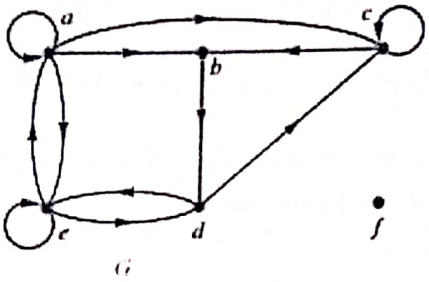
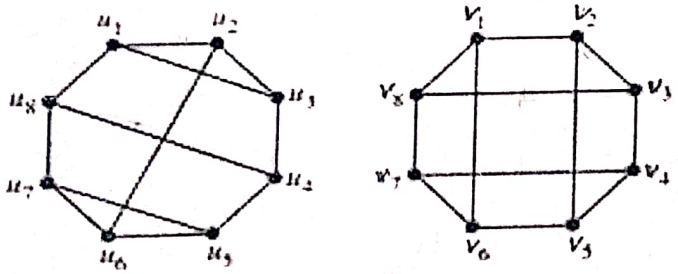
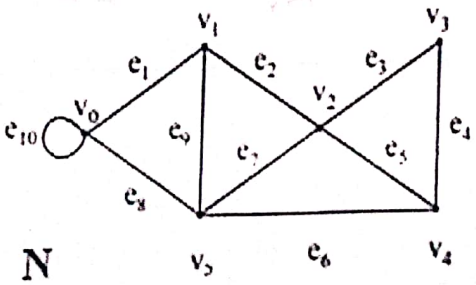
Air University
(Final-Term Examination: Spring 2022)
Department of Mathematics

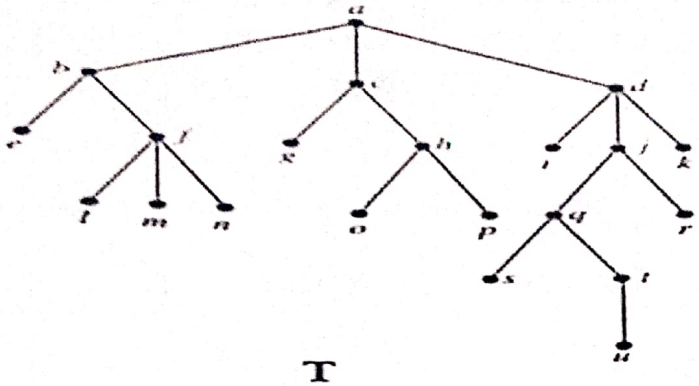
Subject: Discrete Structures
Course Code: MA-216
Class: BS-CYS (II)
Sections: (A, B)
Special Instructions: Attach Question Paper with answer sheet.

Total Marks: 100 (Weightage 45 %)
Date: 10/06/2022
Duration: 3 Hours
FM Name: Noor Zeb Khan

Q. No	Questions	Marks	CLO
(01)	**A).** Show that $f(x) = 7x^2$ is $O(x^3)$. Is it also true that x^3 is $O(7x^2)$?	(05)	CLO 2 C3
	B). Find the Big-O estimate for $f(x) = (x+1)\log(x^2+1) + 3x^2$?	(05)	
	C). Prove the sum of G.P $a + ar + \cdots + ar^n = \frac{ar^{n+1}-a}{r-1}, r \neq 1$ and n is non-negative integers by Mathematical Induction.	(10)	
(02)	**A).** Let $A = \{2,4\}$ & $B = \{6,8,10\}$ and defined relations R and S from A to B as follows: for all $(x,y) \in A \times B$, $xRy \Leftrightarrow x \mid y$, for all $(x,y) \in A \times B$, $xSy \Leftrightarrow y - 4 = x$. State explicitly which ordered pairs are in $A \times B, S, R, R \cup S,$ & $R \cap S$.	(10)	CLO 2 C3
	B). For the relation matrix: $\mathbf{M} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$ Write down the ordered pairs represented by M and also draw the directed graph.	(05)	
	C). Let $R = \{(1,1), (2,2), (2,4), (3,3), (4,2), (4,4)\}$. Determine whether R is and Equivalent relation.	(05)	

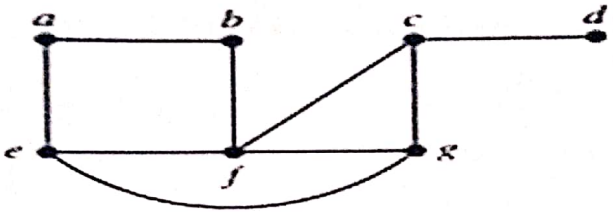
(03)	**A).** Express $GCD(252,198) = 18$ as a linear combination of 252 and 198. **B).** Assign a memory location to the record of the customer with social security number 107405723 by Hashing function with **mod** 111? **C).** What is the secret message produced from the message "**MEET YOU IN THE PARK**" using the Caesar Cipher?	(10) (05) (05)	CLO 3 C3
(04)	**A).** Find the Adjacency Matrix of Graph G?  G **B).** Determine whether the graphs H and I are Path isomorphic?  G H **C).** Determine whether the walk is path, closed path, trail, circuit, walk, and closed walk. $v_1e_2v_2e_3v_3e_4v_4e_5v_2e_2v_1e_1v_0$  	(05) (10) (05)	CLO 4 C4

A). Find root vertex, internal vertices, and Leave vertices of T. (05)



T

B). Find at least one spanning trees of the simple graph. (05)



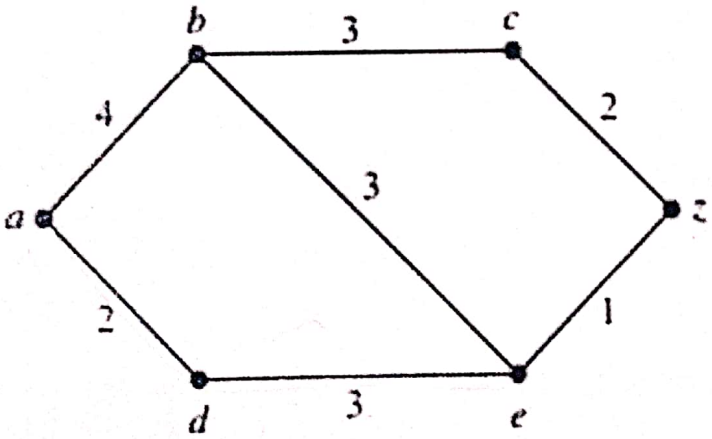
G

(05)

CLO 1

C2

C). Use Dijkstra's algorithm to find the length of a shortest path from source "a" to each vertex. (10)



********** End of Examination Paper **********

Final Paper

(05) (C3 CLO-2)

Q:1# a). → We need witnesses "c" and "k"

So; $7x^2 \leq 7x^3 \quad \forall \; x \geq k.$

$x^2 \leq x^3 \quad \forall \; x \geq 1.$

So; $k = 1, \; c = 1.$

It is true that $7x^2$ is $O(x^3)$.

→ Let c and k are witnesses

then $x^3 \leq c(7x^2)$; where $x \geq k$

$x \leq 7c$; ÷ing by x^2

$x \leq 7c$ does not hold

$\forall \; x \geq k$, which is contradiction.

Thus x^3 is not $O(7x^2)$.

(05)

b). $f(x) = (x+1)\log(x^2+1) + 3x^2$

Sol; $x+1 \Rightarrow \quad x+1 \leq x+x.$

$x+1 \leq 2x.$

→ $(x+1)$ is $O(x)$; $k = 1, \; c = 2$

$\log(x^2+1) \Rightarrow$

$x^2+1 \leq x^2+x^2$

$x^2+1 \leq 2x^2$; $x \geq 1.$

$\log(x^2+1) \leq \log(2x^2)$

$< \log 2 + \log x^2$

$\leq \log 2 + 2\log x.$

$\leq \log x + 2\log x$ ∵ $\log 2 \leq \log x$

$\log(x^2+1) \leq 3\log x$; $x \geq 2$

→ $\log(x^2+1)$ is $O(\log x)$; $k = 2, \; c = 3$

$3x^2 \Rightarrow \quad 3x^2 \leq 3x^2$; $1 \geq k$

$3x^2$ is $O(x^2)$; $k = 1, \; c = 3$

By Theorm # 03:-

$(x+1)\log(x^2+1)$ is $O(x\log x)$.

By Theorem # 02:-

$(x+1)\log(x^2+1)$ is $O(x\log x)$

$3x^2$ is $O(x^2)$.

$(x+1)\log(x^2+1) + 3x^2$ is $O(\max|(x\log x), (x^2)|)$

$= O(x^2)$

(10)

c).

So.P#

$a + ar + \cdots + ar^n = \frac{ar^{n+1} - a}{r-1}$; $r \neq 1$ $n \in Z^+ \cup 0$

M. Induction

Let $P(n) = a + ar + \cdots + ar^n = \frac{ar^{n+1} - a}{r-1}$.

i)# Basis Step# put $n = 0$

$P(0)$: $ar^0 = \frac{ar^1 - a}{r-1}$.

$a = \frac{a(r-1)}{(r-1)}$

$a = a$

$(1 = 1)$ proved.

2). Inductive Step#

i)- Hypothesis # $P(n)$ is true for $n = k$.

$P(k)$: $a + \cdots + ar^k = \frac{ar^{k+1} - a}{r-1}$ → (i).

ii). Must show # Put $n = k+1$.

$P(k+1)$: $a + \cdots + ar^k + ar^{k+1} = \frac{ar^{k+2} - a}{r-1}$ → (ii)

Substitute (i) in (ii)

$P(k+1)$: $\frac{ar^{k+1}-a}{r-1} + ar^{k+1} = \frac{ar^{k+2}-a}{r-1}$.

L.H.S $= \frac{ar^{k+1} - a + ar^{k+1}(r-1)}{r-1}$

$\frac{ar^{k+1} - a + ar^{k+2} - ar^{k+1}}{r-1}$

$\frac{ar^{k+2} - a}{r-1} =$ R.H.S.

$P(k+1)$ is also true

Thus $a + ar + \cdots + ar^n = \frac{ar^{n+1}-a}{r-1}$

is true for all non-negative integers.

(10) (CLO-2, C3)

Q2: a): $A = \{2, 4\}$, $B = \{6, 8, 10\}$

$R = x|y = \{(2,6), (2,8), (2,10), (4,8)\}$

$S \Rightarrow y - 4 = x \Rightarrow \{(2,6), (4,8)\}$.

$R \cup S = \{(2,6), (2,8), (2,10), (4,8)\} = R$

$R \cap S = \{(2,6), (4,8)\} = S$

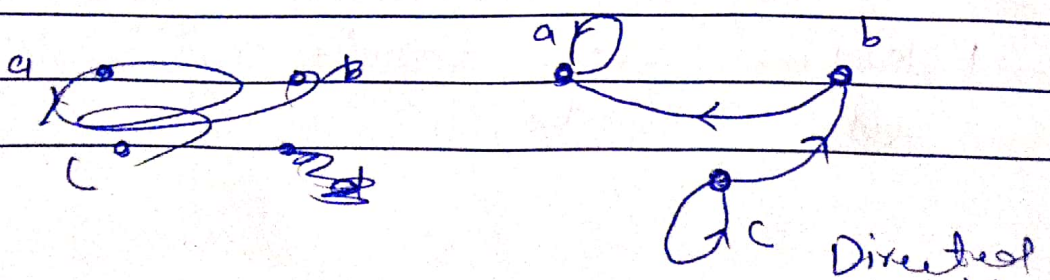
$A \times B = \{(2,6), (2,8), (2,10), (4,6), (4,8), (4,10)\}$.

(05)

(b):-

$$M = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}.$$

$R = \{(a,a), (b,a), (c,b), (c,c)\}$.



Directed graph.

(05)

C). $R = \{(1,1), (2,2), (2,4), (3,3), (4,2), (4,4)\}$.

Reflexive: R is Reflexive because each number/model satisfies $a = a$ property i.e aRa.

Symmetric: R is Symmetric Relation bcz $2R4 \Rightarrow 4R2 \in R$.

Transitive Relation:- R is also transitive Relation bcz $2R4$ and $4R2 \Rightarrow (2,2) \in R$.

All the 3-properties are satisfied by R thus R is Eq. Relation.

(10) (C.LO-3 C3)

Q3: A). $GCD(252, 198) = 18$.

Linear Combination of 252 & 198.

Sol

$198 \overline{)252}\ 1$

198

$54 \overline{)198}\ 3$

162

$36 \overline{)54}\ 1$

36

$18 \overline{)36}\ 2$

36

x

$252 = 1 \cdot 198 + 54$

$198 = 3 \cdot 54 + 36$

$54 = 1 \cdot 36 + 18$

$36 = 2 \cdot 18 + 0$.

Linear Combination:

$18 = 54 - 1 \cdot 36$

~~$36 = 198 - 3 \cdot 54$~~

$18 = 4 \cdot 54 - 1 \cdot 98$

$18 = 4 \cdot 252 - 5 \cdot 198$

05

Q3

B:-

$107405723 \bmod 111$.

Hashing Function?

Sol#

```
          967619
    111 | 107405723
          0999
           750
           666
            845
            777
             687
             666
              212
              111
              1013
               999
                14
```

$f(107405723) = 107405723 \bmod 111$

$= 14$

↓

Memory location

Q3 (05)

C). Encode "MEET YOU IN THE PARK" By Caesar Cipher.

Sol

$f(k) = (k+3) \bmod 26$.

→ "PHHW BRX LQ WKH SDUN"

Q4: A). (05) (CLO4 C4)

Adjacency Matrix.

	a	b	c	d	e	f	out degree
a	1	1	1	0	1	0	$\deg^-(a) = 4$
b	0	0	0	1	0	0	(b) = 1
c	0	1	1	0	0	0	(c) = 2
d	0	0	1	0	1	0	(d) = 2
e	1	0	0	1	1	0	(e) = 3
f	0	0	0	0	0	0	(f) = 0

Indegree ← $\deg^+(a)$ (b) (c) (d) (e) (f)
= 2 2 3 2 3 0.

B). (10)

Isomorphism?

$V(G) = 8$ $\quad$ $V(H) = 8$.

$E(G) = 12$ $\quad$ $E(H) = 12$.

$D(u_1) = 3$ $\quad$ $D(v_1) = 3$

$D(u_2) = 3$ $\quad$ $D(v_2) = 3$

$D(u_3) = 3$ $\quad$ $D(v_3) = 3$

$D(u_4) = 3$ $\quad$ $D(v_4) = 3$

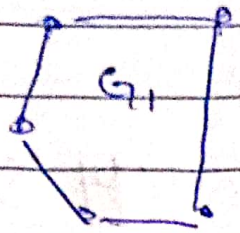
$D(u_5) = 3$ $\quad$ $D(v_5) = 3$

$D(u_6) = 3$ $\quad$ $D(v_6) = 3$

$D(u_7) = 3$ $\quad$ $D(v_7) = 3$

$D(u_8) = 3$ $\quad$ $D(v_8) = 3$

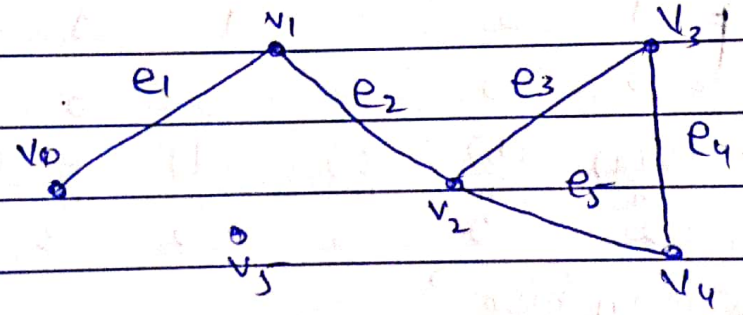
Circuits



Legth(G_1) = 5 But there no circuit of Legth 5 in "H" thus G & H are not Iso-phic.

Q4 (5)

c) $v_1 e_2 v_2 e_3 v_3 e_4 v_4 e_5 v_5 e_2 v_1 e_1 v_0$



It is just a walk Bcz $e_2 v_2 v_2$ are repeated.

(05)

Q5: (A) Root, Internal & Leaves. (CLO-1 C-2)

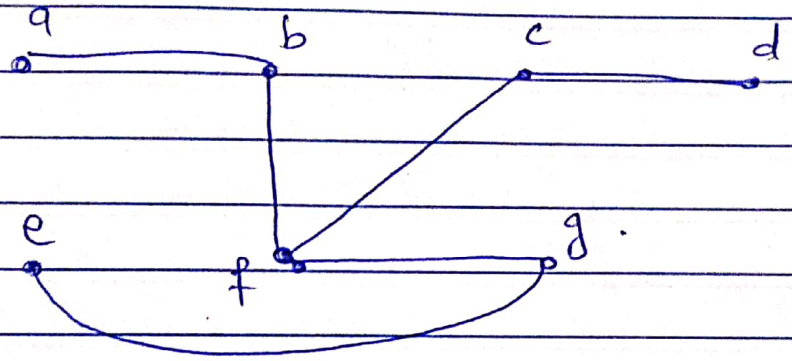
Leaves # e, l, m, n, g, o, p, i, s, k, u, t

Root # a.

Internal # a, b, c, d, f, h, j, q, t.

(05)

Q5: (B) Spinning tree (CLO-1 C-2)



(10)

Q5: C). Dijkstra's Algo... (CLO-1 C-2)

Source	Distinations				
a	b	c	d	e	z
	∞	∞	∞	∞	∞
a, d	4	∞	2 fix	∞	∞
a, d, b	4 fix	∞	2	5	∞
a, d, b, e	4	7	2	5 fix	∞
a, d, b, e, z	4	7	2	5	6 fix
a, d, b, e, z, c	4	7 fix	2	5	6
	4	7	2	5	6