

Bài Kiểm tra lấy điểm Giữa kỳ, môn MA003

Câu 1

$$A = \begin{pmatrix} -1 & 4 & -3 \\ -1 & 3 & -2 \\ 1 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -5 & -2 & 4 \\ 1 & -3 & 6 \\ -1 & 0 & -2 \end{pmatrix}, \quad C = \begin{pmatrix} -2 & 5 & -1 \\ 4 & 0 & -3 \\ -2 & 9 & 8 \end{pmatrix}$$

a/ Tìm ma trận $B^T A + 3C$

T/ có.

$$\Rightarrow \begin{pmatrix} -5 & 1 & -1 \\ -2 & -3 & 0 \\ 4 & 6 & -2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 4 & -3 \\ -1 & 3 & -2 \\ 1 & -1 & 1 \end{pmatrix} + 3 \begin{pmatrix} -2 & 5 & -1 \\ 4 & 0 & -3 \\ -2 & 9 & 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & -16 & 12 \\ 5 & -17 & 12 \\ -12 & 36 & -26 \end{pmatrix} + \begin{pmatrix} -6 & 15 & -3 \\ 12 & 0 & -9 \\ -6 & 27 & 24 \end{pmatrix} = \begin{pmatrix} -3 & -1 & 9 \\ 17 & -17 & 3 \\ -18 & 63 & -2 \end{pmatrix}$$

b/ $C^T B - 7A$

T/ có

$$\Rightarrow \begin{pmatrix} -2 & 4 & -2 \\ 5 & 0 & 9 \\ -1 & -3 & 8 \end{pmatrix} \cdot \begin{pmatrix} -5 & -2 & 4 \\ 1 & -3 & 6 \\ -1 & 0 & -2 \end{pmatrix} - 7 \begin{pmatrix} -1 & 4 & -3 \\ -1 & 3 & -2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 16 & -8 & 20 \\ -34 & -10 & 2 \\ -6 & 11 & -38 \end{pmatrix} + \begin{pmatrix} 7 & -28 & 21 \\ 7 & -21 & 14 \\ -7 & 7 & -7 \end{pmatrix} = \begin{pmatrix} 23 & -36 & 41 \\ -27 & -31 & 16 \\ -13 & 18 & -45 \end{pmatrix}$$

c/ $XA = 2C$

$$\Leftrightarrow X = 2C \cdot A^{-1} = 2 \begin{pmatrix} -2 & 5 & -1 \\ 4 & 0 & -3 \\ -2 & 9 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -2 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -10 & 18 & 4 \\ 20 & -26 & 2 \\ -54 & 88 & 30 \end{pmatrix}$$

$$\begin{array}{l} A^T \cdot A = I \\ \frac{1}{A} \cdot A = I \end{array}$$

Câu 2.

Ghi rõ và biến luân hpt v/t:

$$\begin{cases} -2x_3 - x_1 - 2x_2 = 1 \\ 2x_1 + (5-m)x_3 - (m-2)x_2 = -2 \\ x_2 + mx_1 + (m+1)x_3 = -2 \end{cases} \quad (A|B) = \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 2 & 2-m & 5-m & -2 \\ m & 1 & m+1 & -2 \end{array} \right)$$

$$\begin{array}{l} (3) \rightarrow (3) + m(1) \\ (2) \rightarrow (2) + 2(1) \end{array} \rightarrow \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & -2-m & 1+m & 0 \\ 0 & 1-2m & 1-m & m-2 \end{array} \right) (*)$$

TH1. $-2-m \neq 0 \Leftrightarrow m \neq -2$.

$$(*) \xrightarrow{(3) \rightarrow (3) - \left(\frac{1-2m}{-2-m}\right)(2)} \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & -m-2 & 1-m & 0 \\ 0 & 0 & \frac{m^2-4m+3}{m+2} & m-2 \end{array} \right) (\#*)$$

TH1.1: $m \neq -2, m^2-4m+3 \neq 0$

$$\begin{cases} -1x_1 - 2x_2 - 2x_3 = 1 \\ (-m-2)x_2 - (m-1)x_3 = 0 \quad (1) \\ \frac{m^2-4m+3}{m+2}x_3 = m-2 \end{cases}$$

$$x_3 = \frac{m^2 - 4}{m^2 - 4m + 3}$$

$$x_2 = (-m-2)^{-1} \cdot (m-1) \cdot \frac{m^2 - 4}{m^2 - 4m + 3} = (-m-2)^{-1} \cdot \frac{m^2 - 4}{m-3} = \frac{-m+2}{m-3}$$

$$-x_1 = 1 + 2 \left(\frac{-m+2}{m-3} \right) + 2 \left(\frac{m^2 - 4}{m^2 - 4m + 3} \right) = \frac{m^2 + 2m - 9}{m^2 - 4m + 3} = \frac{-m^2 - 2m + 9}{m^2 - 4m + 3}$$

$$\text{TH1.2. } m^2 - 4m + 3 = 0.$$

$$\left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & -m-2 & -m+1 & 0 \\ 0 & 0 & 0 & m-2 \end{array} \right)$$

\Rightarrow Pt/4 vô ng m/m.

$$\text{TH2. } m = -2.$$

$$(*) \Rightarrow \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & 0 & -m+1 & 0 \\ 0 & -2m+1 & -m+1 & m-2 \end{array} \right) \xrightarrow{(3) \leftrightarrow (2)} \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & -2m+1 & -m+1 & m-2 \\ 0 & 0 & -m+1 & 0 \end{array} \right)$$

$$\left\{ \begin{array}{l} -1x_1 - 2x_2 - 2x_3 = 1 \\ (-2m+1)x_2 - (m-1)x_3 = m-2 \\ (-m+1)x_3 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_3 = 0 \\ x_2 = \frac{-m+2}{2m-1} \\ x_1 = \frac{-3}{2m-1} \end{array} \right.$$

$$\sqrt[n]{m-2}$$

$$\left\{ \begin{array}{l} x_3 = 0 \\ x_2 = \frac{4}{-5} \\ x_1 = \frac{-3}{-5} \end{array} \right.$$

Kết luận:

Khi $m \neq -2$ và $m^2 - 4m + 3 \neq 0$ hệ pt có nghiệm duy nhất

$$\left\{ \begin{array}{l} x_1 = \frac{-m^2 - 2m + 9}{m^2 - 4m + 3}, \quad x_2 = \frac{-m + 2}{m - 3}, \quad x_3 = \frac{m^2 - 4}{m^2 - 4m + 3} \end{array} \right.$$

Khi $m = -2$ và $m^2 - 4m + 3 = 0$ hệ pt vô nghiệm.

Khi $m = -2$, và $2m - 1 \neq 0$ hệ pt có nghiệm duy nhất
 $x_1 = \frac{+3}{5}, \quad x_2 = \frac{-4}{5}, \quad x_3 = 0$

Bài 3.

$$\begin{pmatrix} -7 & -9 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} -9 & -7 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ -5 & 1 \end{pmatrix}. \text{ T/c}$$

$$\Leftrightarrow X = \begin{pmatrix} -7 & -9 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 8 & -2 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} -9 & -7 \\ 5 & 4 \end{pmatrix}^{-1} =$$

T/c:

$$\det(A) = \begin{vmatrix} -7 & -9 \\ 3 & 4 \end{vmatrix} = -7.4 - -9.3 = -1 \neq 0 \Rightarrow \det(A) \neq 0 \text{ (không nghịch)}$$

$$\begin{pmatrix} -7 & -9 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{\det(A)} \begin{pmatrix} 4 & 9 \\ -3 & -7 \end{pmatrix} = \begin{pmatrix} -4 & -9 \\ 3 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 8 & -2 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} -9 & -7 \\ 5 & 4 \end{pmatrix}^{-1}$$

$$\text{Tacđ: } \det(B) = \begin{vmatrix} -9 & -7 \\ 5 & 4 \end{vmatrix} = -9.4 - -7.5 = -1.$$

$$\begin{pmatrix} -9 & -7 \\ 5 & 4 \end{pmatrix}^{-1} = \frac{1}{\det(B)} \begin{pmatrix} 4 & 7 \\ -5 & -9 \end{pmatrix} = \begin{pmatrix} -4 & -7 \\ 5 & 9 \end{pmatrix}$$

$$X = \begin{pmatrix} -4 & -9 \\ 3 & 7 \end{pmatrix} \cdot \begin{pmatrix} 8 & -2 \\ -5 & 1 \end{pmatrix} \cdot \begin{pmatrix} -4 & -7 \\ 5 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} -57 & -100 \\ 49 & 86 \end{pmatrix}$$

Câu 4.

$$V = \mathbb{R}^5, W = \{(x, y, z, t, k) | 2x - 3y + 4z - 5t - 8k = 0\}$$

$$\text{Hàng }\gamma' = (2, -3, 4, -5, -8) \in W$$

$$W = \{(x, y, z, t, k) \in \mathbb{R}^5 | 2x - 3y + 4z - 5t - 8k = 0\}.$$

$$\because T/\text{cô } \gamma' = (0, 0, 0, 0, 0) \in W \text{ do } 0 = 0 \text{ (*)}$$

. C/m dưới phép cộng

$$u = (x_1, y_1, z_1, t_1, k_1) \in W \Rightarrow 2x_1 - 3y_1 + 4z_1 - 5t_1 - 8k_1 = 0 \quad (1)$$

$$v = (x_2, y_2, z_2, t_2, k_2) \in W \Rightarrow 2x_2 - 3y_2 + 4z_2 - 5t_2 - 8k_2 = 0 \quad (2)$$

(Cộng (1), (2)) + (*)

$$(2x_1 - 3y_1 + 4z_1 - 5t_1 - 8k_1) + (2x_2 - 3y_2 + 4z_2 - 5t_2 - 8k_2) = 0$$

$$\Leftrightarrow 2(x_1 + x_2) - 3(y_1 + y_2) + 4(z_1 + z_2) - 5(t_1 + t_2) - 8(k_1 + k_2) = 0$$

$$\Rightarrow u + v \in W \text{ (***)}$$

. C/m dưới phép nhân với hằng

$$G/3: u = (x, y, z, t, k) \in W, \text{ do } 2x - 3y + 4z - 5t - 8k = 0$$

$$\text{Và } 1/s \text{ rõ hằng c, xét } cu = (cx, cy, cz, ct, ck).$$

. Thay vào pt.

$$2(cx) - 3(cy) + 4(cz) - 5(ct) - 8(ck) = c(2x - 3y + 4z - 5t - 8k) = c \cdot 0 = 0$$

Do đó, $cu \in W \text{ (****)}$

$$\text{Từ (***), (****), (*****)} \Rightarrow W \leq V$$