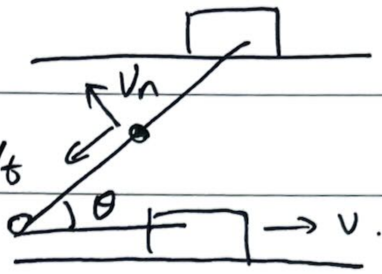
HW2.

Solution 1.

$|\vec{v_t}| = v$. $\quad |\vec{v_n}| = l\dot{\theta}$

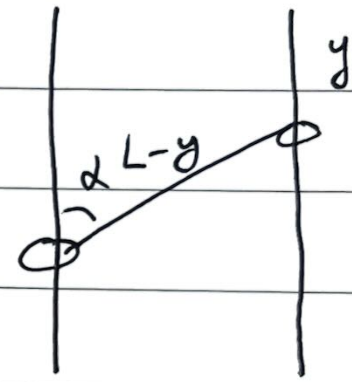
$$\begin{cases} v_B\cdot\cos\frac{\pi}{6} = v. \\ v_B\cdot\sin\frac{\pi}{6} = 2l\dot{\theta} \end{cases} \Rightarrow |\vec{v_n}| = \frac{\sqrt{3}}{6}v.$$



Solution 2.

$$\begin{cases} \frac{dy}{dt} = v \\ \frac{d}{dt}[(L-y)\sin\alpha] = 0 \end{cases}$$

$$\Rightarrow \dot{\alpha} = \frac{v\tan\alpha}{L-y}$$



$$v_0 = v + (-v\cos\alpha - (L-y)\sin\alpha\,\dot{\alpha}) = v\left(1 - \frac{1}{\cos\alpha}\right).$$

negative value implies upwards direction.

Solution 3.

a) $x = \omega R t + R\sin\omega t$, $\quad y = \begin{cases} R(1+\cos\omega t) & [\text{top}] \\ R(-1+\cos\omega t) & [\text{bot}] \end{cases}$

b) $\left(x = R\arccos\left(1+\frac{y}{R}\right) + \sqrt{-y(2R+y)}\right)$ [bot].

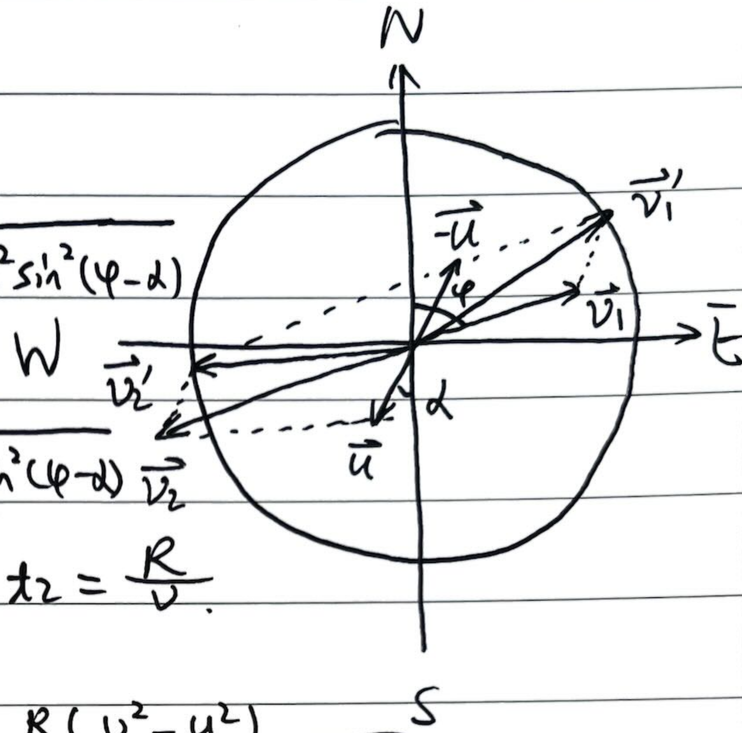
form can be different.

c) $\dot{x} = \omega R + \omega R\cos\omega t$. $\quad \dot{y} = -\omega R\sin\omega t$.

d) $dS = \sqrt{2}R\omega\sqrt{1+\cos\omega t}\,dt$

$$\Rightarrow S = 8kR + 4R\left(\cos k\pi - \cos\frac{\omega t}{2}\right). \quad t \in \left[k\frac{2\pi}{\omega}, (k+1)\frac{2\pi}{\omega}\right)$$

Solution 4:



$|\vec{v_1}'| = |\vec{v_2}'| = v.$

$$v_1 = -u\cos(\varphi-\alpha) + \sqrt{v^2 - u^2\sin^2(\varphi-\alpha)}$$

$$v_2 = u\cos(\varphi-\alpha) + \sqrt{v^2 - u^2\sin^2(\varphi-\alpha)}$$

$$v_1 t_1 = v_2 t_2, \quad t_1 + t_2 = \frac{R}{v}.$$

$$\Rightarrow t_1 = \frac{R v_2}{v(v_1+v_2)}$$

$$S_{max} = v_1 t_1 = \frac{R(v^2-u^2)}{2v\sqrt{v^2-u^2\sin^2(\varphi-\alpha)}}.$$

Solution 5:

The envelope is $y = -\frac{g}{2u^2}x^2 + \frac{u^2}{2g}$

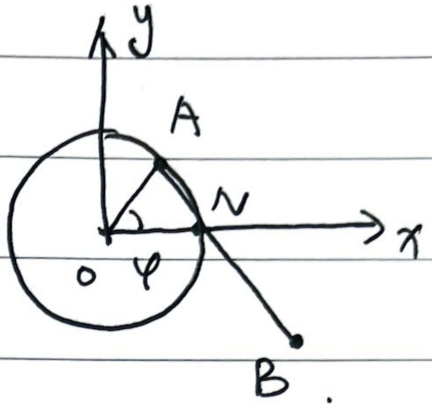
Let $y = -H$. then $x = \frac{u}{g}\sqrt{u^2+2gH}$.

The method of calculating the envelope is in RC 1. You can check the slides.

Solution 6:

Any methods are fine.

Solution 7:



for the position of B:

$$\begin{cases} x = r\cos\varphi + l\sin\frac{\varphi}{2} \\ y = r\sin\varphi - l\cos\frac{\varphi}{2} \end{cases}$$

$$\begin{cases} \dot{x} = -r\omega\sin\varphi + \frac{l}{2}\omega\cos\frac{\varphi}{2} \\ \dot{y} = r\omega\cos\varphi + \frac{l}{2}\omega\sin\frac{\varphi}{2} \end{cases}$$

$$\Rightarrow v = \left[(r\omega)^2 + \left(\frac{l}{2}\omega\right)^2 - rl\omega^2\sin\frac{\varphi}{2}\right]^{1/2}$$

$$\begin{cases} \ddot{x} = -r\omega^2\cos\varphi - \frac{l}{4}\omega^2\sin\frac{\varphi}{2} \\ \ddot{y} = -r\omega^2\sin\varphi + \frac{l}{4}\omega^2\cos\frac{\varphi}{2} \end{cases}$$

$$\Rightarrow a = \left[(r\omega^2)^2 + \left(\frac{l}{4}\omega^2\right)^2 - \frac{l}{2}r\omega^4\sin\frac{\varphi}{2}\right]^{1/2}.$$

$$a_r = \frac{dv}{dt} = -\frac{1}{4v}rl\omega^3\cos\frac{\varphi}{2}.$$

When $\varphi = 0$.

$$v = \omega\left(r^2 + \frac{l^2}{4}\right)^{1/2}, \quad a = \frac{1}{4}\omega^2(16r^2 + l^2)^{1/2}.$$

$$a_\tau = -\frac{rl\omega^2}{2(4r^2+l^2)^{1/2}}, \quad a_n = \frac{(8r^2+l^2)\omega^2}{4(4r^2+l^2)^{1/2}}$$

$$\rho = \frac{v^2}{a_n} = \frac{(4r^2+l^2)^{3/2}}{8r^2+l^2}.$$

Solution 8:

$$\vec{\omega} = \omega \hat{i}.$$

$$\dot{\hat{e}}_r = (\dot{\varphi}\hat{k} + \omega\hat{i}) \times \hat{e}_r = \dot{\varphi}\hat{e}_\varphi + \omega\sin\varphi\,\hat{k}.$$

$$\dot{\hat{e}}_\varphi = (\dot{\varphi}\hat{k} + \omega\hat{i}) \times \hat{e}_\varphi = -\dot{\varphi}\hat{e}_r + \omega\cos\varphi\,\hat{k}.$$

$$\dot{\hat{k}} = \omega\hat{i} \times \hat{k} = -\omega\hat{j} = -\omega\sin\varphi\,\hat{e}_r - \omega\cos\varphi\,\hat{e}_\varphi.$$

$$\vec{v} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt} = ce^{ct}\hat{e}_r + be^{ct}\hat{e}_\varphi + e^{ct}\omega\sin\varphi\,\hat{k}$$

$$\vec{a} = \dot{\vec{v}} = c^2e^{ct}\hat{e}_r + ce^{ct}\dot{\hat{e}}_r + bce^{ct}\hat{e}_\varphi + be^{ct}\dot{\hat{e}}_\varphi$$

$$+ (ce^{ct}\omega\sin\varphi + be^{ct}\omega\cos\varphi)\hat{k} + e^{ct}\omega\sin\varphi\,\dot{\hat{k}}$$

$$= e^{ct}\left[(c^2 - b^2 - \omega^2\sin\varphi)\hat{e}_r + (2bc - \omega^2\sin\varphi\cos\varphi)\hat{e}_\varphi\right.$$

$$\left. + 2\omega(b\cos\varphi + c\sin\varphi)\hat{k}\right].$$