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Algorithms for Big Data

Fall Semester 2019

Exercise Set 9

Recall Hadamard transform, given by a matrix H, such that $H_{i,j} = \frac{1}{\sqrt{n}} \cdot (-1)^{\mathrm{bc(i\ \&\ j)}}$, where $i\&j$ is bit-wise AND of binary representations, and $\mathrm{bc}(x)$ returns number of 1's in binary representation. Remember: $H = H^{-1}$, and assume n is power of two.

Exercise 1:
Recall the tests for bits of u in algorithm for $k = 1$ of Fourier transform:

$$b_i = 0 \qquad \text{iff} \qquad |a_r - a_{r+n/2^{i+1}}| \leq |a_r + a_{r+n/2^{i+1}}|$$

where r is randomly picked. Design analogous test for Hadamard transform.

Exercise 2: (2 pts)
Let $(\hat{a}_0, \ldots, \hat{a}_{n-1})$ be a Hadamard transform of $(a_0, \ldots, a_{n-1})$. Let $m \leq n$ be power of two as well. Let $(b_0, \ldots, b_{n-1})$ be a sequence such that for any $0 \leq i < n/m$, $(b_{im}, b_{im+1}, \ldots, b_{im+m-1})$ is a Hadamard transform of $(a_{im}, a_{im+1}, \ldots, a_{im+m-1})$.
Show that for any $0 \leq j < m$, $(b_j, b_{m+j}, b_{2m+j}, b_{n-m+j})$ is a Hadamard transform of $(\hat{a}_j, \hat{a}_{m+j}, \hat{a}_{2m+j}, \hat{a}_{n-m+j})$. (Keep in mind those transforms are of smaller dimension.)

Exercise 3:
Using previous exercise, design sparse Hadamard transform algorithm (it's almost 1-1 equivalent to one from lecture).