# EC591 Lab 5 Report Notes

**5.1A** The light pattern on the screen after the diffraction grating consists of an horizontal series of bright spots. The 1st order of diffraction is the 1st spot displaced from the center. Its angle of propagation relative to the rail is

$$\theta \approx \frac{\lambda}{\Lambda}$$

$\lambda \rightarrow$ wavelength (633 nm)

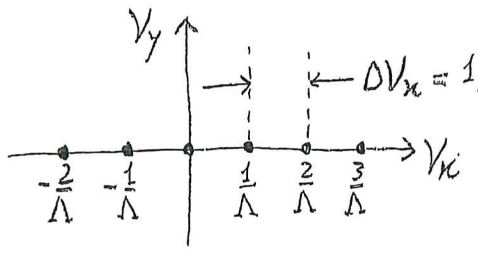
$\Lambda \rightarrow$ grating period

| $\Lambda$ | $\theta$ |
|---|---|
| $\frac{1\,mm}{600}$ | 21.8° |
| $\frac{1\,mm}{300}$ | 10.9° |

**5.2A** Here the field distribution of the object $f(x,y)$ is constant with $y$ and periodic with $x$ with period $\Lambda = \frac{1\,mm}{80}$ (because of the $x$-periodic transmittance of the grating)

$\Rightarrow$ the Fourier components of $f(x,y)$ have frequencies $\begin{cases} \nu_x = m/\Lambda & \forall \text{ integer } m \\ \nu_y = 0 \end{cases}$

On the $\nu_x$-$\nu_y$ plane, $|F(\nu_x,\nu_y)|^2$ looks like this:



The intensity distribution on the image plane is $I(x,y) \propto \left|F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)\right|^2$

Therefore, it consists of an horizontal series of bright spots separated by $\Delta x = \lambda f \Delta \nu_x = \frac{\lambda f}{\Lambda} = 15.2$ mm for $f = 300$ mm

5.2B| Here $f(x,y)$ is a periodic function of $x$ and $y$ with the same period $\Lambda$ along both directions.

By the same arguments above, $I(x,y)$ consists of a square periodic array of spots with period $P = \frac{\lambda f}{\Lambda}$

| wire screen | P1 | P2 | P3 |
|---|---|---|---|
| $\Lambda$ ($\mu$m) | 127 | 78 | 51 |
| P (mm) | 1.5 | 2.4 | 3.7 |

NOTE: the relative brigthness of the different spots depends on the detailed $x$ and $y$ variations of $f(x,y)$.

5.2C| Here $f(x,y) = \begin{cases} f_0 \neq 0 & \text{for } \rho = \sqrt{x^2+y^2} < \frac{D}{2} \\ 0 & \text{otherwise} \end{cases}$

Its Fourier transform is $F(\nu_x, \nu_y) \propto \frac{J_1(\pi D \nu_\rho)}{\nu_\rho}$

→ Bessel function of order 1

$\nu_\rho \to \sqrt{\nu_x^2+\nu_y^2}$

$\Rightarrow |F(\nu_x, \nu_y)|^2$ consists of a center disc of radius $\nu_\rho|_0 = \frac{1.22}{D}$

[because the first zero of $J_1(2\pi x)$ occurs at $x_0 = 0.61$]

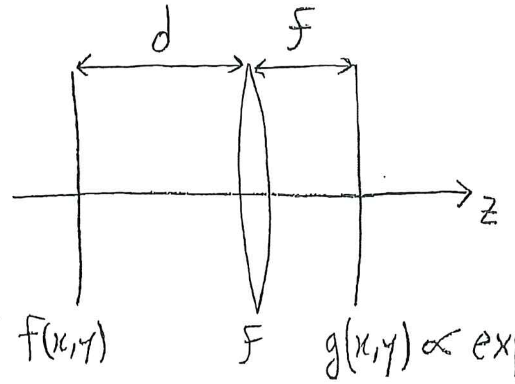
surrounded by weaker concentric rings

$\Rightarrow$ the center disc of the intensity distribution on the image plane

is $\rho_0 = \lambda f \nu_\rho|_0 = \frac{1.22 \lambda f}{D} = 2.3$ mm for $D = 100\,\mu m$

5.3 Why does the distance d between the object and the lens matter in a 4f system?

From the lecture:




$g(x,y) \propto \exp\left[j\pi \frac{(x^2+y^2)(d-f)}{\lambda f^2}\right] F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$

If $d \neq f$, $g(x,y)$ is no longer proportional to $F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$ and therefore $\mathcal{F}[g(x,y)]$ (the field distribution on the output plane of the 4f system without a mask) is no longer proportional to $f(-x,-y)$

5.3A] Typical results look as follows:

Image of P0 without mask

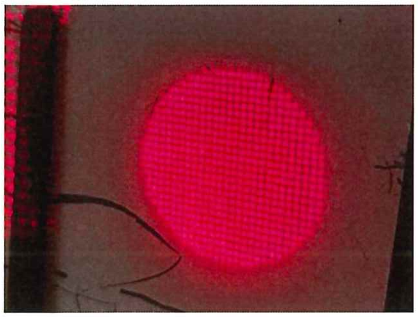


Image of P0 with mask

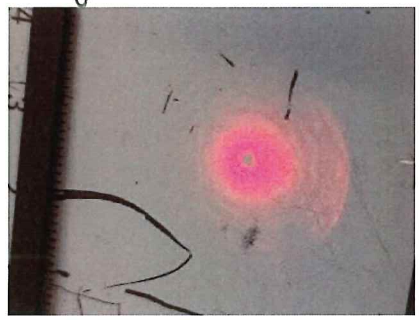


Image of P4 without mask

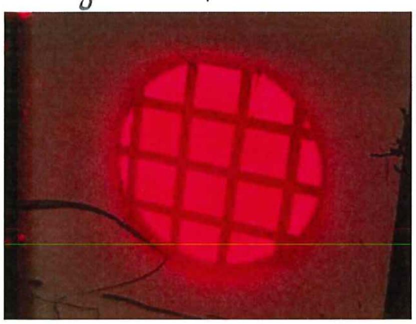
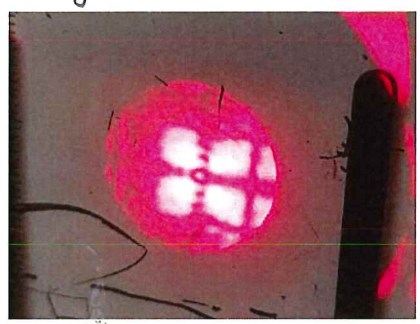


Image of P4 with mask



A pinhole of diameter D in the Fourier plane of a 4-f system acts as a low-pass filter with cutoff frequency $\nu_c = \frac{D}{2\lambda f}$ (where $f$ is the focal length of the first lens in the system) $\Rightarrow$ it only transmits frequency components of the object for which $\nu_x^2 + \nu_y^2 < \nu_c^2$

(*) For P0, the periodic pattern is completely lost in the image with the mask, indicating that all its frequency components are larger than $\nu_c$

$$\Rightarrow q\frac{1}{\Lambda} > \frac{D}{2\lambda f} \qquad \Lambda < \frac{2\lambda f}{D} = 1.1\text{ mm}$$

$q$: 1, 2, 3, ...

$\Lambda$: period of P0

$f$: 90 mm

$D$: 100 $\mu$m

(*) For P4, the periodic pattern is blurred but still recognizable in the image with the mask, indicating that its lowest frequency component $\left(\frac{1}{\Lambda}\right)$ is smaller than $\nu_c$ $\Rightarrow \Lambda > 1.1$ mm

5.3B

Image of P4 without mask

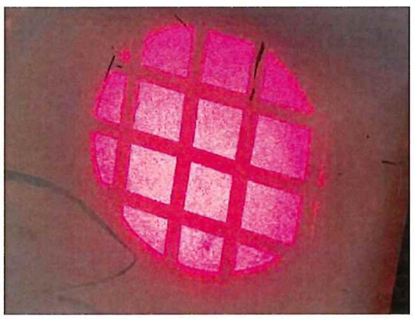
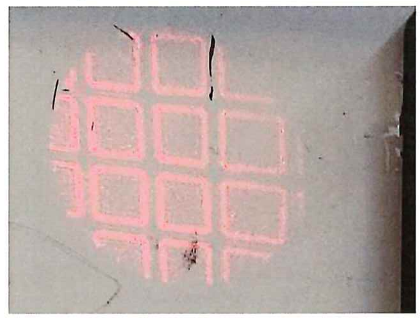


Image of P4 with mask



A transparent slide with an opaque disc of diameter D at its center acts as a high-pass filter with cutoff frequency $\nu_c = \frac{D}{2\lambda f}$ $\Rightarrow$ it only transmits high-frequency components with $\nu_x^2 + \nu_y^2 > \nu_c^2$

$\Rightarrow$ in the filtered image, the edges are enhanced relative to the slowly varying features, as illustrated in the pictures above

5.3.c)

Image of PO without mask

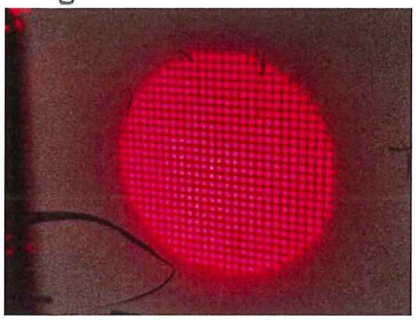
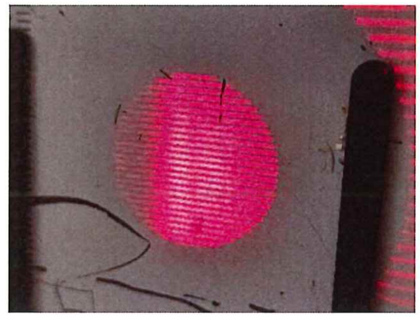


Image of PO with mask



A vertically oriented slit in an otherwise opaque screen filters out the periodic variations in the object field distribution with $x$ and transmits the periodic variations with $y$ (where $x$ and $y$ are in the horizontal and vertical directions, respectively)

$\Rightarrow$ therefore, the filter in this measurement blocks the vertical lines in PO and only the horizontal lines are observed