Unit 4: Elementary Combinatorics

1 marks:

1. Find the Recurrence relation of the sequence 2, 6, 18, 54, 162,….
2. Calculate the permutation 9P8.
3. A computer system considers a string of decimal digits to be a valid code word if and only if it contains an even number of zero digits. For instance, 1230407869 is valid, whereas 3141529046 is not. Let Vn be the number of valid n digit code words; find a recurrence for Vn.
4. Define generating functions.
5. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not contain two consecutive 0s. How many such bit strings are there of length five?
6. State at least one usage of counting principle.

2 marks:

1. 10 integers are chosen from 1 to 100 inclusively. Using pigeon principle proves that we can find 2 disjoint non-empty subsets of the chosen integers such that the 2 subsets give the same sum of elements.
2. Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters. There are more than 100 cookies available for each type of cookie.
3. Identify the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D and F.
4. From a club consisting of a 8 men and 7 women, in how many ways can we select a committee of:
   (a) 3 men and 4 women?
   (b) 4 persons which has at-least one woman?
5. What is Pigeon Hole principle? Write two different real world situations where Pigeon Hole principle is used.

5 marks:

1. Briefly discuss any one application of Mathematical Induction in Computer Science domain. In a program, student has used the formula $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n(n+1)(2n+1)$. Prove that formula used by student is correct by using Mathematical Induction.
2. Verify that for all $n \geq 1$, the sum of the squares of the first 2n positive integers is given by the
3. formula $1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$ for every positive integer.

4. Prove by mathematical induction, that $1\circ 2\circ 3 + 2\circ 3\circ 4 + 5\circ 6\circ 7 + \ldots + n(n+1)(n+2) = \frac{1}{4} n(n+1)(n+2)$

5. In how many ways can a photographer at a wedding arrange a six people in a row, including the bride and groom, if
   a) The bride must be next to the groom?
   b) The bride is not next to the groom?
6. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair Die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he win/looses.
7. Suppose there are 9 faculty members in mathematics department and 11 in the computer science department.
   a) How many ways are there to select an event committee if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?
   b) How many ways to select an event committee of seven members with at-least three members from mathematics department?
   c) How many ways to select event committee of seven members with at-most four members from computer science department.
8. In how many ways can 20 students out of a class of 30 be selected for an extra-curricular activity, if
   a) Krishang refuses to be selected?
   b) Urmi insists on being selected?
   c) Shivansh and Krisha insist on being selected?
   d) Either Shivansh or Krisha or both get selected?
9. There are 3 piles of identical red, blue and green balls, where each pile contains at least 10 balls. In how many ways can 10 balls be selected:
   a) If there is no restriction?
   b) If at least one red ball must be selected?
   c) If at least one red ball, at least 2 blue balls and at least 3 green balls must be selected?
   d) If exactly one red ball must be selected?
   e) If exactly one red ball and at least one blue ball must be selected?

Unit 5: Analytical Geometry

1. Ram and Shyam are going to college. For this, Ram has selected the line-path with equation $x + y + 1 = 0$. Whereas Shyam has selected the line-path $3x + y - 5 = 0$. Find the co-ordinate point of the college. After reaching to the college, they are planning to celebrate birthday in party plot which has co-ordinate (1,-3). Find the distance between the college and party plot. Also find the equation of line-path selected by them to reach to the college to the party plot.

2. Find the equation to the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$.

3. Explain the straight line equation with a straight line makes intercepts 3 and -5 on X and Y axe respectively. Find its equation

4. Find the equation of the line passing from the point P (1, – 3) and permendicular to the line $2y - 3x = 4$.

5. Find the equation of the straight line passing through (1, 2) and perpendicular to the line $x + y + 7 = 0$.

6. Find the distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$.

7. Explain and show in what ratio the line joining A (5, 12) and B (2,9) is divided by a point P(3,10).

8. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). Find the coordinates of the point A.

9. Consider a line passing through vertices (2, 5) and (-4, -2). Find the equation of the line. Also find its intersection point with the line passing through vertices (2, -3) and (-5, 1).

10. A cricket ground is to be developed by cricket association of a small village. The standard radius of cricket ground is 137.16 meters. What should be the length of square plot to accommodate the cricket ground?

11. In a puzzle game, points of parallel lines are given. Points of line AB are (k, 3) and (-2, 1). Points of line CD are (-3, 2) and (1, 0). Find the value of k and equations of both parallel lines.

12. Determine the slope and the y-intercept of the line whose equation is $8x + 3y = 5$. Also find the equation of line perpendicular to the given line.

Unit 6: Determinant

1 marks:

1. What do you mean by symmetric matrix?
2. What is the value of the determinant of 2x2 order and having same value in all rows?
3. What is the value of the determinant for Identity matrix?
4. Write an example of scalar matrix.
5. Find the determinant of the given matrix: $\begin{vmatrix} 1 & 2 \\ & 3 \end{vmatrix}$.
6. Find the determinant of the given matrix $\begin{vmatrix} 6 & 3 \\ 2 & 4 \end{vmatrix}$.

2 marks:

1. Using example, verify whether the value of determinant remains same or not if the order of any two rows is changed in determinant.
2. Find the value of the equation 3I+5O+5 where determinant is of 3 x 3 order matrix. I indicates the determinant of identity matrix and O indicates determinant of zero matrix.
3. Evaluate
$$\begin{vmatrix} 20 & 11 & 31 \\ 11 & -7 & 4 \\ 19 & 11 & 30 \end{vmatrix}$$
4. If $A = \begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & 3 \end{pmatrix}$ what is the order of the matrix and find A^T
5. Find $\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$.
6. Perform any two operations on Determinant of 2 x 2 orders.

5 marks:

1. Prove that $\begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} = (p-q)(q-r)(r-p)$
2. a. Prove that: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

   b. Solve the equation: $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 0$

3. Prove that $\begin{vmatrix} (a-1)^2 & (b-1)^2 & (c-1)^2 \\ 1 & 1 & 1 \\ a+1 & b+1 & c+1 \end{vmatrix} = (a-b)(b-c)(c-a)$

4. Solve the equations $4x + 10y = 2xy$ and $5x + 16y = 3xy$ using Cramer's rule.

5. Solve the equations using Cramer's rule:

$$3x+5y+6z=4$$
$$x+2y+3z=2$$
$$2x+4y+5z=3$$