

Algorithms for Big Data

Fall Semester 2019

Exercise Set 6

Definition 1 (Hadamard matrix) *We define $H_1 = [1]$ and $H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix}$. We will write $F = \frac{1}{\sqrt{n}}H_n$, dropping n from the index (and assuming n is a power of two).*

Exercise 1:

Show that $\|Fx\|_2 = \|x\|_2$ for any $x \in \mathbb{R}^n$.

Exercise 2:

Show that $F \times F = I$.

Exercise 3:

Show algorithm that given $x \in \mathbb{R}^n$ computes Fx in time $\mathcal{O}(n \log n)$.