

**Algorithms for Big Data**

Fall Semester 2019

**Exercise Set 8**

Consider a regression problem of

$$\arg \min_X \|AX - B\|_F \quad (1)$$

where  $A \in \mathbb{R}^{n \times d}$ ,  $B \in \mathbb{R}^{n \times m}$  and  $X \in \mathbb{R}^{d \times m}$ .

**Exercise 1:**

Show that  $X = A^\dagger B$  is a solution to (1).

**Exercise 2:**

Show that  $X$  from previous exercise minimizes  $\|X\|_F$  among all the solutions.

We move to low-rank approximation:

$$A_k = \arg \min_{B: \text{rank}(B) \leq k} \|A - B\|_F$$

**Exercise 3:**

Show that  $\Sigma_k$  (as defined on the lecture) is a low-rank approximation to  $\Sigma$  wrt to Frobenius norm (that is it solves the problem for diagonal matrices).

**Exercise 4:**

Use previous exercise to show that  $A_k = U\Sigma_k V^T$  is indeed low-rank approximation to  $A = U\Sigma V^T$ .

We move to Fourier transform. Let  $\omega = e^{-\frac{2\pi}{n}}$ . Let  $F$  be such that  $F_{ij} = \frac{1}{\sqrt{n}}\omega^{ij}$ . Then  $\hat{a} = Fa$  is a (Discrete) Fourier transform of  $a$ .

**Exercise 5:**

Show how to compute  $\hat{a}$  in time  $\mathcal{O}(n \log n)$  (you can assume  $n$  is a power of two).

**Exercise 6:**

Show that  $\|a\|_2 = \|\hat{a}\|_2$ .

**Exercise 7:**

Let  $\hat{a}_k$  is  $\hat{a}$  with all but  $k$  largest-magnitude coefficients zeroed. Show that  $a_k = F^{-1}\hat{a}_k$  is a solution to

$$\arg \min_{x: fs(x) \leq k} \|a - x\|_2$$

where  $fs(x) = \|\hat{x}\|_0$  is the size of Fourier support.