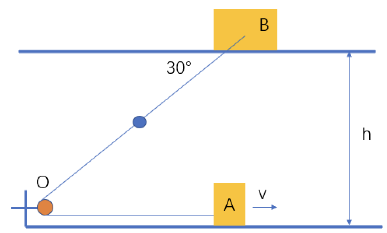
Due: 11:59 pm on June 4, 2024

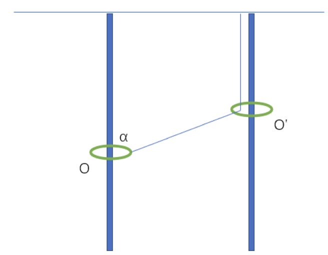
Problem 1 (10pts)

A and B are two small boxes (can be treated as particles). They are connected by a rope that goes over a wheel (denoted in orange). A and B are standing on two parallel tracks, separated by a distance of h. A is moving at a constant velocity v. At a certain moment, the angle between the rope (OB part) and B's track is 30 degrees, and a drop of water (denoted by the blue circle) at the midpoint of OB falls from the rope. Before the drop leaves, it is relatively static to the rope OB. When the drop falls, what is its velocity?



Problem 2 (10pts)

Two rings (O and O$'$) hanging on rods, which stand vertically. A rope is mounted on the top of the rod holding O$'$, it passes through O$'$ and connects O. Assuming O$'$ moves downwards at the constant velocity v. What is the velocity of O when the angle between the rope and the rod holding O is α.



Problem 3 (15pts)

A wheel of radius R moves forward, rolling (without sliding) on a flat surface with constant angular velocity ω. For a point P on the circumference of the wheel:
(a) Find parametric equations of the trajectory of P.
(b) Find the trajectory of P, which should be a cycloid[1].
(b) Find the velocity of point P.
(c) Find the distance traveled by point P as a function of time.

Problem 4 (10pts)

An airplane carrying fuel, flying at a speed of v in the absence of wind, can fly a total distance of R. Now, in the case of wind, it performs the task in the direction of north-east angle φ and then flies back to the base. If the wind is in the direction of north-east α and the wind speed is u. How far can the airplane fly from the base?

Problem 5 (10pts)

A bomb explodes at a height of H into many small fragments. It is given that after the explosion the fragments have the same speed u and a uniform angular distribution in all directions. After some time all fragments hit the ground. Find the radius R of the distribution of the debris.

[1] Cycloid. In *Wikipedia*. `https://en.wikipedia.org/wiki/Cycloid`.

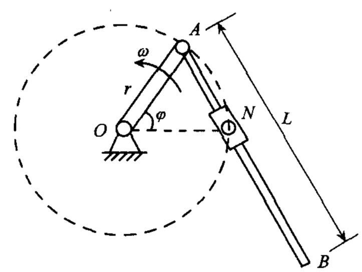
Problem 6 (15pts)

A particle moves in the $x - y$ plane with the trajectory $y = y(x)$. Prove the formula of the radius of curvature[2]:

$$\rho = \frac{(1 + {y'}^2)^{3/2}}{|y''|}$$

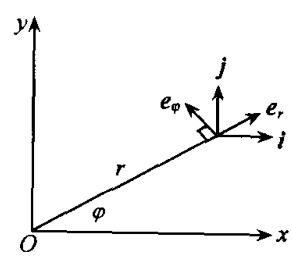
Problem 7 (15pts)

The crank $OA = r$ rotates around the fixed axis O at a uniform angular speed ω. The connecting rod AB is connected to the endpoint A of the crank with a hinge and can slide in the sliding sleeve N with a hinge. When $\varphi = 0$, end A is located in the sliding sleeve at N. It is known that $AB = L > 2r$. When $\varphi = 0$, find the magnitude of velocity v and acceleration a, the tangential acceleration a_τ, the normal acceleration a_n of the point B on the connecting rod, and the radius of curvature of the track ρ.



Problem 8 (15pts)

A particle moves in the $x - y$ plane with the trajectory $r = e^{ct}, \varphi = bt$, c, b are constants. This plane rotates around the fixed x-axis with constant angular velocity $\vec{\omega} = \omega\hat{i}$. Find the velocity and the acceleration of the particle using the unit vectors $(\hat{e_r}, \hat{e_\varphi}, \hat{k})$.



[2] Radius of curvature. In *Wikipedia*. `https://en.wikipedia.org/wiki/Radius_of_curvature`.