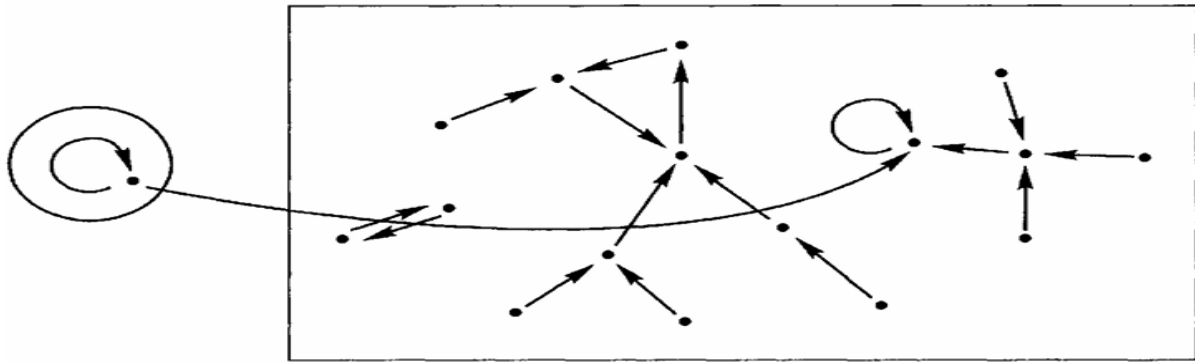
Theorem: Suppose That C is any category and both T_1 and T_2 are terminal objects. Then T_1 and T_2 are isomorphic and there exist only one isomorphism.

II. Points of an Object:

As mentioned above, the concern is to describe everything as maps or compositions, including the object itself. This is where points of objects come into the picture.

To execute this, we use terminal object concept

1. Sets: Every element in a set is essentially a map from terminal (singleton) set called point (1). $x: 1 \rightarrow X$, X is the objects whose elements are in consideration. This map x is called the element of object X . (The map for the element of the terminal object is identity on the terminal set because terminal set is the primary and universal idea).
2. Category of Endomap: The terminal endomap here is a singleton set with identity map. This terminal endomap is mapped to only fixed points of the other endomaps (considered object) as it should preserve the structure.



3. Irreflexive graphs: Both the pair of sets in the objects must have one element and the two maps (source and target) must be identity. This terminal graph is mapped only to the looped elements (for an arrow, target and source are same) in order to preserve the structure.

Category	Terminal object	'Points of X ' means...
\mathcal{C}	T	map $T \rightarrow X$
\mathcal{S}	$\boxed{\bullet}$	element of X
$\mathcal{S}^{\circlearrowright}$ endomaps of sets	$\boxed{\bullet \circlearrowright}$	fixed point or equilibrium state