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4 P(n) be the statement that

$1^3 + 2^3 + \dots + n^3 = (n(n+1)/2)^2 \rightarrow$ for the positive integer n

a) p(1) ?

$\rightarrow P(1) = (1)^3 = \left(\frac{1(1+1)}{2}\right)^2$

b) proved that p(1) is true!

$n = 1$

$1^3 = \left(\frac{1(1+1)}{2}\right)^2$

$1^3 = \left(\frac{2}{2}\right)^2$

$1 = 1 \longrightarrow$ PROVED!

c) The inductive hypothesis :

$\rightarrow P(k) : 1^3 + 2^3 + \dots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$

d) what we need to prove in the inductive step is P(k+1) :

e) inductive step :

$1^3 + 2^3 + \dots + k^3 + (k+1)^3 = \left(\frac{(k+1)((k+1)((k+1)+1)}{2}\right)^2$

$= \boxed{\frac{k^4 + 6k^3 + 13k^2 + 12k + 4}{4}}$

identifying :

$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \left(\frac{k^2+k}{2}\right)^2 + (k+1)^3$

$= \frac{k^4 + 2k^3 + k^2 + 4(k+1)^3}{4}$

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$$= \frac{k^4 + 2k^3 + k^2 + 4k^3 + 12k^2 + 12k + 4}{4}$$

PROVED!

$$\boxed{= \frac{k^4 + 6k^3 + 13k^2 + 12k + 4}{4}}$$

f) Explain why these steps show that this formula is true whenever n is a positive integer.

↳ because to test whether the formula is correct when n is a positive integer, first we need to test n = 1 in p(n) if it is true, then we change n to k and we need to test p(k+1). If All of that proved true then that formula is true whenever n is a positive integer.

6 Prove that $1.1! + 2.2! + \dots + n.n! = (n+1)! - 1$ whenever n is a positive integer.

↳ n = 1

$p(1) : 1.1! = (1+1)! - 1$

$1 = 2! - 1$

$1 = 1$ (PROVED)

$p(k) : 1.1! + 2.2! + \dots + k.k! = (k+1)! - 1$

$p(k+1) : 1.1! + 2.2! + \dots + k.k! + (k+1).(k+1)! = \boxed{((k+1)+1)! - 1}$

$(k+1)! - 1 + (k+1).(k+1)! = 1.(k+1)! + (k+1).(k+1)! - 1$

$= (1+k+1)(k+1)! - 1$

$= (k+2)(k+1)! - 1$

$= (k+2)(k+1)(k)! - 1$

$= (k+2)! - 1 = \boxed{(k+1)+1)! - 1}$

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6 Find an inverse of a modulo m for each of these pairs of relatively prime integers using the method followed in Ex. 2.

a) $a = 2, m = 17$

$ab \equiv \bmod m$

$17 = 8.2 + 1$

$2 = 2.1$

$\gcd(a, m) = 1$

$\gcd(a, m) = 17 - 8.2$

$\gcd(a, m) = 1.17 - 8.2 \rightarrow a$

→ the inverse is right next a, -8

∴ $-8 \bmod 17 = 9$, so 9 also inverse a mod m.

b) $a = 34, m = 89$

$ab = \bmod m$

$89 = 2.34 + 21 \leftrightarrow 21 = 89 - 2.34$

$34 = 1.21 + 13 \leftrightarrow 13 = 34 - 1.21$

$21 = 1.13 + 8 \leftrightarrow 8 = 21 - 1.13$

$13 = 1.8 + 5 \leftrightarrow 5 = 13 - 1.8$

$8 = 1.5 + 3 \leftrightarrow 3 = 8 - 1.5$

$5 = 1.3 + 2 \leftrightarrow 2 = 5 - 1.3$

$3 = 1.2 + 1 \leftrightarrow 1 = 3 - 1.2$

$2 = 2.1$

$\gcd(a, m) = 1$

$1 = 3 - 1.2$

$1 = 3 - 1(5 - 1.3)$

$1 = 2.3 - 1.5$

$1 = 2.(8 - 1.5) - 1.5$

$1 = 2.8 - 3.5$

$1 = 2.8 - 3(13 - 18)$

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$1 = 5.8 - 3.13$

$1 = 5.(21 - 1.13) - 3.13$

$1 = 5.21 - 8.13$

$1 = 5.21 - 8.(34 - 1.21)$

$1 = 13.21 - 8.34$

$1 = 13.(89 - 2.34) - 8.34$

$1 = 13.89 - 34.34 \rightarrow a$

+ the inverse is right next a, -34

+ $-34 \bmod 89 = -34 + 89 \bmod 89 = 55 \bmod 89 = 55$, so 55 also inverse a mod n