## Due: 11:59 pm on July 23, 2024

## Problem 1 (10pts)

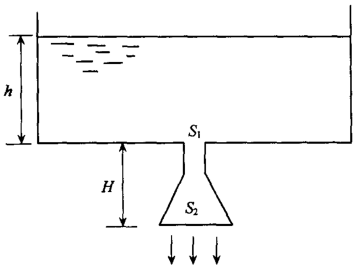
If a spring oscillator system is placed on a smooth horizontal surface, the mass of the spring cannot be ignored, proving that the period of the system is

$$T = 2\pi\sqrt{\frac{M + m/3}{k}}$$

where $m$ is the mass of the spring, $k$ is the spring constant, and $M$ is the mass of the object attached to the spring.

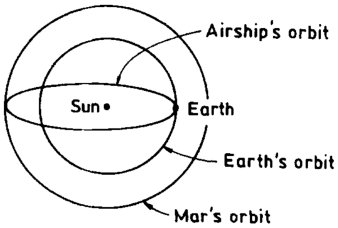
## Problem 2 (10pts)

Water with a density of $\rho$ flows into a trumpet-shaped conduit from a large container, with cross-sectional areas of $S_1$ and $S_2$ at the inlet and outlet, a conduit length of $H$, atmospheric pressure $p$. The fluid motion is the steady flow. Find the height $h$ when the pressure at the inlet of the conduit is zero.



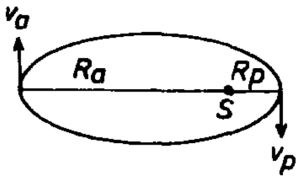

## Problem 3 (15pts)

In a few weeks Mariner 9 will be launched from Cape Kennedy on a mission to Mars. Assume that this spacecraft is launched into an elliptical orbit about the sun with perihelion at the earth's orbit and aphelion at Mar's orbit.
(a) Find the values of the parameters $\lambda$ and $\varepsilon$ of the orbit equation $r = \lambda(1 + \varepsilon)/(1 + \varepsilon\cos\theta)$ and sketch the north.
(b) Use Kepler's third law to calculate the time duration of the mission to Mars on this orbit.
(c) In what direction should the launch be made from the earth for minimum expenditure of fuel?
Mean distance of Mars from the sun = 1.5 A.U.
Mean distance of the earth from the sun = 1 A.U.



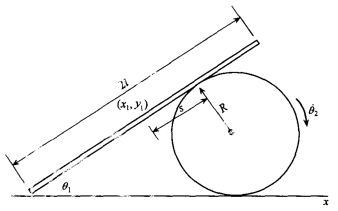

## Problem 4 (10pts)

A comet in am orbit about the sun has a velocity 10 km/sec at aphelion and 80 km/sec at perihelion. If the earth's velocity in a circular orbit is 30 km/sec and the radius of its orbit is $1.5 \times 10^8$ km, find the aphelion distance $R_a$ for the comet.



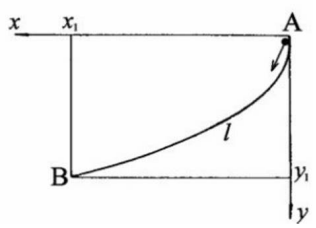
## Problem 5 (10pts)

A homogeneous disk with a radius of $R$ rolls purely along the horizontal x-axis, while a homogeneous thin rod with a length of $2l$ maintains non-sliding contact with the disk and slides along the x-axis at one end. The disk and rod remain in the same vertical plane. Find how many independent coordinates are required to describe the system.



## Problem 6 (15pts)

***Brachistochrone Problem:*** A bead with a wire threaded through a hole in it so that the bead can slide with no friction along the wire. How to curve the wire in such a way that the bead descents under gravity (from rest) as quickly as possible?




## Problem 7 (15pts)

A particle with a mass of $m$ is subjected to a force with a potential energy of $V(r)$, where $r$ is in spherical coordinates.
(1) Write the Lagrange function of a spherical coordinate system rotating at an angular velocity $\omega$ around the z-axis;
(2) Prove that the Lagrange function written above is the same as the Lagrange function using the same rotational reference frame with a generalized potential related to velocity, which comes from the centrifugal force and Coriolis force.

## Problem 8 (15pts)

Two thin beams of mass $m$ and length $l$ axe connected by a frictionless hinge and a thread. The system rests on a smooth surface. At $t = 0$ the thread is cut. In the following you may neglect the thread and the mass of the hinge.
(a) Find the speed of the hinge when it hits the floor.
(b) Find the time it takes for the hinge to hit the floor, expressing this in terms of a concrete integral which you need not evaluate explicitly.