Week Ten

Siva Sundar, EE23B151

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2^{nd} December

- **Graphs**: consists of the following:
  - Set V which contains **vertices** and set A with **arrows**,
  - s and t are the **source** and **target** functions respectively.

  Note: From every graph we can get a *preorder*. **Hasse Diagram** is a graph that gives a *presentation* of a preorder $(P, \leqslant)$. (See page.14)
- *Total order*: They are *posets* (partially ordered sets), with an additional condition: "*for all* x, y, either $x \leqslant y$ or $y \leqslant x$". (They should be *comparable*)
- **Partitions** can be made from preorders. (See page.16)
- Preorder of **upper sets** ($U(X)$ contains q, if $p, q \in X$ and $p \leqslant q$) on a *discrete preorder* on set X is same as power set $P(X)$.
- **Product Preorder:** Given $(P, \leqslant)$ and $(Q, \leqslant)$, we define $(P \times Q, \leqslant)$ such that:

$$(p, q) \leqslant (p', q') \iff p \leqslant p' \ \& \ q \leqslant q'$$

4^{th} December

- **Monotone map** is a *structure preserving* function $f : A \to B$, such that:

$$\forall x, y \in A, \text{ if } x \leqslant_A y \text{ then } f(x) \leqslant_B f(y).$$

  Cardinality is a function which maps a set to a natural number (which is the number of elements in the set). This function is a monotone map, as:

$$\text{if } X \subseteq Y, \text{ then } n(X) \leqslant n(Y).$$

  If a map $f : X \to Y$ exists, then there exists a monotone map $g; \mathrm{Prt}(Y) \to \mathrm{Prt}(X)$. ($\mathrm{Prt}(X)$ gives the set of all partitions on X).

  If f and g are monotones, then $f \circ g$ is also monotone.

  Let P be a preorder. Monotone maps $P \to \mathcal{B}$ are in one-to-one correspondence with upper sets of P. (See page.22).
- *Yoneda Lemma:* to know an element is the same as knowing its upper set (the relationships it has with other elements). (see page.20).
- **Pullback map**: Let P and Q be preorders, and $f : P \to Q$ be a monotone map. Then we can define a montone map $g : U(Q) \to U(P)$ which is called the *pullback along f*. (U(X) is the set of all uppersets of X).