

Python code for Artificial Intelligence: Foundations of Computational Agents (CPSC 322 Edition)

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Version 0.8.4 of October 20, 2020.

<http://aipython.org> <http://artint.info>

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Chapter 1

Python for Artificial Intelligence

1.1 Why Python?

We use Python because Python programs can be close to pseudo-code. It is designed for humans to read.

Python is reasonably efficient. Efficiency is usually not a problem for small examples. If your Python code is not efficient enough, a general procedure to improve it is to find out what is taking most the time, and implement just that part more efficiently in some lower-level language. Most of these lower-level languages interoperate with Python nicely. This will result in much less programming and more efficient code (because you will have more time to optimize) than writing everything in a low-level language. You will not have to do that for the code here if you are using it for course projects.

1.2 Getting Python

You need Python 3 (<http://python.org/>) and matplotlib (<http://matplotlib.org/>) that runs with Python 3. This code is *not* compatible with Python 2 (e.g., with Python 2.7).

Download and install the latest Python 3 release from <http://python.org/>. This should also install *pip3*. You can install matplotlib using

```
pip3 install matplotlib
```

in a terminal shell (not in Python). That should “just work”. If not, try using *pip* instead of *pip3*.

The command `python` or `python3` should then start the interactive python shell. You can quit Python with a control-D or with `quit()`.

To upgrade matplotlib to the latest version (which you should do if you install a new version of Python) do:

```
pip3 install --upgrade matplotlib
```

We recommend using the enhanced interactive python **ipython** (<http://ipython.org/>). To install ipython after you have installed python do:

```
pip3 install ipython
```

1.3 Running Python

We assume that everything is done with an interactive Python shell. You can either do this with an IDE, such as IDLE that comes with standard Python distributions, or just running ipython3 (or perhaps just ipython) from a shell.

Here we describe the most simple version that uses no IDE. If you download the zip file, and cd to the “aipython.322” folder where the .py files are, you should be able to do the following, with user input following : . The first ipython3 command is in the operating system shell (note that the -i is important to enter interactive mode), with user input in bold:

```
$ ipython -i searchGeneric.py
```

```
Python 3.6.5 (v3.6.5:f59c0932b4, Mar 28 2018, 05:52:31)
```

```
Type 'copyright', 'credits' or 'license' for more information
```

```
IPython 6.2.1 -- An enhanced Interactive Python. Type '?' for help.
```

```
Testing problem 1:
```

```
7 paths have been expanded and 4 paths remain in the frontier
```

```
Path found: a --> b --> c --> d --> g
```

```
Passed unit test
```

```
In [1]: searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) #A*
```

```
In [2]: searcher2.search() # find first path
```

```
16 paths have been expanded and 5 paths remain in the frontier
```

```
Out[2]: o103 --> o109 --> o119 --> o123 --> r123
```

```
In [3]: searcher2.search() # find next path
```

```
21 paths have been expanded and 6 paths remain in the frontier
```

```
Out[3]: o103 --> b3 --> b4 --> o109 --> o119 --> o123 --> r123
```

```
In [4]: searcher2.search() # find next path
```

```
28 paths have been expanded and 5 paths remain in the frontier
```

```
Out[4]: o103 --> b3 --> b1 --> b2 --> b4 --> o109 --> o119 --> o123 --> r123
```

```
In [5]: searcher2.search() # find next path
```

```
No (more) solutions. Total of 33 paths expanded.
```

In [6]:

You can then interact at the last prompt.

There are many textbooks for Python. The best source of information about python is <https://www.python.org/>. We will be using Python 3; please download the latest release. The documentation is at <https://docs.python.org/3/>.

The rest of this chapter is about what is special about the code for AI tools. We will only use the Standard Python Library and matplotlib. All of the exercises can be done (and should be done) without using other libraries; the aim is for you to spend your time thinking about how to solve the problem rather than searching for pre-existing solutions.

1.4 Pitfalls

It is important to know when side effects occur. Often AI programs consider what would happen or what may have happened. In many such cases, we don't want side effects. When an agent acts in the world, side effects are appropriate.

In Python, you need to be careful to understand side effects. For example, the inexpensive function to add an element to a list, namely *append*, changes the list. In a functional language like Haskell or Lisp, adding a new element to a list, without changing the original list, is a cheap operation. For example if x is a list containing n elements, adding an extra element to the list in Python (using *append*) is fast, but it has the side effect of changing the list x . To construct a new list that contains the elements of x plus a new element, without changing the value of x , entails copying the list, or using a different representation for lists. In the searching code, we will use a different representation for lists for this reason.

1.5 Features of Python

1.5.1 Lists, Tuples, Sets, Dictionaries and Comprehensions

We make extensive uses of lists, tuples, sets and dictionaries (dicts). See <https://docs.python.org/3/library/stdtypes.html>

One of the nice features of Python is the use of list comprehensions (and also tuple, set and dictionary comprehensions).

$(fe \text{ for } e \text{ in } iter \text{ if } cond)$

enumerates the values fe for each e in $iter$ for which $cond$ is true. The “if $cond$ ” part is optional, but the “for” and “in” are not optional. Here e has to be a variable, $iter$ is an iterator, which can generate a stream of data, such as a list, a set, a range object (to enumerate integers between ranges) or a file. $cond$

is an expression that evaluates to either True or False for each e , and fe is an expression that will be evaluated for each value of e for which $cond$ returns True.

The result can go in a list or used in another iteration, or can be called directly using *next*. The procedure *next* takes an iterator returns the next element (advancing the iterator) and raises a StopIteration exception if there is no next element. The following shows a simple example, where user input is prepended with >>>

```
>>> [e*e for e in range(20) if e%2==0]
[0, 4, 16, 36, 64, 100, 144, 196, 256, 324]
>>> a = (e*e for e in range(20) if e%2==0)
>>> next(a)
0
>>> next(a)
4
>>> next(a)
16
>>> list(a)
[36, 64, 100, 144, 196, 256, 324]
>>> next(a)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
StopIteration
```

Notice how *list(a)* continued on the enumeration, and got to the end of it.

Comprehensions can also be used for dictionaries. The following code creates an index for list *a*:

```
>>> a = ["a","f","bar","b","a","aaaaa"]
>>> ind = {a[i]:i for i in range(len(a))}
>>> ind
{'a': 4, 'f': 1, 'bar': 2, 'b': 3, 'aaaaa': 5}
>>> ind['b']
3
```

which means that 'b' is the 3rd element of the list.

The assignment of *ind* could have also be written as:

```
>>> ind = {val:i for (i,val) in enumerate(a)}
```

where *enumerate* returns an iterator of (*index,value*) pairs.

1.5.2 Functions as first-class objects

Python can create lists and other data structures that contain functions. There is an issue that tricks many newcomers to Python. For a local variable in a function, the function uses the last value of the variable when the function is

called, not the value of the variable when the function was defined (this is called “late binding”). This means if you want to use the value a variable has when the function is created, you need to save the current value of that variable. Whereas Python uses “late binding” by default, the alternative that newcomers often expect is “early binding”, where a function uses the value a variable had when the function was defined, can be easily implemented.

Consider the following programs designed to create a list of 5 functions, where the *i*th function in the list is meant to add *i* to its argument:¹

```
pythonDemo.py — Some tricky examples
11 fun_list1 = []
12 for i in range(5):
13     def fun1(e):
14         return e+i
15     fun_list1.append(fun1)
16
17 fun_list2 = []
18 for i in range(5):
19     def fun2(e,iv=i):
20         return e+iv
21     fun_list2.append(fun2)
22
23 fun_list3 = [lambda e: e+i for i in range(5)]
24
25 fun_list4 = [lambda e,iv=i: e+iv for i in range(5)]
26
27 i=56
```

Try to predict, and then test to see the output, of the output of the following calls, remembering that the function uses the latest value of any variable that is not bound in the function call:

```
pythonDemo.py — (continued)
29 # in Shell do
30 ## ipython -i pythonDemo.py
31 # Try these (copy text after the comment symbol and paste in the Python prompt):
32 # print([f(10) for f in fun_list1])
33 # print([f(10) for f in fun_list2])
34 # print([f(10) for f in fun_list3])
35 # print([f(10) for f in fun_list4])
```

In the first for-loop, the function *fun* uses *i*, whose value is the last value it was assigned. In the second loop, the function *fun2* uses *iv*. There is a separate *iv* variable for each function, and its value is the value of *i* when the function was defined. Thus *fun1* uses late binding, and *fun2* uses early binding. *fun_list3* and *fun_list4* are equivalent to the first two (except *fun_list4* uses a different *i* variable).

¹Numbered lines are Python code available in the code-directory, aipython_322. The name of the file is given in the gray text above the listing. The numbers correspond to the line numbers in that file.

One of the advantages of using the embedded definitions (as in *fun1* and *fun2* above) over the *lambda* is that it is possible to add a `__doc__` string, which is the standard for documenting functions in Python, to the embedded definitions.

1.5.3 Generators and Coroutines

Python has generators which can be used for a form of coroutines.

The *yield* command returns a value that is obtained with *next*. It is typically used to enumerate the values for a *for* loop or in generators.

A version of the built-in *range*, with 2 or 3 arguments (and positive steps) can be implemented as:

```
pythonDemo.py — (continued)
37 def myrange(start, stop, step=1):
38     """enumerates the values from start in steps of size step that are
39     less than stop.
40     """
41     assert step>0, "only positive steps implemented in myrange"
42     i = start
43     while i<stop:
44         yield i
45         i += step
46
47 print("myrange(2,30,3):", list(myrange(2,30,3)))
```

Note that the built-in *range* is unconventional in how it handles a single argument, as the single argument acts as the second argument of the function. Note also that the built-in *range* also allows for indexing (e.g., *range(2,30,3)[2]* returns 8), which the above implementation does not. However *myrange* also works for floats, which the built-in *range* does not.

Exercise 1.1 Implement a version of *myrange* that acts like the built-in version when there is a single argument. (Hint: make the second argument have a default value that can be recognized in the function.)

Yield can be used to generate the same sequence of values as in the example of Section 1.5.1:

```
pythonDemo.py — (continued)
49 def ga(n):
50     """generates square of even nonnegative integers less than n"""
51     for e in range(n):
52         if e%2==0:
53             yield e*e
54 a = ga(20)
```

The sequence of *next(a)*, and *list(a)* gives exactly the same results as the comprehension in Section 1.5.1.

It is straightforward to write a version of the built-in *enumerate*. Let's call it *myenumerate*:

```
pythonDemo.py — (continued)
56 def myenumerate(enum):
57     for i in range(len(enum)):
58         yield i,enum[i]
```

Exercise 1.2 Write a version of *enumerate* where the only iteration is “for val in enum”. Hint: keep track of the index.

1.6 Useful Libraries

1.6.1 Timing Code

In order to compare algorithms, we often want to compute how long a program takes; this is called the **runtime** of the program. The most straightforward way to compute runtime is to use *time.perf_counter()*, as in:

```
import time
start_time = time.perf_counter()
compute_for_a_while()
end_time = time.perf_counter()
print("Time:", end_time - start_time, "seconds")
```

Note that *time.perf_counter()* measures clock time; so this should be done without user interaction between the calls. On the console, you should do:

```
start_time = time.perf_counter(); compute_for_a_while(); end_time = time.perf_counter()
```

If this time is very small (say less than 0.2 second), it is probably very inaccurate, and it may be better to run your code many times to get a more accurate count. For this you can use *timeit* (<https://docs.python.org/3/library/timeit.html>). To use *timeit* to time the call to *foo.bar(aaa)* use:

```
import timeit
time = timeit.timeit("foo.bar(aaa)",
                    setup="from __main__ import foo,aaa", number=100)
```

The setup is needed so that Python can find the meaning of the names in the string that is called. This returns the number of seconds to execute *foo.bar(aaa)* 100 times. The variable *number* should be set so that the runtime is at least 0.2 seconds.

You should not trust a single measurement as that can be confounded by interference from other processes. *timeit.repeat* can be used for running *timeit* a few (say 3) times. Usually the minimum time is the one to report, but you should be explicit and explain what you are reporting.

1.6.2 Plotting: Matplotlib

The standard plotting for Python is matplotlib (<http://matplotlib.org/>). We will use the most basic plotting using the pyplot interface.

Here is a simple example that uses everything we will use.

```
pythonDemo.py — (continued)
60 import matplotlib.pyplot as plt
61
62 def myplot(min,max,step,fun1,fun2):
63     plt.ion() # make it interactive
64     plt.xlabel("The x axis")
65     plt.ylabel("The y axis")
66     plt.xscale('linear') # Makes a 'log' or 'linear' scale
67     xvalues = range(min,max,step)
68     plt.plot(xvalues,[fun1(x) for x in xvalues],
69             label="The first fun")
70     plt.plot(xvalues,[fun2(x) for x in xvalues], linestyle='--',color='k',
71             label=fun2.__doc__) # use the doc string of the function
72     plt.legend(loc="upper right") # display the legend
73
74 def slin(x):
75     """y=2x+7"""
76     return 2*x+7
77 def sqfun(x):
78     """y=(x-40)^2/10-20"""
79     return (x-40)**2/10-20
80
81 # Try the following:
82 # from pythonDemo import myplot, slin, sqfun
83 # import matplotlib.pyplot as plt
84 # myplot(0,100,1,slin,sqfun)
85 # plt.legend(loc="best")
86 # import math
87 # plt.plot([41+40*math.cos(th/10) for th in range(50)],
88 #          [100+100*math.sin(th/10) for th in range(50)])
89 # plt.text(40,100,"ellipse?")
90 # plt.xscale('log')
```

At the end of the code are some commented-out commands you should try in interactive mode. Cut from the file and paste into Python (and remember to remove the comments symbol and leading space).

1.7 Utilities

1.7.1 Display

In this distribution, to keep things simple and to only use standard Python, we use a text-oriented tracing of the code. A graphical depiction of the code could

override the definition of *display* (but we leave it as a project).

The method *self.display* is used to trace the program. Any call

```
self.display(level,to_print...)
```

where the *level* is less than or equal to the value for *max_display_level* will be printed. The *to_print...* can be anything that is accepted by the built-in *print* (including any keyword arguments).

The definition of *display* is:

```

display.py — A simple way to trace the intermediate steps of algorithms.
11 class Displayable(object):
12     """Class that uses 'display'.
13     The amount of detail is controlled by max_display_level
14     """
15     max_display_level = 1 # can be overridden in subclasses
16
17     def display(self,level,*args,**nargs):
18         """print the arguments if level is less than or equal to the
19         current max_display_level.
20         level is an integer.
21         the other arguments are whatever arguments print can take.
22         """
23         if level <= self.max_display_level:
24             print(*args, **nargs) ##if error you are using Python2 not Python3

```

Note that *args* gets a tuple of the positional arguments, and *nargs* gets a dictionary of the keyword arguments). This will not work in Python 2, and will give an error.

Any class that wants to use *display* can be made a subclass of *Displayable*.

To change the maximum display level to say 3, for a class do:

```
Classname.max_display_level = 3
```

which will make calls to *display* in that class print when the value of *level* is less than-or-equal to 3. The default display level is 1. It can also be changed for individual objects (the object value overrides the class value).

The value of *max_display_level* by convention is:

- 0 display nothing
- 1 display solutions (nothing that happens repeatedly)
- 2 also display the values as they change (little detail through a loop)
- 3 also display more details
- 4 and above even more detail

In order to implement more sophisticated visualizations of the algorithm, we add a **visualize** “decorator” to the methods to be visualized. The following code ignores the decorator:

```

display.py — (continued)
26 def visualize(func):
27     """A decorator for algorithms that do interactive visualization.
28     Ignored here.
29     """
30     return func

```

1.7.2 Argmax

Python has a built-in *max* function that takes a generator (or a list or set) and returns the maximum value. The *argmax* method returns the index of an element that has the maximum value. If there are multiple elements with the maximum value, one of the indexes to that value is returned at random. This assumes a generator of (*element*, *value*) pairs, as for example is generated by the built-in *enumerate*.

```

utilities.py — AIPython useful utilities
11 import random
12
13 def argmax(gen):
14     """gen is a generator of (element,value) pairs, where value is a real.
15     argmax returns an element with maximal value.
16     If there are multiple elements with the max value, one is returned at random.
17     """
18     maxv = float('-Infinity') # negative infinity
19     maxvals = [] # list of maximal elements
20     for (e,v) in gen:
21         if v>maxv:
22             maxvals,maxv = [e], v
23         elif v==maxv:
24             maxvals.append(e)
25     return random.choice(maxvals)
26
27 # Try:
28 # argmax(enumerate([1,6,3,77,3,55,23]))

```

Exercise 1.3 Change *argmax* to have an optional argument that specifies whether you want the “first”, “last” or a “random” index of the maximum value returned. If you want the first or the last, you don’t need to keep a list of the maximum elements.

1.7.3 Probability

For many of the simulations, we want to make a variable True with some probability. *flip*(*p*) returns True with probability *p*, and otherwise returns False.

```

utilities.py — (continued)
30 def flip(prob):

```

```

31 | """return true with probability prob"""
32 | return random.random() < prob

```

1.7.4 Dictionary Union

The function `dict_union(d1,d2)` returns the union of dictionaries `d1` and `d2`. If the values for the keys conflict, the values in `d2` are used. This is similar to `dict(d1,**d2)`, but that only works when the keys of `d2` are strings.

```

_____utilities.py — (continued)_____
34 | def dict_union(d1,d2):
35 |     """returns a dictionary that contains the keys of d1 and d2.
36 |     The value for each key that is in d2 is the value from d2,
37 |     otherwise it is the value from d1.
38 |     This does not have side effects.
39 |     """
40 |     d = dict(d1) # copy d1
41 |     d.update(d2)
42 |     return d

```

1.8 Testing Code

It is important to test code early and test it often. We include a simple form of **unit test**. The value of the current module is in `__name__` and if the module is run at the top-level, it's value is `"__main__"`. See https://docs.python.org/3/library/__main__.html.

The following code tests `argmax` and `dict_union`, but only when if `utilities` is loaded in the top-level. If it is loaded in a module the test code is not run.

In your code you should do more substantial testing than we do here, in particular testing the boundary cases.

```

_____utilities.py — (continued)_____
44 | def test():
45 |     """Test part of utilities"""
46 |     assert argmax(enumerate([1,6,55,3,55,23])) in [2,4]
47 |     assert dict_union({1:4, 2:5, 3:4},{5:7, 2:9}) == {1:4, 2:9, 3:4, 5:7}
48 |     print("Passed unit test in utilities")
49 |
50 | if __name__ == "__main__":
51 |     test()

```


Chapter 2

Searching for Solutions

2.1 Representing Search Problems

A search problem consists of:

- a start node
- a neighbors function that given a node, returns an enumeration of the arcs from the node
- a specification of a goal in terms of a Boolean function that takes a node and returns true if the node is a goal
- a (optional) heuristic function that, given a node, returns a non-negative real number. The heuristic function defaults to zero.

As far as the searcher is concerned a node can be anything. If multiple-path pruning is used, a node must be hashable. In the simple examples, it is a string, but in more complicated examples (in later chapters) it can be a tuple, a frozen set, or a Python object.

In the following code raise `NotImplementedError()` is a way to specify that this is an abstract method that needs to be overridden to define an actual search problem.

```
searchProblem.py — representations of search problems
11 class Search_problem(object):
12     """A search problem consists of:
13     * a start node
14     * a neighbors function that gives the neighbors of a node
15     * a specification of a goal
16     * a (optional) heuristic function.
```

```

17     The methods must be overridden to define a search problem."""
18
19     def start_node(self):
20         """returns start node"""
21         raise NotImplementedError("start_node") # abstract method
22
23     def is_goal(self,node):
24         """is True if node is a goal"""
25         raise NotImplementedError("is_goal") # abstract method
26
27     def neighbors(self,node):
28         """returns a list of the arcs for the neighbors of node"""
29         raise NotImplementedError("neighbors") # abstract method
30
31     def heuristic(self,n):
32         """Gives the heuristic value of node n.
33         Returns 0 if not overridden."""
34         return 0

```

The neighbors is a list of arcs. A (directed) arc consists of a *from_node* node and a *to_node* node. The arc is the pair $\langle from_node, to_node \rangle$, but can also contain a non-negative *cost* (which defaults to 1) and can be labeled with an *action*.

searchProblem.py — (continued)

```

36 class Arc(object):
37     """An arc has a from_node and a to_node node and a (non-negative) cost"""
38     def __init__(self, from_node, to_node, cost=1, action=None):
39         assert cost >= 0, ("Cost cannot be negative for"+
40                             str(from_node)+"->"+str(to_node)+", cost: "+str(cost))
41         self.from_node = from_node
42         self.to_node = to_node
43         self.action = action
44         self.cost=cost
45
46     def __repr__(self):
47         """string representation of an arc"""
48         if self.action:
49             return str(self.from_node)+" --"+str(self.action)+"--> "+str(self.to_node)
50         else:
51             return str(self.from_node)+" --> "+str(self.to_node)

```

2.1.1 Explicit Representation of Search Graph

The first representation of a search problem is from an explicit graph (as opposed to one that is generated as needed).

An **explicit graph** consists of

- a list or set of nodes
- a list or set of arcs

- a start node
- a list or set of goal nodes
- (optionally) a dictionary that maps a node to a heuristic value for that node

To define a search problem, we need to define the start node, the goal predicate, the neighbors function and the heuristic function.

```

53 class Search_problem_from_explicit_graph(Search_problem):
54     """A search problem consists of:
55     * a list or set of nodes
56     * a list or set of arcs
57     * a start node
58     * a list or set of goal nodes
59     * a dictionary that maps each node into its heuristic value.
60     * a dictionary that maps each node into its (x,y) position
61     """
62
63     def __init__(self, nodes, arcs, start=None, goals=set(), hmap={}, positions={}):
64         self.neighs = {}
65         self.nodes = nodes
66         for node in nodes:
67             self.neighs[node]=[]
68         self.arcs = arcs
69         for arc in arcs:
70             self.neighs[arc.from_node].append(arc)
71         self.start = start
72         self.goals = goals
73         self.hmap = hmap
74         self.positions = positions
75
76     def start_node(self):
77         """returns start node"""
78         return self.start
79
80     def is_goal(self,node):
81         """is True if node is a goal"""
82         return node in self.goals
83
84     def neighbors(self,node):
85         """returns the neighbors of node"""
86         return self.neighs[node]
87
88     def heuristic(self,node):
89         """Gives the heuristic value of node n.
90         Returns 0 if not overridden in the hmap."""
91         if node in self.hmap:
92             return self.hmap[node]

```

```

93         else:
94             return 0
95
96     def __repr__(self):
97         """returns a string representation of the search problem"""
98         res=""
99         for arc in self.arcs:
100             res += str(arc)+" "
101         return res

```

The following is used for the depth-first search implementation below.

```

searchProblem.py — (continued)
103     def neighbor_nodes(self,node):
104         """returns an iterator over the neighbors of node"""
105         return (path.to_node for path in self.neighs[node])

```

2.1.2 Paths

A searcher will return a path from the start node to a goal node. A Python list is not a suitable representation for a path, as many search algorithms consider multiple paths at once, and these paths should share initial parts of the path. If we wanted to do this with Python lists, we would need to keep copying the list, which can be expensive if the list is long. An alternative representation is used here in terms of a recursive data structure that can share subparts.

A path is either:

- a node (representing a path of length 0) or
- a path, *initial* and an arc, where the *from_node* of the arc is the node at the end of *initial*.

These cases are distinguished in the following code by having *arc = None* if the path has length 0, in which case *initial* is the node of the path.

```

searchProblem.py — (continued)
107 class Path(object):
108     """A path is either a node or a path followed by an arc"""
109
110     def __init__(self,initial,arc=None):
111         """initial is either a node (in which case arc is None) or
112         a path (in which case arc is an object of type Arc)"""
113         self.initial = initial
114         self.arc=arc
115         if arc is None:
116             self.cost=0
117         else:
118             self.cost = initial.cost+arc.cost
119
120     def end(self):

```

```

121     """returns the node at the end of the path"""
122     if self.arc is None:
123         return self.initial
124     else:
125         return self.arc.to_node
126
127     def nodes(self):
128         """enumerates the nodes for the path.
129         This starts at the end and enumerates nodes in the path backwards."""
130         current = self
131         while current.arc is not None:
132             yield current.arc.to_node
133             current = current.initial
134         yield current.initial
135
136     def initial_nodes(self):
137         """enumerates the nodes for the path before the end node.
138         This starts at the end and enumerates nodes in the path backwards."""
139         if self.arc is not None:
140             for nd in self.initial.nodes(): yield nd # could be "yield from"
141
142     def __repr__(self):
143         """returns a string representation of a path"""
144         if self.arc is None:
145             return str(self.initial)
146         elif self.arc.action:
147             return (str(self.initial)+"\n --"+str(self.arc.action)
148                 + "--> "+str(self.arc.to_node))
149         else:
150             return str(self.initial)+" --> "+str(self.arc.to_node)

```

2.1.3 Example Search Problems

The first search problem is one with 5 nodes where the least-cost path is one with many arcs. See Figure 2.1. Note that this example is used for the unit tests, so the test (in `searchGeneric`) will need to be changed if this is changed.

```

searchProblem.py — (continued)
152 problem1 = Search_problem_from_explicit_graph(
153     {'a','b','c','d','g'},
154     [Arc('a','c',1), Arc('a','b',3), Arc('c','d',3), Arc('c','b',1),
155       Arc('b','d',1), Arc('b','g',3), Arc('d','g',1)],
156     start = 'a',
157     goals = {'g'},
158     positions={'a': (0, 0), 'b': (1, 1), 'c': (0,1), 'd': (1,2), 'g': (2,2)})

```

The second search problem is one with 8 nodes where many paths do not lead to the goal. See Figure 2.2.

```

searchProblem.py — (continued)

```

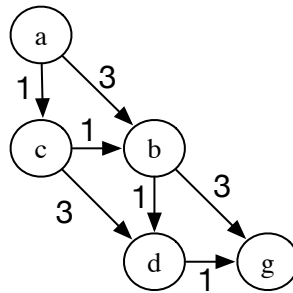


Figure 2.1: problem1

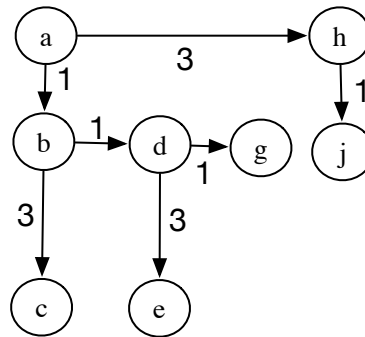


Figure 2.2: problem2

```

159 | problem2 = Search_problem_from_explicit_graph(
160 |     {'a','b','c','d','e','g','h','j'},
161 |     [Arc('a','b',1), Arc('b','c',3), Arc('b','d',1), Arc('d','e',3),
162 |       Arc('d','g',1), Arc('a','h',3), Arc('h','j',1)],
163 |     start = 'a',
164 |     goals = {'g'},
165 |     positions={'a': (0, 0), 'b': (0, 1), 'c': (0,4), 'd': (1,1), 'e': (1,4),
166 |               'g': (2,1), 'h': (3,0), 'j': (3,1)})

```

The third search problem is a disconnected graph (contains no arcs), where the start node is a goal node. This is a boundary case to make sure that weird cases work.

searchProblem.py — (continued) —

```

168 | problem3 = Search_problem_from_explicit_graph(
169 |     {'a','b','c','d','e','g','h','j'},
170 |     [],
171 |     start = 'g',
172 |     goals = {'k','g'})

```

The `acyclic_delivery_problem` is the delivery problem described in Example 3.4 and shown in Figure 3.2 of the textbook.

```

searchProblem.py — (continued)
174 acyclic_delivery_problem = Search_problem_from_explicit_graph(
175     {'mail', 'ts', 'o103', 'o109', 'o111', 'b1', 'b2', 'b3', 'b4', 'c1', 'c2', 'c3',
176     'o125', 'o123', 'o119', 'r123', 'storage'},
177     [Arc('ts', 'mail', 6),
178       Arc('o103', 'ts', 8),
179       Arc('o103', 'b3', 4),
180       Arc('o103', 'o109', 12),
181       Arc('o109', 'o119', 16),
182       Arc('o109', 'o111', 4),
183       Arc('b1', 'c2', 3),
184       Arc('b1', 'b2', 6),
185       Arc('b2', 'b4', 3),
186       Arc('b3', 'b1', 4),
187       Arc('b3', 'b4', 7),
188       Arc('b4', 'o109', 7),
189       Arc('c1', 'c3', 8),
190       Arc('c2', 'c3', 6),
191       Arc('c2', 'c1', 4),
192       Arc('o123', 'o125', 4),
193       Arc('o123', 'r123', 4),
194       Arc('o119', 'o123', 9),
195       Arc('o119', 'storage', 7)],
196     start = 'o103',
197     goals = {'r123'},
198     hmap = {
199         'mail' : 26,
200         'ts' : 23,
201         'o103' : 21,
202         'o109' : 24,
203         'o111' : 27,
204         'o119' : 11,
205         'o123' : 4,
206         'o125' : 6,
207         'r123' : 0,
208         'b1' : 13,
209         'b2' : 15,
210         'b3' : 17,
211         'b4' : 18,
212         'c1' : 6,
213         'c2' : 10,
214         'c3' : 12,
215         'storage' : 12
216     }
217 )

```

The cyclic_delivery_problem is the delivery problem described in Example 3.8 and shown in Figure 3.6 of the textbook. This is the same as acyclic_delivery_problem, but almost every arc also has its inverse.

```

219 cyclic_delivery_problem = Search_problem_from_explicit_graph(
220     {'mail', 'ts', 'o103', 'o109', 'o111', 'b1', 'b2', 'b3', 'b4', 'c1', 'c2', 'c3',
221      'o125', 'o123', 'o119', 'r123', 'storage'},
222     [ Arc('ts', 'mail', 6), Arc('mail', 'ts', 6),
223       Arc('o103', 'ts', 8), Arc('ts', 'o103', 8),
224       Arc('o103', 'b3', 4),
225       Arc('o103', 'o109', 12), Arc('o109', 'o103', 12),
226       Arc('o109', 'o119', 16), Arc('o119', 'o109', 16),
227       Arc('o109', 'o111', 4), Arc('o111', 'o109', 4),
228       Arc('b1', 'c2', 3),
229       Arc('b1', 'b2', 6), Arc('b2', 'b1', 6),
230       Arc('b2', 'b4', 3), Arc('b4', 'b2', 3),
231       Arc('b3', 'b1', 4), Arc('b1', 'b3', 4),
232       Arc('b3', 'b4', 7), Arc('b4', 'b3', 7),
233       Arc('b4', 'o109', 7),
234       Arc('c1', 'c3', 8), Arc('c3', 'c1', 8),
235       Arc('c2', 'c3', 6), Arc('c3', 'c2', 6),
236       Arc('c2', 'c1', 4), Arc('c1', 'c2', 4),
237       Arc('o123', 'o125', 4), Arc('o125', 'o123', 4),
238       Arc('o123', 'r123', 4), Arc('r123', 'o123', 4),
239       Arc('o119', 'o123', 9), Arc('o123', 'o119', 9),
240       Arc('o119', 'storage', 7), Arc('storage', 'o119', 7)],
241     start = 'o103',
242     goals = {'r123'},
243     hmap = {
244         'mail' : 26,
245         'ts' : 23,
246         'o103' : 21,
247         'o109' : 24,
248         'o111' : 27,
249         'o119' : 11,
250         'o123' : 4,
251         'o125' : 6,
252         'r123' : 0,
253         'b1' : 13,
254         'b2' : 15,
255         'b3' : 17,
256         'b4' : 18,
257         'c1' : 6,
258         'c2' : 10,
259         'c3' : 12,
260         'storage' : 12
261     }
262 )

```


2.2 Generic Searcher and Variants

To run the search demos, in folder “aipython_322”, load “searchGeneric.py” , using e.g., `ipython -i searchGeneric.py`, and copy and paste the example queries at the bottom of that file. This requires Python 3.

2.2.1 Searcher

A *Searcher* for a problem can be asked repeatedly for the next path. To solve a problem, we can construct a *Searcher* object for the problem and then repeatedly ask for the next path using *search*. If there are no more paths, *None* is returned.

```

searchGeneric.py — Generic Searcher, including depth-first and A*
11 from display import Displayable, visualize
12
13 class Searcher(Displayable):
14     """returns a searcher for a problem.
15     Paths can be found by repeatedly calling search().
16     This does depth-first search unless overridden
17     """
18     def __init__(self, problem):
19         """creates a searcher from a problem
20         """
21         self.problem = problem
22         self.initialize_frontier()
23         self.num_expanded = 0
24         self.add_to_frontier(Path(problem.start_node()))
25         super().__init__()
26
27     def initialize_frontier(self):
28         self.frontier = []
29
30     def empty_frontier(self):
31         return self.frontier == []
32
33     def add_to_frontier(self, path):
34         self.frontier.append(path)
35
36     @visualize
37     def search(self):
38         """returns (next) path from the problem's start node
39         to a goal node.
40         Returns None if no path exists.
41         """
42         while not self.empty_frontier():
43             path = self.frontier.pop()
44             self.display(2, "Expanding:", path, "(cost:", path.cost, ")")
45             self.num_expanded += 1

```

```

46         if self.problem.is_goal(path.end()): # solution found
47             self.display(1, self.num_expanded, "paths have been expanded and",
48                         len(self.frontier), "paths remain in the frontier")
49             self.solution = path # store the solution found
50             return path
51         else:
52             neighs = self.problem.neighbors(path.end())
53             self.display(3, "Neighbors are", neighs)
54             for arc in reversed(list(neighs)):
55                 self.add_to_frontier(Path(path, arc))
56             self.display(3, "Frontier:", self.frontier)
57     self.display(1, "No (more) solutions. Total of",
58                 self.num_expanded, "paths expanded.")

```

Note that this reverses the neighbours so that it implements depth-first search in an intuitive manner (expanding the first neighbor first), and *list* is needed if the neighbours are generated. Reversing the neighbours might not be required for other methods. The calls to *reversed* and *list* can be removed, and the algorithm still implements depth-first search.

Exercise 2.1 When it returns a path, the algorithm can be used to find another path by calling *search()* again. However, it does not find other paths that go through one goal node to another. Explain why, and change the code so that it can find such paths when *search()* is called again.

2.2.2 Frontier as a Priority Queue

In many of the search algorithms, such as A^* and other best-first searchers, the frontier is implemented as a priority queue. Here we use the Python's built-in priority queue implementations, *heapq*.

Following the lead of the Python documentation, <http://docs.python.org/3.3/library/heapq.html>, a frontier is a list of triples. The first element of each triple is the value to be minimized. The second element is a unique index which specifies the order when the first elements are the same, and the third element is the path that is on the queue. The use of the unique index ensures that the priority queue implementation does not compare paths; whether one path is less than another is not defined. It also lets us control what sort of search (e.g., depth-first or breadth-first) occurs when the value to be minimized does not give a unique next path.

The variable *frontier_index* is the total number of elements of the frontier that have been created. As well as being used as a unique index, it is useful for statistics, particularly in conjunction with the current size of the frontier.

```

searchGeneric.py — (continued)
60 import heapq          # part of the Python standard library
61 from searchProblem import Path
62
63 class FrontierPQ(object):

```

```

64     """A frontier consists of a priority queue (heap), frontierpq, of
65         (value, index, path) triples, where
66         * value is the value we want to minimize (e.g., path cost + h).
67         * index is a unique index for each element
68         * path is the path on the queue
69         Note that the priority queue always returns the smallest element.
70     """
71
72     def __init__(self):
73         """constructs the frontier, initially an empty priority queue
74         """
75         self.frontier_index = 0 # the number of items ever added to the frontier
76         self.frontierpq = [] # the frontier priority queue
77
78     def empty(self):
79         """is True if the priority queue is empty"""
80         return self.frontierpq == []
81
82     def add(self, path, value):
83         """add a path to the priority queue
84         value is the value to be minimized"""
85         self.frontier_index += 1 # get a new unique index
86         heapq.heappush(self.frontierpq, (value, -self.frontier_index, path))
87
88     def pop(self):
89         """returns and removes the path of the frontier with minimum value.
90         """
91         (_,_,path) = heapq.heappop(self.frontierpq)
92         return path

```

The following methods are used for finding and printing information about the frontier.

```

searchGeneric.py — (continued)
94     def count(self, val):
95         """returns the number of elements of the frontier with value=val"""
96         return sum(1 for e in self.frontierpq if e[0]==val)
97
98     def __repr__(self):
99         """string representation of the frontier"""
100        return str([(n,c,str(p)) for (n,c,p) in self.frontierpq])
101
102    def __len__(self):
103        """length of the frontier"""
104        return len(self.frontierpq)
105
106    def __iter__(self):
107        """iterate through the paths in the frontier"""
108        for (_,_,path) in self.frontierpq:
109            yield path

```

2.2.3 A* Search

For an A* **Search** the frontier is implemented using the FrontierPQ class.

```

searchGeneric.py — (continued)
111 class AStarSearcher(Searcher):
112     """returns a searcher for a problem.
113     Paths can be found by repeatedly calling search().
114     """
115
116     def __init__(self, problem):
117         super().__init__(problem)
118
119     def initialize_frontier(self):
120         self.frontier = FrontierPQ()
121
122     def empty_frontier(self):
123         return self.frontier.empty()
124
125     def add_to_frontier(self, path):
126         """add path to the frontier with the appropriate cost"""
127         value = path.cost + self.problem.heuristic(path.end())
128         self.frontier.add(path, value)

```

Code should always be tested. The following provides a simple **unit test**, using problem1 as the the default problem.

```

searchGeneric.py — (continued)
130 import searchProblem as searchProblem
131
132 def test(SearchClass, problem=searchProblem.problem1, solutions=[['g','d','b','c','a']] ):
133     """Unit test for aipython searching algorithms.
134     SearchClass is a class that takes a problem and implements search()
135     problem is a search problem
136     solutions is a list of optimal solutions
137     """
138     print("Testing problem 1:")
139     schr1 = SearchClass(problem)
140     path1 = schr1.search()
141     print("Path found:", path1)
142     assert path1 is not None, "No path is found in problem1"
143     assert list(path1.nodes()) in solutions, "Shortest path not found in problem1"
144     print("Passed unit test")
145
146 if __name__ == "__main__":
147     #test(Searcher)
148     test(AStarSearcher)
149
150 # example queries:
151 # searcher1 = Searcher(searchProblem.acyclic_delivery_problem) # DFS
152 # searcher1.search() # find first path

```

```

153 # searcher1.search() # find next path
154 # searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) # A*
155 # searcher2.search() # find first path
156 # searcher2.search() # find next path
157 # searcher3 = Searcher(searchProblem.cyclic_delivery_problem) # DFS
158 # searcher3.search() # find first path with DFS. What do you expect to happen?
159 # searcher4 = AStarSearcher(searchProblem.cyclic_delivery_problem) # A*
160 # searcher4.search() # find first path

```

Exercise 2.2 Change the code so that it implements (i) best-first search and (ii) lowest-cost-first search. For each of these methods compare it to A^* in terms of the number of paths expanded, and the path found.

Exercise 2.3 In the *add* method in *FrontierPQ* what does the "-" in front of *frontier_index* do? When there are multiple paths with the same f -value, which search method does this act like? What happens if the "-" is removed? When there are multiple paths with the same value, which search method does this act like? Does it work better with or without the "-"? What evidence did you base your conclusion on?

Exercise 2.4 The searcher acts like a Python iterator, in that it returns one value (here a path) and then returns other values (paths) on demand, but does not implement the iterator interface. Change the code so it implements the iterator interface. What does this enable us to do?

2.2.4 Multiple Path Pruning

To run the multiple-path pruning demo, in folder "aipython.322", load "searchMPP.py", using e.g., `ipython -i searchMPP.py`, and copy and paste the example queries at the bottom of that file.

The following implements A^* with multiple-path pruning. It overrides *search()* in *Searcher*.

```

searchMPP.py — Searcher with multiple-path pruning
11 from searchGeneric import AStarSearcher, visualize
12 from searchProblem import Path
13
14 class SearcherMPP(AStarSearcher):
15     """returns a searcher for a problem.
16     Paths can be found by repeatedly calling search().
17     """
18     def __init__(self, problem):
19         super().__init__(problem)
20         self.explored = set()
21
22     @visualize
23     def search(self):
24         """returns next path from an element of problem's start nodes
25         to a goal node.
26         Returns None if no path exists.

```

```

27     """
28     while not self.empty_frontier():
29         path = self.frontier.pop()
30         if path.end() not in self.explored:
31             self.display(2, "Expanding:", path, "(cost:", path.cost, ")")
32             self.explored.add(path.end())
33             self.num_expanded += 1
34             if self.problem.is_goal(path.end()):
35                 self.display(1, self.num_expanded, "paths have been expanded and",
36                             len(self.frontier), "paths remain in the frontier")
37                 self.solution = path # store the solution found
38                 return path
39             else:
40                 neighs = self.problem.neighbors(path.end())
41                 self.display(3, "Neighbors are", neighs)
42                 for arc in neighs:
43                     self.add_to_frontier(Path(path, arc))
44                 self.display(3, "Frontier:", self.frontier)
45             self.display(1, "No (more) solutions. Total of",
46                         self.num_expanded, "paths expanded.")
47
48 from searchGeneric import test
49 if __name__ == "__main__":
50     test(SearcherMPP)
51
52 import searchProblem
53 # searcherMPPcdp = SearcherMPP(searchProblem.cyclic_delivery_problem)
54 # print(searcherMPPcdp.search()) # find first path

```

Exercise 2.5 Implement a searcher that implements cycle pruning instead of multiple-path pruning. You need to decide whether to check for cycles when paths are added to the frontier or when they are removed. (Hint: either method can be implemented by only changing one or two lines in SearcherMPP. Hint: there is a cycle if `path.end()` in `path.initial_nodes()`) Compare no pruning, multiple path pruning and cycle pruning for the cyclic delivery problem. Which works better in terms of number of paths expanded, computational time or space?

2.3 Branch-and-bound Search

To run the demo, in folder "aipython_322", load "searchBranchAndBound.py", and copy and paste the example queries at the bottom of that file.

Depth-first search methods do not need an a priority queue, but can use a list as a stack. In this implementation of branch-and-bound search, we call *search* to find an optimal solution with cost less than bound. This uses depth-first search to find a path to a goal that extends *path* with cost less than the

bound. Once a path to a goal has been found, that path is remembered as the *best_path*, the bound is reduced, and the search continues.

```

_____searchBranchAndBound.py — Branch and Bound Search _____
11 from searchProblem import Path
12 from searchGeneric import Searcher
13 from display import Displayable, visualize
14
15 class DF_branch_and_bound(Searcher):
16     """returns a branch and bound searcher for a problem.
17     An optimal path with cost less than bound can be found by calling search()
18     """
19     def __init__(self, problem, bound=float("inf")):
20         """creates a searcher than can be used with search() to find an optimal path.
21         bound gives the initial bound. By default this is infinite - meaning there
22         is no initial pruning due to depth bound
23         """
24         super().__init__(problem)
25         self.best_path = None
26         self.bound = bound
27
28     @visualize
29     def search(self):
30         """returns an optimal solution to a problem with cost less than bound.
31         returns None if there is no solution with cost less than bound."""
32         self.frontier = [Path(self.problem.start_node())]
33         self.num_expanded = 0
34         while self.frontier:
35             path = self.frontier.pop()
36             if path.cost+self.problem.heuristic(path.end()) < self.bound:
37                 # if path.end() not in path.initial_nodes(): # for cycle pruning
38                 self.display(3,"Expanding:",path,"cost:",path.cost)
39                 self.num_expanded += 1
40                 if self.problem.is_goal(path.end()):
41                     self.best_path = path
42                     self.bound = path.cost
43                     self.display(2,"New best path:",path," cost:",path.cost)
44                 else:
45                     neighs = self.problem.neighbors(path.end())
46                     self.display(3,"Neighbors are", neighs)
47                     for arc in reversed(list(neighs)):
48                         self.add_to_frontier(Path(path, arc))
49         self.display(1,"Number of paths expanded:",self.num_expanded,
50                     "(optimal" if self.best_path else "(no", "solution found)")
51         self.solution = self.best_path
52         return self.best_path

```

Note that this code used *reversed* in order to expand the neighbors of a node in the left-to-right order one might expect. It does this because *pop()* removes the rightmost element of the list. The call to *list* is there because *reversed* only works on lists and tuples, but the neighbours can be generated.

Here is a unit test and some queries:

```

searchBranchAndBound.py — (continued)
54 from searchGeneric import test
55 if __name__ == "__main__":
56     test(DF_branch_and_bound)
57
58 # Example queries:
59 import searchProblem
60 # searcherb1 = DF_branch_and_bound(searchProblem.acyclic_delivery_problem)
61 # print(searcherb1.search()) # find optimal path
62 # searcherb2 = DF_branch_and_bound(searchProblem.cyclic_delivery_problem, bound=100)
63 # print(searcherb2.search()) # find optimal path

```

Exercise 2.6 Implement a branch-and-bound search uses recursion. Hint: you don't need an explicit frontier, but can do a recursive call for the children.

Exercise 2.7 After the branch-and-bound search found a solution, Sam ran search again, and noticed a different count. Sam hypothesized that this count was related to the number of nodes that an A^* search would use (either expand or be added to the frontier). Or maybe, Sam thought, the count for a number of nodes when the bound is slightly above the optimal path case is related to how A^* would work. Is there relationship between these counts? Are there different things that it could count so they are related? Try to find the most specific statement that is true, and explain why it is true.

To test the hypothesis, Sam wrote the following code, but isn't sure it is helpful:

```

searchTest.py — code that may be useful to compare A* and branch-and-bound
11 from searchGeneric import Searcher, AStarSearcher
12 from searchBranchAndBound import DF_branch_and_bound
13 from searchMPP import SearcherMPP
14
15 DF_branch_and_bound.max_display_level = 1
16 Searcher.max_display_level = 1
17
18 def run(problem,name):
19     print("\n\n*****",name)
20
21     print("\nA*:")
22     asearcher = AStarSearcher(problem)
23     print("Path found:",asearcher.search()," cost=",asearcher.solution.cost)
24     print("there are",asearcher.frontier.count(asearcher.solution.cost),
25           "elements remaining on the queue with f-value=",asearcher.solution.cost)
26
27     print("\nA* with MPP:"),
28     msearcher = SearcherMPP(problem)
29     print("Path found:",msearcher.search()," cost=",msearcher.solution.cost)
30     print("there are",msearcher.frontier.count(msearcher.solution.cost),
31           "elements remaining on the queue with f-value=",msearcher.solution.cost)
32

```



```

33 bound = asearcher.solution.cost+0.01
34 print("\nBranch and bound (with too-good initial bound of", bound,")")
35 tbb = DF_branch_and_bound(problem,bound) # cheating!!!
36 print("Path found:",tbb.search()," cost=",tbb.solution.cost)
37 print("Rerunning B&B")
38 print("Path found:",tbb.search())
39
40 bbound = asearcher.solution.cost*2+10
41 print("\nBranch and bound (with not-very-good initial bound of", bbound, ")")
42 tbb2 = DF_branch_and_bound(problem,bbound) # cheating!!!
43 print("Path found:",tbb2.search()," cost=",tbb2.solution.cost)
44 print("Rerunning B&B")
45 print("Path found:",tbb2.search())
46
47 print("\nDepth-first search: (Use ^C if it goes on forever)")
48 tsearcher = Searcher(problem)
49 print("Path found:",tsearcher.search()," cost=",tsearcher.solution.cost)
50
51
52 import searchProblem
53 from searchTest import run
54 if __name__ == "__main__":
55     run(searchProblem.problem1,"Problem 1")
56     # run(searchProblem.acyclic_delivery_problem,"Acyclic Delivery")
57     # run(searchProblem.cyclic_delivery_problem,"Cyclic Delivery")
58 # also test some graphs with cycles, and some with multiple least-cost paths

```


Chapter 3

Reasoning with Constraints

3.1 Constraint Satisfaction Problems

3.1.1 Constraints

A **variable** is a string or any value that is printable and can be the key of a Python dictionary.

A **constraint** consists of a list (or tuple) of variables and a condition.

- The tuple (or list) of variables is called the **scope**.
- The **condition** is a Boolean function that takes the same number of arguments as there are variables in the scope. The condition must have a `__name__` property that gives a printable name of the function; built-in functions and functions that are defined using *def* have such a property; for other functions you may need to define this property.

```
_____cspProblem.py — Representations of a Constraint Satisfaction Problem _____
11 class Constraint(object):
12     """A Constraint consists of
13     * scope: a tuple of variables
14     * condition: a function that can applied to a tuple of values
15     * string: a string for printing the constraints. All of the strings must be unique.
16     for the variables
17     """
18     def __init__(self, scope, condition, string=None):
19         self.scope = scope
20         self.condition = condition
21         if string is None:
22             self.string = self.condition.__name__ + str(self.scope)
23         else:
```

```

24         self.string = string
25
26     def __repr__(self):
27         return self.string

```

An **assignment** is a *variable:value* dictionary.

If *con* is a constraint, *con.holds(assignment)* returns True or False depending on whether the condition is true or false for that assignment. The assignment *assignment* must assigns a value to every variable in the scope of the constraint *con* (and could also assign values other variables); *con.holds* gives an error if not all variables in the scope of *con* are assigned in the assignment. It ignores variables in *assignment* that are not in the scope of the constraint.

In Python, the *** notation is used for unpacking a tuple. For example, *F(*(1,2,3))* is the same as *F(1,2,3)*. So if *t* has value (1,2,3), then *F(*t)* is the same as *F(1,2,3)*.

```

_____cspProblem.py — (continued)_____
29     def holds(self, assignment):
30         """returns the value of Constraint con evaluated in assignment.
31
32         precondition: all variables are assigned in assignment
33         """
34         return self.condition(*tuple(assignment[v] for v in self.scope))

```

3.1.2 CSPs

A constraint satisfaction problem (CSP) requires:

- *domains*: a dictionary that maps variables to the set of possible values. Thus *domains[var]* is the domain of variable *var*.
- *constraints*: a set or list of constraints.

Other properties are inferred from these:

- *variables* is the set of variables. The variables can be enumerated by using “for var in domains” because iterating over a dictionary gives the keys, which in this case are the variables.
- *var_to_const* is a mapping from variables to set of constraints, such that *var_to_const[var]* is the set of constraints with *var* in the scope.

```

_____cspProblem.py — (continued)_____
36 class CSP(object):
37     """A CSP consists of
38     * domains, a dictionary that maps each variable to its domain
39     * constraints, a list of constraints
40     * variables, a set of variables

```

```

41 * var_to_const, a variable to set of constraints dictionary
42 """
43 def __init__(self, domains, constraints, positions={}):
44     """domains is a variable:domain dictionary
45     constraints is a list of constraints
46     """
47     self.variables = set(domains)
48     self.domains = domains
49     self.constraints = constraints
50     self.positions = positions
51     self.var_to_const = {var:set() for var in self.variables}
52     for con in constraints:
53         for var in con.scope:
54             self.var_to_const[var].add(con)
55
56 def __str__(self):
57     """string representation of CSP"""
58     return str(self.domains)
59
60 def __repr__(self):
61     """more detailed string representation of CSP"""
62     return "CSP("+str(self.domains)+", "+str([str(c) for c in self.constraints])+")"

```

`csp.consistent(assignment)` returns true if the assignment is consistent with each of the constraints in `csp` (i.e., all of the constraints that can be evaluated evaluate to true). Note that this is a local consistency with each constraint; it does *not* imply the CSP is consistent or has a solution.

```

cspProblem.py — (continued)
64 def consistent(self, assignment):
65     """assignment is a variable:value dictionary
66     returns True if all of the constraints that can be evaluated
67     evaluate to True given assignment.
68     """
69     return all(con.holds(assignment)
70               for con in self.constraints
71               if all(v in assignment for v in con.scope))

```

3.1.3 Examples

In the following code `ne_`, when given a number, returns a function that is true when its argument is not that number. For example, if $f = ne_ (3)$, then $f(2)$ is True and $f(3)$ is False. That is, $ne_ (x)(y)$ is true when $x \neq y$. Allowing a function of multiple arguments to use its arguments one at a time is called **currying**, after the logician Haskell Curry. Functions used as conditions in constraints require names (so they can be printed).

```

cspExamples.py — Example CSPs
11 from cspProblem import CSP, Constraint

```

```

12 from operator import lt,ne,eq,gt
13
14 def ne_(val):
15     """not equal value"""
16     # nev = lambda x: x != val # alternative definition
17     # nev = partial(neq,val) # another alternative definition
18     def nev(x):
19         return val != x
20     nev.__name__ = str(val)+"!=" # name of the function
21     return nev

```

Similarly $is_x(y)$ is true when $x = y$.

```

cspExamples.py — (continued)
23 def is_(val):
24     """is a value"""
25     # isv = lambda x: x == val # alternative definition
26     # isv = partial(eq,val) # another alternative definition
27     def isv(x):
28         return val == x
29     isv.__name__ = str(val)+"=="
30     return isv

```

The CSP, *csp0* has variables X , Y and Z , each with domain $\{1,2,3\}$. The constraints are $X < Y$ and $Y < Z$.

```

cspExamples.py — (continued)
32 csp0 = CSP({'X':{1,2,3}, 'Y':{1,2,3}, 'Z':{1,2,3}},
33           [ Constraint(['X','Y'],lt),
34             Constraint(['Y','Z'],lt)])

```

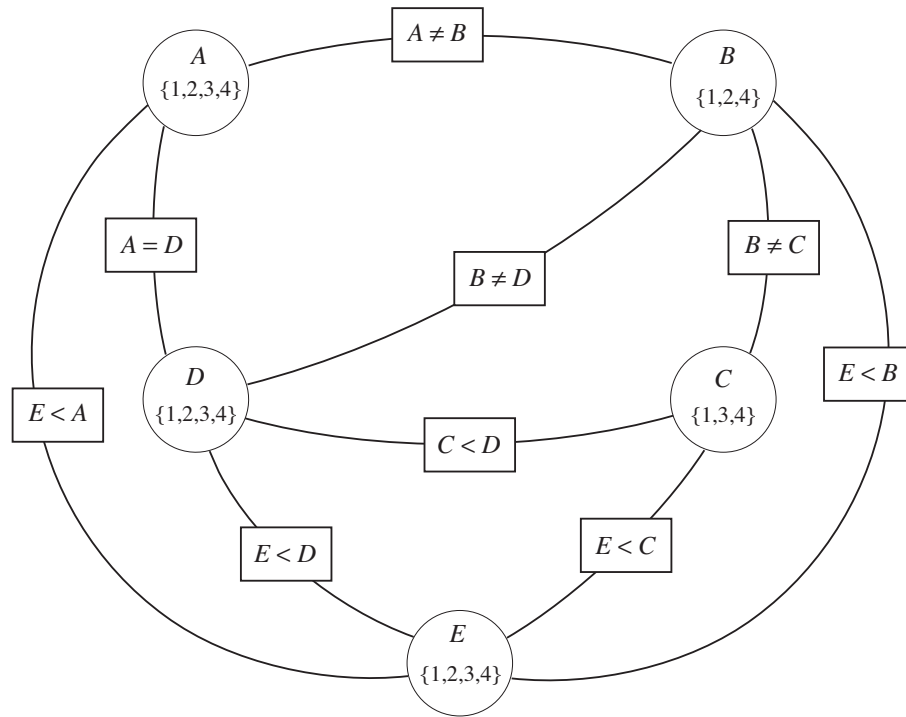
The CSP, *csp1* has variables A , B and C , each with domain $\{1,2,3,4\}$. The constraints are $A < B$, $B \neq 2$ and $B < C$. This is slightly more interesting than *csp0* as it has more solutions. This example is used in the unit tests, and so if it is changed, the unit tests need to be changed.

```

cspExamples.py — (continued)
36 C0 = Constraint(['A','B'], lt, "A < B")
37 C1 = Constraint(['B'], ne_(2), "B != 2")
38 C2 = Constraint(['B','C'], lt, "B < C")
39 csp1 = CSP({'A':{1,2,3,4}, 'B':{1,2,3,4}, 'C':{1,2,3,4}},
40           [C0, C1, C2],
41           positions={"A": (1, 0),
42                     "B": (3, 0),
43                     "C": (5, 0),
44                     "A < B": (2, 1),
45                     "B < C": (4, 1),
46                     "B != 2": (3, 2)})

```

The next CSP, *csp2* is Example 4.9 of the textbook; the domain consistent network (after applying the unary constraints) is shown in Figure 3.1.

Figure 3.1: Domain-consistent constraint network (*csp2*).

cspExamples.py — (continued)

```

48 csp2 = CSP({'A':{1,2,3,4}, 'B':{1,2,3,4}, 'C':{1,2,3,4},
49           'D':{1,2,3,4}, 'E':{1,2,3,4}},
50           [ Constraint(['B'], ne_(3), "B != 3"),
51             Constraint(['C'], ne_(2), "C != 2"),
52             Constraint(['A', 'B'], ne, "A != B"),
53             Constraint(['B', 'C'], ne, "A != C"),
54             Constraint(['C', 'D'], lt, "C < D"),
55             Constraint(['A', 'D'], eq, "A = D"),
56             Constraint(['A', 'E'], gt, "A > E"),
57             Constraint(['B', 'E'], gt, "B > E"),
58             Constraint(['C', 'E'], gt, "C > E"),
59             Constraint(['D', 'E'], gt, "D > E"),
60             Constraint(['B', 'D'], ne, "B != D")])

```

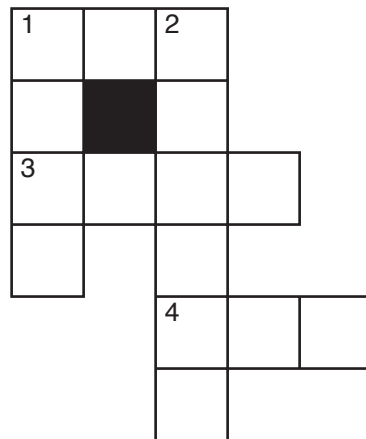
The following example is another scheduling problem (but with multiple answers). This is the same a scheduling 2 in the original AIspace.org consistency app.

cspExamples.py — (continued)

```

62 csp3 = CSP({'A':{1,2,3,4}, 'B':{1,2,3,4}, 'C':{1,2,3,4},
63           'D':{1,2,3,4}, 'E':{1,2,3,4}},
64           [Constraint(['A', 'B'], ne, "A != B"),
65             Constraint(['A', 'D'], lt, "A < D"),

```

**Words:**

ant, big, bus, car, has,
book, buys, hold, lane,
year, ginger, search,
symbol, syntax.

Figure 3.2: A crossword puzzle to be solved

```

66     Constraint(['A','E'], lambda a,e: (a-e)%2 == 1, "A-E is odd"), # A-E is odd
67     Constraint(['B','E'], lt, "B < E"),
68     Constraint(['D','C'], lt, "D < C"),
69     Constraint(['C','E'], ne, "C != E"),
70     Constraint(['D','E'], ne, "D != E"))

```

The following example is another abstract scheduling problem. What are the solutions?

```

cspExamples.py — (continued)
72 def adjacent(x,y):
73     """True when x and y are adjacent numbers"""
74     return abs(x-y) == 1
75
76 csp4 = CSP({'A':{1,2,3,4,5}, 'B':{1,2,3,4,5}, 'C':{1,2,3,4,5},
77           'D':{1,2,3,4,5}, 'E':{1,2,3,4,5}},
78           [Constraint(['A','B'], adjacent, "adjacent(A,B)"),
79             Constraint(['B','C'], adjacent, "adjacent(B,C)"),
80             Constraint(['C','D'], adjacent, "adjacent(C,D)"),
81             Constraint(['D','E'], adjacent, "adjacent(D,E)"),
82             Constraint(['A','C'], ne, "A != C"),
83             Constraint(['B','D'], ne, "A != D"),
84             Constraint(['C','E'], ne, "C != E")])

```

The following examples represent the crossword shown in Figure 3.2.

In the first representation, the variables represent words. The constraint imposed by the crossword is that where two words intersect, the letter at the intersection must be the same. The method `meet_at` is used to test whether two words intersect with the same letter. For example, the constraint `meet_at(2, 0)` means that the third letter (at position 2) of the first argument is the same as the first letter of the second argument.

cspExamples.py — (continued)


```

86 def meet_at(p1,p2):
87     """returns a function of two words that is true when the words intersect at postions p1, p2.
88     The positions are relative to the words; starting at position 0.
89     meet_at(p1,p2)(w1,w2) is true if the same letter is at position p1 of word w1
90     and at position p2 of word w2.
91     """
92     def meets(w1,w2):
93         return w1[p1] == w2[p2]
94     meets.__name__ = "meet_at("+str(p1)+','+str(p2)+')'
95     return meets
96
97 crossword1 = CSP({'one_across':{'ant', 'big', 'bus', 'car', 'has'},
98                  'one_down':{'book', 'buys', 'hold', 'lane', 'year'},
99                  'two_down':{'ginger', 'search', 'symbol', 'syntax'},
100                  'three_across':{'book', 'buys', 'hold', 'land', 'year'},
101                  'four_across':{'ant', 'big', 'bus', 'car', 'has'}},
102                  [Constraint(['one_across','one_down'], meet_at(0,0)),
103                  Constraint(['one_across','two_down'], meet_at(2,0)),
104                  Constraint(['three_across','two_down'], meet_at(2,2)),
105                  Constraint(['three_across','one_down'], meet_at(0,2)),
106                  Constraint(['four_across','two_down'], meet_at(0,4))])

```

In an alternative representation of a crossword (the “dual” representation), the variables represent letters, and the constraints are that adjacent sequences of letters form words.

```

cspExamples.py — (continued)
108 words = {'ant', 'big', 'bus', 'car', 'has', 'book', 'buys', 'hold',
109          'lane', 'year', 'ginger', 'search', 'symbol', 'syntax'}
110
111 def is_word(*letters, words=words):
112     """is true if the letters concatenated form a word in words"""
113     return "".join(letters) in words
114
115 letters = ["a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k", "l",
116           "m", "n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y",
117           "z"]
118
119 crossword1d = CSP({'p00':letters, 'p10':letters, 'p20':letters, # first row
120                  'p01':letters, 'p21':letters, # second row
121                  'p02':letters, 'p12':letters, 'p22':letters, 'p32':letters, # third row
122                  'p03':letters, 'p23':letters, #fourth row
123                  'p24':letters, 'p34':letters, 'p44':letters, # fifth row
124                  'p25':letters # sixth row
125                  },
126                  [Constraint(['p00', 'p10', 'p20'], is_word), #1-across
127                  Constraint(['p00', 'p01', 'p02', 'p03'], is_word), # 1-down
128                  Constraint(['p02', 'p12', 'p22', 'p32'], is_word), # 3-across
129                  Constraint(['p20', 'p21', 'p22', 'p23', 'p24', 'p25'], is_word), # 2-down
130                  Constraint(['p24', 'p34', 'p44'], is_word) # 4-across
131                  ])

```

Unit tests

The following defines a **unit test** for solvers, by default using example `csp1`.

```

133 def test(CSP_solver, csp=csp1,
134         solutions=[{'A': 1, 'B': 3, 'C': 4}, {'A': 2, 'B': 3, 'C': 4}]):
135     """CSP_solver is a solver that takes a csp and returns a solution
136     csp is a constraint satisfaction problem
137     solutions is the list of all solutions to csp
138     This tests whether the solution returned by CSP_solver is a solution.
139     """
140     print("Testing csp with",CSP_solver.__doc__)
141     sol0 = CSP_solver(csp)
142     print("Solution found:",sol0)
143     assert sol0 in solutions, "Solution not correct for "+str(csp)
144     print("Passed unit test")

```

Exercise 3.1 Modify *test* so that instead of taking in a list of solutions, it checks whether the returned solution actually is a solution.

Exercise 3.2 Propose a test that is appropriate for CSPs with no solutions. Assume that the test designer knows there are no solutions. Consider what a CSP solver should return if there are no solutions to the CSP.

Exercise 3.3 Write a unit test that checks whether all solutions (e.g., for the search algorithms that can return multiple solutions) are correct, and whether all solutions can be found.

3.2 Solving a CSP using Search

To run the demo, in folder "aipython.322", load "cspSearch.py", and copy and paste the example queries at the bottom of that file.

The first solver searches through the space of partial assignments. This takes in a CSP problem and an optional variable ordering, which is a list of the variables in the CSP. It then constructs a search space that can be solved using the search methods of the previous chapter. In this search space:

- A node is a *variable : value* dictionary which does not violate any constraints (so that dictionaries that violate any constraints are not added).
- An arc corresponds to an assignment of a value to the next variable. This assumes a static ordering; the next variable chosen to split does not depend on the context. If no variable ordering is given, this makes no attempt to choose a good ordering.

```

cspSearch.py — Representations of a Search Problem from a CSP.
11 from cspProblem import CSP, Constraint
12 from searchProblem import Arc, Search_problem
13 from utilities import dict_union
14
15 class Search_from_CSP(Search_problem):
16     """A search problem directly from the CSP.
17
18     A node is a variable:value dictionary"""
19     def __init__(self, csp, variable_order=None):
20         self.csp=csp
21         if variable_order:
22             assert set(variable_order) == set(csp.variables)
23             assert len(variable_order) == len(csp.variables)
24             self.variables = variable_order
25         else:
26             self.variables = list(csp.variables)
27
28     def is_goal(self, node):
29         """returns whether the current node is a goal for the search
30         """
31         return len(node)==len(self.csp.variables)
32
33     def start_node(self):
34         """returns the start node for the search
35         """
36         return {}

```

The *neighbors(node)* method uses the fact that the length of the node, which is the number of variables already assigned, is the index of the next variable to split on. Note that we do not need to check whether there are no more variables to split on, as the nodes are all consistent, by construction, and so when there are no more variables we have a solution, and so don't need the neighbours.

```

cspSearch.py — (continued)
38 def neighbors(self, node):
39     """returns a list of the neighboring nodes of node.
40     """
41     var = self.variables[len(node)] # the next variable
42     res = []
43     for val in self.csp.domains[var]:
44         new_env = dict_union(node,{var:val}) #dictionary union
45         if self.csp.consistent(new_env):
46             res.append(Arc(node,new_env))
47     return res

```

The unit tests relies on a solver. The following procedure creates a solver using search that can be tested.

```

cspSearch.py — (continued)
49 from cspExamples import csp1,csp2,test, crossword1, crossword1d

```

```

50 from searchGeneric import Searcher
51
52 def dfs_solver(csp):
53     """depth-first search solver"""
54     path = Searcher(Search_from_CSP(csp)).search()
55     if path is not None:
56         return path.end()
57     else:
58         return None
59
60 if __name__ == "__main__":
61     test(dfs_solver)
62
63 ## Test Solving CSPs with Search:
64 searcher1 = Searcher(Search_from_CSP(csp1))
65 #print(searcher1.search()) # get next solution
66 searcher2 = Searcher(Search_from_CSP(csp2))
67 #print(searcher2.search()) # get next solution
68 searcher3 = Searcher(Search_from_CSP(crossword1))
69 #print(searcher3.search()) # get next solution
70 searcher4 = Searcher(Search_from_CSP(crossword1d))
71 #print(searcher4.search()) # get next solution (warning: slow)

```

Exercise 3.4 What would happen if we constructed the new assignment by assigning $node[var] = val$ (with side effects) instead of using dictionary union? Give an example of where this could give a wrong answer. How could the algorithm be changed to work with side effects? (Hint: think about what information needs to be in a node).

Exercise 3.5 Change neighbors so that it returns an iterator of values rather than a list. (Hint: use *yield*.)

3.3 Consistency Algorithms

To run the demo, in folder "aipython.322", load "cspConsistency.py", and copy and paste the commented-out example queries at the bottom of that file.

A *Con_solver* is used to simplify a CSP using arc consistency.

```

_____cspConsistency.py — Arc Consistency and Domain splitting for solving a CSP_____
11 from display import Displayable
12
13 class Con_solver(Displayable):
14     """Solves a CSP with arc consistency and domain splitting
15     """
16     def __init__(self, csp, **kwargs):
17         """a CSP solver that uses arc consistency
18         * csp is the CSP to be solved

```

```

19         * kwargs is the keyword arguments for Displayable superclass
20         """
21         self.csp = csp
22         super().__init__(**kwargs) # Or Displayable.__init__(self,**kwargs)

```

The following implementation of arc consistency maintains the set *to_do* of (variable, constraint) pairs that are to be checked. It takes in a domain dictionary and returns a new domain dictionary. It needs to be careful to avoid side effects (by copying the *domains* dictionary and the *to_do* set).

```

cspConsistency.py — (continued)
24 def make_arc_consistent(self, orig_domains=None, to_do=None):
25     """Makes this CSP arc-consistent using generalized arc consistency
26     orig_domains is the original domains
27     to_do is a set of (variable,constraint) pairs
28     returns the reduced domains (an arc-consistent variable:domain dictionary)
29     """
30     if orig_domains is None:
31         orig_domains = self.csp.domains
32     if to_do is None:
33         to_do = {(var, const) for const in self.csp.constraints
34                 for var in const.scope}
35     else:
36         to_do = to_do.copy() # use a copy of to_do
37     domains = orig_domains.copy()
38     self.display(2, "Performing AC with domains", domains)
39     while to_do:
40         var, const = self.select_arc(to_do)
41         self.display(3, "Processing arc (", var, ",", const, ")")
42         other_vars = [ov for ov in const.scope if ov != var]
43         new_domain = {val for val in domains[var]
44                       if self.any_holds(domains, const, {var: val}, other_vars)}
45         if new_domain != domains[var]:
46             self.display(4, "Arc: (", var, ",", const, ") is inconsistent")
47             self.display(3, "Domain pruned", "dom(", var, ") =", new_domain,
48                           " due to ", const)
49             domains[var] = new_domain
50             add_to_do = self.new_to_do(var, const) - to_do
51             to_do |= add_to_do # set union
52             self.display(3, " adding", add_to_do if add_to_do else "nothing", "to to_do.")
53             self.display(4, "Arc: (", var, ",", const, ") now consistent")
54         self.display(2, "AC done. Reduced domains", domains)
55     return domains
56
57 def new_to_do(self, var, const):
58     """returns new elements to be added to to_do after assigning
59     variable var in constraint const.
60     """
61     return {(nvar, nconst) for nconst in self.csp.var_to_const[var]
62             if nconst != const
63             for nvar in nconst.scope}

```

```
64 |         if nvar != var}
```

The following selects an arc. Any element of *to_do* can be selected. The selected element needs to be removed from *to_do*. The default implementation just selects which ever element *pop* method for sets returns. A user interface could allow the user to select an arc. Alternatively a more sophisticated selection could be employed (or just a stack or a queue).

```

cspConsistency.py — (continued)
66 |     def select_arc(self, to_do):
67 |         """Selects the arc to be taken from to_do .
68 |         * to_do is a set of arcs, where an arc is a (variable,constraint) pair
69 |         the element selected must be removed from to_do.
70 |         """
71 |         return to_do.pop()
```

The value of *new_domain* is the subset of the domain of *var* that is consistent with the assignment to the other variables. It might be easier to understand the following code, which treats unary (with no other variables in the constraint) and binary (with one other variables in the constraint) constraints as special cases (this can replace the assignment to *new_domain* in the above code):

```

        if len(other_vars)==0:          # unary constraint
            new_domain = {val for val in domains[var]
                           if const.holds({var:val})}
        elif len(other_vars)==1:        # binary constraint
            other = other_vars[0]
            new_domain = {val for val in domains[var]
                           if any(const.holds({var: val,other:other_val})
                                  for other_val in domains[other])}
        else:                            # general case
            new_domain = {val for val in domains[var]
                           if self.any_holds(domains, const, {var: val}, other_vars)}
```

any_holds is a recursive function that tries to find an assignment of values to the other variables (*other_vars*) that satisfies constraint *const* given the assignment in *env*. The integer variable *ind* specifies which index to *other_vars* needs to be checked next. As soon as one assignment returns *True*, the algorithm returns *True*. Note that it has side effects with respect to *env*; it changes the values of the variables in *other_vars*. It should only be called when the side effects have no ill effects.

```

cspConsistency.py — (continued)
73 |     def any_holds(self, domains, const, env, other_vars, ind=0):
74 |         """returns True if Constraint const holds for an assignment
75 |         that extends env with the variables in other_vars[ind:]
76 |         env is a dictionary
77 |         Warning: this has side effects and changes the elements of env
78 |         """
```

```

79         if ind == len(other_vars):
80             return const.holds(env)
81         else:
82             var = other_vars[ind]
83             for val in domains[var]:
84                 # env = dict_union(env,{var:val}) # no side effects!
85                 env[var] = val
86                 if self.any_holds(domains, const, env, other_vars, ind + 1):
87                     return True
88             return False

```

3.3.1 Direct Implementation of Domain Splitting

The following is a direct implementation of domain splitting with arc consistency that uses recursion. It finds one solution if one exists or returns False if there are no solutions.

```

_____cspConsistency.py — (continued)_____
90 def solve_one(self, domains=None, to_do=None):
91     """return a solution to the current CSP or False if there are no solutions
92     to_do is the list of arcs to check
93     """
94     if domains is None:
95         domains = self.csp.domains
96     new_domains = self.make_arc_consistent(domains, to_do)
97     if any(len(new_domains[var]) == 0 for var in domains):
98         return False
99     elif all(len(new_domains[var]) == 1 for var in domains):
100         self.display(2, "solution:", {var: select(
101             new_domains[var]) for var in new_domains})
102         return {var: select(new_domains[var]) for var in domains}
103     else:
104         var = self.select_var(x for x in self.csp.variables if len(new_domains[x]) > 1)
105         if var:
106             dom1, dom2 = partition_domain(new_domains[var])
107             self.display(3, "...splitting", var, "into", dom1, "and", dom2)
108             new_doms1 = copy_with_assign(new_domains, var, dom1)
109             new_doms2 = copy_with_assign(new_domains, var, dom2)
110             to_do = self.new_to_do(var, None)
111             self.display(3, " adding", to_do if to_do else "nothing", "to to_do.")
112             return self.solve_one(new_doms1, to_do) or self.solve_one(new_doms2, to_do)
113
114     def select_var(self, iter_vars):
115         """return the next variable to split"""
116         return select(iter_vars)
117
118     def partition_domain(dom):
119         """partitions domain dom into two.
120         """
121         split = len(dom) // 2

```

```

122     dom1 = set(list(dom)[:split])
123     dom2 = dom - dom1
124     return dom1, dom2

```

The domains are implemented as a dictionary that maps each variables to its domain. Assigning a value in Python has side effects which we want to avoid. *copy_with_assign* takes a copy of the domains dictionary, perhaps allowing for a new domain for a variable. It creates a copy of the CSP with an (optional) assignment of a new domain to a variable. Only the domains are copied.

```

_____cspConsistency.py — (continued)_____
126 def copy_with_assign(domains, var=None, new_domain={True, False}):
127     """create a copy of the domains with an assignment var=new_domain
128     if var==None then it is just a copy.
129     """
130     newdoms = domains.copy()
131     if var is not None:
132         newdoms[var] = new_domain
133     return newdoms

```

```

_____cspConsistency.py — (continued)_____
135 def select(iterable):
136     """select an element of iterable. Returns None if there is no such element.
137
138     This implementation just picks the first element.
139     For many of the uses, which element is selected does not affect correctness,
140     but may affect efficiency.
141     """
142     for e in iterable:
143         return e # returns first element found

```

Exercise 3.6 Implement *solve_all* that is like *solve_one* but returns the set of all solutions.

Exercise 3.7 Implement *solve_enum* that enumerates the solutions. It should use Python's *yield* (and perhaps *yield from*).

Unit test:

```

_____cspConsistency.py — (continued)_____
145 from cspExamples import test
146 def ac_solver(csp):
147     "arc consistency (solve_one)"
148     return Con_solver(csp).solve_one()
149 if __name__ == "__main__":
150     test(ac_solver)

```


3.3.2 Domain Splitting as an interface to graph searching

An alternative implementation is to implement domain splitting in terms of the search abstraction of Chapter 2.

A node is domains dictionary.

```

cspConsistency.py — (continued)
152 from searchProblem import Arc, Search_problem
153
154 class Search_with_AC_from_CSP(Search_problem, Displayable):
155     """A search problem with arc consistency and domain splitting
156
157     A node is a CSP """
158     def __init__(self, csp):
159         self.cons = Con_solver(csp) #copy of the CSP
160         self.domains = self.cons.make_arc_consistent()
161
162     def is_goal(self, node):
163         """node is a goal if all domains have 1 element"""
164         return all(len(node[var])==1 for var in node)
165
166     def start_node(self):
167         return self.domains
168
169     def neighbors(self, node):
170         """returns the neighboring nodes of node.
171         """
172         neighs = []
173         var = select(x for x in node if len(node[x])>1)
174         if var:
175             dom1, dom2 = partition_domain(node[var])
176             self.display(2, "Splitting", var, "into", dom1, "and", dom2)
177             to_do = self.cons.new_to_do(var, None)
178             for dom in [dom1, dom2]:
179                 newdoms = copy_with_assign(node, var, dom)
180                 cons_doms = self.cons.make_arc_consistent(newdoms, to_do)
181                 if all(len(cons_doms[v])>0 for v in cons_doms):
182                     # all domains are non-empty
183                     neighs.append(Arc(node, cons_doms))
184             else:
185                 self.display(2, "...", var, "in", dom, "has no solution")
186         return neighs

```

Exercise 3.8 When splitting a domain, this code splits the domain into half, approximately in half (without any effort to make a sensible choice). Does it work better to split one element from a domain?

Unit test:

```

cspConsistency.py — (continued)
188 from cspExamples import test

```

```

189 from searchGeneric import Searcher
190
191 def ac_search_solver(csp):
192     """arc consistency (search interface)"""
193     sol = Searcher(Search_with_AC_from_CSP(csp)).search()
194     if sol:
195         return {v:select(d) for (v,d) in sol.end().items()}
196
197 if __name__ == "__main__":
198     test(ac_search_solver)

```

Testing:

```

_____cspConsistency.py — (continued)_____
200 from cspExamples import csp1, csp2, csp3, csp4, crossword1, crossword1d
201
202 ## Test Solving CSPs with Arc consistency and domain splitting:
203 #Con_solver.max_display_level = 4 # display details of AC (0 turns off)
204 #Con_solver(csp1).solve_one()
205 #searcher1d = Searcher(Search_with_AC_from_CSP(csp1))
206 #print(searcher1d.search())
207 #Searcher.max_display_level = 2 # display search trace (0 turns off)
208 #searcher2c = Searcher(Search_with_AC_from_CSP(csp2))
209 #print(searcher2c.search())
210 #searcher3c = Searcher(Search_with_AC_from_CSP(crossword1))
211 #print(searcher3c.search())
212 #searcher5c = Searcher(Search_with_AC_from_CSP(crossword1d))
213 #print(searcher5c.search())

```

3.4 Solving CSPs using Stochastic Local Search

To run the demo, in folder "aipython_322", load "cspSLS.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3. Some of the queries require matplotlib.

This implements both the two-stage choice, the any-conflict algorithm and a random choice of variable (and a probabilistic mix of the three).

Given a CSP, the stochastic local searcher (*SLSearcher*) creates the data structures:

- *variables_to_select* is the set of all of the variables with domain-size greater than one. For a variable not in this set, we cannot pick another value from that variable.
- *var_to_constraints* maps from a variable into the set of constraints it is involved in. Note that the inverse mapping from constraints into variables is part of the definition of a constraint.

```

cspSLS.py — Stochastic Local Search for Solving CSPs
11 from cspProblem import CSP, Constraint
12 from searchProblem import Arc, Search_problem
13 from display import Displayable
14 import random
15 import heapq
16
17 class SLSearcher(Displayable):
18     """A search problem directly from the CSP..
19
20     A node is a variable:value dictionary"""
21     def __init__(self, csp):
22         self.csp = csp
23         self.variables_to_select = {var for var in self.csp.variables
24                                     if len(self.csp.domains[var]) > 1}
25         # Create assignment and conflicts set
26         self.current_assignment = None # this will trigger a random restart
27         self.number_of_steps = 0 #number of steps after the initialization

```

restart creates a new total assignment, and constructs the set of conflicts (the constraints that are false in this assignment).

```

cspSLS.py — (continued)
29 def restart(self):
30     """creates a new total assignment and the conflict set
31     """
32     self.current_assignment = {var:random_sample(dom) for
33                               (var,dom) in self.csp.domains.items()}
34     self.display(2,"Initial assignment",self.current_assignment)
35     self.conflicts = set()
36     for con in self.csp.constraints:
37         if not con.holds(self.current_assignment):
38             self.conflicts.add(con)
39     self.display(2,"Number of conflicts",len(self.conflicts))
40     self.variable_pq = None

```

The *search* method is the top-level searching algorithm. It can either be used to start the search or to continue searching. If there is no current assignment, it must create one. Note that, when counting steps, a restart is counted as one step.

This method selects one of two implementations. The argument *pob_best* is the probability of selecting a best variable (one involving the most conflicts). When the value of *prob_best* is positive, the algorithm needs to maintain a priority queue of variables and the number of conflicts (using *search_with_var_pq*). If the probability of selecting a best variable is zero, it does not need to maintain this priority queue (as implemented in *search_with_any_conflict*).

The argument *prob_anycon* is the probability that the any-conflict strategy is used (which selects a variable at random that is in a conflict), assuming that it is not picking a best variable. Note that for the probability parameters, any value less than zero acts like probability zero and any value greater than 1 acts

like probability 1. This means that when *prob_anycon* = 1.0, a best variable is chosen with probability *prob_best*, otherwise a variable in any conflict is chosen. A variable is chosen at random with probability $1 - \text{prob_anycon} - \text{prob_best}$ as long as that is positive.

This returns the number of steps needed to find a solution, or *None* if no solution is found. If there is a solution, it is in *self.current_assignment*.

```

cspSLS.py — (continued)
42 def search(self,max_steps, prob_best=0, prob_anycon=1.0):
43     """
44     returns the number of steps or None if there is no solution.
45     If there is a solution, it can be found in self.current_assignment
46
47     max_steps is the maximum number of steps it will try before giving up
48     prob_best is the probability that a best variable (one in most conflict) is selected
49     prob_anycon is the probability that a variable in any conflict is selected
50     (otherwise a variable is chosen at random)
51     """
52     if self.current_assignment is None:
53         self.restart()
54         self.number_of_steps += 1
55         if not self.conflicts:
56             self.display(1,"Solution found:", self.current_assignment, "after restart")
57             return self.number_of_steps
58     if prob_best > 0: # we need to maintain a variable priority queue
59         return self.search_with_var_pq(max_steps, prob_best, prob_anycon)
60     else:
61         return self.search_with_any_conflict(max_steps, prob_anycon)

```

Exercise 3.9 This does an initial random assignment but does not do any random restarts. Implement a searcher that takes in the maximum number of walk steps (corresponding to existing *max_steps*) and the maximum number of restarts, and returns the total number of steps for the first solution found. (As in *search*, the solution found can be extracted from the variable *self.current_assignment*).

3.4.1 Any-conflict

If the probability of picking a best variable is zero, the implementation need to keep track of which variables are in conflicts.

```

cspSLS.py — (continued)
63 def search_with_any_conflict(self, max_steps, prob_anycon=1.0):
64     """Searches with the any_conflict heuristic.
65     This relies on just maintaining the set of conflicts;
66     it does not maintain a priority queue
67     """
68     self.variable_pq = None # we are not maintaining the priority queue.
69                             # This ensures it is regenerated if
70                             # we call search_with_var_pq.

```

```

71     for i in range(max_steps):
72         self.number_of_steps += 1
73         if random.random() < probab_anycon:
74             con = random_sample(self.conflicts) # pick random conflict
75             var = random_sample(con.scope) # pick variable in conflict
76         else:
77             var = random_sample(self.variables_to_select)
78         if len(self.csp.domains[var]) > 1:
79             val = random_sample(self.csp.domains[var] -
80                               {self.current_assignment[var]})
81             self.display(2, self.number_of_steps, ": Assigning", var, "=", val)
82             self.current_assignment[var]=val
83             for varcon in self.csp.var_to_const[var]:
84                 if varcon.holds(self.current_assignment):
85                     if varcon in self.conflicts:
86                         self.conflicts.remove(varcon)
87                 else:
88                     if varcon not in self.conflicts:
89                         self.conflicts.add(varcon)
90             self.display(2, "    Number of conflicts", len(self.conflicts))
91         if not self.conflicts:
92             self.display(1, "Solution found:", self.current_assignment,
93                         "in", self.number_of_steps, "steps")
94             return self.number_of_steps
95         self.display(1, "No solution in", self.number_of_steps, "steps",
96                     len(self.conflicts), "conflicts remain")
97     return None

```

Exercise 3.10 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value that reduces the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

3.4.2 Two-Stage Choice

This is the top-level searching algorithm that maintains a priority queue of variables ordered by (the negative of) the number of conflicts, so that the variable with the most conflicts is selected first. If there is no current priority queue of variables, one is created.

The main complexity here is to maintain the priority queue. This uses the dictionary *var_differential* which specifies how much the values of variables should change. This is used with the updatable queue (page 55) to find a variable with the most conflicts.

```

cspSLS.py — (continued)
99     def search_with_var_pq(self, max_steps, probab_best=1.0, probab_anycon=1.0):
100         """search with a priority queue of variables.
101         This is used to select a variable with the most conflicts.
102         """

```

```

103     if not self.variable_pq:
104         self.create_pq()
105     pick_best_or_con = probabest + probabanycon
106     for i in range(max_steps):
107         self.number_of_steps += 1
108         randnum = random.random()
109         ## Pick a variable
110         if randnum < probabest: # pick best variable
111             var,oldval = self.variable_pq.top()
112         elif randnum < pick_best_or_con: # pick a variable in a conflict
113             con = random_sample(self.conflicts)
114             var = random_sample(con.scope)
115         else: #pick any variable that can be selected
116             var = random_sample(self.variables_to_select)
117         if len(self.csp.domains[var]) > 1: # var has other values
118             ## Pick a value
119             val = random_sample(self.csp.domains[var] -
120                               {self.current_assignment[var]})
121             self.display(2,"Assigning",var,val)
122             ## Update the priority queue
123             var_differential = {}
124             self.current_assignment[var]=val
125             for varcon in self.csp.var_to_const[var]:
126                 self.display(3,"Checking",varcon)
127                 if varcon.holds(self.current_assignment):
128                     if varcon in self.conflicts: #was incons, now consis
129                         self.display(3,"Became consistent",varcon)
130                         self.conflicts.remove(varcon)
131                         for v in varcon.scope: # v is in one fewer conflicts
132                             var_differential[v] = var_differential.get(v,0)-1
133                 else:
134                     if varcon not in self.conflicts: # was consis, not now
135                         self.display(3,"Became inconsistent",varcon)
136                         self.conflicts.add(varcon)
137                         for v in varcon.scope: # v is in one more conflicts
138                             var_differential[v] = var_differential.get(v,0)+1
139             self.variable_pq.update_each_priority(var_differential)
140             self.display(2,"Number of conflicts",len(self.conflicts))
141         if not self.conflicts: # no conflicts, so solution found
142             self.display(1,"Solution found:", self.current_assignment,"in",
143                         self.number_of_steps,"steps")
144             return self.number_of_steps
145     self.display(1,"No solution in",self.number_of_steps,"steps",
146                 len(self.conflicts),"conflicts remain")
147     return None

```

create_pq creates an updatable priority queue of the variables, ordered by the number of conflicts they participate in. The priority queue only includes variables in conflicts and the value of a variable is the *negative* of the number of conflicts the variable is in. This ensures that the priority queue, which picks

the minimum value, picks a variable with the most conflicts.

```

_____cspSLS.py — (continued) _____
149     def create_pq(self):
150         """Create the variable to number-of-conflicts priority queue.
151         This is needed to select the variable in the most conflicts.
152
153         The value of a variable in the priority queue is the negative of the
154         number of conflicts the variable appears in.
155         """
156         self.variable_pq = Updatable_priority_queue()
157         var_to_number_conflicts = {}
158         for con in self.conflicts:
159             for var in con.scope:
160                 var_to_number_conflicts[var] = var_to_number_conflicts.get(var,0)+1
161         for var,num in var_to_number_conflicts.items():
162             if num>0:
163                 self.variable_pq.add(var,-num)

_____cspSLS.py — (continued) _____
165     def random_sample(st):
166         """selects a random element from set st"""
167         return random.sample(st,1)[0]
```

Exercise 3.11 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value that reduces the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

Exercise 3.12 These implementations always select a value for the variable selected that is different from its current value (if that is possible). Change the code so that it does not have this restriction (so it can leave the value the same). Would you expect this code to be faster? Does it work worse (or better)?

3.4.3 Updatable Priority Queues

An **updatable priority queue** is a priority queue, where key-value pairs can be stored, and the pair with the smallest key can be found and removed quickly, and where the values can be updated. This implementation follows the idea of <http://docs.python.org/3.5/library/heapq.html>, where the updated elements are marked as removed. This means that the priority queue can be used unmodified. However, this might be expensive if changes are more common than popping (as might happen if the probability of choosing the best is close to zero).

In this implementation, the equal values are sorted randomly. This is achieved by having the elements of the heap being $[val, rand, elt]$ triples, where the second element is a random number. Note that Python requires this to be a list, not a tuple, as the tuple cannot be modified.

cspSLS.py — (continued)

```

169 class Updatable_priority_queue(object):
170     """A priority queue where the values can be updated.
171     Elements with the same value are ordered randomly.
172
173     This code is based on the ideas described in
174     http://docs.python.org/3.3/library/heapq.html
175     It could probably be done more efficiently by
176     shuffling the modified element in the heap.
177     """
178     def __init__(self):
179         self.pq = [] # priority queue of [val,rand,elt] triples
180         self.elt_map = {} # map from elt to [val,rand,elt] triple in pq
181         self.REMOVED = "*removed*" # a string that won't be a legal element
182         self.max_size=0
183
184     def add(self,elt,val):
185         """adds elt to the priority queue with priority=val.
186         """
187         assert val <= 0, val
188         assert elt not in self.elt_map, elt
189         new_triple = [val, random.random(), elt]
190         heapq.heappush(self.pq, new_triple)
191         self.elt_map[elt] = new_triple
192
193     def remove(self,elt):
194         """remove the element from the priority queue"""
195         if elt in self.elt_map:
196             self.elt_map[elt][2] = self.REMOVED
197             del self.elt_map[elt]
198
199     def update_each_priority(self,update_dict):
200         """update values in the priority queue by subtracting the values in
201         update_dict from the priority of those elements in priority queue.
202         """
203         for elt,incr in update_dict.items():
204             if incr != 0:
205                 newval = self.elt_map.get(elt,[0])[0] - incr
206                 assert newval <= 0, str(elt)+": "+str(newval+incr)+"-"+str(incr)
207                 self.remove(elt)
208                 if newval != 0:
209                     self.add(elt,newval)
210
211     def pop(self):
212         """Removes and returns the (elt,value) pair with minimal value.
213         If the priority queue is empty, IndexError is raised.
214         """
215         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
216         triple = heapq.heappop(self.pq)
217         while triple[2] == self.REMOVED:

```



```

218         triple = heapq.heappop(self.pq)
219         del self.elts_map[triple[2]]
220         return triple[2], triple[0] # elt, value
221
222     def top(self):
223         """Returns the (elt,value) pair with minimal value, without removing it.
224         If the priority queue is empty, IndexError is raised.
225         """
226         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
227         triple = self.pq[0]
228         while triple[2] == self.REMOVED:
229             heapq.heappop(self.pq)
230             triple = self.pq[0]
231         return triple[2], triple[0] # elt, value
232
233     def empty(self):
234         """returns True iff the priority queue is empty"""
235         return all(triple[2] == self.REMOVED for triple in self.pq)

```

3.4.4 Plotting Runtime Distributions

Runtime_distribution uses matplotlib to plot runtime distributions. Here the runtime is a misnomer as we are only plotting the number of steps, not the time. Computing the runtime is non-trivial as many of the runs have a very short runtime. To compute the time accurately would require running the same code, with the same random seed, multiple times to get a good estimate of the runtime. This is left as an exercise.

```

cspSLS.py — (continued)
237 import matplotlib.pyplot as plt
238
239 class Runtime_distribution(object):
240     def __init__(self, csp, xscale='log'):
241         """Sets up plotting for csp
242         xscale is either 'linear' or 'log'
243         """
244         self.csp = csp
245         plt.ion()
246         plt.xlabel("Number of Steps")
247         plt.ylabel("Cumulative Number of Runs")
248         plt.xscale(xscale) # Makes a 'log' or 'linear' scale
249
250     def plot_runs(self, num_runs=100, max_steps=1000, prob_best=1.0, prob_anycon=1.0):
251         """Plots num_runs of SLS for the given settings.
252         """
253         stats = []
254         SLSearcher.max_display_level, temp_mdl = 0, SLSearcher.max_display_level # no display
255         for i in range(num_runs):
256             searcher = SLSearcher(self.csp)

```

```

257         num_steps = searcher.search(max_steps, prob_best, prob_anycon)
258         if num_steps:
259             stats.append(num_steps)
260     stats.sort()
261     if prob_best >= 1.0:
262         label = "P(best)=1.0"
263     else:
264         p_ac = min(prob_anycon, 1-prob_best)
265         label = "P(best)=%.2f, P(ac)=%.2f" % (prob_best, p_ac)
266     plt.plot(stats, range(len(stats)), label=label)
267     plt.legend(loc="upper left")
268     #plt.draw()
269     SLSearcher.max_display_level= temp_mdl #restore display

```

3.4.5 Testing

```

_____cspSLS.py — (continued)_____
271 from cspExamples import test
272 def sls_solver(csp, prob_best=0.7):
273     """stochastic local searcher (prob_best=0.7)"""
274     se0 = SLSearcher(csp)
275     se0.search(1000, prob_best)
276     return se0.current_assignment
277 def any_conflict_solver(csp):
278     """stochastic local searcher (any-conflict)"""
279     return sls_solver(csp, 0)
280
281 if __name__ == "__main__":
282     test(sls_solver)
283     test(any_conflict_solver)
284
285 from cspExamples import csp1, csp2, crossword1
286
287 ## Test Solving CSPs with Search:
288 #se1 = SLSearcher(csp1); print(se1.search(100))
289 #se2 = SLSearcher(csp2); print(se2.search(1000, 1.0)) # greedy
290 #se2 = SLSearcher(csp2); print(se2.search(1000, 0)) # any_conflict
291 #se2 = SLSearcher(csp2); print(se2.search(1000, 0.7)) # 70% greedy; 30% any_conflict
292 #SLSearcher.max_display_level=2 #more detailed display
293 #se3 = SLSearcher(crossword1); print(se3.search(100), 0.7)
294 #p = Runtime_distribution(csp2)
295 #p.plot_runs(1000, 1000, 0) # any_conflict
296 #p.plot_runs(1000, 1000, 1.0) # greedy
297 #p.plot_runs(1000, 1000, 0.7) # 70% greedy; 30% any_conflict

```

Exercise 3.13 Modify this to plot the runtime, instead of the number of steps. To measure runtime use *timeit* (<https://docs.python.org/3.5/library/timeit.html>). Small runtimes are inaccurate, so *timeit* can run the same code multiple times. Stochastic local algorithms give different runtimes each time called.

To make the timing meaningful, you need to make sure the random seed is the same for each repeated call (see `random.getstate` and `random.setstate` in <https://docs.python.org/3.5/library/random.html>). Because the runtime for different seeds can vary a great deal, for each seed, you should start with 1 iteration and multiplying it by, say 10, until the time is greater than 0.2 seconds. Make sure you plot the average time for each run. Before you start, try to estimate the total runtime, so you will be able to tell if there is a problem with the algorithm stopping.

Chapter 4

Planning with Certainty

4.1 Representing Actions and Planning Problems

The STRIPS representation of an action consists of:

- the name of the action
- preconditions: a dictionary of *feature:value* pairs that specifies that the feature must have this value for the action to be possible
- effects: a dictionary of *feature:value* pairs that are made true by this action. In particular, a feature in the dictionary has the corresponding value (and not its previous value) after the action, and a feature not in the dictionary keeps its old value.

```
stripsProblem.py — STRIPS Representations of Actions
11 class Strips(object):
12     def __init__(self, name, preconds, effects, cost=1):
13         """
14         defines the STRIPS representation for an action:
15         * name is the name of the action
16         * preconds, the preconditions, is feature:value dictionary that must hold
17         for the action to be carried out
18         * effects is a feature:value map that this action makes
19         true. The action changes the value of any feature specified
20         here, and leaves other features unchanged.
21         * cost is the cost of the action
22         """
23         self.name = name
```

```

24         self.preconds = preconds
25         self.effects = effects
26         self.cost = cost
27
28     def __repr__(self):
29         return self.name

```

A STRIPS domain consists of:

- A set of actions.
- A dictionary that maps each feature into a set of possible values for the feature.
- A list of the actions

```

_____stripsProblem.py — (continued)_____
31 class STRIPS_domain(object):
32     def __init__(self, feats_vals, actions):
33         """Problem domain
34         feats_vals is a feature:domain dictionary,
35         mapping each feature to its domain
36         actions
37         """
38         self.feats_vals = feats_vals
39         self.actions = actions

```

A planning problem consists of a planning domain, an initial state, and a goal. The goal does not need to fully specify the final state.

```

_____stripsProblem.py — (continued)_____
41 class Planning_problem(object):
42     def __init__(self, prob_domain, initial_state, goal):
43         """
44         a planning problem consists of
45         * a planning domain
46         * the initial state
47         * a goal
48         """
49         self.prob_domain = prob_domain
50         self.initial_state = initial_state
51         self.goal = goal

```

4.1.1 Robot Delivery Domain

The following specifies the robot delivery domain of Section 6.1, shown in Figure 4.1.

```

_____stripsProblem.py — (continued)_____
53 boolean = {True, False}

```

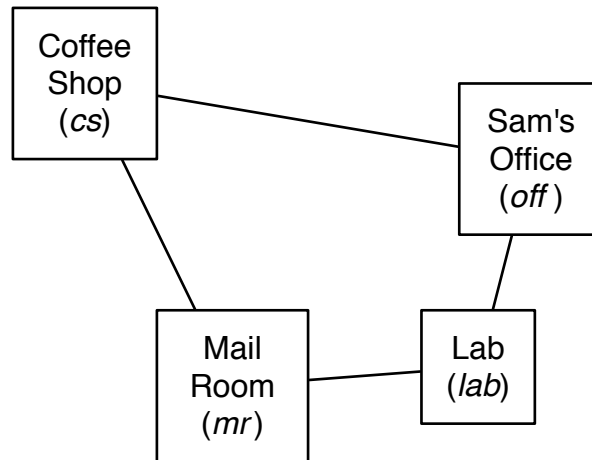
**Features to describe states***RLoc* – Rob's location*RHC* – Rob has coffee*SWC* – Sam wants coffee*MW* – Mail is waiting*RHM* – Rob has mail**Actions***mc* – move clockwise*mcc* – move counterclockwise*puc* – pickup coffee*dc* – deliver coffee*pum* – pickup mail*dm* – deliver mail

Figure 4.1: Robot Delivery Domain

```

54 delivery_domain = STRIPS_domain(
55     {'RLoc':{'cs', 'off', 'lab', 'mr'}, 'RHC':boolean, 'SWC':boolean,
56      'MW':boolean, 'RHM':boolean}, #feature:values dictionary
57     { Strips('mc_cs', {'RLoc':'cs'}, {'RLoc':'off'}),
58       Strips('mc_off', {'RLoc':'off'}, {'RLoc':'lab'}),
59       Strips('mc_lab', {'RLoc':'lab'}, {'RLoc':'mr'}),
60       Strips('mc_mr', {'RLoc':'mr'}, {'RLoc':'cs'}),
61       Strips('mcc_cs', {'RLoc':'cs'}, {'RLoc':'mr'}),
62       Strips('mcc_off', {'RLoc':'off'}, {'RLoc':'cs'}),
63       Strips('mcc_lab', {'RLoc':'lab'}, {'RLoc':'off'}),
64       Strips('mcc_mr', {'RLoc':'mr'}, {'RLoc':'lab'}),
65       Strips('puc', {'RLoc':'cs', 'RHC':False}, {'RHC':True}),
66       Strips('dc', {'RLoc':'off', 'RHC':True}, {'RHC':False, 'SWC':False}),
67       Strips('pum', {'RLoc':'mr', 'MW':True}, {'RHM':True, 'MW':False}),
68       Strips('dm', {'RLoc':'off', 'RHM':True}, {'RHM':False})
69     } )

```

stripsProblem.py — (continued)

```

71 problem0 = Planning_problem(delivery_domain,
72     {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,

```

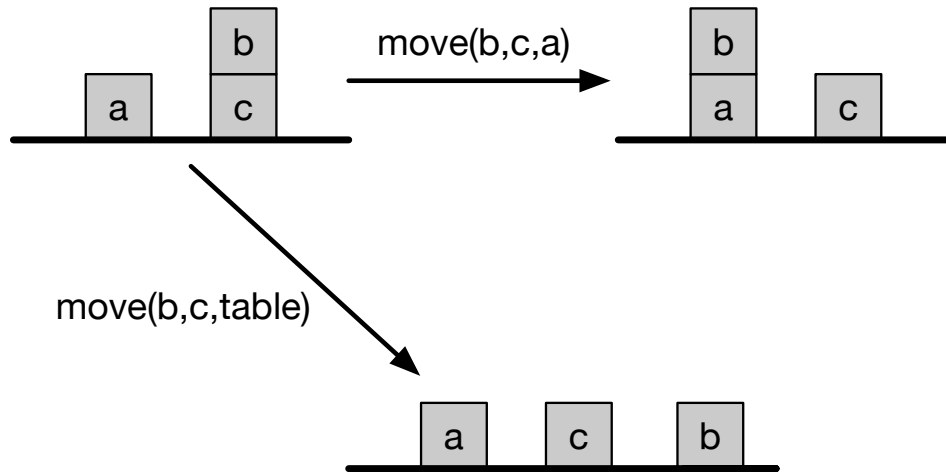


Figure 4.2: Blocks world with two actions

```

73         'RHM':False},
74         {'RLoc':'off'})
75 problem1 = Planning_problem(delivery_domain,
76                             {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
77                             'RHM':False},
78                             {'SWC':False})
79 problem2 = Planning_problem(delivery_domain,
80                             {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
81                             'RHM':False},
82                             {'SWC':False, 'MW':False, 'RHM':False})

```

4.1.2 Blocks World

The blocks world consist of blocks and a table. Each block can be on the table or on another block. A block can only have one other block on top of it. Figure 4.2 shows 3 states with some of the actions between them.

A state is defined by the two features:

- *on* where $on(x) = y$ when block x is on block or table y
- *clear* where $clear(x) = True$ when block x has nothing on it.

There is one parameterized action

- $move(x, y, z)$ move block x from y to z , where y and z could be a block or the table.

To handle parameterized actions (which depend on the blocks involved), the actions and the features are all strings, created for the all combinations of the

blocks. Note that we treat moving to a block separately from moving to the table, because the blocks needs to be clear, but the table always has room for another block.

```

stripsProblem.py — (continued)
84 ### blocks world
85 def move(x,y,z):
86     """string for the 'move' action"""
87     return 'move_'+x+'_from_'+y+'_to_'+z
88 def on(x):
89     """string for the 'on' feature"""
90     return x+'_is_on'
91 def clear(x):
92     """string for the 'clear' feature"""
93     return 'clear_'+x
94 def create_blocks_world(blocks = {'a','b','c','d'}):
95     blocks_and_table = blocks | {'table'}
96     stmap = {Strips(move(x,y,z),{on(x):y, clear(x):True, clear(z):True},
97                     {on(x):z, clear(y):True, clear(z):False})
98             for x in blocks
99             for y in blocks_and_table
100            for z in blocks
101            if x!=y and y!=z and z!=x}
102     stmap.update({Strips(move(x,y,'table'), {on(x):y, clear(x):True},
103                     {on(x):'table', clear(y):True})
104                 for x in blocks
105                 for y in blocks
106                 if x!=y})
107     feats_vals = {on(x):blocks_and_table-{x} for x in blocks}
108     feats_vals.update({clear(x):boolean for x in blocks_and_table})
109     return STRIPS_domain(feats_vals, stmap)

```

The problem *blocks1* is a classic example, with 3 blocks, and the goal consists of two conditions. See Figure 4.3. Note that this example is challenging because we can't achieve one of the goals and then the other; whichever one we achieve first has to be undone to achieve the second.

```

stripsProblem.py — (continued)
111 blocks1dom = create_blocks_world({'a','b','c'})
112 blocks1 = Planning_problem(blocks1dom,
113     {on('a'):'table', clear('a'):True,
114     on('b'):'c', clear('b'):True,
115     on('c'):'table', clear('c'):False}, # initial state
116     {on('a'):'b', on('c'):'a'}) #goal

```

The problem *blocks2* is one to invert a tower of size 4.

```

stripsProblem.py — (continued)
118 blocks2dom = create_blocks_world({'a','b','c','d'})
119 tower4 = {clear('a'):True, on('a'):'b',
120           clear('b'):False, on('b'):'c',

```

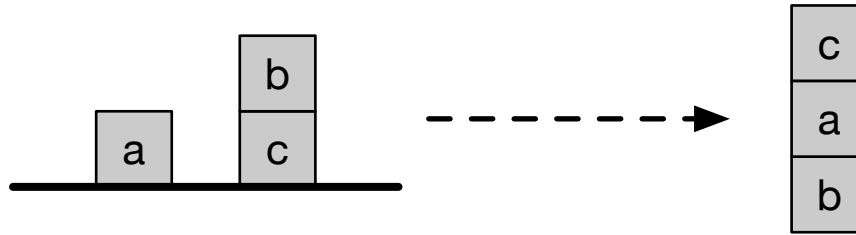


Figure 4.3: Blocks problem blocks1

```

121 |         clear('c'):False, on('c'):'d',
122 |         clear('d'):False, on('d'):'table'}
123 | blocks2 = Planning_problem(blocks2dom,
124 |     tower4, # initial state
125 |     {on('d'):'c',on('c'):'b',on('b'):'a'}) #goal

```

The problem *blocks3* is to move the bottom block to the top of a tower of size 4.

```

stripsProblem.py — (continued)
127 | blocks3 = Planning_problem(blocks2dom,
128 |     tower4, # initial state
129 |     {on('d'):'a', on('a'):'b', on('b'):'c'}) #goal

```

Exercise 4.1 Represent the problem of given a tower of 4 blocks (*a* on *b* on *c* on *d* on table), the goal is to have a tower with the previous top block on the bottom (*b* on *c* on *d* on *a*). Do not include the table in your goal (the goal does not care whether *a* is on the table). [Before you run the program, estimate how many steps it will take to solve this.] How many steps does an optimal planner take?

Exercise 4.2 Represent the domain so that *on*(*x*, *y*) is a Boolean feature that is True when *x* is on *y*. Does the representation of the state need to not include negative *on* facts? Why or why not? (Note that this may depend on the planner; write your answer with respect to particular planners.)

Exercise 4.3 It is possible to write the representation of the problem without using *clear*, where *clear*(*x*) means nothing is on *x*. Change the definition of the blocks world so that it does not use *clear* but uses *on* being false instead. Does this work better for any of the planners?

4.2 Forward Planning

To run the demo, in folder "aipython_322", load "stripsForwardPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a forward planner, a node is a state. A state consists of an assignment, which is a variable:value dictionary. In order to be able to do multiple-path pruning, we need to define a hash function, and equality between states.

```

stripsForwardPlanner.py — Forward Planner with STRIPS actions
11 from searchProblem import Arc, Search_problem
12 from stripsProblem import Strips, STRIPS_domain
13
14 class State(object):
15     def __init__(self, assignment):
16         self.assignment = assignment
17         self.hash_value = None
18     def __hash__(self):
19         if self.hash_value is None:
20             self.hash_value = hash(frozenset(self.assignment.items()))
21         return self.hash_value
22     def __eq__(self, st):
23         return self.assignment == st.assignment
24     def __str__(self):
25         return str(self.assignment)

```

In order to define a search problem (page 17), we need to define the goal condition, the start nodes, the neighbours, and (optionally) a heuristic function. Here *zero* is the default heuristic function.

```

stripsForwardPlanner.py — (continued)
27 def zero(*args,**nargs):
28     """always returns 0"""
29     return 0
30
31 class Forward_STRIPS(Search_problem):
32     """A search problem from a planning problem where:
33     * a node is a state object.
34     * the dynamics are specified by the STRIPS representation of actions
35     """
36     def __init__(self, planning_problem, heur=zero):
37         """creates a forward search space from a planning problem.
38         heur(state,goal) is a heuristic function,
39         an underestimate of the cost from state to goal, where
40         both state and goals are feature:value dictionaries.
41         """
42         self.prob_domain = planning_problem.prob_domain
43         self.initial_state = State(planning_problem.initial_state)
44         self.goal = planning_problem.goal
45         self.heur = heur
46
47     def is_goal(self, state):
48         """is True if node is a goal.
49
50         Every goal feature has the same value in the state and the goal."""
51         return all(state.assignment[prop]==self.goal[prop]

```

```

52         for prop in self.goal)
53
54     def start_node(self):
55         """returns start node"""
56         return self.initial_state
57
58     def neighbors(self, state):
59         """returns neighbors of state in this problem"""
60         return [ Arc(state, self.effect(act, state.assignment), act.cost, act)
61                 for act in self.prob_domain.actions
62                 if self.possible(act, state.assignment)]
63
64     def possible(self, act, state_asst):
65         """True if act is possible in state.
66         act is possible if all of its preconditions have the same value in the state"""
67         return all(state_asst[pre] == act.preconds[pre]
68                   for pre in act.preconds)
69
70     def effect(self, act, state_asst):
71         """returns the state that is the effect of doing act given state_asst
72         Python 3.9: return state_asst | act.effects"""
73         new_state_asst = state_asst.copy()
74         new_state_asst.update(act.effects)
75         return State(new_state_asst)
76
77     def heuristic(self, state):
78         """in the forward planner a node is a state.
79         the heuristic is an (under)estimate of the cost
80         of going from the state to the top-level goal.
81         """
82         return self.heur(state.assignment, self.goal)

```

Here are some test cases to try.

```

stripsForwardPlanner.py — (continued)
84 from searchBranchAndBound import DF_branch_and_bound
85 from searchMPP import SearcherMPP
86 from stripsProblem import problem0, problem1, problem2, blocks1, blocks2, blocks3
87
88 # SearcherMPP(Forward_STRIPS(problem1)).search() #A* with MPP
89 # DF_branch_and_bound(Forward_STRIPS(problem1),10).search() #B&B
90 # To find more than one plan:
91 # s1 = SearcherMPP(Forward_STRIPS(problem1)) #A*
92 # s1.search() #find another plan

```

4.2.1 Defining Heuristics for a Planner

Each planning domain requires its own heuristics. If you change the actions, you will need to reconsider the heuristic function, as there might then be a lower-cost path, which might make the heuristic non-admissible.

Here is an example of defining a (not very good) heuristic for the coffee delivery planning domain.

First we define the distance between two locations, which is used for the heuristics.

```

stripsHeuristic.py — Planner with Heuristic Function
11 def dist(loc1, loc2):
12     """returns the distance from location loc1 to loc2
13     """
14     if loc1==loc2:
15         return 0
16     if {loc1,loc2} in [{'cs','lab'},{'mr','off'}]:
17         return 2
18     else:
19         return 1

```

Note that the current state is a complete description; there is a value for every feature. However the goal need not be complete; it does not need to define a value for every feature. Before checking the value for a feature in the goal, a heuristic needs to define whether the feature is defined in the goal.

```

stripsHeuristic.py — (continued)
21 def h1(state,goal):
22     """ the distance to the goal location, if there is one"""
23     if 'RLoc' in goal:
24         return dist(state['RLoc'], goal['RLoc'])
25     else:
26         return 0
27
28 def h2(state,goal):
29     """ the distance to the coffee shop plus getting coffee and delivering it
30     if the robot needs to get coffee
31     """
32     if ('SWC' in goal and goal['SWC']==False
33         and state['SWC']==True
34         and state['RHC']==False):
35         return dist(state['RLoc'],'cs')+3
36     else:
37         return 0

```

The maximum of the values of a set of admissible heuristics is also an admissible heuristic. The function `maxh` takes a number of heuristic functions as arguments, and returns a new heuristic function that takes the maximum of the values of the heuristics. For example, `h1` and `h2` are heuristic functions and so `maxh(h1,h2)` is also. `maxh` can take an arbitrary number of arguments.

```

stripsHeuristic.py — (continued)
39 def maxh(*heuristics):
40     """Returns a new heuristic function that is the maximum of the functions in heuristics.
41     heuristics is the list of arguments which must be heuristic functions.

```

```

42     """
43     # return lambda state,goal: max(h(state,goal) for h in heuristics)
44     def newh(state,goal):
45         return max(h(state,goal) for h in heuristics)
46     return newh

```

The following runs the example with and without the heuristic.

```

stripsHeuristic.py — (continued)
48 ##### Forward Planner #####
49 from searchMPP import SearcherMPP
50 from stripsForwardPlanner import Forward_STRIPS
51 from stripsProblem import problem0, problem1, problem2, blocks1, blocks2, blocks3
52
53 def test_forward_heuristic(thisproblem=problem1):
54     print("\n***** FORWARD NO HEURISTIC")
55     print(SearcherMPP(Forward_STRIPS(thisproblem)).search())
56
57     print("\n***** FORWARD WITH HEURISTIC h1")
58     print(SearcherMPP(Forward_STRIPS(thisproblem,h1)).search())
59
60     print("\n***** FORWARD WITH HEURISTIC h2")
61     print(SearcherMPP(Forward_STRIPS(thisproblem,h2)).search())
62
63     print("\n***** FORWARD WITH HEURISTICS h1 and h2")
64     print(SearcherMPP(Forward_STRIPS(thisproblem,maxh(h1,h2))).search())
65
66 if __name__ == "__main__":
67     test_forward_heuristic()

```

Exercise 4.4 Try the forward planner with a heuristic function of just h_1 , with just h_2 and with both. Explain how each one prunes or doesn't prune the search space.

Exercise 4.5 Create a better heuristic than $\max(h_1, h_2)$. Try it for a number of different problems.

Exercise 4.6 Create an admissible heuristic for the blocks world.

4.3 Regression Planning

To run the demo, in folder "aipython_322", load "stripsRegressionPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a regression planner a node is a subgoal that need to be achieved.

A *Subgoal* object consists of an assignment, which is *variable:value* dictionary. We make it hashable so that multiple path pruning can work. The hash is only computed when necessary (and only once).

```

stripsRegressionPlanner.py — Regression Planner with STRIPS actions
11 from searchProblem import Arc, Search_problem
12
13 class Subgoal(object):
14     def __init__(self, assignment):
15         self.assignment = assignment
16         self.hash_value = None
17     def __hash__(self):
18         if self.hash_value is None:
19             self.hash_value = hash(frozenset(self.assignment.items()))
20         return self.hash_value
21     def __eq__(self, st):
22         return self.assignment == st.assignment
23     def __str__(self):
24         return str(self.assignment)

```

A regression search has subgoals as nodes. The initial node is the top-level goal of the planner. The goal for the search (when the search can stop) is a subgoal that holds in the initial state.

```

stripsRegressionPlanner.py — (continued)
26 from stripsForwardPlanner import zero
27
28 class Regression_STRIPS(Search_problem):
29     """A search problem where:
30     * a node is a goal to be achieved, represented by a set of propositions.
31     * the dynamics are specified by the STRIPS representation of actions
32     """
33
34     def __init__(self, planning_problem, heur=zero):
35         """creates a regression search space from a planning problem.
36         heur(state,goal) is a heuristic function;
37         an underestimate of the cost from state to goal, where
38         both state and goals are feature:value dictionaries
39         """
40         self.prob_domain = planning_problem.prob_domain
41         self.top_goal = Subgoal(planning_problem.goal)
42         self.initial_state = planning_problem.initial_state
43         self.heur = heur
44
45     def is_goal(self, subgoal):
46         """if subgoal is true in the initial state, a path has been found"""
47         goal_asst = subgoal.assignment
48         return all(self.initial_state[g]==goal_asst[g]
49                   for g in goal_asst)
50
51     def start_node(self):
52         """the start node is the top-level goal"""
53         return self.top_goal
54
55     def neighbors(self, subgoal):

```

```

56     """returns a list of the arcs for the neighbors of subgoal in this problem"""
57     goal_asst = subgoal.assignment
58     return [ Arc(subgoal, self.weakest_precond(act,goal_asst), act.cost, act)
59             for act in self.prob_domain.actions
60             if self.possible(act,goal_asst)]
61
62     def possible(self,act,goal_asst):
63         """True if act is possible to achieve goal_asst.
64
65         the action achieves an element of the effects and
66         the action doesn't delete something that needs to be achieved and
67         the preconditions are consistent with other subgoals that need to be achieved
68         """
69         return ( any(goal_asst[prop] == act.effects[prop]
70                     for prop in act.effects if prop in goal_asst)
71                 and all(goal_asst[prop] == act.effects[prop]
72                        for prop in act.effects if prop in goal_asst)
73                 and all(goal_asst[prop] == act.preconds[prop]
74                        for prop in act.preconds if prop not in act.effects and prop in goal_asst)
75                 )
76
77     def weakest_precond(self,act,goal_asst):
78         """returns the subgoal that must be true so goal_asst holds after act
79         should be: act.preconds | (goal_asst - act.effects)
80         """
81         new_asst = act.preconds.copy()
82         for g in goal_asst:
83             if g not in act.effects:
84                 new_asst[g] = goal_asst[g]
85         return Subgoal(new_asst)
86
87     def heuristic(self,subgoal):
88         """in the regression planner a node is a subgoal.
89         the heuristic is an (under)estimate of the cost of going from the initial state to subgoal.
90         """
91         return self.heur(self.initial_state, subgoal.assignment)

```

stripsRegressionPlanner.py — (continued)

```

93 from searchBranchAndBound import DF_branch_and_bound
94 from searchMPP import SearcherMPP
95 from stripsProblem import problem0, problem1, problem2, blocks1, blocks2, blocks3
96
97 # SearcherMPP(Regression_STRIPS(problem1)).search() #A* with MPP
98 # DF_branch_and_bound(Regression_STRIPS(problem1),10).search() #B&B

```

Exercise 4.7 Multiple path pruning could be used to prune more than the current code. In particular, if the current node contains more conditions than a previously visited node, it can be pruned. For example, if $\{a : \text{True}, b : \text{False}\}$ has been visited, then any node that is a superset, e.g., $\{a : \text{True}, b : \text{False}, d : \text{True}\}$, need not be expanded. If the simpler subgoal does not lead to a solution, the more complicated

one won't either. Implement this more severe pruning. (Hint: This may require modifications to the searcher.)

Exercise 4.8 It is possible that, as knowledge of the domain, that some assignment of values to variables can never be achieved. For example, the robot cannot be holding mail when there is mail waiting (assuming it isn't holding mail initially). An assignment of values to (some of the) variables is incompatible if no possible (reachable) state can include that assignment. For example, $\{ 'MW' : True, 'RHM' : True \}$ is an incompatible assignment. This information may be useful information for a planner; there is no point in trying to achieve these together. Define a subclass of *STRIPS_domain* that can accept a list of incompatible assignments. Modify the regression planner code to use such a list of incompatible assignments. Give an example where the search space is smaller.

Exercise 4.9 After completing the previous exercise, design incompatible assignments for the blocks world. (This should result in dramatic search improvements.)

4.3.1 Defining Heuristics for a Regression Planner

The regression planner can use the same heuristic function as the forward planner. However, just because a heuristic is useful for a forward planner does not mean it is useful for a regression planner, and vice versa. you should experiment with whether the same heuristic works well for both a regression planner and a forward planner.

The following runs the same example as the forward planner with and without the heuristic defined for the forward planner:

```

stripsHeuristic.py — (continued)
69 ##### Regression Planner
70 from stripsRegressionPlanner import Regression_STRIPS
71
72 def test_regression_heuristic(thisproblem=problem1):
73     print("\n***** REGRESSION NO HEURISTIC")
74     print(SearcherMPP(Regression_STRIPS(thisproblem)).search())
75
76     print("\n***** REGRESSION WITH HEURISTICS h1 and h2")
77     print(SearcherMPP(Regression_STRIPS(thisproblem,maxh(h1,h2))).search())
78
79 if __name__ == "__main__":
80     test_regression_heuristic()

```

Exercise 4.10 Try the regression planner with a heuristic function of just *h1* and with just *h2* (defined in Section 4.2.1). Explain how each one prunes or doesn't prune the search space.

Exercise 4.11 Create a better heuristic than *heuristic_fun* defined in Section 4.2.1.

4.4 Planning as a CSP

To run the demo, in folder "aipython_322", load "stripsCSPPlanner.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3.

Here we implement the CSP planner assuming there is a single action at each step. This creates a CSP that can use any of the CSP algorithms to solve (e.g., stochastic local search or arc consistency with domain splitting).

This assumes the same action representation as before; we do not consider factored actions (action features), nor do we implement state constraints.

```

_____stripsCSPPlanner.py — CSP planner where actions are represented using STRIPS_____
11 from cspProblem import CSP, Constraint
12
13 class CSP_from_STRIPS(CSP):
14     """A CSP where:
15     * a CSP variable is constructed by st(var,stage).
16     * the dynamics are specified by the STRIPS representation of actions
17     """
18
19     def __init__(self, planning_problem, number_stages=2):
20         prob_domain = planning_problem.prob_domain
21         initial_state = planning_problem.initial_state
22         goal = planning_problem.goal
23         self.act_vars = [st('action',stage) for stage in range(number_stages)]
24         domains = {av:prob_domain.actions for av in self.act_vars}
25         domains.update({ st(var,stage):dom
26                         for (var,dom) in prob_domain.feats_vals.items()
27                         for stage in range(number_stages+1)})
28         # initial state constraints:
29         constraints = [Constraint((st(var,0),), is_(val))
30                        for (var,val) in initial_state.items()]
31         # goal constraints on the final state:
32         constraints += [Constraint((st(var,number_stages),),
33                                   is_(val))
34                        for (var,val) in goal.items()]
35         # precondition constraints:
36         constraints += [Constraint((st(var,stage), st('action',stage)),
37                                   if_(val,act)) # st(var,stage)==val if st('action',stage)=act
38                        for act in prob_domain.actions
39                        for var,val in act.preconds.items()
40                        for stage in range(number_stages)]
41         # effect constraints:
42         constraints += [Constraint((st(var,stage+1), st('action',stage)),
43                                   if_(val,act)) # st(var,stage+1)==val if st('action',stage)=act
44                        for act in prob_domain.actions
45                        for var,val in act.effects.items()
46                        for stage in range(number_stages)]

```

```

47     # frame constraints:
48     constraints += [Constraint((st(var,stage), st('action',stage), st(var,stage+1)),
49                               eq_if_not_in_({act for act in prob_domain.actions
50                                              if var in act.effects}))
51                               for var in prob_domain.feats_vals
52                               for stage in range(number_stages) ]
53     CSP.__init__(self, domains, constraints)
54
55     def extract_plan(self,soln):
56         return [soln[a] for a in self.act_vars]
57
58     def st(var,stage):
59         """returns a string for the var-stage pair that can be used as a variable"""
60         return str(var)+"_"+str(stage)

```

The following methods return methods which can be applied to the particular environment.

For example, *is_(3)* returns a function that when applied to 3, returns *True* and when applied to any other value returns *False*. So *is_(3)(3)* returns *True* and *is_(3)(7)* returns *False*.

Note that the underscore ('_') is part of the name; here we use it as the convention that it is a function that returns a function. This uses two different styles to define *is_* and *if_*; returning a function defined by *lambda* is equivalent to returning the embedded function, except that the embedded function has a name. The embedded function can also be given a docstring.

```

stripsCSPPlanner.py — (continued)
62 def is_(val):
63     """returns a function that is true when it is it applied to val.
64     """
65     #return lambda x: x == val
66     def is_fun(x):
67         return x == val
68     is_fun.__name__ = "value_is_"+str(val)
69     return is_fun
70
71 def if_(v1,v2):
72     """if the second argument is v2, the first argument must be v1"""
73     #return lambda x1,x2: x1==v1 if x2==v2 else True
74     def if_fun(x1,x2):
75         return x1==v1 if x2==v2 else True
76     if_fun.__name__ = "if x2 is "+str(v2)+" then x1 is "+str(v1)
77     return if_fun
78
79 def eq_if_not_in_(actset):
80     """first and third arguments are equal if action is not in actset"""
81     # return lambda x1, a, x2: x1==x2 if a not in actset else True
82     def eq_if_not_fun(x1, a, x2):
83         return x1==x2 if a not in actset else True
84     eq_if_not_fun.__name__ = "first and third arguments are equal if action is not in "+str(actset)

```

```
85 | return eq_if_not_fun
```

Putting it together, this returns a list of actions that solves the problem *prob* for a given horizon. If you want to do more than just return the list of actions, you might want to get it to return the solution. Or even enumerate the solutions (by using *Search_with_AC_from_CSP*).

stripsCSPPlanner.py — (continued)

```
87 def con_plan(prob,horizon):
88     """finds a plan for problem prob given horizon.
89     """
90     csp = CSP_from_STRIPS(prob, horizon)
91     sol = Con_solver(csp).solve_one()
92     return csp.extract_plan(sol) if sol else sol
```

The following are some example queries.

stripsCSPPlanner.py — (continued)

```
94 from searchGeneric import Searcher
95 from stripsProblem import delivery_domain
96 from cspConsistency import Search_with_AC_from_CSP, Con_solver
97 from stripsProblem import Planning_problem, problem0, problem1, problem2, blocks1, blocks2, blocks3
98
99 # Problem 0
100 # con_plan(problem0,1) # should it succeed?
101 # con_plan(problem0,2) # should it succeed?
102 # con_plan(problem0,3) # should it succeed?
103 # To use search to enumerate solutions
104 #searcher0a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem0, 1)))
105 #print(searcher0a.search())
106
107 ## Problem 1
108 # con_plan(problem1,5) # should it succeed?
109 # con_plan(problem1,4) # should it succeed?
110 ## To use search to enumerate solutions:
111 #searcher15a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem1, 5)))
112 #print(searcher15a.search())
113
114 ## Problem 2
115 #con_plan(problem2, 6) # should fail??
116 #con_plan(problem2, 7) # should succeed???
117
118 ## Example 6.13
119 problem3 = Planning_problem(delivery_domain,
120                              {'SWC':True, 'RHC':False}, {'SWC':False})
121 #con_plan(problem3,2) # Horizon of 2
122 #con_plan(problem3,3) # Horizon of 3
123
124 problem4 = Planning_problem(delivery_domain,{'SWC':True},
125                              {'SWC':False, 'MW':False, 'RHM':False})
126
```

```

127 | # For the stochastic local search:
128 | #from cspSLS import SLSearcher, Runtime_distribution
129 | # cspplanning15 = CSP_from_STRIPS(problem1, 5) # should succeed
130 | #se0 = SLSearcher(cspplanning15); print(se0.search(100000,0.5))
131 | #p = Runtime_distribution(cspplanning15)
132 | #p.plot_run(1000,1000,0.7) # warning will take a few minutes

```

4.5 Partial-Order Planning

To run the demo, in folder "aipython_322", load "stripsPOP.py", and copy and paste the commented-out example queries at the bottom of that file.

A partial order planner maintains a partial order of action instances. An action instance consists of a name and an index. We need action instances because the same action could be carried out at different times.

```

_____stripsPOP.py — Partial-order Planner using STRIPS representation_____
11 | from searchProblem import Arc, Search_problem
12 | import random
13 |
14 | class Action_instance(object):
15 |     next_index = 0
16 |     def __init__(self, action, index=None):
17 |         if index is None:
18 |             index = Action_instance.next_index
19 |             Action_instance.next_index += 1
20 |         self.action = action
21 |         self.index = index
22 |
23 |     def __str__(self):
24 |         return str(self.action)+"#"+str(self.index)
25 |
26 |     __repr__ = __str__ # __repr__ function is the same as the __str__ function

```

A node (as in the abstraction of search space) in a partial-order planner consists of:

- *actions*: a set of action instances.
- *constraints*: a set of (a_1, a_2) pairs, where a_1 and a_2 are action instances, which represents that a_1 must come before a_2 in the partial order. There are a number of ways that this could be represented. Here we represent the set of pairs that are in transitive closure of the *before* relation. This lets us quickly determine whether some before relation is consistent with the current constraints.

- *agenda*: a list of (s, a) pairs, where s is a (var, val) pair and a is an action instance. This means that variable var must have value val before a can occur.
- *causal_links*: a set of (a_0, g, a_1) triples, where a_1 and a_2 are action instances and g is a (var, val) pair. This holds when action a_0 makes g true for action a_1 .

```

stripsPOP.py — (continued)
28 class POP_node(object):
29     """a (partial) partial-order plan. This is a node in the search space."""
30     def __init__(self, actions, constraints, agenda, causal_links):
31         """
32         * actions is a set of action instances
33         * constraints a set of (a0,a1) pairs, representing a0<a1,
34           closed under transitivity
35         * agenda list of (subgoal,action) pairs to be achieved, where
36           subgoal is a (variable,value) pair
37         * causal_links is a set of (a0,g,a1) triples,
38           where ai are action instances, and g is a (variable,value) pair
39         """
40         self.actions = actions # a set of action instances
41         self.constraints = constraints # a set of (a0,a1) pairs
42         self.agenda = agenda # list of (subgoal,action) pairs to be achieved
43         self.causal_links = causal_links # set of (a0,g,a1) triples
44
45     def __str__(self):
46         return ("actions: "+str({str(a) for a in self.actions})+
47             "\nconstraints: "+
48             str({(str(a1),str(a2)) for (a1,a2) in self.constraints})+
49             "\nagenda: "+
50             str([(str(s),str(a)) for (s,a) in self.agenda])+
51             "\ncausal_links:"+
52             str({(str(a0),str(g),str(a2)) for (a0,g,a2) in self.causal_links})
53         )

```

extract_plan constructs a total order of action instances that is consistent with the partial order.

```

stripsPOP.py — (continued)
54 def extract_plan(self):
55     """returns a total ordering of the action instances consistent
56     with the constraints.
57     raises IndexError if there is no choice.
58     """
59     sorted_acts = []
60     other_acts = set(self.actions)
61     while other_acts:
62         a = random.choice([a for a in other_acts if
63             all(((a1,a) not in self.constraints) for a1 in other_acts)])

```

```

64         sorted_acts.append(a)
65         other_acts.remove(a)
66     return sorted_acts

```

POP_search_from_STRIPS is an instance of a search problem. As such, we need to define the start nodes, the goal, and the neighbors of a node.

```

stripsPOP.py — (continued)
68 from display import Displayable
69
70 class POP_search_from_STRIPS(Search_problem, Displayable):
71     def __init__(self, planning_problem):
72         Search_problem.__init__(self)
73         self.planning_problem = planning_problem
74         self.start = Action_instance("start")
75         self.finish = Action_instance("finish")
76
77     def is_goal(self, node):
78         return node.agenda == []
79
80     def start_node(self):
81         constraints = {(self.start, self.finish)}
82         agenda = [(g, self.finish) for g in self.planning_problem.goal.items()]
83         return POP_node([self.start, self.finish], constraints, agenda, [])

```

The *neighbors* method is a coroutine that enumerates the neighbors of a given node.

```

stripsPOP.py — (continued)
85 def neighbors(self, node):
86     """enumerates the neighbors of node"""
87     self.display(3, "finding neighbors of\n", node)
88     if node.agenda:
89         subgoal, act1 = node.agenda[0]
90         self.display(2, "selecting", subgoal, "for", act1)
91         new_agenda = node.agenda[1:]
92         for act0 in node.actions:
93             if (self.achieves(act0, subgoal) and
94                 self.possible((act0, act1), node.constraints)):
95                 self.display(2, "reusing", act0)
96                 consts1 = self.add_constraint((act0, act1), node.constraints)
97                 new_clink = (act0, subgoal, act1)
98                 new_cls = node.causal_links + [new_clink]
99                 for consts2 in self.protect_cl_for_actions(node.actions, consts1, new_clink):
100                     yield Arc(node,
101                               POP_node(node.actions, consts2, new_agenda, new_cls),
102                               cost=0)
103         for a0 in self.planning_problem.prob_domain.actions: #a0 is an action
104             if self.achieves(a0, subgoal):
105                 #a0 achieves subgoal
106                 new_a = Action_instance(a0)

```

```

107         self.display(2, " using new action", new_a)
108         new_actions = node.actions + [new_a]
109         consts1 = self.add_constraint((self.start, new_a), node.constraints)
110         consts2 = self.add_constraint((new_a, act1), consts1)
111         new_agenda1 = new_agenda + [(pre, new_a) for pre in a0.preconds.items()]
112         new_clink = (new_a, subgoal, act1)
113         new_cls = node.causal_links + [new_clink]
114         for consts3 in self.protect_all_cls(node.causal_links, new_a, consts2):
115             for consts4 in self.protect_cl_for_actions(node.actions, consts3, new_clink):
116                 yield Arc(node,
117                           POP_node(new_actions, consts4, new_agenda1, new_cls),
118                           cost=1)

```

Given a casual link $(a0, subgoal, a1)$, the following method protects the causal link from each action in *actions*. Whenever an action deletes *subgoal*, the action needs to be before *a0* or after *a1*. This method enumerates all constraints that result from protecting the causal link from all actions.

```

stripsPOP.py — (continued)
120 def protect_cl_for_actions(self, actions, constrs, clink):
121     """yields constraints that extend constrs and
122     protect causal link (a0, subgoal, a1)
123     for each action in actions
124     """
125     if actions:
126         a = actions[0]
127         rem_actions = actions[1:]
128         a0, subgoal, a1 = clink
129         if a != a0 and a != a1 and self.deletes(a, subgoal):
130             if self.possible((a, a0), constrs):
131                 new_const = self.add_constraint((a, a0), constrs)
132                 for e in self.protect_cl_for_actions(rem_actions, new_const, clink): yield e
133             # could be "yield from"
134             if self.possible((a1, a), constrs):
135                 new_const = self.add_constraint((a1, a), constrs)
136                 for e in self.protect_cl_for_actions(rem_actions, new_const, clink): yield e
137             else:
138                 for e in self.protect_cl_for_actions(rem_actions, constrs, clink): yield e
139         else:
140             yield constrs

```

Given an action *act*, the following method protects all the causal links in *clinks* from *act*. Whenever *act* deletes *subgoal* from some causal link $(a0, subgoal, a1)$, the action *act* needs to be before *a0* or after *a1*. This method enumerates all constraints that result from protecting the causal links from *act*.

```

stripsPOP.py — (continued)
141 def protect_all_cls(self, clinks, act, constrs):
142     """yields constraints that protect all causal links from act"""
143     if clinks:
144         (a0, cond, a1) = clinks[0] # select a causal link

```



```

145         rem_clinks = clinks[1:] # remaining causal links
146         if act != a0 and act != a1 and self.deletes(act,cond):
147             if self.possible((act,a0),constrs):
148                 new_const = self.add_constraint((act,a0),constrs)
149                 for e in self.protect_all_cls(rem_clinks,act,new_const): yield e
150             if self.possible((a1,act),constrs):
151                 new_const = self.add_constraint((a1,act),constrs)
152                 for e in self.protect_all_cls(rem_clinks,act,new_const): yield e
153         else:
154             for e in self.protect_all_cls(rem_clinks,act,constrs): yield e
155     else:
156         yield constrs

```

The following methods check whether an action (or action instance) achieves or deletes some subgoal.

```

stripsPOP.py — (continued)
158 def achieves(self,action,subgoal):
159     var,val = subgoal
160     return var in self.effects(action) and self.effects(action)[var] == val
161
162 def deletes(self,action,subgoal):
163     var,val = subgoal
164     return var in self.effects(action) and self.effects(action)[var] != val
165
166 def effects(self,action):
167     """returns the variable:value dictionary of the effects of action.
168     works for both actions and action instances"""
169     if isinstance(action, Action_instance):
170         action = action.action
171     if action == "start":
172         return self.planning_problem.initial_state
173     elif action == "finish":
174         return {}
175     else:
176         return action.effects

```

The constraints are represented as a set of pairs closed under transitivity. Thus if (a,b) and (b,c) are the list, then (a,c) must also be in the list. This means that adding a new constraint means adding the implied pairs, but querying whether some order is consistent is quick.

```

stripsPOP.py — (continued)
178 def add_constraint(self, pair, const):
179     if pair in const:
180         return const
181     todo = [pair]
182     newconst = const.copy()
183     while todo:
184         x0,x1 = todo.pop()
185         newconst.add((x0,x1))

```

```

186         for x,y in newconst:
187             if x==x1 and (x0,y) not in newconst:
188                 todo.append((x0,y))
189             if y==x0 and (x,x1) not in newconst:
190                 todo.append((x,x1))
191         return newconst
192
193     def possible(self,pair,constraint):
194         (x,y) = pair
195         return (y,x) not in constraint

```

Some code for testing:

```

stripsPOP.py — (continued)
197 from searchBranchAndBound import DF_branch_and_bound
198 from searchMPP import SearcherMPP
199 from stripsProblem import problem0, problem1, problem2, blocks1, blocks2, blocks3
200
201 rplanning0 = POP_search_from_STRIPS(problem0)
202 rplanning1 = POP_search_from_STRIPS(problem1)
203 rplanning2 = POP_search_from_STRIPS(problem2)
204 searcher0 = DF_branch_and_bound(rplanning0,5)
205 searcher0a = SearcherMPP(rplanning0)
206 searcher1 = DF_branch_and_bound(rplanning1,10)
207 searcher1a = SearcherMPP(rplanning1)
208 searcher2 = DF_branch_and_bound(rplanning2,10)
209 searcher2a = SearcherMPP(rplanning2)
210 # Try one of the following searchers
211 # a = searcher0.search()
212 # a = searcher0a.search()
213 # a.end().extract_plan() # print a plan found
214 # a.end().constraints # print the constraints
215 # SearcherMPP.max_display_level = 0 # less detailed display
216 # DF_branch_and_bound.max_display_level = 0 # less detailed display
217 # a = searcher1.search()
218 # a = searcher1a.search()
219 # a = searcher2.search()
220 # a = searcher2a.search()

```

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