EC591 Lab 7 Report Notes

7.1A Malus' law:

$$\vec{J}_{out} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \sqrt{2\eta_0 I_{in}} \\ 0 \end{bmatrix} = \sqrt{2\eta_0 I_{in}} \begin{bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{bmatrix}$$

Jones matrix of 2^{nd} polarizer (with transmission axis at θ with respect to $\hat{x}$)

Jones vector of light after 1^{st} polarizer (with transmission axis along $\hat{x}$)

η_0: impedance of free space

I_{in}: input intensity

Output intensity: $I_{out} = \dfrac{J_{out,1}^2 + J_{out,2}^2}{2\eta_0} = I_{in}\cos^2\theta\,[\cancel{\cos^2\theta + \sin^2\theta}]$ 1

Transmittance: $T = \dfrac{I_{out}}{I_{in}} = \cos^2\theta$ ✓

7.1B If the 1^{st} polarizer transmits vertical polarization and the angle between the 1^{st}-polarizer transmission axis and the $\frac{1}{2}$-wave-plate fast axis is 45°, the optical wave after the wave plate has horizontal polarization. Therefore, the output intensity in this measurement should again vary with the orientation angle of the 2^{nd}-polarizer transmission axis according to Malus' law (same as in 7.1A, but with the angle of maximum transmission shifted by 90°)

7.1 C) If the angle between the 1^{st} polarizer transmission axis and the $\frac{1}{4}$-wave-plate fast axis is $45°$, the optical wave after the wave plate has circular polarization.

Therefore, the output intensity in this measurement should not vary with the orientation angle of the 2^{nd} polarizer transmission axis (because for circularly polarized light, the x and y components of the electric field have equal intensity)

To discriminate between circularly polarized and unpolarized light, you can use a $\frac{1}{4}$-wave plate followed by a linear polarizer. In this system, the optical wave after the wave plate is $\begin{Bmatrix}\text{linearly polarized}\\ \text{unpolarized}\end{Bmatrix}$ if the input light is $\begin{Bmatrix}\text{circularly polarized}\\ \text{unpolarized}\end{Bmatrix}$, and therefore its transmitted intensity through the polarizer $\begin{Bmatrix}\text{varies}\\ \text{remains constant}\end{Bmatrix}$ as the polarizer is rotated.

A wave plate with fast and slow axes along $\hat{x}$ and $\hat{y}$, respectively, consists of an anisotropic crystal with $n_x < n_y$

$n_{\genfrac{}{}{0pt}{}{x}{y}}$: refractive index for $\genfrac{}{}{0pt}{}{x}{y}$-polarized light

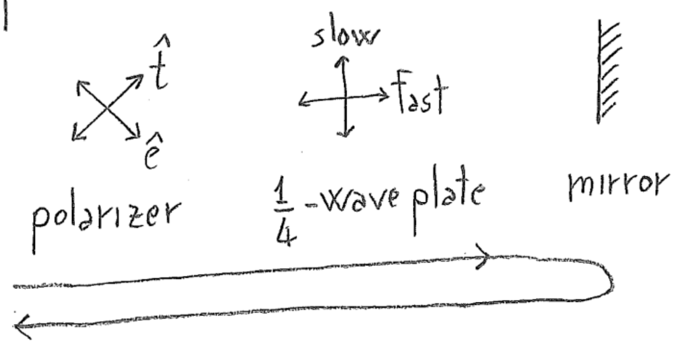
As light propagates through the wave plate, a phase difference $\Gamma = \frac{2\pi}{\lambda}(n_y - n_x)L$ is introduced between its y and x components

$\hookrightarrow$ wave-plate thickness

For a $\left\{\begin{array}{c} 1/4 \\ 1/2 \end{array}\right\}$-wave plate, $\Gamma = \left\{\begin{array}{c} \pi/2 \\ \pi \end{array}\right\}$ at the design wavelength λ

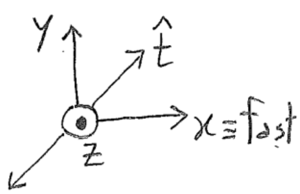
If you launch into either device a linearly polarized wave at a drastically different wavelength, the transmitted light will generally be elliptically polarized

7.2 | Optical isolator:

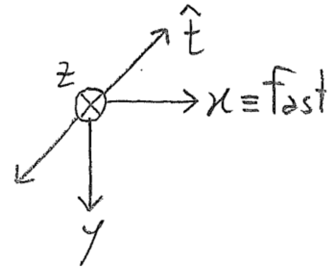


Here it is convenient to select the x and y axes along the fast and slow axes of the $\frac{1}{4}$-wave plate, and to flip the direction of the z axis for the reflected wave (then the direction of the y axis must be flipped too to maintain the correct handedness of the coordinate system)

$\Rightarrow$ for the incident light:



for the reflected light:



Jones matrix for the whole system: $T_{tot} = T_5 T_4 T_3 T_2 T_1$

$$T_1 = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}_{\theta=45^\circ} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$: forward transmission through polarizer

$$T_2 = T_4 = \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}$$: forward and backward transmission through $\frac{1}{4}$-wave plate

$$T_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$: mirror reflection (the sign of the y component is flipped because of the different coordinate system)

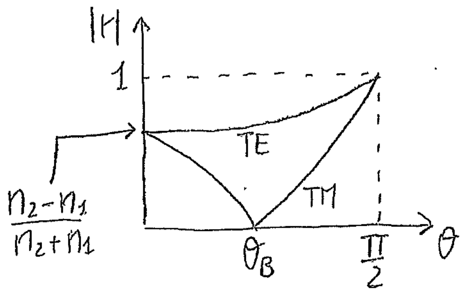
$$T_5 = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}_{\theta=-45^\circ} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$: backward transmission through polarizer

$\theta = -45^\circ$ because of the flipped y axis

$$T_{tot} = T_5 T_4 T_3 \begin{bmatrix} 1/2 & 1/2 \\ j/2 & j/2 \end{bmatrix} = T_5 T_4 \begin{bmatrix} 1/2 & 1/2 \\ -j/2 & -j/2 \end{bmatrix} = T_5 \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ for any $\vec{J}_{in}$, $\vec{J}_{out} = T_{tot} \vec{J}_{in} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$ the reflected intensity is zero

7.3A As shown in class, for "external" reflection $(n_2 > n_1)$ the amplitude reflection coefficients vary with angle of incidence as follows:



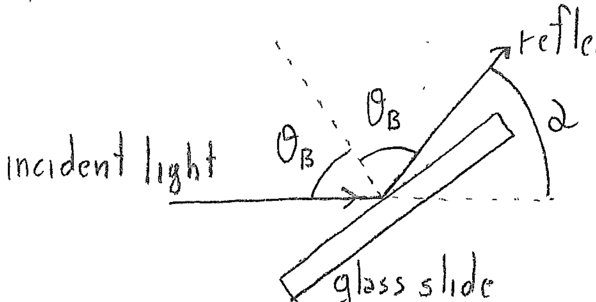
$\theta_B = \operatorname{atan}\frac{n_2}{n_1}$

If the incident light is unpolarized (like the output light of the HeNe lasers in the lab), $|r|^2 = \frac{1}{2}\left(|r_{TE}|^2 + |r_{TM}|^2\right)$, and the variations in reflected light intensity with angle of incidence are too small to be observable with the naked eye

7.3B If the light incident on the glass slide is vertically (i.e., TE) polarized, the reflected intensity is proportional to $|r_{TE}|^2$ and the resulting variations with θ_1 are again too small to observe

7.3C For horizontal (i.e., TM) polarization, the reflected light intensity is proportional to $|r_{TM}|^2$ and therefore vanishes for $\theta_1 = \theta_B$

$\Rightarrow$ the reflected beam disappears when the angle α (defined below) is equal to $180° - 2\theta_B$



For $n_1 = 1$ (air) and $n_2 = 1.52$ (glass), $\theta_B = \operatorname{atan} \frac{n_2}{n_1} = 56.7°$

$\Rightarrow$ the measured value of α where the reflected beam disappears should be 67°

If the incident light is unpolarized, the reflected light at the Brewster angle is TE polarized (because the TM component is not reflected)