upsara

Date: ____

Resource Theories: Monoidal Preorders and Enrichment

Resource theories: (Takeaway)

The ideas of how to go from what we have to what we want.

We will study two components (majorly) to understand how questioning: given what I have,

i) Is it possible to get what I want?

ii) What is the minimum cost to get what I want?

iii) set of ways to get what I want?

will help in transformation.

[getting from A to B]

Date:

I Symmetric Monoidal preorder : [additional structure]

→ Defⁿ: (Symmetric Monoidal structure on a preorder) consists of two constituents : $(X, \leq)$

(i) Monoidal unit : an element $I \in X$

(ii) a function $\otimes : X \times X \longrightarrow X$ called monoidal product $[\otimes(x_1, x_2) = x_1 \otimes x_2]$

For which the following conditions must satisfy:

monotonicity a) $\forall x_1, x_2, y_1, y_2 \in X$ if $x_1 \leq y_1$ and $x_2 \leq y_2$ then $\cancel{x_1 \otimes}$ $\otimes(x_1, x_2) \leq \otimes(y_1, y_2)$

unitality b) $\forall x$, the equations $\otimes(I, x) = x$ and $\otimes(x, I) = x$

Associativity c) $\forall x, y, z \in X$ $(x \otimes y) \otimes z = x \otimes (y \otimes z)$

symmetry d) $\forall x, y \in X$ $x \otimes y = y \otimes x$

then this monoidal preorder = $(\underbrace{X}_{\text{preordered set}}, \underbrace{\leq}_{\text{order relation}}, \underbrace{I}_{\text{monoidal unit}}, \underbrace{\otimes}_{\text{monoidal product}})$

- If we replace $=$ with $\cong$ it is called weak monoidal structure

- Main Monoidal (symmetric) preorders :

1) The Booleans:

$(B, \leq, \text{true}, \wedge)$

$B = \{\text{True}, \text{false}\}$

True $\geq$ false

$I =$ true

$\wedge =$ AND

2) The cost:

$([0, \infty], \geq, 0, +)$

usual meaning

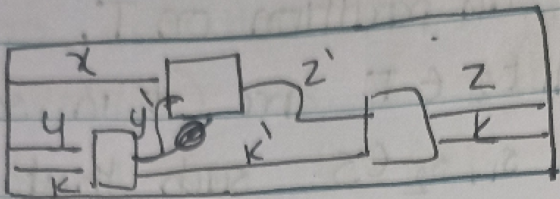
- The opposite of Monoidal pre-orders:

Suppose $\mathcal{X} = (X, \leq)$ is a preorder $\mathcal{X}^{op} = (X, \geq)$ is its opposite. If $(X, \leq, I, \otimes)$ is a symmetrical monoidal pre-order means $(X, \geq, I, \otimes)$ is also a ~~pr~~ symmetric Monoidal preorder

II Wiring Diagram:

Wiring diagrams are visual representations for building new relationships from old.

consider:

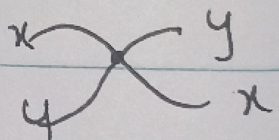


components

How does a wiring diagram represent ' symmetric Monoidal preorder

- the wires are the elements of the preordered set
- the boxes are ~~relation~~ order relations.
- the parallel lines (like x & y' or y & k) represent monoidal product.
- the nothing wire/line represents monoidal unit so,

Symmetry can be shown like this.

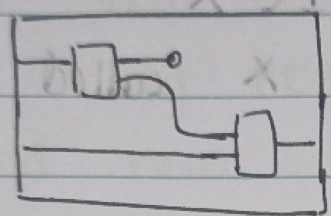


Associativity :

$$\frac{x}{y} \quad \frac{}{z} \;=\; \frac{x}{} \quad \frac{y}{z}$$

• some other components of a wiring diagram:

- If we need to discard an element after its relation then we can use



e) Discard axiom

$x \le I \quad \forall x \in X$

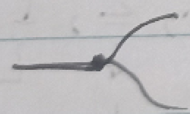
[X is preordered set]

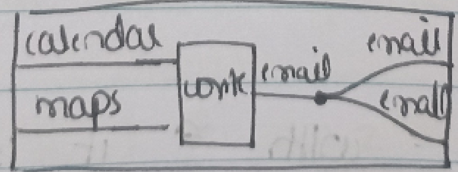
(like in manufacturing $a+b \to c+d$) (and we don't need this d)

- Incase of information (only) an element can be copied (like an email)

f) Copy axiom

$x \le x+x$ for all $x \in X$





III Monoidal Monotone maps:

Def: let $\mathcal{P} = (P, \leq_P, I_P, \otimes_P)$ and $\mathcal{Q} = (Q, \leq_Q, I_Q, \otimes_Q)$ be monoidal preorders.

A monoidal monotone map from $\mathcal{P}$ to $\mathcal{Q}$ is a monotone map $f: (P, \leq_P) \longrightarrow (Q, \leq_Q)$ such that:

a) $I_Q \leq_Q f(I_P)$

b) $f(p_1) \otimes_Q f(p_2) \leq f(p_1 \otimes_P p_2)$

for $\forall\ p_1\ p_2 \in P$

- strong monotone maps:

a') $I_Q \cong f(I_P)$

b') $f(p_1) \otimes_Q f(p_2) \cong f(p_1 \otimes_P p_2)$

- Strict monotone maps:

a'') $I_Q = f(I_P)$

b'') $f(p') \otimes_Q f(p_2) = f(p_1 \otimes_P p_2)$