Przemysław Uznański 16/12/2019

Algorithms for Big Data

Fall Semester 2019

Exercise Set 10

Below we assume that $k \ll m \ll n$ to avoid annoying border-cases.

Exercise 1:
Show equivalence between k-disjoint set families and k-separable set families.

Exercise 2:
Show that any k-separable set family also separates I_1, I_2 such that $I_1 \neq I_2$ and I_2 can be arbitrarily large, while $|I_1| \leq k$.

Exercise 3:
Describe a decoding procedure for k-separable set family: given $\bigcup_{i \in I} F_i$, output I if $|I| \leq k$, and otherwise outputs that its not the case.

Exercise 4:
Let A be a k-separable matrix. Show a decoding procedure, that given Ax outputs x if x is k-sparse, and otherwise outputs that its not the case. Assume $x \geq 0$.

Exercise 5: (2 pts)
Assume k-separable family which has slow decoding. Show that it can be transformed into (suboptimal) k-separable family with $m' = \mathcal{O}(m \log n)$, and decoding time $\mathrm{poly}(m, k, \log n)$.