$\mathcal{V}$-Profunctors.

We will see that Feasibility relation is just a special case of $\mathcal{V}$-profunctors where $\mathcal{V}$ = Bool.

In order to understand the interpretation of co-design problem [formalizing and operating] we need understand pro-functors.

~~Profunctors~~: (Intuitively)

Bool- Category	Preorder
Bool- functor	Monotone map
Bool- profunctor	feasibility relation

$\mathcal{V}$-Profunctors:

Def: let $\mathcal{V} = (V, \leq, I, \otimes)$ be a quantale (unital commutative) and let $\mathcal{X}$ and $\mathcal{Y}$ be $\mathcal{V}$-categories. A $\mathcal{V}$-profunctor from $\mathcal{X}$ to $\mathcal{Y}$ denoted $\Phi : \mathcal{X} \nrightarrow \mathcal{Y}$, is a $\mathcal{V}$-functor $\Phi : \mathcal{X}^{op} \times \mathcal{Y} \to \mathcal{V}$

Because this is the codomain ($\mathcal{V}$ is enriched in itself.

- By def of $\mathcal{V}$-product and $\mathcal{X}^{op}$ definition we can say that,

$\mathcal{V}$-profunctor is same as a function $\Phi : ob(\mathcal{X}) \times ob(\mathcal{Y}) \to V$ such that for any $x, x' \in \mathcal{X}$ and $y, y' \in \mathcal{Y}$ the following inequality holds in $\mathcal{V}$:

$$\mathcal{X}(x', x) \otimes \Phi(x, y) \otimes \mathcal{Y}(y, y') \leq \Phi(x', y')$$

Proof: $\mathcal{V}$-profuntor is $\Phi : \mathcal{X}^{op} \times \mathcal{Y} \to \mathcal{V}$

so, by definition of $\mathcal{V}$-product

~~$ob(\mathcal{X}^{op} \times \mathcal{Y}) = \mathcal{X}^{op} \otimes \mathcal{Y}((x, y), (x$~~

(x, y) & (x', y') are objects in $\mathcal{X}^{op} \times \mathcal{Y}$

so, $\mathcal{X}^{op} \times \mathcal{Y}((x, y), (x', y')) = \mathcal{X}^{op}(x, x') \otimes \mathcal{Y}(y, y')$ — (i)

$\mathcal{X}^{op}(x, x') = \mathcal{X}(x', x)$ [By def of $\mathcal{X}^{op}$] — (ii)

$\mathcal{V}[\Phi(x, y), \Phi(x', y')] := \Phi(x, y) \multimap \Phi(x', y')$ — (iii)

(Remark 2.89)

Now according to Φ

$$\mathcal{X}^{op} \times \mathcal{Y}((x,y),(x',y')) \leq \mathcal{V}(\Phi(x,y), \Phi(x',y'))$$

$$\mathcal{X}(x',x) \otimes \mathcal{Y}(y,y') \leq \Phi(x,y) \multimap \Phi(x',y')$$

Now according to def 2.79

$\mathcal{V}$ = Bool profunctor

$$\underline{\mathcal{X}(x',x)} \otimes \underline{\Phi(x,y)} \otimes \underline{\mathcal{Y}(y,y')} \leq \underline{\Phi(x',y')}$$

$$\times \qquad \times \qquad \times$$

- Feasibility Matrix : Φ can be expressed as a matrix the (m,n)th entry is the value of $\Phi(m,n) \in B$.

~~Fill out the Bool math.~~

Ex: 4.11 & Ex 4.12 for better understanting $\mathcal{V}$-profunctors

Interpreting the co-design diagram:

W.R.T

Box : feasibility relation

L.H.S: produces (set of ports, each indicating a resource)

R.H.S: requirements (set " ")

Now, each of these resources (produce or requirements) are taken as pre-orders.

Then we take product pre-order of preorders on the left and similary for those on the right. The box is then the feasibility relation

Box: $\underset{\text{product of produces}}{(\quad)} \longrightarrow \underset{\text{product of requirements}}{(\quad)}$

Now, feasibility relation says is it possible to get to the source from the target (can we get produce from the requirements)

(Understand movie example)

The ~~cottage~~ above statement is valid for the whole collage