

SEIR Model Ext.

⇒ Assmp : Recovered people are immune to disease, exposed pop. can't spread disease, no birth or death rates, people are not dying due to disease

* Basic eq:

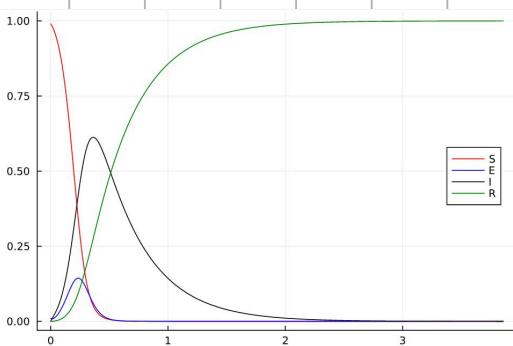
$$\frac{ds}{dt} = -\beta is \quad \beta = 1$$

$$\frac{de}{dt} = \beta is - \sigma e \quad \gamma = 0.1$$

$$\frac{di}{dt} = \sigma e - \gamma i \quad S_0 = 0.99$$

$$\frac{dr}{dt} = \gamma i \quad e_0 = 0.01$$

$$i_0, r_0 = 0$$



Since no one is dying due to disease \Leftrightarrow no birth & death rates are included, $S, e, i \rightarrow 0 \Leftrightarrow r \rightarrow 1$

- * Changing β just changes the graph to shrink or stretch
- * changing σ only changes the peak value of E graph & oppositely affects peak of infected.
- * $\gamma \rightarrow$ as expected if more recovery happens fast

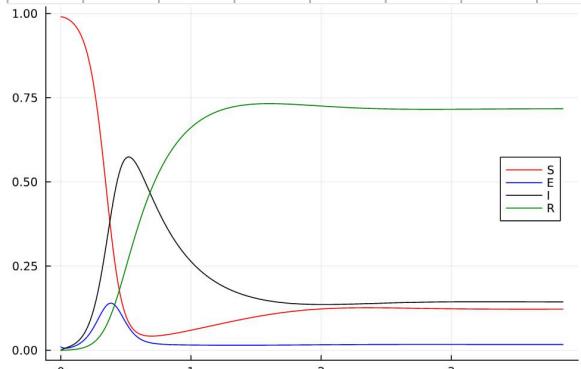
1. Adding birth & death rates :

$$\mu$$

→ if $b.r \neq d.r$ are same

$$\begin{aligned}\dot{S} &= -\beta IS + \mu - \mu S \\ \dot{E} &= +\beta IS - \sigma E - \mu E \\ \dot{I} &= \sigma E - \gamma I - \mu I \\ \dot{R} &= \gamma I - \mu R\end{aligned}$$

$$\mu = 0.02$$

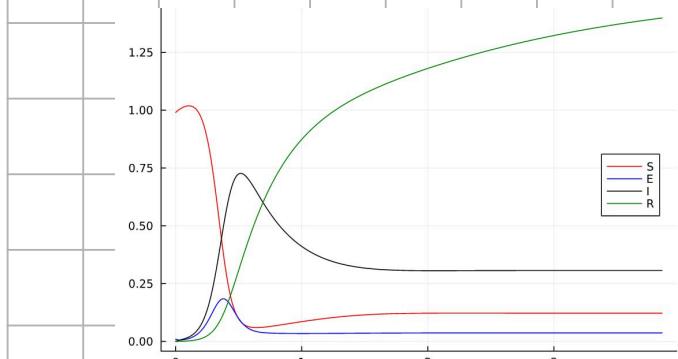


\Rightarrow The disease doesn't stop (there is I at equilibrium)

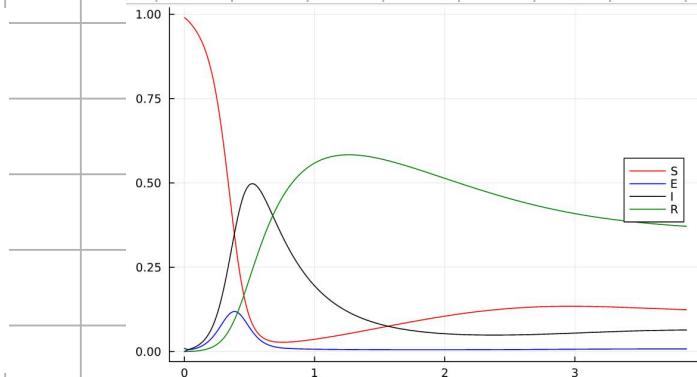
$$(R_0 = \frac{\beta}{\mu + \gamma} > 1 \text{ here so})$$

\rightarrow If I change b_r : $b_r > d_r$, then $\frac{dN}{dt} > 0$ & the recovery rate seems to grow over t , remaining at some equilibrium:

and then reaches.



$$b_r > d_r$$



$$d_r > b_r$$

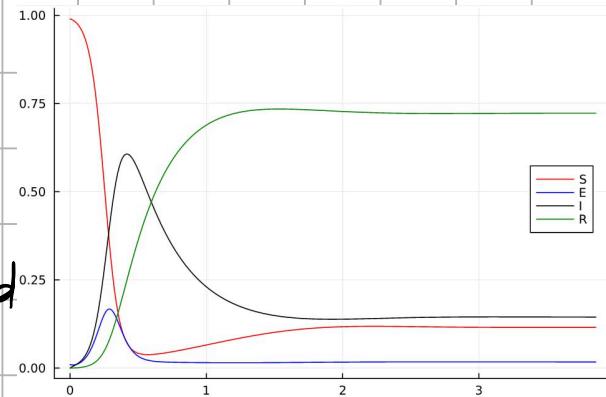
(recovered pop. goes down & reach an equilibrium)
Also $S_{eq} > I_{eq}$ in this case

- 2) Adding the fact that exposed pop. can also cause disease but at a lower rate comp. to infected $\&$ ($\mu = b_r = d_r$)

$$\begin{aligned}
 \dot{s} &= -\beta is + \mu - \mu s - \delta es \\
 \dot{e} &= \beta is + \delta es - \mu e - \sigma e \\
 \dot{i} &= \sigma e - \gamma i - \mu i \\
 \dot{r} &= \gamma i - \mu r
 \end{aligned}$$

$$\delta = 0.5$$

S_{eq} is slightly decreased
 γ_{eq} is slightly increased
in this case



3) SEIRD model with $b \cdot r = d \cdot r$:

↳ deceased from infected

so, they are put in some sep. compartment

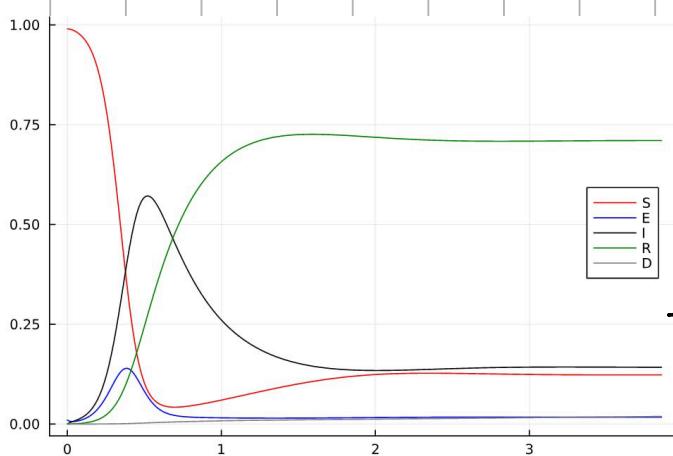
$$\begin{aligned}
 \dot{s} &= -\beta is + \mu - \mu s \\
 \dot{e} &= \beta is - \sigma e - \mu e \\
 \dot{i} &= \sigma e - \gamma i - \mu i - \kappa i \\
 \dot{r} &= \gamma i - \mu r \\
 \dot{d} &= \kappa i
 \end{aligned}$$

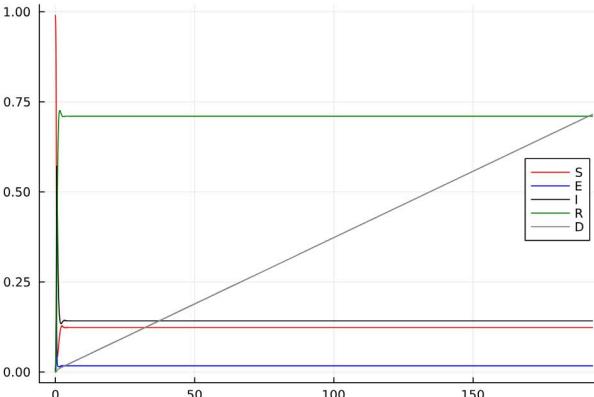
κ = death rate
due to disease alone

for $\kappa = 0.01$ itself
which is $\ll \gamma = 0.1$
the graph of d
increases linearly &
so on

→ Initially
 D is very slowly
increasing

But... after times:

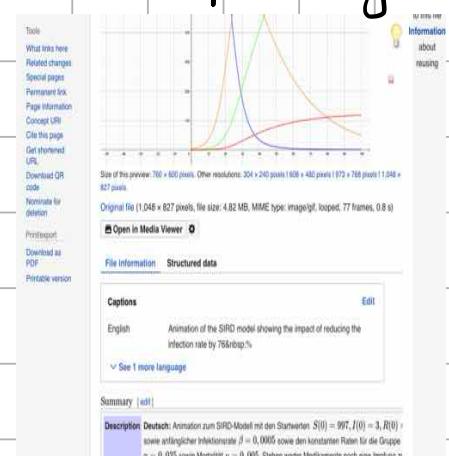




It crosses γ & goes straight over 1 and so on.

But can adjust parameters like β to get a reasonable graph see this :

https://commons.wikimedia.org/wiki/File:SIRD_model_anim.gif



* Other models to think of but too much to write & execute

* SEIS model (recovered has no immunity so goes to sus. directly)

* SEIRS model (when recovered temporary immune or only few people are immune)

* SEIRV & SEIRVD models ($V \rightarrow$ vaccination)

* $SEIR_{SS} \Rightarrow SEIR$ model + Social Stress

At beginning of epidemic people might be ignorant & careless, after time they get alarmed & epidemics may decline, and when this happens, still

overtime, people get tired of restrictions if new cases no. drops down, so it may increase again this can happen