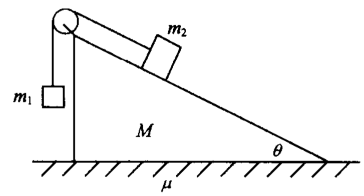
Homework #3 PHYS1600J 24SU

Due: 11:59 pm on June 11, 2024

Problem 1 (10pts)

A wedge of mass M is placed on a rough horizontal surface with a friction coefficient of μ. An object of mass m_1 is suspended by a massless, inextensible string and connected to an object of mass $m_2 < m_1$ sliding on the frictionless wedge surface across a smooth pulley fixed to the wedge. The wedge surface has an inclination angle of θ.



a) Find the accelerations of m_1 and m_2 and the tension of the string when μ is very large.

b) Find the minimum friction coefficient μ_{min} that allows the wedge to remain stationary.

Problem 2 (10pts)

For a canal with a width of b and water flowing north at a speed of v, at λ north latitude, prove the water level on the east bank is

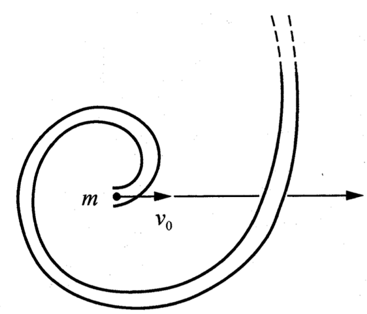
$$\Delta h = \frac{2bv\omega \sin\lambda}{g}$$

higher than that on the west bank, where ω is the angular velocity of the earth's rotation and g is the local gravitational acceleration.

Problem 3 (15pts)

A horizontal fixed slender pipe has the shape of an Archimedean spiral $r = r_0\frac{\theta}{\pi}$ with a smooth inner wall. A particle with a mass of m moves in the pipe at a speed of v_0, starting from $r = 0, \theta = 0$, until $\theta = 2K\pi (K = 1, 2, 3, ...)$. Let N be the magnitude of the elastic force exerted on the particle during its motion. Find the average value of N during the entire process $\bar{N}$, and find an approximate expression for $\bar{N}$ when K is very large.

Hint: $\int \sqrt{a^2 + x^2}dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln(x + \sqrt{a^2 + x^2}) + C$



Problem 4 (10pts)

A particle is thrown vertically upward with an initial velocity v_0. If the air drag $\vec{f} = -kv\vec{v}$, where k is an unknown constant. Find the velocity when returning to the initial position, and express it using v_0 and the terminal velocity[1] v_t.

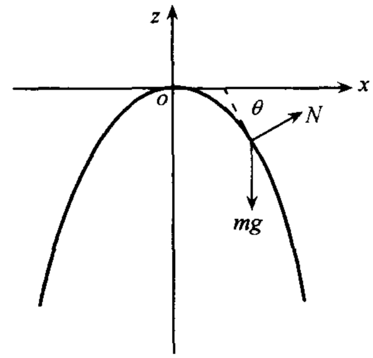
[1] Terminal velocity. In *Wikipedia*. `https://en.wikipedia.org/wiki/Terminal_velocity`.

Problem 5 (10pts)

A smooth paraboloid of revolution[2]

$$z = -\frac{1}{2}(x^2 + y^2)$$

has a vertically upward z-axis. At its vertex is a particle with a mass of m. After a slight disturbance, it starts to slide down from rest. Find the force exerted by the paraboloid of revolution on the particle $N(x)$.

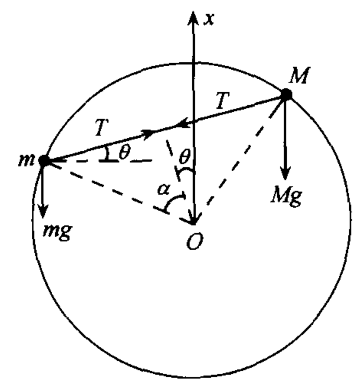


Problem 6 (15pts)

A smooth horizontal disk rotates with an angular velocity ω about a vertical axis passing through the center of the disk. A person on the disk at a distance R from the center of the disk pushes a smooth coin of mass m (ignore the size) toward the center of the disk, causing the coin to have an initial velocity v relative to the disk. Prove that from the perspective of the person on the disk, the motion is a parabola for a short time [$(\omega t)^2$ can be ignored]. Give the equation of this parabola.

Problem 7 (15pts)

Two small rings of mass m and M are moving in a circle in a smooth vertical plane. The two rings are connected by a massless, inextensible rope. As long as the rope is kept taut, find the tension in the rope T. The angle between the rope and the center of the circle when the rope is kept taut is 2α, the angle between the rope and the horizontal line is θ, and the gravitational acceleration is g.



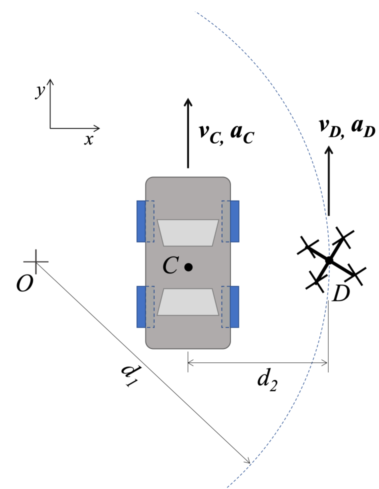
Problem 8 (15pts)

A camera drone D, flies over a car race in a curved trajectory (center O) with a constant ground-speed velocity of $v_D = 9m/s$. At the moment shown, car C is traveling with velocity of $v_C = 12m/s$ and an acceleration of $a_C = 2m/s^2$ as shown. Assume $d_1 = 7.5m, d_2 = 3m$.

a) Find the velocity of the car as observed by the camera on drone D at this instant.

b) Find the acceleration of the car as observed by the camera on drone D at this instant.

Hint: The drone D is a rotating reference frame.



[2] Paraboloid. In *Wikipedia*. https://en.wikipedia.org/wiki/Paraboloid.