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Algorithms for Big Data

Fall Semester 2019

Exercise Set 6

Definition 1 (Hadamard matrix) *We define* $H_1 = \begin{bmatrix}1\end{bmatrix}$ *and* $H_{2n} = \begin{bmatrix}H_n & H_n \\ H_n & -H_n\end{bmatrix}$. *We will write* $F = \frac{1}{\sqrt{n}} H_n$, *dropping* n *from the index (and assuming* n *is a power of two).*

Exercise 1:
Show that $\|Fx\|_2 = \|x\|_2$ for any $x \in \mathbb{R}^n$.

Exercise 2:
Show that $F \times F = I$.

Exercise 3:
Show algorithm that given $x \in \mathbb{R}^n$ computes Fx in time $\mathcal{O}(n \log n)$.