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Anisotropic non-gaussianity with noncommutative spacetime

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ABSTRACT

We study single field inflation in noncommutative spacetime and compute two-point and three-point correlation functions for the curvature perturbation. We find that both power spectrum and bispectrum for comoving curvature perturbation are statistically anisotropic and the bispectrum is also modified by a phase factor depending upon the noncommutative parameters. The non-linearity parameter f_{NL} is small for small statistical anisotropic corrections to the bispectrum coming from the noncommutative geometry and is consistent with the recent PLANCK bounds. There is a scale dependence of f_{NL} due to the noncommutative spacetime which is different from the standard single field inflation models and statistically anisotropic vector field inflation models. Deviations from statistical isotropy of CMB, observed by PLANCK can tightly constraint the effects due to noncommutative geometry on power spectrum and bispectrum.

1. Introduction

Inflation [1] not only solves the various puzzles of the Big-Bang theory, but it also provides seeds for the temperature anisotropy of the cosmic microwave background (CMB) radiation and structures in the universe. In the standard inflationary scenario, the potential energy of a scalar field called "inflaton" dominates the energy density of the universe and its quantum fluctuations generate perturbations in the metric causing small inhomogeneities in the early universe which give rise to CMB anisotropy and structures in the universe.

Inflation predicts nearly scale invariant, adiabatic and gaussian perturbations. The first two are in excellent agreement with the observations of CMB anisotropy and polarization by COBE [2], WMAP [3–5] and other ground based and satellite based experiments, but the test of gaussian statistics of the perturbations is controversial and is the major goal of ongoing and future observations like PLANCK [6], CMBPOL [7] and Euclid satellite [8]. Recently released PLANCK data has tightened the bounds on non-gaussianity [9].

The non-gaussianity in CMB can be primordial or can be generated due to secondary sources (see [10] for detailed review). The primordial non-gaussianity arises due to the interaction terms in the scalar potential and non-linearities of the gravity, where the latter effect is dominant than the former. The magnitude of the non-gaussianity in standard single-field inflation comes out to be small and of the order of slow-roll parameters [11].

Inflation occurs at a very high energy and it stretches out very small scales, of the order of Planck length, to the current hubble scale due to superluminal expansion of the universe. So, it provides window to see the new physics at the Planck scale at which quantum corrections to the gravity becomes important. These new effects can significantly change the predictions of inflation that can be tested precisely by PLANCK experiment.

Spacetime noncommutativity (see [12] for review) is one of such modifications at high-energy, which is well motivated by quantum gravity and string theory. Modifications to the power spectrum of scalar perturbations during inflation and its effects on CMB due to noncommutative geometry has been studied in many places [13–17]. In this Letter we compute the three-point correlation functions of the curvature perturbation and hence the non-linearity parameter f_{NL} determining the primordial non-gaussianity using the noncommutative quantum field theories related to deformed Poincare symmetry [14]. In this approach of noncommutative geometry quantum fields follow twisted statistics, as implied by the deformed Poincare symmetry in quantum theories. Non-gaussianity due to noncommutative geometry has been studied earlier in [18,19], where the former is based on the models motivated by string theory and has considered the spacetime components of noncommutativity parameter to be zero to keep unitarily, while the latter has used the noncommutative spacetime with deformed Poincare symmetry as described in [14]. The computation of three-point function by Koivisto et al. [19] is based on the δN formalism [20], which is used to calculate the local non-gaussianity and treats the comoving curvature perturbations

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as classical. But, here we compute the two-point and three-point function using Maldacena approach [11] which is based on the second order perturbation theory and takes the gravitational back reaction into account.

As described in [14,19], the power spectrum of inflaton with noncommutative spacetime is direction dependent and can lead to the violation of statistical isotropy of CMB. PLANCK has seen some anomalies [21], specifically dipolar power modulation and hemispherical power asymmetry. Although the model studied in [14] cannot account for these anomalies, but generalization of it can lead to hemispherical power asymmetry [22].

The Letter is organized as follows. In Section 2, after discussing spacetime noncommutativity we review the expressions for deformed quantum fields and $\star$-product, described by Akofor et al. [14], that are used to compute two and three-point correlation functions. In Section 3, we review the calculation of second and third-order action for comoving curvature perturbation using ADM formalism and compute the power spectrum and three-point correlation function for the same in noncommutative Groenewold–Moyal plane. The expressions for the bispectrum and non-linearity parameter f_{NL} with the three-point function obtained in Section 3 are derived in Section 4 and there observational implications are also discussed. The conclusions are drawn in Section 5.

2. Quantum fields in noncommutative spacetime

At the energy scale of inflation, the noncommutativity of spacetime, which is motivated by Heisenberg uncertainty principle and Einstein's general relativity, can play a crucial role. The spacetime noncommutativity can be represented by the commutation relations [12]

$$[\tilde{x}_\mu, \tilde{x}_\nu] = i\theta_{\mu\nu} \tag{1}$$

where $\theta_{\mu\nu}$ is a real antisymmetric matrix with constant elements and $\tilde{x}_\mu$ are the coordinate functions of the chosen coordinate system:

$$\tilde{x}_\mu(x) = x_\mu. \tag{2}$$

The relation (1) holds only in special coordinate systems and looks quite complicated in other coordinates. The natural choice of the coordinate for cosmological applications is the comoving frame, where the galaxies are freely falling. This choice makes the time coordinate as the proper time measured by a clock at rest in any typical freely falling galaxy ($\vec{x}$ and t are thus comoving coordinates) and also simplifies the calculations.

Due to spacetime noncommutativity, the usual quantum fields are deformed and can be given in terms of undeformed quantum fields as [12]

$$\phi_\theta = \phi_0 e^{\frac{1}{2}\overleftarrow{\partial}\wedge P} \tag{3}$$

where $\overleftarrow{\partial} \wedge P = \overleftarrow{\partial_\mu}\theta^{\mu\nu}P_\nu$ and P_ν represents the field momentum operator. The product of the deformed (twisted) quantum fields at the same spacetime point is represented by the star product given as

$$(\phi_\theta \star \phi_\theta)(x) = \phi_\theta(x) e^{\frac{i}{2}\overleftarrow{\partial_x}\wedge\overrightarrow{\partial_y}} \phi_\theta(y)\big|_{x=y}. \tag{4}$$

In the following sections we will make use of these relations to calculate two and three-point correlation functions of the comoving curvature perturbations.

3. Two-point and Three-point correlation functions with noncommutative spacetime

3.1. Background

The action of a single scalar field minimally coupled with gravity is

$$S = \int d^4x \sqrt{-g}\left(\frac{{M_p}^2}{2}R + \mathcal{L}\right). \tag{5}$$

Here R is the Ricci scalar and $\mathcal{L}$ is the Lagrangian for the scalar field, i.e.

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{6}$$

Noncommutativity doesn't change the classical background so all the background dynamics will be similar to the standard case. We take the metric signature $(-,+,+,+)$ and work in the units where $M_p = 1$. The background geometry of the homogeneous isotropic universe is described by the FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right). \tag{7}$$

For a scalar field dominated universe the Friedmann equations are given as

$$\begin{aligned} &3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \\ &\dot{\rho} + 3H(\rho + p) = 0, \\ &\dot{H} = -\frac{1}{2}(\rho + p). \end{aligned} \tag{8}$$

The equation of motion for the scalar field is given as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{9}$$

During inflation, the potential energy of the scalar field dominates the total energy density of the universe and the dynamics of the scalar field is governed by slow-roll parameters defined as

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H}. \tag{10}$$

Here we follow the definition of η as in [23], which is different from the definition using scalar field potential ($\eta_V = \frac{d^2V}{d\phi^2}/V$) and $\eta = -2\eta_V + 4\epsilon$.

3.2. Perturbations and ADM formalism

The quantum fluctuations in the scalar field $\delta\phi(x,t)$ generated during inflation are coupled to the perturbations in the metric through Einstein's equation. Inflation gives rise to scalar and tensor perturbations in the metric and the scalar part is written as

$$\begin{aligned} ds^2 = {} & -(1+2\Phi)\,dt^2 + 2a^2(t)B_{,i}\,dx^i\,dt \\ & + a^2(t)\left((1-2\Psi)\delta_{ij} + 2E_{,ij}\right)dx^2\,dx^j. \end{aligned} \tag{11}$$

Here we have four scalar degrees of freedom in the metric and one in the scalar field which can be reduced to three by using gauge transformations. We can again use the constraint equations derived from the perturbed Einstein's equation and describe the scalar perturbations in terms of the curvature perturbation defined as [24]

$$\zeta = -\Psi - \frac{H}{\dot{\phi}}\delta\phi. \tag{12}$$

This variable is gauge invariant and is conserved on super-horizon scales. One can write the action (5) in terms of ζ and it turns out quadratic in ζ. To do the perturbation theory in higher order it is convenient to use ADM formalism where the metric can be written as [25]

$$ds^2 = -N^2\,dt^2 + h_{ij}\big(dx^i + N^i\,dt\big)\big(dx^j + N^j\,dt\big) \tag{13}$$

where N is laps function, N_i, N_j are shift vectors and h_{ij} is metric of three-dimensional hypersurface of constant time. Here N and N_i appear as Lagrangian multipliers in the action so one can solve their constraint equations and substitute the solution back into the action. This simplifies the tedious calculations needed while working with (11). Now we chose comoving gauge $\delta\phi = 0$ to do our calculation and in this gauge we can use non-linear generalization of ζ [26] and define the gauge as [11,23]

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij}, \qquad \delta\phi = 0. \tag{14}$$

With this gauge choice the action (5) with the metric (13) becomes

$$S = \frac{1}{2}\int dt\,d^3x\,\sqrt{h} \times \big(NR^{(3)} - 2NV(\phi) + N^{-1}\dot\phi^2 + N^{-1}\big(E_{ij}E^{ij} - E^2\big)\big). \tag{15}$$

Here $R^{(3)}$ represents the Ricci scalar calculated using the three-dimensional metric h_{ij} and E_{ij} is related to the extrinsic curvature of the constant time hypersurface and is given as

$$E_{ij} = \frac{1}{2}(\dot h_{ij} - \nabla_j N_i - \nabla_i N_j). \tag{16}$$

Varying the action (15) we get the constraint equation for N and N^i as

$$R^{(3)} - 2V - N^{-2}\big(E_{ij}E^{ij} - E^2\big) - N^{-2}\dot\phi^2 = 0,$$
$$\nabla_j\big[N^{-1}\big(E_i^j - \delta_i^j E\big)\big] = 0. \tag{17}$$

Now we can decompose N_i into irrotational and incompressible parts as $N_i = \tilde N_i + \partial_i\psi$ where $\partial_i\tilde N^i = 0$ and expand N, ψ and $\tilde N^i$ into powers of ζ as

$$N = 1 + \alpha_1 + \alpha_2 + \cdots,$$
$$\tilde N_i = \tilde N_i^{(1)} + \tilde N_i^{(2)} + \cdots,$$
$$\psi = \psi_1 + \psi_2 + \cdots. \tag{18}$$

Using these expansions, the constraint equations (17) can be solved order by order with metric (14) and at first order one gets

$$\alpha_1 = \frac{\dot\zeta}{H}, \qquad \tilde N_i^{(1)} = 0, \qquad \psi_1 = -\frac{\zeta}{H} + \chi,$$
$$\partial^2\chi = a^2\epsilon\dot\zeta. \tag{19}$$

Here $\partial^2 = \delta^{ij}\partial_i\partial_j$ and the use of suitable choice of boundary conditions has been made to put $N_i^{(1)} = 0$. As mentioned in [11,23] to calculate the action up to nth order in ζ, we need to calculate N and N_i only up to the order-ζ^{n-1} and here the terms of order-ζ^2 also drop out from the third order action, so Eq. (19) is sufficient to compute the action up to third order. So, after putting these solutions in (15) we get the action for second and third order in ζ as [11,27,23]

$$S_2 = \int dt\,d^3x\big[a^3\epsilon\dot\zeta^2 - a\epsilon(\partial\zeta)^2\big] \tag{20}$$

$$S_3 = \int dt\,d^3x\bigg[-a\epsilon\zeta(\partial\zeta)^2 - a^3\epsilon\dot\zeta^3 + 3a^3\epsilon\zeta\dot\zeta^2 + \frac{1}{2a}\bigg(3\zeta - \frac{\dot\zeta}{H}\bigg)\big(\partial_i\partial_j\psi\partial^i\partial^j\psi - \partial^2\psi\partial^2\psi\big) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi\bigg]. \tag{21}$$

3.3. Two-point correlation function and power spectrum

Now to calculate the two-point correlation function the quadratic part (20) of the action is considered, which in conformal time ($d\tau = \frac{dt}{a}$) can be written as

$$S_2 = \int d\tau\,d^3x\,a^2\epsilon\big[\zeta'^2 - (\partial\zeta)^2\big]. \tag{22}$$

Here $'$ denotes derivative w.r.t. conformal time τ. The above action looks like an action of a massless scalar field in conformal space-time and ζ can be considered as the scalar field for quantization. ζ can be written in terms of creation and annihilation operator as

$$\zeta(\vec x,\tau) = \int \frac{d^3k}{(2\pi)^3}\zeta(\vec k,\tau)e^{i\vec k\cdot\vec x} = \int \frac{d^3k}{(2\pi)^3}\big(u(\vec k,\tau)a_{\vec k} + u^\star(-\vec k,\tau)a^\dagger_{-\vec k}\big)e^{i\vec k\cdot\vec x}. \tag{23}$$

The equation of motion for ζ can be obtained by varying the action (22) and is given by

$$\zeta'' + 2\frac{z'}{z}\zeta' - \partial^2\zeta = 0. \tag{24}$$

Here $z^2 = 2a^2\epsilon$ and we can define $v_{\vec k} = z\zeta(\vec k,\tau)$ and use Eq. (23) to get

$$v''_{\vec k} + \bigg(k^2 - \frac{z''}{z}\bigg)v_{\vec k} = 0. \tag{25}$$

The solution for the mode functions $v_{\vec k}$ can be obtained assuming Bunch Davies initial conditions and is given as

$$v_{\vec k} = \frac{1}{\sqrt{2k}}\bigg(1 - \frac{i}{k\tau}\bigg)e^{-ik\tau}. \tag{26}$$

Hence the basis function $u(\vec k,\tau)$ is

$$u(\vec k,\tau) = \frac{v_{\vec k}}{z} = \frac{iH}{\sqrt{4\epsilon k^3}}(1 + ik\tau)e^{-ik\tau}. \tag{27}$$

The two point correlation function of the field ζ in position space can be expressed as

$$\big\langle\zeta(\vec x,\tau)\zeta(\vec y,\tau)\big\rangle = \int \frac{d^3k\,d^3k'}{(2\pi)^6}\langle 0|\zeta(\vec k,\tau)\zeta\big(\vec k',\tau\big)|0\rangle e^{i(\vec k\cdot\vec x + \vec k'\cdot\vec y)} \tag{28}$$

$$= \int \frac{d^3k}{(2\pi)^3}\big|u(\vec k,\tau)\big|^2 e^{i\vec k\cdot(\vec x - \vec y)}, \tag{29}$$

where we have used the relation $\langle 0|\zeta(\vec k,\tau)\zeta(\vec k',\tau)|0\rangle = (2\pi)^3\delta^3(\vec k + \vec k')u(\vec k,\tau)u^\star(-\vec k',\tau)$. The power spectrum for ζ is defined by

$$\langle 0|\zeta(\vec k,\tau)\zeta\big(\vec k',\tau\big)|0\rangle = (2\pi)^3\delta^3\big(\vec k + \vec k'\big)P_\zeta(k). \tag{30}$$

So

$$P_\zeta(k) = \left|u(\vec{k},\tau)\right|^2. \tag{31}$$

The another convention for the power spectrum, that is commonly used for data analysis, is

$$\Delta_\zeta^2 = \frac{k^3}{2\pi^2}\left|u(\vec{k},\tau)\right|^2. \tag{32}$$

In this case Δ_ζ represents the variance of the classical fluctuations and the two-point correlation in position space becomes

$$\left\langle \zeta(\vec{x},\tau)\zeta(\vec{y},\tau)\right\rangle = \int \frac{dk}{k}\Delta_\zeta^2 e^{i\vec{k}\cdot(\vec{x}-\vec{y})}. \tag{33}$$

The power spectrum is calculated on super-horizon limit, i.e. $-k\tau \ll 1$ in which $v_{\vec{k}} = \frac{1}{\sqrt{2k}}(-\frac{i}{k\tau})e^{-ik\tau}$ and we get the power spectrum as

$$P_\zeta(k) = \frac{H^2}{4\epsilon}\frac{1}{k^3}. \tag{34}$$

Now due to noncommutativity of spacetime the two point correlation function for field ζ gets modified [14]. We will denote the field in noncommutative spacetime with a subscript θ. Since here ζ represents our quantum field, hence similar to (3) the twisted quantum field ζ_θ can be expressed in terms of the untwisted field ζ as

$$\zeta_\theta(\vec{x},t) = \zeta(\vec{x},t)e^{\frac{1}{2}\overleftarrow{\partial}_\mu\wedge P_\nu}. \tag{35}$$

With the twisted quantum field one can compute the two-point correlation function in position space as

$$\begin{aligned}\left\langle \zeta_\theta(\vec{x},t)\zeta_\theta\left(\vec{y},t'\right)\right\rangle &= \left\langle \zeta(\vec{x},t)e^{\frac{1}{2}\overleftarrow{\partial}_{x_\mu}\wedge P_\nu}\zeta\left(\vec{y},t'\right)e^{\frac{1}{2}\overleftarrow{\partial}_{y_\mu}\wedge P_\nu}\right\rangle \\ &= \left\langle \zeta(\vec{x},t)\zeta\left(\vec{y},t'\right)\right\rangle e^{-\frac{i}{2}\overleftarrow{\partial}_{x_\mu}\wedge\vec{\partial}_{y_\nu}},\end{aligned} \tag{36}$$

where we have used the commutation relations between the field and the momentum operator $[P_\mu,\zeta] = -i\partial_\mu\zeta$. Now taking the Fourier transform on the right hand side we get

$$\begin{aligned}&\left\langle \zeta_\theta(\vec{x},t)\zeta_\theta\left(\vec{y},t'\right)\right\rangle \\ &= \int \frac{d^3k\,d^3k'}{(2\pi)^6}\langle 0|\zeta(\vec{k},t)\zeta\left(\vec{k}',t'\right)|0\rangle e^{-\frac{i}{2}\overleftarrow{\partial}_{x_\mu}\wedge\vec{\partial}_{y_\nu}}e^{i(\vec{k}\cdot\vec{x}+\vec{k}'\cdot\vec{y})} \\ &= \int \frac{d^3k\,d^3k'}{(2\pi)^6}\langle 0|\zeta(\vec{k},t)\zeta\left(\vec{k}',t'\right)|0\rangle e^{-\frac{i}{2}(\partial_t\theta^{0i}\partial_{\vec{y}}+\partial_{\vec{x}}\theta^{io}\partial_{t'}+\partial_{\vec{x}}\wedge\partial_{\vec{y}})} \\ &\quad\times e^{i(\vec{k}\cdot\vec{x}+\vec{k}'\cdot\vec{y})} \\ &= \int \frac{d^3k\,d^3k'}{(2\pi)^6}\langle 0|\zeta(\vec{k},t)\zeta\left(\vec{k}',t'\right)|0\rangle e^{(\frac{i}{2}\vec{k}\wedge\vec{k}'+\frac{\vec{\theta^0}\cdot\vec{k}'}{2}\partial_t-\frac{\vec{\theta^0}\cdot\vec{k}}{2}\partial_{t'})} \\ &\quad\times e^{i(\vec{k}\cdot\vec{x}+\vec{k}'\cdot\vec{y})} \\ &= \int \frac{d^3k\,d^3k'}{(2\pi)^6}\langle 0|\zeta\left(\vec{k},t+\frac{\vec{\theta^0}\cdot\vec{k}'}{2}\right)\zeta\left(\vec{k}',t'-\frac{\vec{\theta^0}\cdot\vec{k}}{2}\right)|0\rangle \\ &\quad\times e^{\frac{i}{2}\vec{k}\wedge\vec{k}'}e^{i(\vec{k}\cdot\vec{x}+\vec{k}'\cdot\vec{y})}.\end{aligned} \tag{37}$$

Here $\vec{\theta^0} = \theta^{0i}$. So the two-point correlation function in momentum space can be expressed as

$$\begin{aligned}&\langle 0|\zeta_\theta(\vec{k},t)\zeta_\theta\left(\vec{k}',t'\right)|0\rangle \\ &= e^{\frac{i}{2}\vec{k}\wedge\vec{k}'}\langle 0|\zeta\left(\vec{k},t+\frac{\vec{\theta^0}\cdot\vec{k}'}{2}\right)\zeta\left(\vec{k}',t'-\frac{\vec{\theta^0}\cdot\vec{k}}{2}\right)|0\rangle.\end{aligned} \tag{38}$$

Now since in de Sitter space

$$\tau(t) = \frac{1}{aH}e^{-Ht}. \tag{39}$$

So in conformal time and in the limit $t' \to t$

$$\zeta\left(\vec{k},t+\frac{\vec{\theta^0}\cdot\vec{k}'}{2}\right) \to \zeta\left(\vec{k},\tau e^{-H\frac{\vec{\theta^0}\cdot\vec{k}'}{2}}\right), \tag{40}$$

$$\zeta\left(\vec{k},t'-\frac{\vec{\theta^0}\cdot\vec{k}}{2}\right) \to \zeta\left(\vec{k},\tau e^{H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right). \tag{41}$$

Hence the two-point function will be

$$\begin{aligned}\left\langle \zeta_\theta(\vec{k},\tau)\zeta_\theta\left(\vec{k}',\tau\right)\right\rangle &= \langle 0|\zeta\left(\vec{k},\tau e^{-H\frac{\vec{\theta^0}\cdot\vec{k}'}{2}}\right)\zeta\left(\vec{k}',\tau e^{H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)|0\rangle e^{\frac{i}{2}\vec{k}\wedge\vec{k}'} \\ &= \left|u\left(\vec{k},\tau e^{H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2(2\pi)^3\delta^3\left(\vec{k}+\vec{k}'\right).\end{aligned} \tag{42}$$

Now we take the self-adjoint part of two-point correlation function defined as [14]

$$\begin{aligned}&\langle 0|\zeta_\theta(\vec{k},\tau)\zeta_\theta\left(\vec{k}',\tau\right)|0\rangle_M \\ &= \frac{1}{2}\left(\langle 0|\zeta_\theta(\vec{k},\tau)\zeta_\theta\left(\vec{k}',\tau\right)|0\rangle + \langle 0|\zeta_\theta(-\vec{k},\tau)\zeta_\theta\left(-\vec{k}',\tau\right)\right).\end{aligned} \tag{43}$$

So the power spectrum can be obtained from (30) as

$$P_{\zeta_\theta}(k) = \frac{1}{2}\left(\left|u\left(\vec{k},\tau e^{H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2 + \left|u\left(-\vec{k},\tau e^{-H\frac{\vec{\theta^0}\cdot\vec{k}}{2}}\right)\right|^2\right). \tag{44}$$

Now since $v_{\vec{k}} = z\zeta(\vec{k},\tau)$, the argument of $v_{\vec{k}}$ is shifted due to deformation of $\zeta(\vec{k},\tau)$ and the argument of the scale factor $a(\tau)$ and hence z is not shifted.

Since on super-horizon limit $v_{\vec{k}} = \frac{1}{\sqrt{2k}}(\frac{-i}{k\tau}e^{-H\frac{\vec{\theta^0}\cdot\vec{k}}{2}})$ so

$$P_{\zeta_\theta}(k) = P_\zeta(k)\cosh\left(H\vec{\theta^0}\cdot\vec{k}\right). \tag{45}$$

This power spectrum was derived in [14,19] and they showed that it can lead to the breaking of statistical isotropy of the CMB. Akofor et al. [15] tested the above power spectrum with WMAP5 [28], ACBAR [29] and CBI [30] data sets considering only the effects on *Cls* and ignoring the off-diagonal terms in $\langle a_{lm}a_{l'm'}\rangle$ correlations. As the effects of modifications to the power spectrum due to noncommutativity increase at small scales, it was concluded in [15] that WMAP5 data, which gives the power spectra for *Cls* up to $l = 1000$, is not sufficient to constrain the scale of noncommutativity. Doing a one-parameter χ^2 analysis with ACBAR and CBI data, which give CMB power spectra up to $l = 2958$ and $l = 3500$ respectively (but only for small scales), they claimed that $H\theta^0 < 0.01$ MPc (where θ^0 is the magnitude of the noncommutativity parameter $\vec{\theta^0}$). Recently PLANCK has released data for the CMB power spectra up to $l = 2500$ [31] with better precision and less systematic errors. Since there may be parameter degeneracy (for e.g. due to spectral index), we are planning to reanalyze the power spectrum (45) with the recently released PLANCK data by varying all parameters along with $H\theta^0$ to constraint the scale of noncommutativity.

The above power spectrum can be expanded in terms of $(H\vec{\theta^0}\cdot\vec{k})$ and keeping only the leading order term we get,

$$P_{\zeta_\theta}(k) = P_\zeta(k)\left(1 + \frac{(H\theta^0 k)^2}{2}\left(\hat{\theta^0}\cdot\hat{k}\right)^2\right), \tag{46}$$

here k denote the magnitudes of the wavenumber and $\hat{\theta^0}$ is a unit vector in the direction of $\vec{\theta^0}$ along which the rotational invariance is broken. A power spectrum of similar form was considered in [32] where a small non-zero vector was introduced to break the rotational invariance and the coefficient of the direction dependent term (denoted by $g(k)$ in [32]) was scale invariant. Groeneboom et al. [33] analyzed the power spectrum of [32] with WMAP5 data and obtained the bound $g = 0.29 \pm 0.031$ with the exclusion of $g = 0$ at 9σ by including the CMB multipoles up to $l = 400$. The result was contradicted by Hanson et al. [34] and they argued that the detection of non-zero g can be due to the beam asymmetry. Pullen and Hirata [35] re-analyzed the power spectrum of [32] with the large scale structure surveys and they obtained $g = 0.007 \pm 0.037$. The power spectra (45) and of Ackerman et al. [32] give rise to multipole alignments along the preferred direction. The quadrupole–octopole alignment was first reported by Tegmark et al. [36] using the WMAP first year data and was less significant in WMAP. With recently released PLANCK data, the significance for multipole alignment is even smaller than WMAP [21]. The off-diagonal terms in $\langle a_{lm} a_{l'm'}\rangle$ arising due to the power spectrum (45) can be described by bipolar spherical harmonics (BipoSH) [37] representing the modulation of the CMB power spectrum. PLANCK claims 3.7 to 2.9σ detection of dipole modulation (non-zero $L = 1$ BipoSH) but null result for higher multipoles of BipoSH. The power spectrum (45) can only give rise to even multipole BiopoSH so it cannot account for the observed dipole modulation of CMB. We will describe the modified three-point correlation function due to noncommutativity and its observational implications in the next sections.

3.4. Three-point function

The primordial non-gaussianity in CMB arises due to the non-zero three-point and four-point correlation functions of curvature perturbations. These correlation functions were calculated for non-commutative spacetime in [19], where they have used δN formalism which ignores modifications to the correlation functions at Hubble crossing and also interaction between quantum fluctuations on sub-hubble scales with the super-hubble scale fluctuations at non-linear label.

The third order action (21) obtained for ζ using ADM formalism is

$$S_3 = \int dt\, d^3x \bigg[-a\epsilon\zeta(\partial\zeta)^2 - a^3\epsilon\dot{\zeta}^3 + 3a^3\epsilon\zeta\dot{\zeta}^2 + \frac{1}{2a}\left(3\zeta - \frac{\dot{\zeta}}{H}\right)\left(\partial_i\partial_j\psi\partial^i\partial^j\psi - \partial^2\psi\partial^2\psi\right) - 2a^{-1}\partial_i\psi\partial_i\zeta\partial^2\psi\bigg]. \tag{47}$$

We put the value of ψ from Eq. (19) in this action, integrate by parts and use background Friedmann equations to get terms proportional to ϵ^2

$$S_3 = \int dt\, d^3x \bigg[a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}(\partial\zeta)(\partial\chi) + \frac{a^3\epsilon}{2}\frac{d\eta}{dt}\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)(\partial^2\chi) + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + \frac{1}{2}a\mathcal{F}\frac{\delta L}{\delta\zeta}\bigg|_1\bigg] \tag{48}$$

where $\mathcal{F} = (\eta\zeta^2 + \text{terms with derivatives of } \zeta)$ and $\frac{\delta L}{\delta\zeta}$ represents the terms proportional to the Gaussian action S_2. We can again integrate by parts the above action to remove the terms involving $\partial\chi$ and use the Gaussian field equation (24) to get

$$S_3 = \int dt\, d^3x \bigg[4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta} + \frac{1}{2}a\mathcal{F}\frac{\delta L}{\delta\zeta}\bigg|_1\bigg]. \tag{49}$$

Now $\mathcal{F} = (\eta - \epsilon)\zeta^2 + 2\epsilon\partial^{-2}(\zeta\partial^2\zeta)$ and ∂^{-2} is the inverse of ∂^2 and we have ignored the terms containing the derivatives of ζ in $\mathcal{F}$ as they are negligible on super-horizon scales. One can get rid of the second term in the above action following field redefinition

$$\zeta \to \zeta_n + \frac{\mathcal{F}}{4}\zeta_n^2. \tag{50}$$

After this field redefinition the three-point function becomes

$$\begin{aligned}\big\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\big\rangle &= \big\langle\zeta_n(x_1)\zeta_n(x_2)\zeta_n(x_3)\big\rangle \\ &\quad + \frac{(\eta-\epsilon)}{4}\Big(\big\langle\zeta_n(x_1)\zeta_n(x_2)\big\rangle\big\langle\zeta_n(x_1)\zeta_n(x_3)\big\rangle + \text{permutations}\Big) \\ &\quad + \frac{\epsilon}{2}\partial_{x_1}^{-2}\Big(\big\langle\zeta(x_1)\zeta(x_2)\big\rangle\partial_{x_1}^2\big\langle\zeta_n(x_1)\zeta_n(x_3)\big\rangle + \text{permutations}\Big).\end{aligned} \tag{51}$$

The first term in above expression represents the three-point function while the last two terms represents the corrections to the three-point function due to field redefinition. We will omit the subscript n in the following calculations. Now the interaction Hamiltonian to calculate the three-point function can be obtained from action (49), i.e.

$$\mathcal{H}(t') = -\int d^3x\, 4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta}. \tag{52}$$

As mentioned earlier, we use ζ as the quantum field to compute the various correlation functions. Hence, to see the effects of noncommutative geometry on three-point correlation function we replace the usual quantum field ζ with the twisted field ζ_θ both in the interaction Hamiltonian and in (51). Since the product of the twisted fields at the same spacetime point is given by the star-product [12], the interaction Hamiltonian will be given as

$$\mathcal{H}(t') = -\int d^3x\, 4a^5\epsilon^2 H\dot{\zeta}_\theta \star \dot{\zeta}_\theta \star \partial^{-2}\dot{\zeta}_\theta. \tag{53}$$

One important point to be mentioned here is that, in principle, we should replace ζ with ζ_θ and the product between them as star-product in Eq. (21), but since $\theta_{\mu\nu}$ is constant in comoving coordinates and hence the star-product of the deformed fields is associative [12], all the steps to reach to interaction Hamiltonian from the third-order action can be performed as in standard case and ζ can be replaced with ζ_θ in the final interaction Hamiltonian.

Using the relation (3) between the twisted and untwisted quantum field and expression for star-product (4), the interaction Hamiltonian becomes

$$\mathcal{H}(t') = -\int d^3x\, 4a^5\epsilon^2 H\dot{\zeta}^2\partial^{-2}\dot{\zeta}e^{\frac{1}{2}\overleftarrow{\partial_\mu}\wedge P_\nu}, \tag{54}$$

where $\overleftarrow{\partial_\mu} \wedge P_\nu = \overleftarrow{\partial_{x_\mu}}\theta^{\mu\nu}P_\nu$. The first term in the RHS of (51) is computed using the in–in formalism [38] and is given by

$$\begin{aligned}\big\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\big\rangle &= -i\int_{t_0}^{t} dt'\,\langle 0|\big[\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3), \mathcal{H}(t')\big]|0\rangle \\ &= -i\int_{t_0}^{t} dt'\,\Big(\langle 0|\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\mathcal{H}(t')|0\rangle \\ &\quad - \langle 0|\mathcal{H}(t')\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle\Big)\end{aligned} \tag{55}$$

Here the three-point function is calculated at equal time, i.e. $t_1 = t_2 = t_3 = t$. Initially we will write them differently for simplification but will put them equal before integration w.r.t. t'. Let us now consider the first term of above equation with (54) and name it as (a). So

$$(a) = 4i\epsilon^2 \int dt' a^5 H \int d^3x \, \langle 0|\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\dot{\zeta}^2\partial^{-2}\dot{\zeta}\big|_{t',\vec{x}} \times e^{\frac{1}{2}\overleftarrow{\partial_{x_\mu}}\wedge P_\nu}|0\rangle. \tag{56}$$

Using the relation (35) we can replace the twisted quantum fields in terms of the untwisted quantum fields and it gives

$$\begin{aligned}(a) &= 4i\epsilon^2 \int dt'\, a^5 H \int d^3x \, \langle 0|\zeta(x_1)\zeta(x_2)\zeta(x_3) \\ &\quad\times e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overleftarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_3}})} e^{\frac{1}{2}\overleftarrow{\partial_{x_1}}\wedge P} e^{\frac{1}{2}\overleftarrow{\partial_{x_2}}\wedge P} \\ &\quad\times e^{\frac{1}{2}\overleftarrow{\partial_{x_3}}\wedge P}\dot{\zeta}^2\partial^{-2}\dot{\zeta}\big|_{t',\vec{x}} e^{\frac{1}{2}\overleftarrow{\partial_x}\wedge P}|0\rangle \end{aligned} \tag{57}$$

$$\begin{aligned} &= 4i\epsilon^2 \int dt'\, a^5 H \int d^3x \, \langle 0|\zeta(x_1)\zeta(x_2)\zeta(x_3) \\ &\quad\times e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overleftarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_3}})} e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}})\wedge\overrightarrow{\partial_x}}\dot{\zeta} \\ &\quad\times e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}})\wedge\overrightarrow{\partial_x}}\dot{\zeta} e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}+\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_3}})\wedge\overrightarrow{\partial_x}}\partial^{-2}\dot{\zeta}|0\rangle. \end{aligned} \tag{58}$$

The above equation in Fourier space becomes

$$\begin{aligned}(a) &= -4i\epsilon^2 \int dt' a^5 H \int d^3x \int \prod_{i=1}^{6} \frac{d^3k_i}{k_6^2(2\pi)^{18}} e^{i(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2+\vec{k}_3\cdot\vec{x}_3)} \\ &\quad\times \langle 0|\zeta\left(\vec{k}_1, t_1 + \frac{\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\ &\quad\times \zeta\left(\vec{k}_2, t_2 + \frac{-\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_3+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\ &\quad\times \zeta\left(\vec{k}_3, t_3 + \frac{-\vec{\theta^0}\cdot\vec{k}_1-\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}\right) \\ &\quad\times \dot{\zeta}\left(\vec{k}_4, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}\right) \\ &\quad\times \dot{\zeta}\left(\vec{k}_5, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}\right) \\ &\quad\times \dot{\zeta}\left(\vec{k}_6, t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}\right)|0\rangle \\ &\quad\times e^{i(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x})} e^{\frac{i}{2}\mathcal{P}}. \end{aligned} \tag{59}$$

Here

$$\mathcal{P} = \big(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3 + (\vec{k}_1+\vec{k}_2+\vec{k}_3)(\vec{k}_4+\vec{k}_5+\vec{k}_6)\big). \tag{60}$$

Since we will express the three-point correlation function in momentum space, we can take the Fourier transform on both side of Eq. (55) and take the limit $t_1 = t_2 = t_3 = t$ to get

$$\begin{aligned}(a) &= -i\int_{t_0}^{t} dt' \big(\langle 0|\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\mathcal{H}(t')\big) \\ &= -4i\epsilon^2 \int dt' a^5 H \int d^3x \int \prod_{i=4}^{6} \frac{d^3k_i}{k_6^2(2\pi)^9} \\ &\quad\times \langle 0|\zeta(\vec{k}_1,t_1)\zeta(\vec{k}_2,t_2)\zeta(\vec{k}_3,t_3)\dot{\zeta}(\vec{k}_4,t_4)\dot{\zeta}(\vec{k}_5,t_5)\dot{\zeta}(\vec{k}_6,t_6)|0\rangle \\ &\quad\times e^{i(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x})} e^{\frac{i}{2}\mathcal{P}} \end{aligned} \tag{61}$$

where

$$\begin{aligned} t_1 &= t + \frac{\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}, \\ t_2 &= t + \frac{-\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_3+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}, \\ t_3 &= t + \frac{-\vec{\theta^0}\cdot\vec{k}_1-\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_4+\vec{\theta^0}\cdot\vec{k}_5+\vec{\theta^0}\cdot\vec{k}_6}{2}, \\ t_4 &= t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}, \\ t_5 &= t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}, \\ t_6 &= t' - \frac{\vec{\theta^0}\cdot\vec{k}_1+\vec{\theta^0}\cdot\vec{k}_2+\vec{\theta^0}\cdot\vec{k}_3}{2}. \end{aligned} \tag{62}$$

A detailed calculation of this term is presented in Appendix A and it is given as (A.11)

$$(a) = \epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2} \times \prod_{i=1}^{3}\frac{1}{k_i^3}\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}\big(k_1^2k_2^2+\text{perm.}\big). \tag{63}$$

Here $K = k_1 + k_2 + k_3$. Now similar calculations can be done for the second term in the three-point function (55). Let us represent it as (b),

$$(b) = i\int_{t_0}^{t} dt' \, \langle 0|\mathcal{H}(t')\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle. \tag{64}$$

The contribution due to this term in momentum space is given by (A.16)

$$(b) = \epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2} \times \prod_{i=1}^{3}\frac{1}{k_i^3}\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}\big(k_1^2k_2^2+\text{perm.}\big).$$

So the contribution to the three-point function of ζ due to the first term of (51) in Fourier space is given as

$$\begin{aligned}&\big\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\big\rangle \\ &\quad= 2\epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2} \\ &\quad\times \prod_{i=1}^{3}\frac{1}{k_i^3}\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}\big(k_1^2k_2^2+\text{perm.}\big). \end{aligned} \tag{65}$$

This concludes the calculations of the three-point function of the redefined field ζ_n. Now to get the final three-point function of the field ζ we need to consider the second and third term of Eq. (51) coming due to field redefinitions. The contribution to the three-point function due to first of these terms can be obtained using Wick's theorem and is given as

$$\begin{aligned}&\big\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\\ &\quad= \frac{\eta-\epsilon}{4}\big(\big\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\big\rangle\big\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\big\rangle + \text{perm.}\big). \end{aligned} \tag{66}$$

Now

$$\big\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\big\rangle = \int \frac{d^3k_2}{(2\pi)^3}\frac{H^2}{4\epsilon}\frac{1}{k_2^3}e^{-H\vec{\theta^0}\cdot\vec{k}_2}e^{i\vec{k}_2\cdot(\vec{x}_1-\vec{x}_2)}. \tag{67}$$

So

$$\begin{aligned}&\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle\\&\quad=\int\frac{d^3k_2\,d^3k_3}{(2\pi)^9}\frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3^3}e^{-H\overrightarrow{\theta^0}\cdot(\vec{k}_2+\vec{k}_3)}e^{i(\vec{k}_2+\vec{k}_3)\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}\\&\quad=(2\pi)^3\int\frac{d^3k_1\,d^3k_2\,d^3k_3}{(2\pi)^9}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3^3}\\&\qquad\times e^{H\overrightarrow{\theta^0}\cdot\vec{k}_1}e^{-i\vec{k}_1\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}.\end{aligned}\tag{68}$$

Here in the second step we have introduced a δ function with integral over k_1 so that it matches with the results of the rest of the terms. So the contribution to the three-point function due to first field redefinition term in momentum space will be

$$\begin{aligned}&\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle\\&\quad=\frac{\eta-\epsilon}{2}(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left(\sum_i k_i^3e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right).\end{aligned}\tag{69}$$

Now consider the second field redefinition term in Eq. (51) the contribution due to that is given as

$$\begin{aligned}&\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)\rangle\\&\quad=\frac{\epsilon}{2}\left(\partial_{x_1}^{-2}\left(\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\partial_{x_1}^2\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle\right)+\text{perm.}\right).\end{aligned}\tag{70}$$

Now

$$\partial_{x_1}^2\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle=-\int\frac{d^3k_3}{(2\pi)^3}\frac{H^2}{4\epsilon}\frac{1}{k_3}e^{-H\overrightarrow{\theta^0}\cdot\vec{k}_3}e^{i\vec{k}_3\cdot(\vec{x}_1-\vec{x}_3)}.\tag{71}$$

So

$$\begin{aligned}&\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\partial_{x_1}^2\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle\\&\quad=-\int\frac{d^3k_2\,d^3k_3}{(2\pi)^6}\frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3}e^{-H\overrightarrow{\theta^0}\cdot(\vec{k}_2+\vec{k}_3)}\\&\qquad\times e^{i(\vec{k}_2+\vec{k}_3)\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}\\&\quad=-(2\pi)^3\int\frac{d^3k_1\,d^3k_2\,d^3k_3}{(2\pi)^9}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\frac{1}{k_2^3k_3}\\&\qquad\times e^{H\overrightarrow{\theta^0}\cdot\vec{k}_1}e^{-i\vec{k}_1\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}.\end{aligned}\tag{72}$$

So

$$\begin{aligned}&\partial_{x_1}^{-2}\left(\langle\zeta_\theta(x_1)\zeta_\theta(x_2)\rangle\partial_{x_1}^2\langle\zeta_\theta(x_1)\zeta_\theta(x_3)\rangle\right)\\&\quad=(2\pi)^3\int\frac{d^3k_1\,d^3k_2\,d^3k_3}{(2\pi)^9}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\frac{1}{k_1^2k_2^3k_3}\\&\qquad\times e^{H\overrightarrow{\theta^0}\cdot\vec{k}_1}e^{-i\vec{k}_1\cdot\vec{x}_1-i\vec{k}_2\cdot\vec{x}_2-i\vec{k}_3\cdot\vec{x}_3}.\end{aligned}\tag{73}$$

So the contribution due to this term in Fourier space will be

$$\begin{aligned}&\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle\\&\quad=\frac{\epsilon}{2}(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left(\sum_{i\neq j}k_ik_j^2e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right).\end{aligned}\tag{74}$$

Now combining all the results from (65), (69), (74) for the various contributions to the three-point function of ζ, we get the final three-point function using Eq. (51) in momentum space as

$$\begin{aligned}&\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle\\&\quad=(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\mathcal{A},\end{aligned}\tag{75}$$

where

$$\begin{aligned}\mathcal{A}&=4\epsilon\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}\left(\sum_{i<j}k_i^2k_j^2\right)\\&\quad+\frac{\eta-\epsilon}{2}\left(\sum_i k_i^3e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right)+\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2e^{H\overrightarrow{\theta^0}\cdot\vec{k}_i}\right).\end{aligned}\tag{76}$$

This is the main result of this Letter. In the limit $\theta^{\mu\nu}\to0$ the above expression becomes similar to expression for the three-point function in commutative spacetime (Eqs. (4.5) and (4.6) of Maldacena [11]). Now due to translational invariance $\vec{k}_1+\vec{k}_2+\vec{k}_3=0$ and on comparing our results with the commutative case [11,23] we see that the first term in (76) is modified due to a phase factor that depends on θ_{ij}, while the second and the last terms are modified by exponential factors. These modifications in the three-point function are due to the non-gaussian nature of noncommutativity. As also mentioned by [14], the n-point correlation functions for noncommutative fields are, in general, non-gaussian and cannot be expressed as sums of products of two-point correlation function even in the absence of interactions. The three-point correlation function here is complex so to see its observational effects we again take its self adjoint given as [19]

$$\begin{aligned}&\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle_M\\&\quad=\frac{1}{2}\big(\langle\zeta_\theta(\vec{k}_1,t)\zeta_\theta(\vec{k}_2,t)\zeta_\theta(\vec{k}_3,t)\rangle\\&\qquad+\langle\zeta_\theta(-\vec{k}_1,t)\zeta_\theta(-\vec{k}_2,t)\zeta_\theta(-\vec{k}_3,t)\rangle\big)\\&\quad=(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\\&\qquad\times\prod_{i=1}^{3}\frac{1}{k_i^3}\left[\frac{4\epsilon\cos(\frac{\vec{k}_1\wedge\vec{k}_2}{2})}{K}\left(\sum_{i<j}k_i^2k_j^2\right)\right.\\&\qquad+\frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right)\\&\qquad\left.+\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2\cosh\left(H\overrightarrow{\theta^0}\cdot\vec{k}_i\right)\right)\right].\end{aligned}\tag{77}$$

4. Implications for observations

The non-gaussianity in CMB is described in terms of the angular three-point correlation functions in harmonic space called as "angular bispectrum", which is related to the three-dimensional bispectrum of the primordial curvature perturbations defined as [39,40]

$$\begin{aligned}&\langle\zeta(\vec{k}_1,t)\zeta(\vec{k}_2,t)\zeta(\vec{k}_3,t)\rangle\\&\quad=(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_\zeta(k_1,k_2,k_3).\end{aligned}\tag{78}$$

We can generalize the above definition of bispectrum for the twisted quantum fields in noncommutative space time and it can be expressed using (77) as

$$B_{\zeta_\theta}(\vec{k}_1,\vec{k}_2,\vec{k}_3)=\frac{H^4}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3}\left[\frac{4\epsilon\cos(\frac{\vec{k}_1\wedge\vec{k}_2}{2})}{K}\left(\sum_{i<j}k_i^2k_j^2\right)+\frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)+\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)\right]. \tag{79}$$

Here the bispectrum also breaks the statistical isotropy. The anisotropic bispectrum also arises in the cases where the vector fields are also present during inflation [41,40]. In [42] the method to analyze these models in the light of new CMB data is derived. Current observational limits on non-gaussianity are given in terms of a non-linearity parameter f_{NL} that determines the amplitude and scale dependence of non-gaussianity. We define f_{NL} in a similar way as [41,40] where it is assumed that the corrections to the standard power spectrum due to statistical anisotropy are very small. So

$$f_{NL}=\frac{5}{6}\frac{B_{\zeta_\theta}(\vec{k}_1,\vec{k}_2,\vec{k}_3)}{P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_1)P_\zeta(k_3)}. \tag{80}$$

Using the power spectrum (34) it becomes

$$f_{NL}=\frac{5}{6}\frac{1}{\sum_i k_i^3}\left[4\epsilon\frac{\cos(\frac{\vec{k}_1\wedge\vec{k}_2}{2})}{K}\sum_{i<j}(k_i^2k_j^2)+\frac{\eta-\epsilon}{2}\left(\sum_i k_i^3\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)+\frac{\epsilon}{2}\left(\sum_{i\neq j}k_ik_j^2\cosh\left(H\vec{\theta^0}\cdot\vec{k}_i\right)\right)\right]. \tag{81}$$

This kind of f_{NL} generally arises where the curvature perturbation is expressed as $\zeta_g=\zeta_g+\frac{3}{5}\zeta_g^2$ and f_{NL} peaks at the so called squeezed triangle limit defined as $|\vec{k}_1|=|\vec{k}_2|=k$ and $|\vec{k}_3|\ll k$. So in a similar fashion the f_{NL} for noncommutative case in the above limit is given as

$$f_{NL}=\frac{5}{12}\left[2\epsilon\cos\left(\frac{\vec{k}_1\wedge\vec{k}_2}{2}\right)+\frac{\eta}{2}\left(\cosh\left(H\vec{\theta^0}\cdot\vec{k}_1\right)+\cosh\left(H\vec{\theta^0}\cdot\vec{k}_2\right)\right)\right]. \tag{82}$$

It is clear from the above expression that the amplitude of f_{NL} is very small and of the order of slow-roll parameters for the case of small statistical anisotropy. But it has scale dependence and direction dependence that can help us to distinguish it from the commutative case. The current limits on the amplitude of f_{NL} for squeezed triangle limit are $f_{NL}=2.7\pm5.8$ from the recently released PLANCK data [9] and $f_{Nl}=48\pm20$ from large scale structure probes [43] at 68% confidence level. One can define the scale dependence of f_{NL} by a parameter n_{NG} analogous to the spectral index [27]

$$n_{NG}=\frac{d\ln|f_{NL}|}{d\ln k}. \tag{83}$$

To quantify the scale dependence coming due to noncommutativity, we assume $\hat{\theta}^0$ along $\vec{k}_1$ and hence n_{NG} due to first term of Eq. (82) (term depending on θ_{ij}) can be obtained as

$$n_{NG}=-k_1^i\theta_{ij}k_2^j\tan\left(\frac{k_1^i\theta_{ij}k_2^j}{2}\right), \tag{84}$$

and similarly for the second term of Eq. (82), terms depending on θ^0, n_{NG} is given as

$$n_{NG}=H\theta^0k\tanh\left(H\theta^0k\right). \tag{85}$$

The running of the non-gaussianity n_{NG} for $\vec{\theta^0}=0$ in our case is similar to [19] (their $n_{f_{NL}}=n_{NG}$) and they argued that the detection of n_{NG} could put strong bounds on θ_{ij}. The constraints on the running of the non-gaussianity with ongoing and future large scale structure surveys and CMB observations were studied in [44,45] and they showed that we will be able to constraint n_{NG} with a $1-\sigma$ uncertainty of $\Delta n_{NG}\sim0.1$. Taking into account the bounds on noncommutativity scale $H\theta^0<0.01$ claimed by Akofor et al. [15], the running of non-gaussianity arising due to the term depending on θ^0 is of the order of 10^{-7} for the pivot scale $k=0.05\ \mathrm{MPc}^{-1}$ which is far beyond the current reachable limit. Since the amplitude of f_{NL} with the noncommutative geometry is of the order of slow-roll parameters, the scale dependence of f_{Nl} due to noncommutativity with ongoing and planned observations of CMB and LSS is undetectable.

5. Conclusions

Detection of primordial non-gaussianity in the CMB anisotropy and large scale structure is the main challenge of current and future observations and it can play an important role in discriminating various models of inflation. In this Letter we have calculated the primordial non-gaussianity in single field inflation with space-time noncommutativity. We have used Maldacena's approach [11] to compute the two-point and three-point correlation functions for the comoving curvature perturbation ζ for the noncommutative case described by [14]. Both the power spectrum and the bispectrum for this model are direction dependent and breaks the statistical isotropy due to the preferred direction of $\hat{\theta}$. This direction dependent power spectrum was analyzed by [15] to put constraints on the scale of noncommutativity in the light of WMAP5, ACBAR and CBI data and it was concluded that the WMAP5 data at high l is not sufficient to constraint the noncommutative scale θ and using one-parameter χ^2 analysis they claimed that $H\theta^0<0.01$ MPc. Since recently released PLANCK data gives the CMB temperature anisotropy power spectra up to $l\geqslant2500$ with better precision, the author and collaborators plan to analyze the power spectrum (45) with the PLANCK and other LSS data. The breaking of statistical isotropy detected by PLANCK, i.e. dipolar modulation and hemispherical power asymmetry cannot be explained with the power spectrum (45) as it is parity conserving. But with some modifications, as in [19], the hemispherical power asymmetry can be generated with noncommutative spacetime [19].

The statistical anisotropic bispectrum can be extracted from the three-point correlation function of CMB [42] and for $f_{NL}\approx30$, future experiments could be sensitive to a ratio of the anisotropic to the isotropic amplitudes of the bispectrum up to 10%. The amplitude of the non-linearity parameter f_{NL} for our case is very small for small statistical anisotropy but it has a scale dependence different then commutative case. Ongoing PLANCK and future CMB and large scale structure observations would be able to measure the running of non-gaussianity up-to $1-\sigma$ uncertainty of $\Delta n_{NG}\sim0.1$ [45]. Since the effects on the scale dependence of f_{NL} due to noncommutativity are very small, it is difficult to distinguish these effects from the commutative case in the light of current observations.

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Appendix A. Some calculation details

The integral appearing in the calculation of the first term in the three-point function (55) in Fourier space can be read from Eq. (61) as

$$(a) = -4i\epsilon^2 \int dt'\, a^5 H \int d^3x \int \prod_{i=4}^{6} \frac{d^3k_i}{k_6^2(2\pi)^9} \times \langle 0|\zeta(\vec{k}_1,t_1)\zeta(\vec{k}_2,t_2)\zeta(\vec{k}_3,t_3)\dot{\zeta}(\vec{k}_4,t_4)\dot{\zeta}(\vec{k}_5,t_5)\dot{\zeta}(\vec{k}_6,t_6)|0\rangle \times e^{i(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x})} e^{\frac{i}{2}\mathcal{P}}, \tag{A.1}$$

where t_is are given by Eq. (62). Now we will calculate the six-point function entering in the above integrand separately and denote it as A. So

$$A = \langle 0|\zeta(\vec{k}_1,t_1)\zeta(\vec{k}_2,t_2)\zeta(\vec{k}_3,t_3)\dot{\zeta}(\vec{k}_4,t_4)\dot{\zeta}(\vec{k}_5,t_5)\dot{\zeta}(\vec{k}_6,t_6)|0\rangle. \tag{A.2}$$

Now using Wick's theorem and leaving the disconnected diagrams we will get 6 terms in above expression. Let us consider one of them and denote it by A_1 so

$$A_1 = \langle 0|\big[\zeta^+(\vec{k}_1,t_1),\dot{\zeta}^-(\vec{k}_4,t_4)\big]\big[\zeta^+(\vec{k}_2,t_2),\dot{\zeta}^-(\vec{k}_5,t_5)\big] \times \big[\zeta^+(\vec{k}_3,t_3),\dot{\zeta}^-(\vec{k}_6,t_6)\big]|0\rangle, \tag{A.3}$$

where the ζ^+ and ζ^- denote the positive and negative frequency part of the quantum field ζ (see (23)). Now since

$$\big[\zeta^+(\vec{k}_1,t_1),\dot{\zeta}^-(\vec{k}_4,t_4)\big] = (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_4)u(\vec{k}_1,t_1)\dot{u}^\star(-\vec{k}_4,t_4) \tag{A.4}$$

we have

$$A_1 = (2\pi)^9\delta^3(\vec{k}_1+\vec{k}_4)\delta^3(\vec{k}_2+\vec{k}_5)\delta^3(\vec{k}_3+\vec{k}_6) \times u(\vec{k}_1,t_1)u(\vec{k}_2,t_2)u(\vec{k}_3,t_3) \times \dot{u}^\star(-\vec{k}_4,t_4)\dot{u}^\star(-\vec{k}_5,t_5)\dot{u}^\star(-\vec{k}_6,t_6). \tag{A.5}$$

Putting this back to the integral (59) and denoting the contribution due to this term as $(a)_1$ and doing the delta integrals we get

$$\begin{aligned}(a)_1 &= -4i\epsilon^2 \int_{t_0}^{t} dt'\, a^5 H \int d^3x \frac{1}{k_3^2} u(\vec{k}_1,t_1)u(\vec{k}_2,t_2)u(\vec{k}_3,t_3) \\ &\quad\times \dot{u}^\star(\vec{k}_1,t_4)\dot{u}^\star(\vec{k}_2,t_5)\dot{u}^\star(\vec{k}_3,t_6)e^{i(\vec{k}_4\cdot\vec{x}+\vec{k}_5\cdot\vec{x}+\vec{k}_6\cdot\vec{x})}e^{\frac{i}{2}\mathcal{P}_1} \\ &= -4i\epsilon^2(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3) \\ &\quad\times \int_{t_0}^{t} dt'\, a^5 H \frac{1}{k_3^2} u(\vec{k}_1,t_1)u(\vec{k}_2,t_2)u(\vec{k}_3,t_3) \\ &\quad\times \dot{u}^\star(\vec{k}_1,t_4)\dot{u}^\star(\vec{k}_2,t_5)\dot{u}^\star(\vec{k}_3,t_6)e^{\frac{i}{2}\mathcal{P}_1},\end{aligned} \tag{A.6}$$

where $\mathcal{P}_1 = \mathcal{P}|_{\vec{k}_4=-\vec{k}_1,\vec{k}_5=-\vec{k}_2,\vec{k}_6=-\vec{k}_3}$ and t_is are also calculated using these values of momenta. Here the limit of integration goes from $t_0=-\infty$ to $t=\infty$. To solve the integral we will go to conformal time where the above limits correspond to $\tau \to (-\infty, 0)$. Now from Eq. (27) we have

$$u(\vec{k},\tau) = \frac{v_{\vec{k}}}{z} = \frac{iH}{\sqrt{4\epsilon k^3}}(1+ik\tau)e^{-ik\tau}. \tag{A.7}$$

Since from Eq. (39) we know that the conformal time corresponding to t_is will be $\tau \times e^{\theta\text{ dependent term}}$ and from Eq. (62) we have terms like ($t+\theta$ dependent term) for t_1,t_2,t_3 so for $t\to\infty$ or $\tau\to 0$ conformal time corresponding to t_1,t_2,t_3 will be zero. So in conformal time $u(\vec{k}_1,t_1)u(\vec{k}_2,t_2)u(\vec{k}_3,t_3) \to u(\vec{k}_1,0)u(\vec{k}_2,0)u(\vec{k}_3,0)$. Now we denote the conformal time corresponding to t' by τ so $\dot{u}^\star(\vec{k}_1,t_4) \to \frac{1}{a}\frac{du^\star(\vec{k}_1,\tau_4)}{d\tau}$ and from (39) and (62) we get $\tau_4=\tau_5=\tau_6=\tau e^{\frac{H\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2}}$. Now

$$\frac{du^\star(\vec{k}_1,\tau_4)}{d\tau} = \frac{-iH}{\sqrt{4\epsilon k^3}}k_1^2\tau e^{H\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}e^{ik_1\tau e^{\frac{H\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2}}}. \tag{A.8}$$

Now due to translational invariance of de Sitter space $\vec{k}_1+\vec{k}_2+\vec{k}_3=0$. So, the integral (A.6) becomes

$$\begin{aligned}(a)_1 &= -i\epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^7}{16\epsilon^2}\prod_{i=1}^{3}\frac{1}{k_i^3} \\ &\quad\times \int_{-\infty}^{0} a^3\tau^3k_1^2k_2^2e^{\frac{i}{2}\mathcal{P}_1}e^{iK\tau} \\ &= \epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2} \\ &\quad\times \prod_{i=1}^{3}\frac{1}{k_i^3}\frac{k_1^2k_2^2}{K}e^{\frac{i}{2}\mathcal{P}_1}e^{\frac{5H\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2}}.\end{aligned} \tag{A.9}$$

Here we have rotated the contour from $(-\infty,0)$ to $i(\infty,0)$ and $K=k_1+k_2+k_3$. Now to calculate $\mathcal{P}_1$ let us recall (60)

$$\mathcal{P} = \vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3+\vec{k}_1\wedge(\vec{k}_4+\vec{k}_5+\vec{k}_6) + \vec{k}_2\wedge(\vec{k}_4+\vec{k}_5+\vec{k}_6)+\vec{k}_3\wedge(\vec{k}_4+\vec{k}_5+\vec{k}_6). \tag{A.10}$$

Hence $\mathcal{P}_1 = \vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3$. Now rest of the terms in (A.2) can be found be different permutations of k_4,k_5,k_6 and the phase factors will be same as $\mathcal{P}_1$ after imposing the different conditions due to delta function integrals. So from Eq. (61) we get the first term of the right hand side of Eq. (55) in Fourier space as

$$(a) = \epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2} \times \prod_{i=1}^{3}\frac{1}{k_i^3}\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}(k_1^2k_2^2+\text{perm.}). \tag{A.11}$$

Now the second term in the three-point function (55) is denoted as (b) and can be read from Eq. (A.13) as

$$\begin{aligned}(b) &= i\int_{t_0}^{t} dt'\,\langle 0|\mathcal{H}(t')\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle \\ &= -4i\epsilon^2\int dt'\,a^5H\int d^3x\,\langle 0|\dot{\zeta}^2\partial^{-2}\dot{\zeta}\big|_{t',\vec{x}} \\ &\quad\times e^{\frac{1}{2}\overleftarrow{\partial_{x_\mu}}\wedge P_\nu}\zeta_\theta(x_1)\zeta_\theta(x_2)\zeta_\theta(x_3)|0\rangle \\ &= -4i\epsilon^2\int dt'\,a^5H\int d^3x\,\langle 0|\dot{\zeta}^2\partial^{-2}\dot{\zeta}\big|_{t',\vec{x}}e^{-\frac{i}{2}\overleftarrow{\partial_x}\wedge(\overrightarrow{\partial_{x_1}}+\overrightarrow{\partial_{x_2}}+\overrightarrow{\partial_{x_3}})} \\ &\quad\times \zeta(x_1)\zeta(x_2)\zeta(x_3)|0\rangle e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}\wedge\overrightarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overrightarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overrightarrow{\partial_{x_3}})}.\end{aligned} \tag{A.12}$$

Here we have used (35). Now in the Fourier space we get

$$(b) = -4i\epsilon^2\int dt'\,a^5H\int d^3x\int\prod_{i=1}^{6}\frac{d^3k}{-k_6^2(2\pi)^{18}}e^{i(\vec{k}_4+\vec{k}_5+\vec{k}_6)\cdot\vec{x}} \times e^{-\frac{i}{2}\overleftarrow{\partial_x}\wedge(\overrightarrow{\partial_{x_1}}+\overrightarrow{\partial_{x_2}}+\overrightarrow{\partial_{x_3}})}$$

$$\times\,\langle 0|\dot{\zeta}(\vec{k}_4,t')\dot{\zeta}(\vec{k}_5,t')\dot{\zeta}(\vec{k}_6,t')\zeta(\vec{k}_1,t_1)\zeta(\vec{k}_2,t_2)\zeta(\vec{k}_3,t_3)|0\rangle$$
$$\times\, e^{i(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2+\vec{k}_3\cdot\vec{x}_3)}e^{-\frac{i}{2}(\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_2}}+\overleftarrow{\partial_{x_2}}\wedge\overleftarrow{\partial_{x_3}}+\overleftarrow{\partial_{x_1}}\wedge\overleftarrow{\partial_{x_3}})}$$
$$= 4i\epsilon^2\int dt'\,a^5 H\int d^3x\int\prod_{i=1}^{6}\frac{d^3k}{k_6^2(2\pi)^{18}}$$
$$\times\, e^{i(\vec{k}_4+\vec{k}_5+\vec{k}_6)\cdot\vec{x}}e^{i(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2+\vec{k}_3\cdot\vec{x}_3)}$$
$$\times\,\langle 0|\dot{\zeta}(\vec{k}_4,t_4)\dot{\zeta}(\vec{k}_5,t_5)\dot{\zeta}(\vec{k}_6,t_6)$$
$$\times\,\zeta(\vec{k}_1,t_1)\zeta(\vec{k}_2,t_2)\zeta(\vec{k}_3,t_3)|0\rangle e^{\frac{i}{2}\tilde{\mathcal{P}}}. \tag{A.13}$$

Here

$$\tilde{\mathcal{P}} = \vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3 - (\vec{k}_1+\vec{k}_2+\vec{k}_3)\wedge(\vec{k}_4+\vec{k}_5+\vec{k}_6), \tag{A.14}$$

$$t_1 = t+\frac{\vec{\theta^0}\cdot(\vec{k}_2+\vec{k}_3-\vec{k}_4-\vec{k}_5-\vec{k}_6)}{2},$$
$$t_2 = t+\frac{\vec{\theta^0}\cdot(-\vec{k}_1+\vec{k}_3-\vec{k}_4-\vec{k}_5-\vec{k}_6)}{2},$$
$$t_3 = t+\frac{\vec{\theta^0}\cdot(\vec{k}_1-\vec{k}_2-\vec{k}_4-\vec{k}_5-\vec{k}_6)}{2},$$
$$t_4 = t'+\frac{\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2},$$
$$t_5 = t'+\frac{\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2},$$
$$t_6 = t'+\frac{\vec{\theta^0}\cdot(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{2}. \tag{A.15}$$

Now all the calculations can be done for (b) as earlier and the final answer is

$$(b) = \epsilon(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{H^4}{16\epsilon^2}\times\prod_{i=1}^{3}\frac{1}{k_i^3}\frac{e^{\frac{i}{2}(\vec{k}_1\wedge\vec{k}_2+\vec{k}_2\wedge\vec{k}_3+\vec{k}_1\wedge\vec{k}_3)}}{K}\left(k_1^2k_2^2+\text{perm.}\right). \tag{A.16}$$

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