Date:

Application of Generative effects (Galois connections)

V Closure operation: (Endomap with conditions)

Def: A closure operation $j: P \to P$ on a preorder P is a monotone map such that ~~for~~ $\forall\ p \in P$ we have

a) $p \leq j(p)$

b) $j(j(p)) \cong j(p)$

- Composition of Galois connections as closure operation:

Given: $f: P \to Q$ (Left adjoint)

$g: Q \to P$ (Right adjoint)

To prove: $f;g: P \to P$ is a closure operation so,

a) $p \leq (f;g)(p)$

b) $(f;g;f;g)(p) \cong \cancel{f}(f;g)(p)$

Proof:

a) w.k.t $p \leq g(f(p))$ [Galois connections]

b) let $p' = g(f(p))$

so, $p' \leq g(f(p'))$ [GC]

So, $g(f(p)) \leq g(f(g(f(p))))$ — ①

$f(g(q)) \leq q$ ($\because$ GC)

let $q = f(p)$

$f(g(f(p))) \leq f(p)$

applying g on both sides (g is monotone)

$g(f(g(f(p)))) \leq g(f(p))$ — ②

① and ② say that $f;g(p) \cong (f;g;f;g)(p)$

- Application of closure operation computation (Ex 1.121)
- Adjunctions from closure operators (Ex 1.122)