CS2040S Tutorial 1

Group T40

Week 3

Admin

Tutorial

- Experience will be awarded for **preparation** and **participation**
- Usual flow:
    - Sit in groups (5 people/group)
    - Discuss selected problems
    - Present solution
        - Goal: convince people that your solution is correct
        - Even if you do not have the full solution, it's alright

PS

- Important: Submit before the deadline!
    - Penalty will be applied
    - Extension will be granted on a case-by-case basis
- Adhere to standard style guide
- Write clean code (at least readable)
- Write comments on your algorithm
    - *If other person read this code, is it easy to understand/clear?*
- Write-up will be provided as supplementary material
- If it is not graded within 4 days, remind me :(

Ice Breaker

Picture of The Day

Tutorial

Problem 1: Java Review

Part (a)

What is the difference between a class and an object? Illustrate with an example.

Solution

A class acts as a 'blueprint', i.e. the structure of an object. An object is the real manifest of the class itself (i.e. the value).

Note: you can say that a human has a name and age. That is the class specification. The real human itself has its own concrete *value* for the name and age.

Example

```java
class Human {
  // declares that any human has name and age, but not specific values
  private String name;
  private int age;

  public Human(String name, int age) {
    this.name = name;
    this.age = age;
  }
}

Human anya = Human("Anya Forger", 6); // initialize object; giving values!
```

Part (b)

Why does the `main` method come with a `static` modifier?

Solution

`main` function is intended to be executed regardless whether an object is created or not (think of how we run a Java program). The `static` modifier indicates a variable/method can be called from class directly.

Imagine that you run `java Main`. You do not instantiate any objects, yet you want the program to run. Therefore it has to find the class method `main`. Therefore it has to be declared as static.

Part (c)

Give an example class (or classes) that uses the modifier `private` incorrectly (i.e. the program will not compile as it is, but would compile if `private` was changed to `public`)

Solution

```java
class A {
  private int a; // change this to public

  public A(int a) {
    this.a = a;
  }
}

class Main {
  public static void main(String args[]) {
    A a = new A(10);

    System.out.println(A.a); // error, but compiles if changed to public!
  }
}
```

More about this on CS2030S :)

Part (d)

The following questions is about interfaces

1. Why do we use interfaces
2. Give an example of using an interface
3. Can a method return an interface?

Solution

1. We use interface to create a "contract" of what an object is able to do. This is useful
   in design stage where we are only concerned with high level of what this object *can do*.
2. PS1 :)
3. Yes

Extra reference: The Flaws of Inheritance

Part (e)

Refer to `IntegerExamination.java`. Without running the code, predict the output of the `main` method. Explain.

Demo

```
javac IntegerExamination.java
java IntegerExamination
```

Solution

```java
static public void addOne(int i) { // (2)
  i = i + 1;
  System.out.println("I am in addOne. The value of i is " + i);
}

// ...
  int i = 0; // (1)
  addOne(i);
```

`int` is a primitive type. When `addOne()` is called, the argument value `i` (1) is copied
over as the function's parameter value `i` (2). Therefore, modification inside the function does not affect the value of `i` from the caller (1).

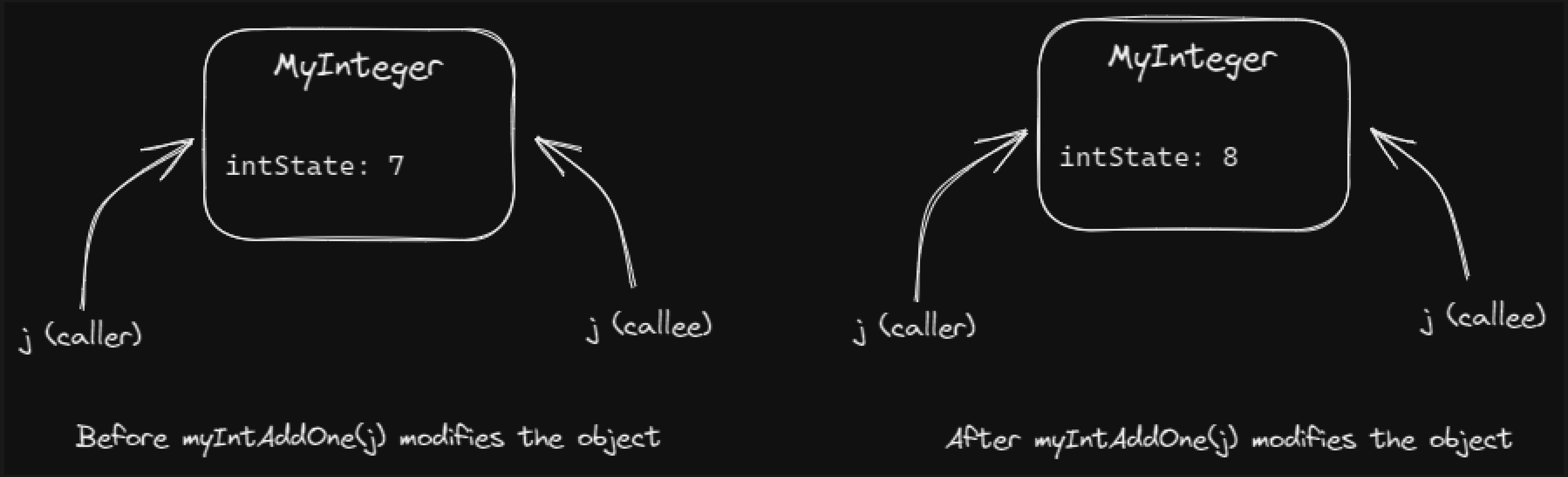
Solution (cont.)

```
static public void myIntAddOne(MyInteger j) { // (2)
  j.intState = j.intState + 1;
  System.out.println("I am in myIntAddOne. The value of j is " + j);
}

// ...
  MyInteger j = new MyInteger(7); // (1)
  myIntAddOne(j);
```

`MyInteger` is a reference type. When `myIntAddOne()` is called, the argument value `j`
(1) is **not** copied over to the function's parameter `j` (2). They refer to the same object.
Hence, modification of internal values of the object by the function affect the value of `j` from the caller (1)

Visualization


MyInteger
intState: 7
j (caller)
j (callee)
Before myIntAddOne(j) modifies the object
MyInteger
intState: 8
j (caller)
j (callee)
After myIntAddOne(j) modifies the object

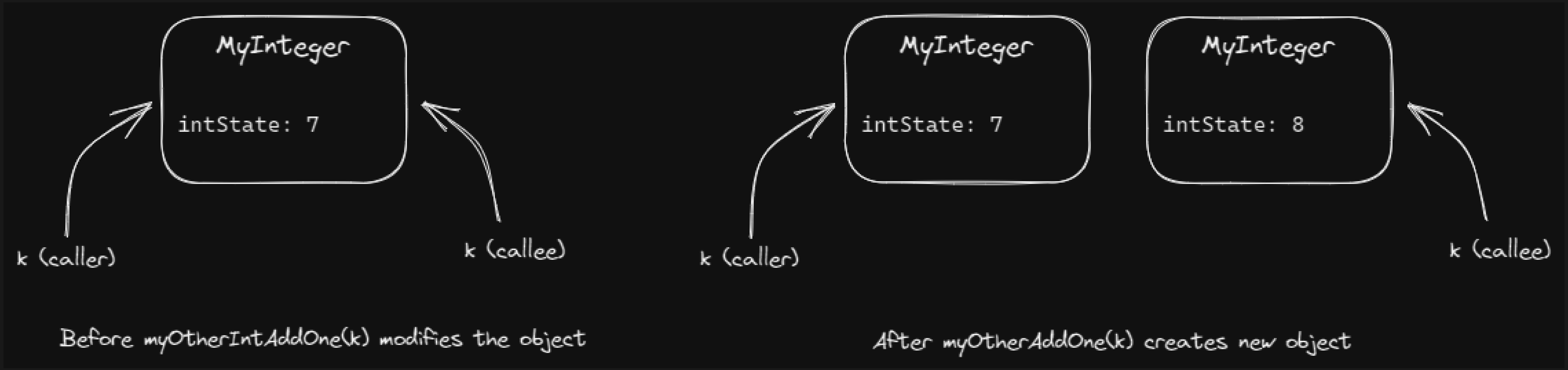
Solution (cont.)

```
static public void myOtherIntAddOne(MyInteger k) { // (2)
  k = new MyInteger(k.intState + 1);
  System.out.println("I am in myOtherIntAddOne. The value of k is " + k);
}

// ...
  MyInteger k = new MyInteger(7); // (1)
  myOtherIntAddOne(k);
```

Similar as before, but now we mutate (i.e. change) the value of the parameter `k` (2)
to point to a new `MyInteger` object. Therefore, it does not modify the original `k` in the caller (1)

Visualization



Part (f)

Can a variable in a parameter list for a method have the same name as a member (or static) variable in the class? If yes, how is the conflict of names resolved?

Solution

Yes, use `this.`. See example answer for part (c)

Problem 2: Asymptotic Analysis

Overview

- Asymptotic analysis is intended to analyze how a program behaves as we increase the input size.
- It measures the order of growth
- *Not* intended to measure the efficiency of two different programs.
  - There are many other factors that affect performance! Constant factor, cache efficiency, etc.

Problem 2: Asymptotic Analysis

Express the following function as the "best" (i.e. tightest) asymptotic upper bound in terms of n.

- $f_1(n) = 7.2 + 34n^3 + 3254n$
- $f_2(n) = n^2 \log n + 25n \log^2 n$
- $f_3(n) = 2^{4 \log n} + 5n^5$
- $f_4(n) = 2^{2n^2+4n+7}$

Note: we assume $\log$ is on base 2 in this module (I believe...)

Solution

- $f_1(n) = \mathcal{O}(n^3)$
- $f_2(n) = \mathcal{O}(n^2 \log n)$
- $f_3(n) = \mathcal{O}(n^5)$
  - $\because\ 2^{4 \log n} = (2^{\log n})^4 = n^4$
- $f_4(n) = \mathcal{O}(2^{2n^2+4n})$
  - $2^{2n^2+4n+7} = 2^{2n^2+4n} \cdot 2^7$
  - You cannot remove 4 from $4n$, as $2^{4n} = (2^n)^4 \neq \mathcal{O}(2^n)$

Problem 3: More Asymptotic Analysis

Overview

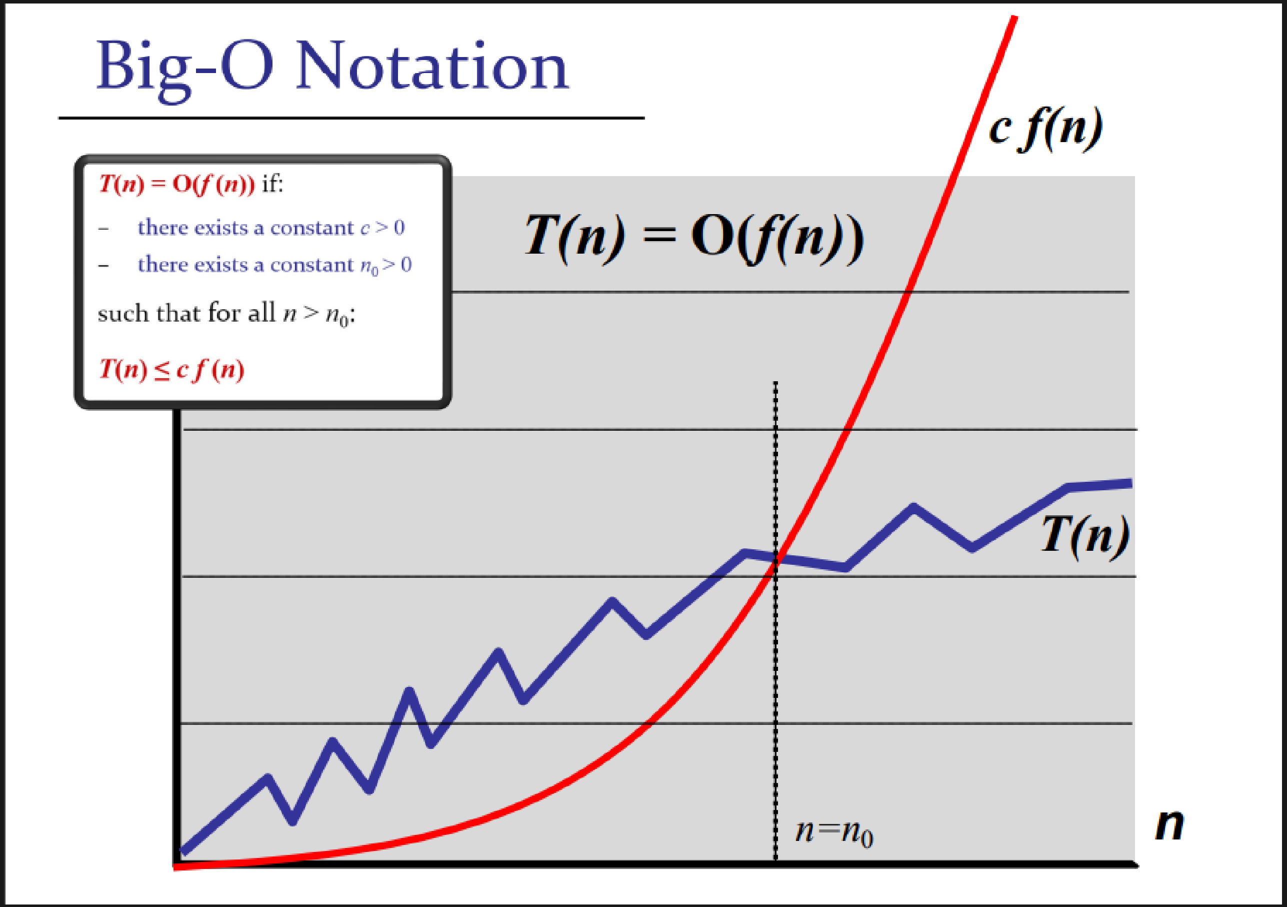
Recall the formal definition of big-O:

A function $T(n)$ is equal to $\mathcal{O}(f(n))$ if there exists a positive constants c and n_0 such that $T(n) \leq cf(n)$ for all $n > n_0$

In layman terms, the function $T(n)$ is upper-bounded by $f(n)$ for sufficiently large value n.

Note that in this definition, it is *okay* for $f(n)$ to be lower than $T(n)$ for small values n.

Example



Big-O Notation

$T(n) = O(f(n))$ if:

- there exists a constant $c > 0$
- there exists a constant $n_0 > 0$

such that for all $n > n_0$:

$T(n) \leq c\, f(n)$

Credit: CS2040S Lecture Slides - 03.Searching

Problem 3: More Asymptotic Analysis!

Let f and g be functions of n where $f(n) = \mathcal{O}(n)$ and $g(n) = \mathcal{O}(\log n)$. Find the best asymptotic bound of the following functions.

Note: We assume the functions are nonnegative functions (i.e. it cannot have negative value for any valid n)

- $h_1(n) = f(n) + g(n)$
- $h_2(n) = f(n) \cdot g(n)$
- $h_3(n) = \max\{f(n), g(n)\}$
- $h_4(n) = f(g(n))$
- $h_5(n) = f(n)^{g(n)}$

Solution

By definition, there exists positive constants c_1, c_2, n_0 such that $f(n) \leq c_1 \cdot n$ and $g(n) \leq c_2 \cdot \log n$ for all $n > n_0$. The following proofs will assume that values of $n > n_0$

Solution (1)

$$\begin{aligned} h_1(n) &= f(n) + g(n) \\ &\leq c_1 n + c_2 \log n \\ &= \mathcal{O}(n) \end{aligned}$$

Solution (2)

$$\begin{aligned} h_2(n) &= f(n) \cdot g(n) \\ &\leq c_1 n \cdot c_2 \log n \\ &= c_1 \cdot c_2 \cdot n \log n \\ &= \mathcal{O}(n \log n) \end{aligned}$$

Solution (3)

$$\begin{aligned} h_3(n) &= \max\{f(n), g(n)\} \\ &\leq f(n) + g(n) \\ &\leq c_1 n + c_2 \log n \\ &= \mathcal{O}(n) \end{aligned}$$

Solution (4)

$$\begin{aligned} h_4(n) &= f(g(n)) \\ &\leq c_1 g(n) \\ &\leq c_1 c_2 \log n \\ &= \mathcal{O}(\log n) \end{aligned}$$

Solution (5)

No appropriate upper bound. Notice that the constant value in exponentiation part cannot be ignored. Since we do not know the exact value of $g(n)$, it is impossible to determine a tight asymptotic upper bound for $f(n)^{g(n)}$.

Problem 4: Application of Binary Search

Overview

- Binary search is a very useful algorithm to search for specific value in a collection of items with certain property.
- In general, you can use this if the property of interest can only change at most once.
    - Suppose you have a predicate (a function that returns a boolean value) $f(x)$.
    - If you evaluate f on an array, it will have all true then all false or vice versa.
    - Example: for peak finding, the predicate f is "is the gradient increasing?"

Issues

- Most people have difficulty in implementing binary search at first. In general, they get the general idea. But the details of +1 and -1 could be confusing sometimes
- Tips:
    - Learn how to reason the correctness of your code
    - Determine what is your pre-condition, loop invariant, and post-condition
    - Takes some time to practice! Don't worry if you don't get it right at first try.

Problem 4: Application of Binary Search

Given a sorted array of $n - 1$ unique integers in the range $[1, n]$, how would you find the missing element? Discuss possible naive solutions and possibly faster solutions.

Solution

- Suppose i-th element is denoted as a_i (0-indexed)
- Predicate f: is $a_i - i = 1$?
- The answer is the first index where $f(a_i)$ evaluates to false.

Implementation

Practice :)

I will release the write-up on this after PS2

Problem 5: Another Application of Binary Search

You have n piles of homework and the ith pile has `piles[i]` pieces of homework. Unfortunately, you realised you have h hours left before all your homework is due. In a moment of panic, you try to figure out the rate k (that is, the “pieces of homework”-per-hour) at which you need to do your homework at in order to finish everything on time.

Problem 5 (cont.)

Here’s how you plan to go about it: At every hour, you choose a pile of homework and start clearing pieces of homework from that pile. If the pile has less than k pieces of homework remaining, you decide to just finish that pile itself, and not start on the next pile yet during the same hour. We all need to take breaks after all, *right*?

Problem 5 (cont.)

To maintain your sanity, you want to minimise the number of pieces of homework you do per hour, i.e. k, while still finishing all piles of homework in time. You can assume that there exists a k that allows you to complete all piles of homework within h hours given the above constraints. Discuss how you can find the minimum integer k such that you can finish all your homework within h hours.

Problem 5 Abridged Statement

- n piles of homework, where i-th pile has `piles[i]` pieces of homework.
- h hours to complete the homework.
- Figure out the minimal number k (i.e. rate of pieces-of-homework / hour) such that you can finish all the work on time
- Execution of doing homework:
  - Take a pile of homework
  - If the pile's pieces of homework $< k$, finish it in one hour and not do another pile until an hour passed
  - Otherwise, complete k pieces in an hour

Solution

Note that the satisfiability of k changes at most once, i.e. cannot finish to can finish (from lower value to higher).

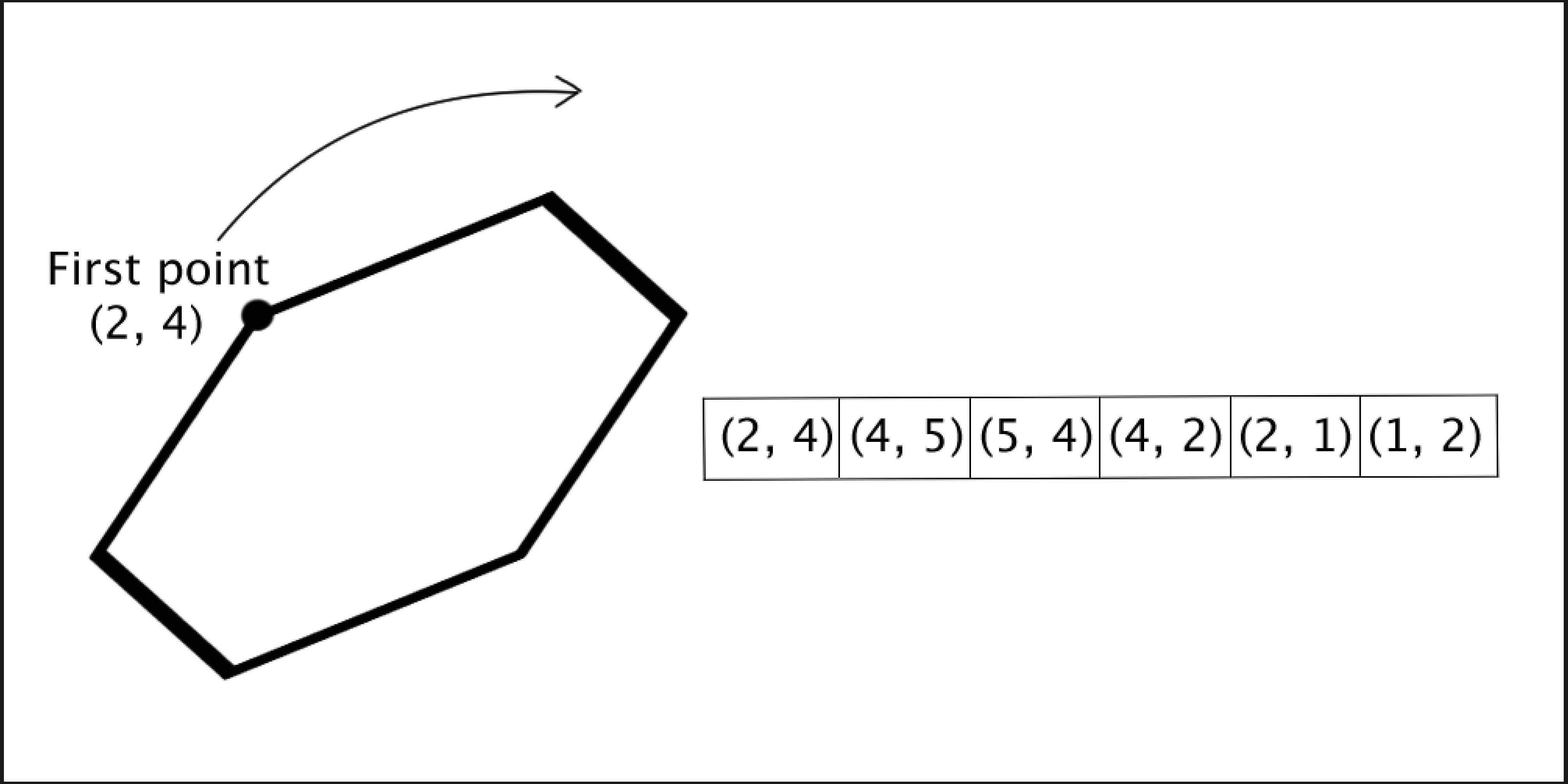
If we cannot satisfy, increase search to right. Otherwise, search left.

Problem 6: Yet Another Application of Binary Search

(Optional) Given an array of n points (in form of (x, y)) of a n-sided *convex* polygon in clockwise order. Find a bounding box around the polygon.

Discuss possible naive solutions and possibly faster solutions. A convex polygon is a polygon where all interior angles are less than 180 degrees.

Example



(2, 4)	(4, 5)	(5, 4)	(4, 2)	(2, 1)	(1, 2)

Solution

- Denote the i-th point of the polygon is p_i, and its coordinates as (x_i, y_i).
- Note that since it is given in clockwise order, the values of x will inflect at most twice, (i.e. inc-dec-inc or dec-inc-dec). The same can be said for values of y as well.
- WLOG, consider the case of inc-dec-inc for x coordinates. Notice that $x_0 \geq x_{n-1}$.
    - Otherwise, it is not convex.
    - Imagine that you do clockwise in circular fashion, but you only consider x values
- The peak and the through are the bounding values for x
- Binary search the peak and through (what's the predicate?)

Solution

- To find peak:
  - The predicate: Is it ascending and $x_m \geq x_0$
- To find through:
  - The predicate: Is it increasing and $x_m \leq x_{n-1}$

Summary

- Java
- Asymptotic Analysis as a tool to analyze program's performance as input size increases
- Asymptotic function properties
- Asymptotic analysis is *not* used to compare performance of two different programs on same input size
- Problem characteristics that can be solved with binary search
    - Determine a property that changes at most once

References / Further Reading

- Binary Search -- Competitive Programmer's Handbook by Antti Laaksonen, pg. 31-34.
- Asymptotic Analysis -- Introduction to Algorithms by CLRS, Ch. 3