Week Six

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Section 19: Terminal Objects

- From any set (even for **null set**), there is **only one map** to a *singleton* set.
- T is an object in a category $\mathcal{C}$, which is said to be **terminal** only if for any object X in $\mathcal{C}$:
  - ⋆ at least one map exists from X to T.
  - ⋆ that map should be the only map from X to T.

  Using these two conditions we can say that: (See page.229)

  "*There exists multiple terminal objects which are isomorphic to each other.*"

- In the category of *endomaps*, we can say that the **singleton set** equipped with an endomap from the **point to itself**, is a terminal '*set-with-endomap*'.

  The mapping from an endomap X to this terminal object also follows the 'structure preserving rule'.

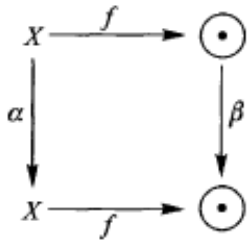

Figure 1: Map from X to T

Section 20: Points

- In the start of this section, we see an example which shows how we can use a *terminal object* (defined in the category) to **select an item** from an object (of the same category). Hence, we can define:

  "*A point of an object X is the map $T \longrightarrow X$*"

  where, T is the terminal object of the category.

- In different categories, the meaning of the word 'point' is different from what we think of. For example, in the category of endomaps, the term 'point' refers to **fixed point** (See page.232). So, if an endomap does **not** have a fixed point, we say it doesnt have '*points*' (which doesn't mean it doesnt have elements!) (See page.233)

Category	Terminal object	'Points of X' means...
$\mathcal{C}$	T	map $T \longrightarrow X$
$\mathcal{S}$	•	element of X
$\mathcal{S}^{\circlearrowright}$ endomaps of sets	•↻	fixed point or equilibrium state