

# Algorithms for Big Data

Fall Semester 2019

## Exercise Set 9

Recall Hadamard transform, given by a matrix  $H$ , such that  $H_{i,j} = \frac{1}{\sqrt{n}} \cdot (-1)^{\text{bc}(i \& j)}$ , where  $i \& j$  is bit-wise AND of binary representations, and  $\text{bc}(x)$  returns number of 1's in binary representation. Remember:  $H = H^{-1}$ , and assume  $n$  is power of two.

### Exercise 1:

Recall the tests for bits of  $u$  in algorithm for  $k = 1$  of Fourier transform:

$$b_i = 0 \quad \text{iff} \quad |a_r - a_{r+n/2^{i+1}}| \leq |a_r + a_{r+n/2^{i+1}}|$$

where  $r$  is randomly picked. Design analogous test for Hadamard transform.

### Exercise 2:

(2 pts)

Let  $(\hat{a}_0, \dots, \hat{a}_{n-1})$  be a Hadamard transform of  $(a_0, \dots, a_{n-1})$ . Let  $m \leq n$  be power of two as well. Let  $(b_0, \dots, b_{n-1})$  be a sequence such that for any  $0 \leq i < n/m$ ,  $(b_{im}, b_{im+1}, \dots, b_{im+m-1})$  is a Hadamard transform of  $(a_{im}, a_{im+1}, \dots, a_{im+m-1})$ .

Show that for any  $0 \leq j < m$ ,  $(b_j, b_{m+j}, b_{2m+j}, \dots, b_{n-m+j})$  is a Hadamard transform of  $(\hat{a}_j, \hat{a}_{m+j}, \hat{a}_{2m+j}, \dots, \hat{a}_{n-m+j})$ . (Keep in mind those transforms are of smaller dimension.)

### Exercise 3:

Using previous exercise, design sparse Hadamard transform algorithm (it's almost 1-1 equivalent to one from lecture).