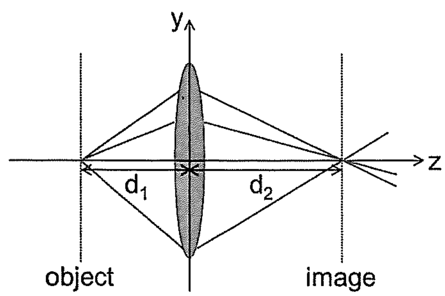
Quiz

PROBLEM 1

(a) Calculate the ray transfer matrix that describes light propagation from the object plane to the image plane in the figure below, in terms of the lens focal length f and the distances d_1 and d_2.



NOTE: the ray transfer matrices of a homogeneous medium of length d and of a thin lens of focal length f are given by the following formulas:

$$M_d = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \qquad M_{lens} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ -1/f & 1 - d_1/f \end{bmatrix}$$

$$M = \begin{bmatrix} 1 - d_2/f & d_1 + d_2 - d_1 d_2/f \\ -1/f & 1 - d_1/f \end{bmatrix}$$

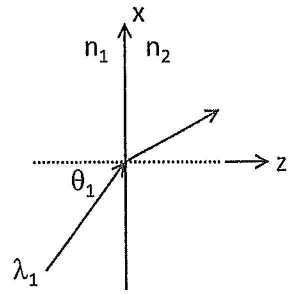
(b) Use the results of (a) to derive the thin-lens equation that relates d_1 and d_2 to f.

$$M \begin{bmatrix} 0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix} \quad \forall \theta_1 \Rightarrow M_{12} = d_1 + d_2 - \frac{d_1 d_2}{f} = 0$$

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

PROBLEM 2

In the figure below, a harmonic plane wave crosses a dielectric interface between two different media of refractive index $n_1 = 2$ and $n_2 = 3$. The incident light is linearly polarized along the y direction (perpendicular to the plane of the plot) and its wavelength, in the medium where it propagates, is $\lambda_1 = 900$ nm. The angle of incidence is $\theta_1 = 30°$.



Free-space wavelength
$\lambda_0 = n_1 \lambda_1 = 1800\,nm$

Compute the following quantities of the transmitted wave:

(a) frequency

$$\nu = \frac{c_0}{\lambda_0} = \frac{c_0}{n_1 \lambda_1} = 1.67 \times 10^{14}\,Hz$$

(b) wavenumber $K_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi n_2}{\lambda_0} = \frac{2\pi n_2}{n_1 \lambda_1} = 10.47\,rad/\mu m$

(c) wavevector (relative to the system of coordinates in the figure)

$$\vec{K}_2 = K_2 \left[\hat{x} \sin\theta_2 + \hat{z} \cos\theta_2\right] \qquad \vec{K}_2 = \hat{x}\,3.49 + \hat{z}\,9.87\,rad/\mu m$$

$$\theta_2 = \arcsin\left(\frac{n_1 \sin\theta_1}{n_2}\right) = 19.47°$$

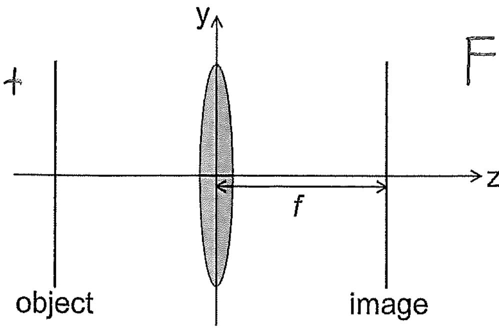
(d) state of polarization (relative to the system of coordinates in the figure)

linear along y (TE)

PROBLEM 3

In the figure below, a harmonic wave of wavelength λ_0 propagates from the object plane to the image plane across a thin lens of focal length f in free space. The complex amplitude on the object plane is $f(x,y) = A[2 + \cos(2\pi x/\Lambda)]$, where A and Λ are two given constants. The image plane coincides with the back focal plane of the lens.

$$f(x,y) = 2A + \frac{A}{2}\left[e^{j2\pi x/\Lambda} + e^{-j2\pi x/\Lambda}\right]$$



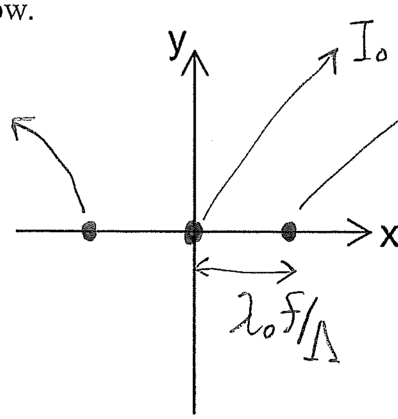
$$F(\nu_x,\nu_y) = 2A\,\delta(\nu_x) + \frac{A}{2}\,\delta\left(\nu_x - \frac{1}{\Lambda}\right) + \frac{A}{2}\,\delta\left(\nu_x + \frac{1}{\Lambda}\right)$$

(a) The intensity distribution on the image plane consists of a set of discrete spots. Draw these spots in the figure below.

$\frac{A^2}{4} = \frac{I_0}{16}$

$I_0 = 4A^2$

$A^2/4 = I_0/16$



only 3 spots

(b) What is the relative distance between neighboring spots?

see figure above

(c) If the central spot has intensity I_0, what are the intensities of all the other spots?

see figure above

PROBLEM 4

Consider a polarization device described by the Jones matrix $T = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. If the incident light is linearly polarized along the direction on the x-y plane at an angle of +45° with respect to the x axis, what is the state of polarization of the transmitted wave?

$$J_{out} \propto \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$: linearly polarized along the y direction

PROBLEM 5

In polarized sunglasses, each lens contains a linear polarizer designed to attenuate sunlight reflected from any large horizontally-oriented flat surface (such as a paved road or the surface of the ocean). What is the orientation of the extinction axis of these polarizers, horizontal or vertical? Make sure to explain your answer.

Reflected sunlight from an horizontally-oriented flat surface is mostly TE (horizontally) polarized

$\Rightarrow$ The extinction axis should be oriented horizontally

PROBLEM 6: What was your favorite lab assignment this semester so far?