CM146, Fall 2018
Problem Set 2: Perceptron and Regression
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Due Nov 1, 2018 at 11:59 pm

1 Problem 1

Solution: Done on CCLE

2 Problem 2

Solution: Done on CCLE

3 Problem 3

(a) Problem 3a

Solution:

Take a hyperplane $H = \vec{w}'^T\vec{x} + \theta'$. The margin m is the distance between the two closest samples, one positive 'A' and one negative 'B', on either side of the hyperplane.

$A = \vec{w}'^T\vec{x}_a + \theta' \geq 0 > B = \vec{w}'^T\vec{x}_b + \theta'$

Since the data is linearly separable, there exists an H that separates the data. An optimal H would lie exactly halfway inside the margin.

The magnitude of m can be represented as:

$\frac{|A-\frac{m}{2}|}{||\vec{w}'||} = \frac{|B-\frac{m}{2}|}{||\vec{w}'||}$

$A - \frac{m}{2} = -B + \frac{m}{2}$

$m = A + B$

Thus,

$A - \frac{m}{2} \geq 0 > B - \frac{m}{2}$

We can rewrite the hyperplane H as $H = \vec{w}'^T\vec{x} + \theta' - \frac{m}{2} = 0$

With an optimal H,

$min\{\vec{w}'^T\vec{x} + \theta' - \frac{m}{2}\} = \frac{A-B}{2}$ where y=1

$B = max\{\vec{w}'^T\vec{x} + \theta' - \frac{m}{2}\} = \frac{B-A}{2}$ where y=-1

Inserting y, it holds true that

$y(\vec{w}'^T\vec{x} + \theta' - \frac{m}{2}) \geq \frac{A-B}{2}$

$y(\frac{2\vec{w}'^T}{m}\vec{x} + \frac{2(\theta' - \frac{m}{2})}{m}) \geq 1$

Thus, from the original equation

$y(\vec{w}^T\vec{x} + \theta) \geq 1 - \delta$

if $\vec{w}^T = \frac{2\vec{w}'^T}{m}$ and $\theta = \frac{2(\theta' - \frac{m}{2})}{m}$

then $\delta = 0$

(b) Problem 3b

Solution:

If $0 < \delta < 1$, the data is linearly separable. If $\delta = 1$, the data may or may not be separable. In this case, if both positive and negative values lie on the line, it is inseparable. If $\delta > 1$, the data is inseparable.

(c) Problem 3c

Solution:

$\delta = 0$ is the optimal value of δ given the $\delta \geq 0$ constraint and the minimization goal. Then $\vec{w}^T$ and θ can be 0 as well. The problem is that this solution would not yield a separating hyperplane.

(d) Problem 3d

Solution:

$y(\vec{w}^T\vec{x} + \theta) \geq 1 - \delta$

Two distinct points are always linearly separable so $\delta = 0$.

$\vec{x_1}^T : 1 * (1 * w_1 + 1 * w_2 + 1 * w_3) + \theta \geq 1$

$\vec{x_1}^T : -1 * ((-1) * w_1 + (-1) * w_2 + (-1) * w_3) + \theta \geq 1$

Simplified:

$\vec{x_1}^T : w_1 + w_2 + w_3 + \theta \geq 1$

$\vec{x_1}^T : w_1 + w_2 + w_3 - \theta \geq 1$

One possible solution of (w, θ, δ) is

$w = [1, 1, 1] \quad \theta = 1 \quad \delta = 0$

which is optimal because δ is minimized under the constraint of $\delta \geq 0$.