Derivative of unilateral quadratic matrix equation

Michel Juillard

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Given equation

$$AX^2 + BX + C = 0,$$

compute $\frac{\partial X}{\partial z}$ given $\frac{\partial A}{\partial z}$, $\frac{\partial B}{\partial z}$, $\frac{\partial C}{\partial z}$.

Derivative of an implicit function:

$$\frac{\partial A}{\partial z}X^2 + A\frac{\partial X}{\partial z}X + AX\frac{\partial X}{\partial z} + B\frac{\partial X}{\partial z} + \frac{\partial B}{\partial z}X + \frac{\partial C}{\partial z} = 0$$

Can be reorganized as

$$A\frac{\partial X}{\partial z}X + (AX + B)\frac{\partial X}{\partial z} + \frac{\partial A}{\partial z}X^2 + \frac{\partial B}{\partial z}X + \frac{\partial C}{\partial z} = 0$$

This can be solved with GeneralizedSylvesterSolver.jl. Function `generalized_sylvester_solver!(a,b,c,` solves

$$ax + bxc = d$$

where

$$\begin{aligned} a &= AX + B \\ b &= A \\ c &= X \\ d &= \frac{\partial A}{\partial z}X^2 + \frac{\partial B}{\partial z}X + \frac{\partial C}{\partial z} \end{aligned}$$

and

```
ws = GeneralizedSylvesterWs(n, n, n, 1)
```