Lab 1

Sam Ionesi Nathaniel Van John Bush

April 16, 2017

1 Introduction

1.1 Question

To what extent does the given spring behave as an ideal spring?

1.2 Hypothesis

As seen previously in physics class, the force exerted by an ideal spring is modeled by Hooke's Law, $F = -kx$, where k is the spring-force constant. This k is considered, in the ideal spring, to be a fixed scalar value and a property intrinsic to the spring. While this model is precise for most practical purposes, a real, non-ideal spring will not conform to this model.

In order to recognize how regularly the spring conforms to the model of the ideal spring, we observe and compare how the spring oscillates over a range of forces and displacements. By suspending weights of varying mass from the spring and displacing those weights, we create an oscillation that can be measured and allow the spring force constant to be calculated. In an ideal spring, the spring-force constant would be constant. Observing a variation in this value would indicate conditions where the spring deviates from the ideal modeled by Hooke's Law.

1.3 Method

1. Setup the rig displayed in the diagram below.
2. Place tape on the bottom of the mass to hold the note card.
3. Pull the mass down directly above the motion detector from the springs equilibrium 2 inches down and release.
4. Record data using the Labquest.
5. Find the period of each mass.
6. Test various weights to see how that affects the k value.
7. Repeat steps as needed.

Equipment	Purpose
Ring stand	Suspend the spring and mass.
Spring	Spring whose characteristics are observed.
Hanging masses	Masses range from 10 g to 1 kg.
Meter stick	To estimate displacement from equilibrium.
Motion detector	Record displacement of spring at regular intervals of time.
Labquest	Control the motion detector and receive data.
Computer	Control the Labquest controller and receive data.
1 Note Card	Act as reflector for motion detector.
Tape	Fix the note card to the suspended mass.

Figure 1.1: Equipment used in experiment.

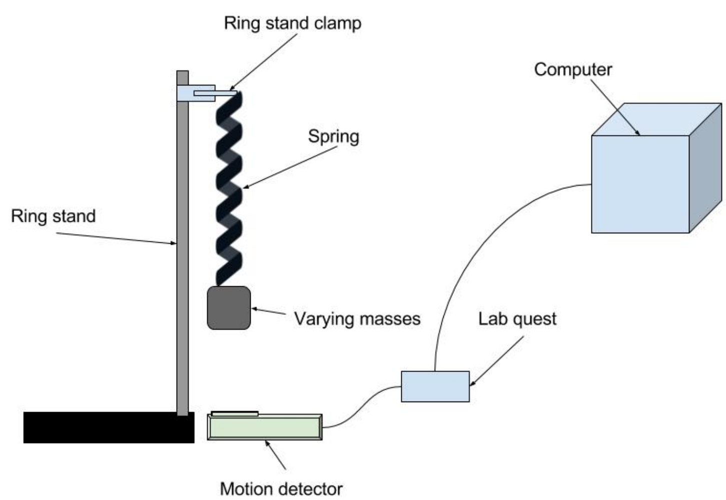

Figure 1.2: Experiment setup

1.4 Predictions

We expect that the frequency of the spring oscillation should be consistent and regular until we add very large masses. With larger masses, at the extremes of force and displacement, the spring will deform and no longer behave as an ideal spring.

2 Analysis

2.1 Method

Per Hooke's Law, the position can be represented as a function of time as a solution to the differential equation,

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx \tag{1}$$

where x is the position of the spring, t is time, m is the mass on the spring, and k is the spring force constant. It can be seen that $x = A\cos(\omega t)$ is a solution to this equation, where $\omega^2 = \frac{k}{m}$. This implies that the frequency of the spring oscillations is a function of the mass and spring force constant, and can easily be obtained by modeling the spring's motion as a sinusoidal function.

It would be difficult to calculate frequency of the spring oscillations directly, as the sampling rate of the motion detector was relatively low, but could be interpolated using the model. The data was fit to a cosine function by least-squares fitting with a Python script.

The data for each trial, as exported by the Labview software, was fir to a cosine function using the SciPy.optimize package's leastsq method. This implements the Levenberg-Marquardt algorithm. The initial paramters were estimated by reviewing the data. Iterating over the data points, the local peaks were idtentified (as local maxima above the mean) and frequency was estimated to be the mean of the distances between them. Amplitude was estimated to be half the distance between minimum and maximum position values. Phase off-set was guessed from the time of the first local maximum in position, and the position off-set was taken from the mean of all position values. For some trials, the bottom of the mass was within the minimum range of the motion detector and yielded a position value near that boundary of range. It was thus necessary to filter out position values in that region.

The curve-fitting algorithm calculated the modeling function's frequency term, ω. As stated above, Hooke's Law would imply that the spring force constant k can be expressed $k = m\omega^2$. Thus, the spring force constant could quickly be calculated from the results of the curve fitting algorithm.

2.2 Observed Results

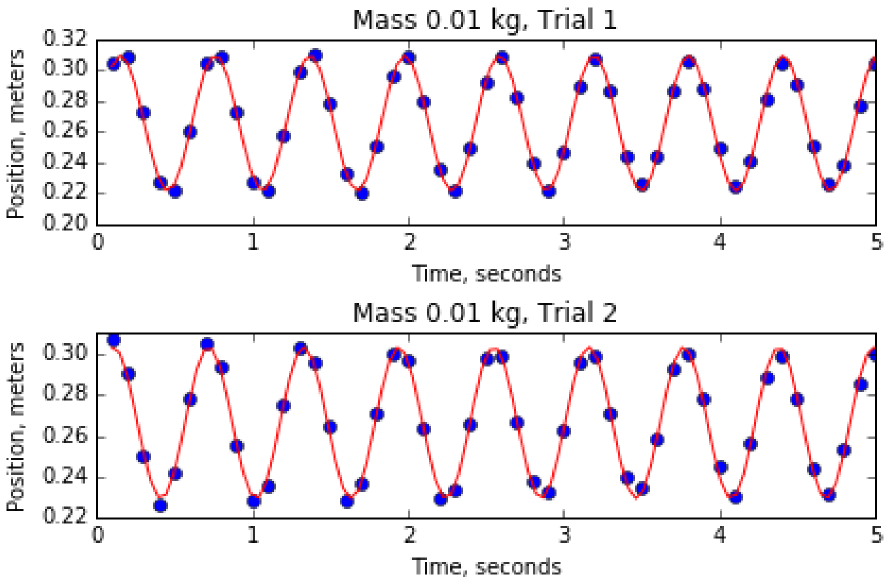

Figure 2.1: Mass 0.01 kg, Position-Time

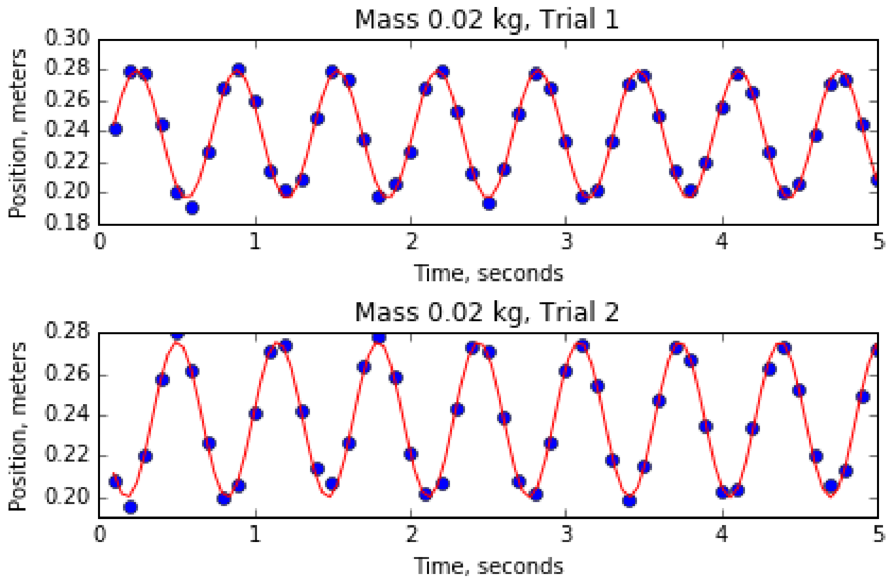

Figure 2.2: Mass 0.02 kg, Position-Time

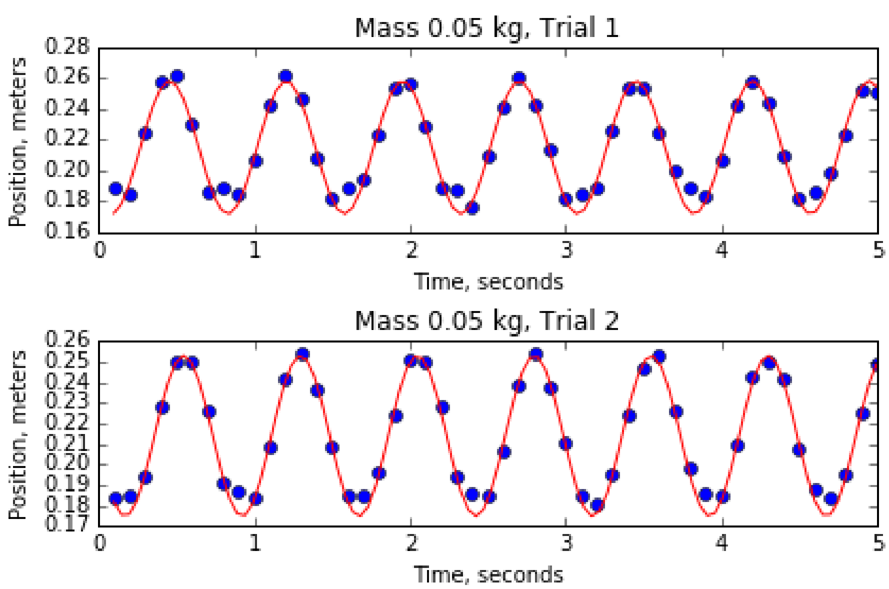

Figure 2.3: Mass 0.05 kg, Position-Time

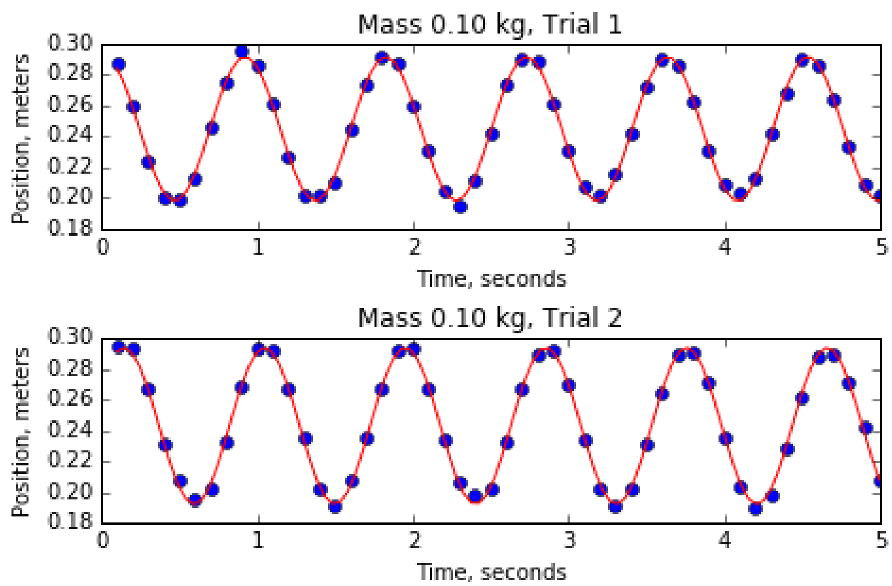

Figure 2.4: Mass 0.10 kg, Position-Time

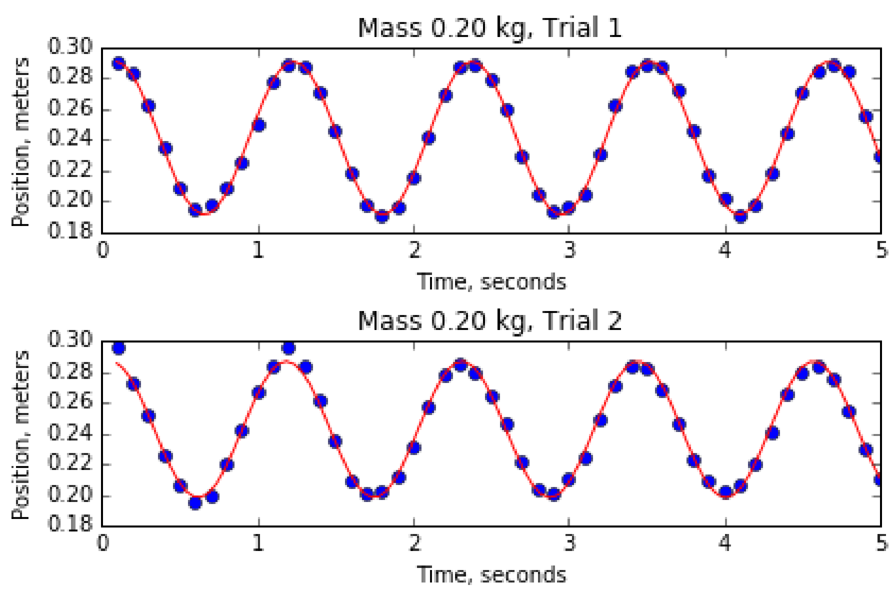

Figure 2.5: Mass 0.20 kg, Position-Time

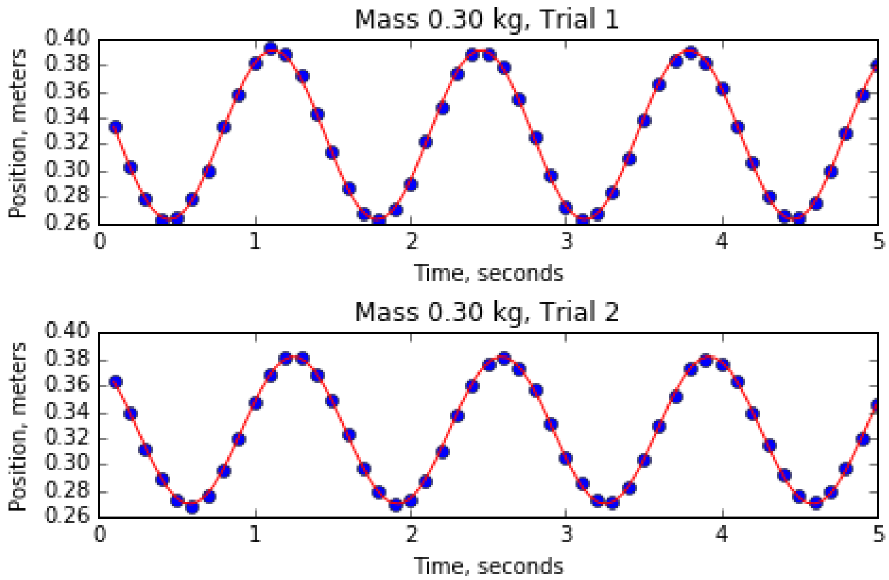

Figure 2.6: Mass 0.30 kg, Position-Time

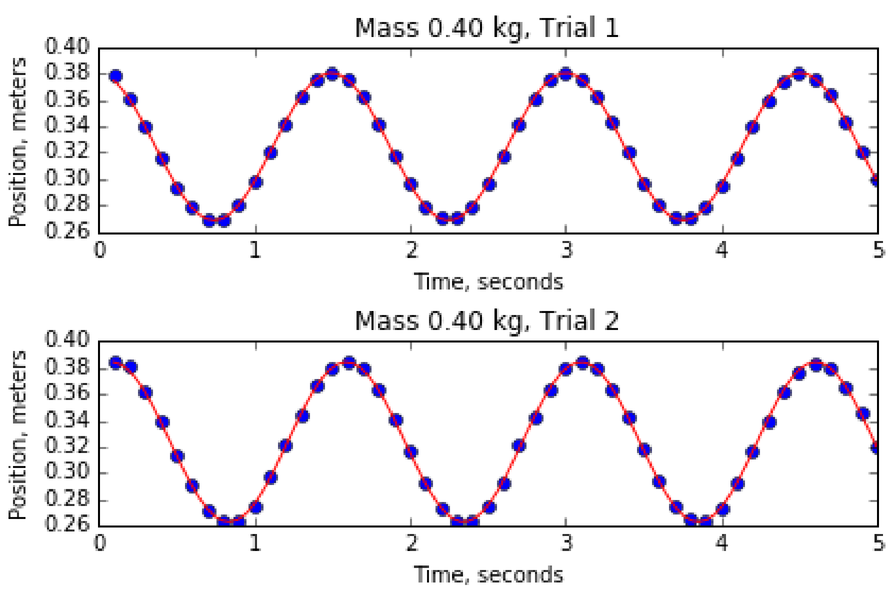

Figure 2.7: Mass 0.40 kg, Position-Time

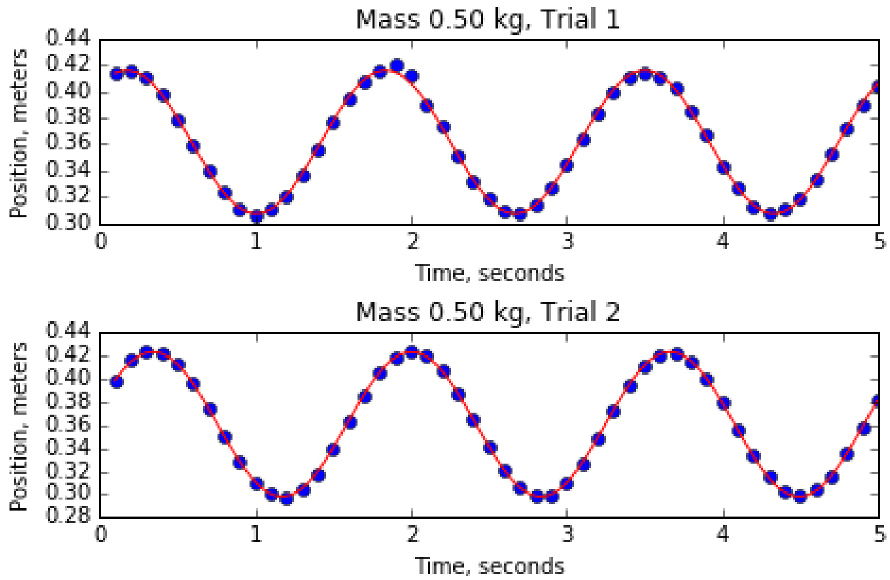

Figure 2.8: Mass 0.50 kg, Position-Time

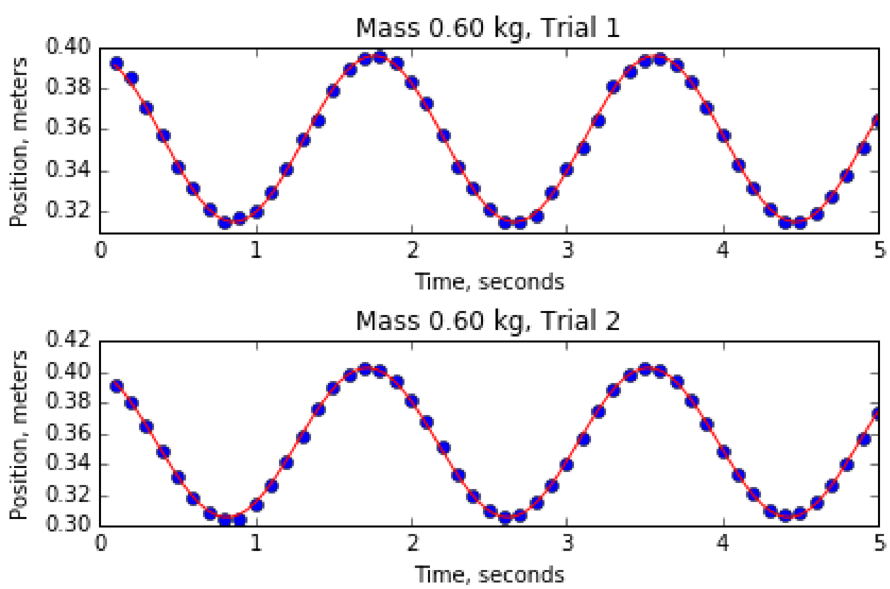

Figure 2.9: Mass 0.60 kg, Position-Time

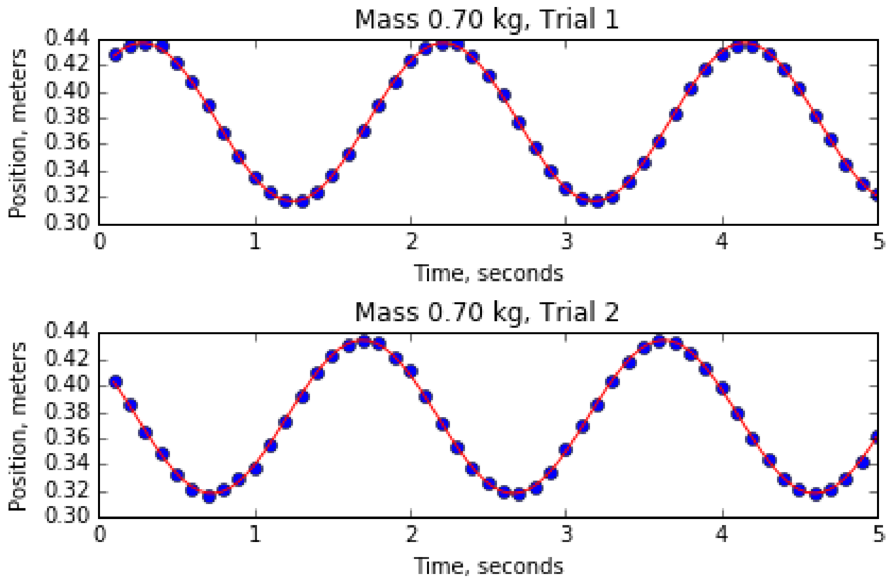

Figure 2.10: Mass 0.70 kg, Position-Time

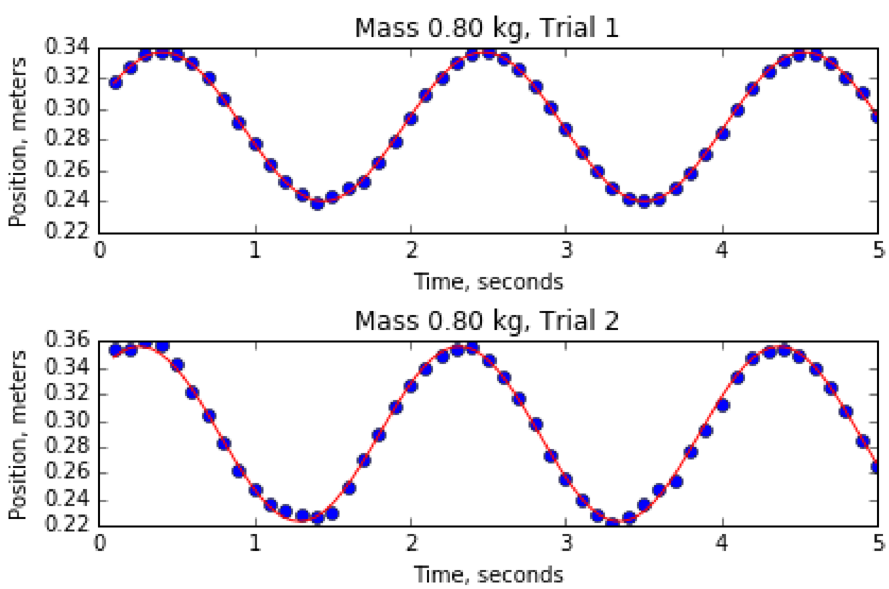

Figure 2.11: Mass 0.80 kg, Position-Time

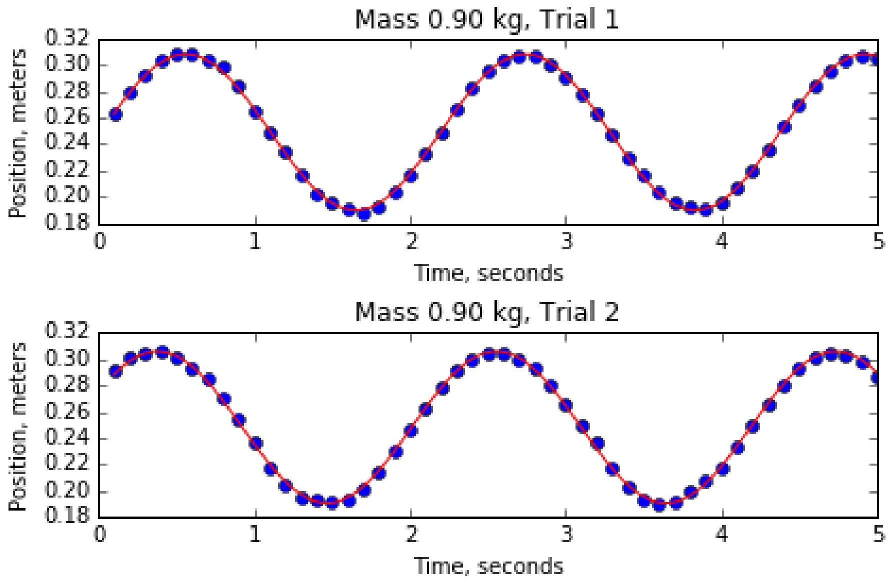

Figure 2.12: Mass 0.90 kg, Position-Time

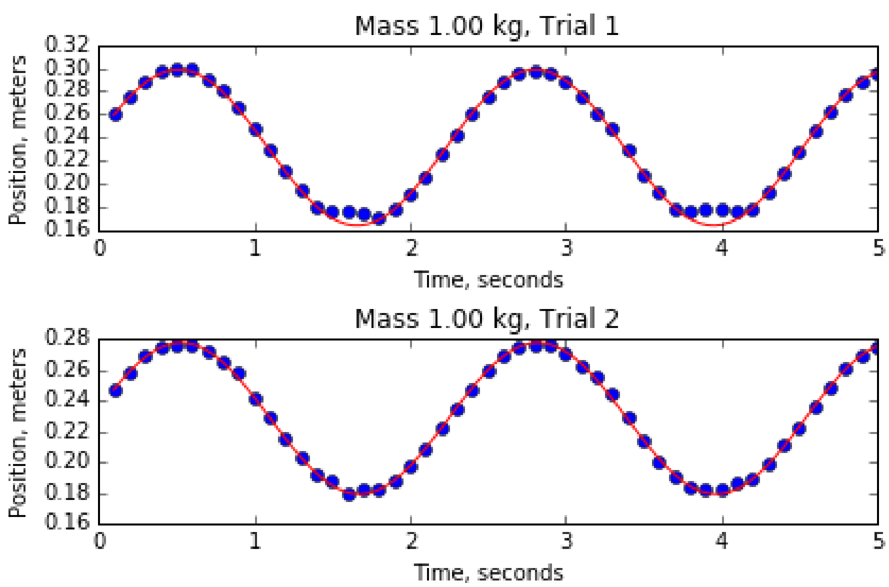

Figure 2.13: Mass 1.00 kg, Position-Time

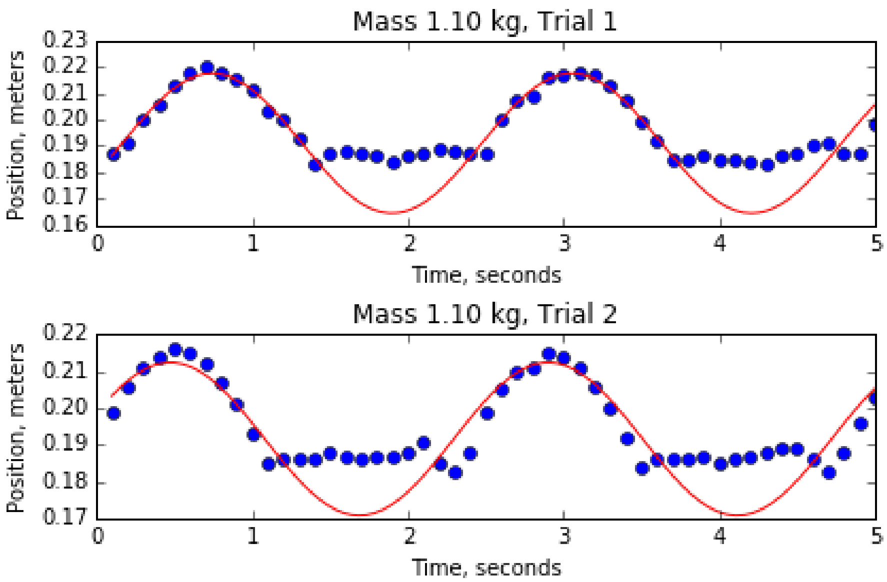

Figure 2.14: Mass 1.10 kg, Position-Time

Mass (kg)	Angular Frequency (ω)	Spring Constant (k)
0.01	10.3310	1.0673
	10.295266	1.059925
0.02	9.7465	1.8999
	9.730597	1.893690
0.05	8.3888	3.5186
	8.367833	3.501032
0.10	6.9505	4.8309
	6.953125	4.834595
0.20	5.4843	6.0154
	5.562774	6.188890
0.30	4.7049	6.6407
	4.709160	6.652855
0.40	4.1710	6.9590
	4.167170	6.946121
0.50	3.7790	7.1403
	3.783115	7.155981
0.60	3.4844	7.2847
	3.482183	7.275358
0.70	3.2472	7.3812
	3.246760	7.379013
0.80	3.0472	7.4283
	3.057934	7.480769
0.90	2.8813	7.4715
	2.885437	7.493171
1.00	2.7425	7.5215
	2.741550	7.516096
1.10	2.7199	8.1374
	2.588541	7.370601

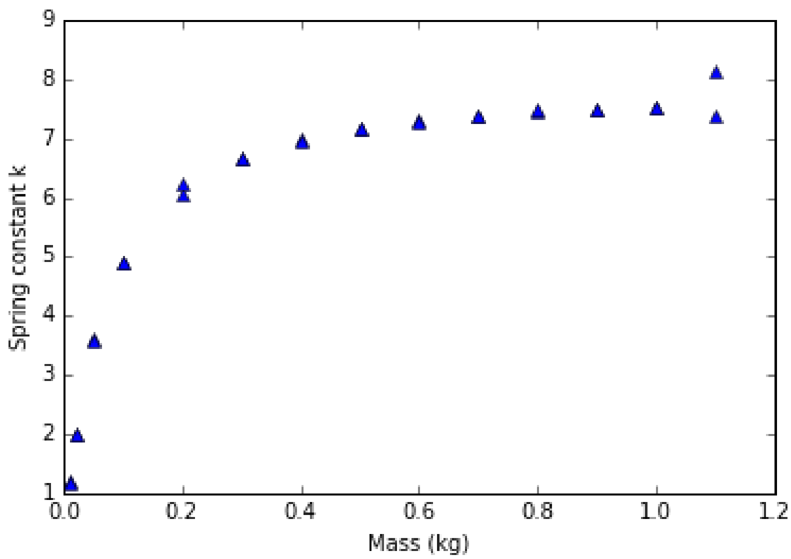

Figure 2.15: Mass (kg) - Spring Constant k