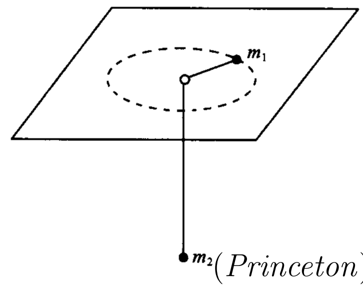
Due: 11:59 pm on July 12, 2024

Problem 1 (10pts)

A massless, inextensible rope passes through a small hole on a smooth horizontal table, with two ends connected to two objects of mass m_1 and m_2. When m_1 makes a circular motion with a radius of r_0 on the table, m_2 is stationary. Now give m_2 a very small downward impulse, prove that m_2 will make a simple harmonic oscillation, and calculate its period (the friction of the small hole can be ignored).



(*Princeton*)

Problem 2 (10pts)

Consider a damped, driven harmonic oscillator (in one dimension) with equation of motion

$$m\ddot{x} = -m\omega_0^2 x - \gamma\dot{x} + A\cos(\omega t).$$

What is the time-averaged rate of energy dissipation?

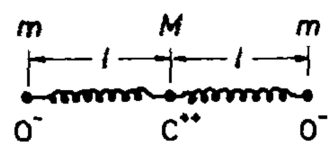
(*Princeton*)

Problem 3 (15pts)

A simple classical model of the CO_2 molecule would be a linear structure of three masses with the electrical forces between the ions represented by two identical springs of equilibrium length l and force constant k. Assume that only motion along the original equilibrium line is possible, i.e. ignore rotations. Let m be the mass of O^- and M be the mass of C^{++}.

(a) Seek a solution to the equations of motion in which all particles oscillate with a common frequency (normal modes) and calculate the possible frequencies.

(b) Calculate the relative amplitudes of the displacements of the particles for each of these modes and describe the nature of the motion for each mode. *You may use a sketch as part of your description.*



(*MIT*)

Problem 4 (10pts)

A violin string, 0.5 m long, has a fundamental frequency of 200 Hz.

(a) At what speed does a transverse pulse travel on this string?

(b) Draw a pulse before and after reflection from one end of the string.

(c) Show a sketch of the string in the next two higher modes of oscillation and give the frequency of each mode.

(*Wisconsin*)

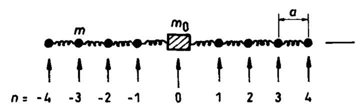
Problem 5 (10pts)

A transverse traveling sinusoidal wave on a long stretched wire of mass per unit length ρ has frequency ω and wave speed c. The maximum amplitude is y_0, where $y_0 \ll A$. The wave travels toward increasing x.
(a) Write an expression for the amplitude y as a function of t and x, where x is distance measured along the wire.
(b)What is the energy density (energy/unit length)?
(c) What is the power transmitted along the wire?
(d) If the wave is generated by a mechanical device at $x = 0$, find the transverse force $F_y(t)$ that it exerts on the wire.

(*Wisconsin*)

Problem 6 (15pts)

Take a very long chain of beads connected by identical springs of spring constant K and equilibrium length a. Each bead is free to oscillate along the x direction. All beads have mass m except for one which has mass $m_0 < m$. The mass of the spring is negligibly small.
(a) Far from the "special" bead, what is the relation between the wave vector and the frequency of the resulting oscillation?
(b) For a wave of wave vector k, what is the reflection probability when the wave hits the special bead?
Hint for part (b): Try a solution of the form



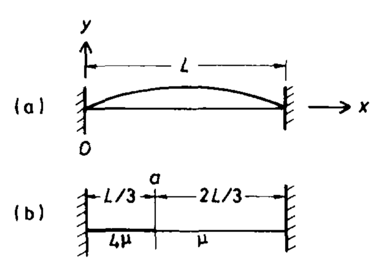
$$x_n = \begin{cases} Ae^{ikan} + Be^{-ikan} & n < 0 \\ Ce^{ikan} & n > 0 \end{cases}$$

where A, B, and C are functions of time.

(*Chicago*)

Problem 7 (15pts)

(a) A violin string of length L with linear density μ kg/m and tension T newtons undergoes small oscillations. Write the solutions for the fundamental and first harmonic, and sketch their x-dependences. Give the angular frequency ω_1 of the fundamental and ω_2 of the first harmonic.
(b) The left-hand 1/3 of the string is wrapped so as to increase its linear density to 4μ kg/m. Repeat part (a), i.e. derive and sketch the new fundamental and first harmonic, and express the new ω_1 and ω_2 in terms of the original ω_1 and ω_2 of (a).



(*UC, Berkeley*)

Problem 8 (15pts)

This problem will lead you through the solution of the equation of motion for a simple pendulum with length l oscillating on Earth at latitude φ (Foucault's pendulum) in the Earth's frame of reference. As we have seen in class, the Foucault pendulum's oscillation plane rotates. We will find the period of this rotation.

(a) Let us start with assuming that the amplitude of oscillations is small, i.e. the pendulum bob moves in a plane, e.g. $z' = 0$. Write down the equations of motion along the axes x' and y'. Remember to include forces of inertia, but ignore the terms of the order of ω^2, where ω is the angular speed of the Earth in its rotational motion.

(b) Rewrite the system of coupled differential equations from part (a) as a single differential equation, by introducing a new (complex) variable $\tilde{\xi} = x' + iy'$. Look for solutions to this equation in the form $\tilde{\xi} = e^{\lambda t}$. Note that $g/l \gg \omega \sin\varphi$ and show that the general solution is given by

$$\tilde{\xi} = e^{-i\omega t \sin\varphi}(Ae^{i\omega_0 t} + Be^{-i\omega_0 t})$$

where $\omega_0 = g/l$ is the natural angular frequency of the pendulum.

(c) Suppose that initially ($t = 0$ s), the pendulum swings in the direction of a line of latitude, i.e. east-west. Show (find the real and the imaginary parts) that this complex solution gives

$$\boldsymbol{r'} = \boldsymbol{k} R \cos\omega_0 t$$

where $\boldsymbol{k} = [\sin(\omega t \sin\varphi), \cos(\omega t \sin\varphi)]$ is a unit vector rotating about the axis z' with angular velocity $\omega \sin\varphi$, and R stands for the radius of the circle, the pendulum's bob moves within in the $x'y'$ plane.

(d) What is the time needed for the vertical oscillation plane to make a full 360° turn? Calculate the actual values for Mohe (Heilongjiang) and Haikou (Hainan).

(UM-SJTU JI)