

**Algorithms for Big Data**

Fall Semester 2019

**Exercise Set 6**

**Definition 1 (Hadamard matrix)** We define  $H_1 = [1]$  and  $H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix}$ . We will write  $F = \frac{1}{\sqrt{n}}H_n$ , dropping  $n$  from the index (and assuming  $n$  is a power of two).

**Exercise 1:**

Show that  $\|Fx\|_2 = \|x\|_2$  for any  $x \in \mathbb{R}^n$ .

**Exercise 2:**

Show that  $F \times F = I$ .

**Exercise 3:**

Show algorithm that given  $x \in \mathbb{R}^n$  computes  $Fx$  in time  $\mathcal{O}(n \log n)$ .