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Algorithms for Big Data

Fall Semester 2019

Exercise Set 7

Exercise 1:
Minimize the value of $\sum_i \frac{\alpha_i^2}{p_i}$ conditioned on $\sum_i p_1 = 1$ and $\forall_i p_i \geq 0$. Find values of p_i that realize this minimum.

Exercise 2: (2+1 pts)
Show existence of ε-net of S_{n-1} with size $c = (\mathcal{O}(1/\varepsilon))^n$. Hint: analyze greedy algorithm, and show that it induces packing of c disjoint balls of radius $\varepsilon/2$ in ball of radius $1 + \varepsilon/2$. Bonus points for showing why a clever geometric construction is not sufficient and produces suboptimal size.

Exercise 3: (1+2+1)
Show following identities:

- $\|A\|_F = \sqrt{\text{trace}(A \cdot A^T)}$
- Sub-multiplicativity: $\|AB\|_F \leq \|A\|_F \|B\|_F$ [use Cauchy-Schwartz]
- For orthonormal U, there is $\|UA\|_F = \|A\|_F$.