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Stats Vocab

1.1 Definition

The mean of a sample of n measured responses $y_1, y_2, \ldots, y_n$ is given by

The corresponding population mean is denoted µ

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

1.2 Definition

The variance of a sample of measurements $y_1, y_2, \ldots, y_n$ is the sum of the square of the differences between the measurements and their mean, divided by n − 1. Symbolically, the sample variance is

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2$$

The corresponding population variance is denoted by the symbol σ2

1.3 Definition

The standard deviation of a sample of measurements is the positive square root of the variance; that is,

$$s = \sqrt{s^2}$$

The corresponding population standard deviation is denoted by σ = √ σ2

2.2 Definition

A simple event is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point. The letter E with a subscript will be used to denote a simple event or the corresponding sample point.

2.3 Definition

The sample space associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S

2.4 Definition

A discrete sample space is one that contains either a finite or a countable number of distinct sample points.

2.5 Definition

An event in a discrete sample space S is a collection of sample points—that is, any subset of S.

2.6 Definition

Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S), we assign a number, P(A), called the probability of A, so that the following axioms hold:

Axiom 1: P(A) ≥ 0.

Axiom 2: P(S) = 1.

Axiom 3: If A_1, A_2, A_3,... form a sequence of pairwise mutually exclusive events in S (that is, $A_i \cap A_j$ = ∅ if i = j), then

$$P(A_1 U A_2 U A_3 \, U \, . \; . \; .) = \sum_{i=1}^{\infty} P(A_i)$$

2.1 Theorem

With m elements A_1, A_2,..., am and n elements B_1, B_2,..., B_n, it is possible to form mn = m × n pairs containing one element from each group.

2.7 Definition

An ordered arrangement of r distinct objects is called a **permutation**. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol p_r^n

2.2 Theorem

$$p_r^n = n(n-1)(n-2) \ldots (n-r+1) = \frac{n!}{(n-r)!}$$

We are concerned with the number of ways of filling r positions with n distinct objects. Applying the extension of the mn rule, we see that the first object can be chosen in one of n ways. After the first is chosen, the second can be chosen in (n − 1) ways, the third in (n − 2), and the rth in (n − r + 1) ways.

2.3 Theorem

The number of ways of partitioning n distinct objects into k distinct groups containing n_1, n_2,..., n_k objects, respectively, where each object appears in exactly one group and $\sum_{i=1}^{k} n_i = n$ is

$$N = \frac{n!}{n^1!\, n^2! \ldots\, n^k!}$$

2.4 Theorem

The number of unordered subsets of size r chosen (without replacement) from n available objects is

$$c_r^n = \frac{p_r^n}{r!} = \frac{n!}{r!\,(n-r)!}$$

2.9 Definition

The conditional probability of an event A, given that an event B has occurred, is equal to

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

provided P(B) > 0. [The symbol P(A|B) is read "probability of A given B."]

2.10 Definition

Two events A and B are said to be independent if any one of the following holds:

P(A|B) = P(A)

P(B|A) = P(B)

P(A ∩ B) = P(A) (P(B)

Otherwise the events are said to be dependent

2.5 Theorem

The Multiplicative Law of Probability The probability of the intersection of two events A and B is

P(A ∩ B) = P(A) P(B|A)

= P(B) P(A|B)

If A and B are independent then

P(A ∩ B) = P(A) (P(B)

2.6 Theorem

The Additive Law of Probability The probability of the union of two events A and B is

P(A ∪ B) = P(A) + P(B) – P(A ⋂ B)

If A and B are mutually exclusive events, P(A ⋂ B) = 0 and

P(A ∪ B) = P(A) + P(B).

2.7 Theorem

If A is an event then

P(A) = 1 – P($\bar{A}$)

2.11 Definition

For some positive integer k, let the sets B_1, B_2 ,..., B_K be such that

1. S = B_1 ∪ B_2 ∪ … ∪ B_K .

2. B_I∩ B_J = ∅, for i ≠ j.

Then the collection of sets {B1, B2,..., Bk } is said to be a partition of S

2.8 Theorem

Assume that { B_1, B_2 ,..., B_K } is a partition of S (see Definition 2.11)

such that P(B_i) > 0, for i = 1, 2,..., k. Then for any event A

$$P(A) = \sum_{i=1}^{n} \mathrm{P}(\mathrm{A}|B_i)\ \mathrm{P}(B_i)$$

2.9 Theorem

Assume that {B_1, B_2,..., B_k } is a partition of S (see Definition 2.11)

such that P(B_i) > 0, for i = 1, 2,..., k. Then,

$$P(B_J \mid A) = \frac{P\left(A\middle|B_J\right)P(B_J)}{\sum_{i=1}^{k} P(A|\,B_I)\,P(B_I)}$$

2.12 Definition

A random variable is a real-valued function for which the domain is a sample space.

2.13 Definition

Let N and n represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the $\binom{N}{n}$ samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a random sample.

3.1 Definition

A random variable Y is said to be discrete if it can assume only a finite or countably infinite1 number of distinct values.

3.1 Definition

The probability that Y takes on the value y, P(Y = y), is defined as the sum of the probabilities of all sample points in S that are assigned the value y. We will sometimes denote P(Y = y) by p(y).

3.1 Definition

3 The probability distribution for a discrete variable Y can be represented by a formula, a table, or a graph that provides p(y) = P(Y = y) for all y.

3.1 Theorem

For any discrete probability distribution, the following must be true:

1. 0 ≤ p(y) ≤ 1 for all y.
2. $\sum$, y p(y) = 1, where the summation is over all values of y with nonzero probability

3.4 Definition

Let Y be a discrete random variable with the probability function p(y). Then the expected value of Y , E(Y), is defined to be^2

E(Y) = $\sum_y yp(y)$

3.2 Theorem

Let Y be a discrete random variable with probability function p(y) and g(Y) be a real-valued function of Y. Then the expected value of g(Y) is given by

$$E[g(y)] = \sum_{all\ y} g(y)p(y)$$

3.5 Definition

If Y is a random variable with mean E(Y) = μ, the variance of a random variable Y is defined to be the expected value of $(Y - \mu)^2$. That is,

$$V(Y) = E[(Y - \mu)^2].$$

The standard deviation of Y is the positive square root of V(Y)

3.3 Theorem

Let Y be a discrete random variable with probability function p(y) and c be a constant. Then E(c) = c.

3.4 Theorem

Let Y be a discrete random variable with probability function p(y), g(Y) be a function of Y , and c be a constant. Then

$$E[g(Y)] = cE[g(Y)]$$

3.5 Theorem

Let Y be a discrete random variable with probability function p(y) and g1(Y), g2(Y), . . . , gk (Y) be k functions of Y . Then

$$E[g_1(Y) + g_2(Y) + \cdots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + \cdots + E[g_k(Y)]$$

3.2 Theorem

Let Y be a discrete random variable with probability function p(y) and mean E(Y) = μ; then

$$V(Y) = \sigma^2 = E[(Y - \mu)^2] = E(y^2) - \mu^2 .$$

3.6 Definition

A binomial experiment possesses the following properties:

1. The experiment consists of a fixed number, n, of identical trials.
2. Each trial results in one of two outcomes: success, S, or failure, F.
3. The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to q = (1 − p).
4. The trials are independent.
5. The random variable of interest is Y , the number of successes observed during the n trials.

3.7 Definition

A random variable Y is said to have a binomial distribution based on n trials with success probability p if and only if

p(y) = $\binom{n}{y} p^y q^{n-y}$,   y = 0, 1, 2,..., n and 0 ≤ p ≤ 1.

3.7 Theorem

Let Y be a binomial random variable based on n trials and success probability p. Then

$\mu = E(Y) = np$ and $\sigma^2 = V(Y) = npq$.

3.8 Definition

A random variable Y is said to have a geometric probability distribution if and only if

p(y) = q^{y-1}p,   y = 1, 2, 3,..., 0 ≤ p ≤ 1.

3.8 Theorem

If Y is a random variable with a geometric distribution

= E(Y) = $\frac{1}{p}$ and σ^2 = V(Y) = $\frac{1-p}{p^2}$.

3.9 Definition

A random variable Y is said to have a negative binomial probability distribution if and only if

p(y) = $\binom{y = 1}{r = 1}$ $p^r q^{y-r}$,   y = r, r + 1, r + 2,..., 0 ≤ p ≤ 1.

3.9 Definition

If Y is a random variable with a negative binomial distribution

μ = E(Y) = $\frac{r}{p}$ and σ^2 = V(Y) = $\frac{r(1-p)}{p^2}$.