

Algorithms for Big Data

Fall Semester 2019

Exercise Set 5

Exercise 1:

Show how to use **CountMin** to support *range queries*: $\text{range}(a, b) = \sum_{i=a}^b x_i \pm \varepsilon |x|_1$. The space usage should be poly log worse than in the vanilla **CountMin**.

Hint: You can use a few **CountMin** structures to obtain datastructure with error guarantee slightly worse than in **CountMin**. You can later just take ε' to be a little smaller than ε to offset this.

Exercise 2:

(Quantiles)

ϕ -quantile of a multiset of size n is $\phi \cdot n$ -smallest element. ε -approximate ϕ -quantile is any element that is between $(\phi - \varepsilon)$ -quantile and $(\phi + \varepsilon)$ -quantile. Use previous exercise to build a sketch that allows a queries of form $\text{quantile}(\phi)$ for a fixed in advance ε .

Exercise 3:

(Sketching for inner product, 2pts)

Let X be a **CountMin** sketch of vector x , and Y be a **CountMin** sketch of vector y , with x and y being non-negative. Both sketches are obtained used the same hashing. Our goal is to approximate $x \odot y = \sum_i x_i y_i$. Show that $\min_j \sum_k X[j][i] \cdot Y[j][i]$ estimates $x \odot y$ up to $\pm \varepsilon |x|_1 |y|_1$ error.