Przemysław Uznański 27/01/2020

Algorithms for Big Data

Fall Semester 2019

Exercise Set 14

Here is a formalization of MPC model (one of many possible, equivalent):

- Input size N, distributed among machines.
- Machine memory is $S = N^\alpha$ for some $0 < \alpha < 1$.
- Machines are numbered with unique ID's, $1 \mathinner{..} \frac{N}{S}$.
- After each round machines send messages addressed to other machines. Each machine can send $\mathcal{O}(S)$ atomic messages in total and receive $\mathcal{O}(S)$ atomic messages in total.

Exercise 1:
Maximum computation: input array $x[1 \mathinner{..} N]$. Output: $\max\{x[i]\}$ (on a single machine), in time $\mathcal{O}(\frac{1}{\alpha})$.

Exercise 2:
Broadcasting: input is one machine has a message m of size $\mathcal{O}(S)$. Output: all machines have m. Show $\mathcal{O}(\frac{1}{\alpha})$ algorithm.

Exercise 3:
Reason that broadcasting cannot be done faster, that is there is no $o(\frac{1}{\alpha})$ algorithm.

Exercise 4:
Prefix sums: input array $x[1 \mathinner{..} N]$. Output: array $y[1 \mathinner{..} N]$ where $y[i] = x[1] + \ldots + x[i]$. Time: $\mathcal{O}(\frac{1}{\alpha})$.

Exercise 5:
Offsets: input array $x[1 \mathinner{..} N]$ and S values $a_1, \ldots, a_S$. Output: values $j_1, \ldots, j_S$ where j_k is the position of a_k in sorted $x[1 \mathinner{..} N]$. Time: $\mathcal{O}(\frac{1}{\alpha})$.

Exercise 6:
Pivot: input array $x[1 \mathinner{..} N]$ and S values $a_1, \ldots, a_{S-1}$. Output: reshuffle x so that some prefix of machines holds all the values from x smaller than a_1, then next batch of machines holds all values from x between a_1 and a_2, etc. Time: $\mathcal{O}(\frac{1}{\alpha})$.

Exercise 7:
Sorting: input array $x[1 \mathinner{..} N]$. Output: x sorted. Time: $\mathcal{O}(\frac{1}{\alpha^2})$. Idea:

- Pick sample of size S.
- Use it as a pivot.
- Show that whp subproblems are of size $\widetilde{\mathcal{O}}(\frac{N}{\sqrt{S}})$.
- Recurse on subproblems.