

HW 8.

Solution 1.

for solid: $\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v$.

$$C_p - C_v = \left(\frac{\partial Q}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v = p \frac{\partial V}{\partial T}$$

From the definition $\alpha = \alpha_{\text{solid}} / \beta = \frac{1}{3V} \frac{\partial V}{\partial T}$.

We obtain $\frac{\partial V}{\partial T} = 3\alpha V = 3\alpha \frac{M}{\rho}$.

Then $C_p - C_v = 3\alpha \frac{M}{\rho} p$.

Solution 2.

(a) $W = \int_A^B p dV = RT_0 \int_{V_0}^{2V_0} dV/V = RT_0 \ln 2$.

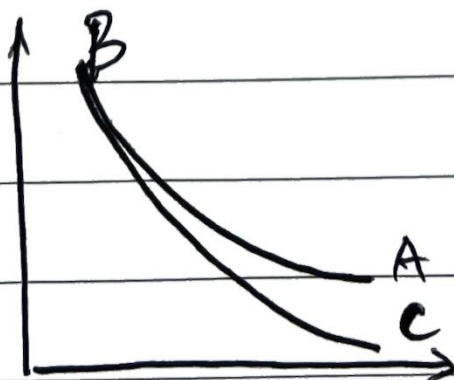
$Q = W = RT_0 \ln 2$.

(b) $W = \int_{V_0}^{2V_0} p dV = RT_0$.

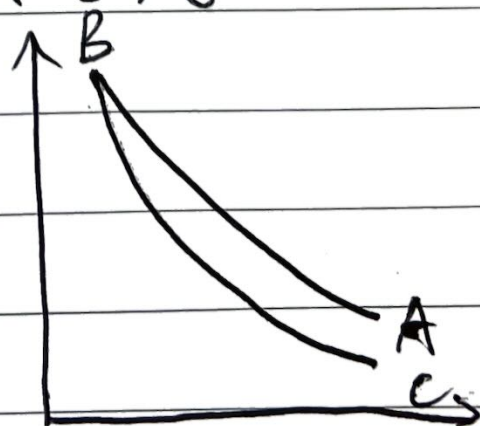
$\Delta U = C_v \Delta T = \frac{3}{2} RT_0$.

$Q = \Delta U + W = \frac{5}{2} RT_0$.

Solution 3. Net work: (a) greater than (b)



(a) monatomic



(b) diatomic.

AB: Same. BC: (a) is lower than (b).

Solution 4.

$$\Delta S_1 = \int_{T_i}^{T_f} \frac{C_p dT}{T} = C_p \ln \frac{T_f}{T_i}$$

$$\Delta S_2 = \frac{\Delta Q}{T_f} = C_p \left(\frac{T_i}{T_f} - 1 \right)$$

$$\Delta S = \Delta S_1 + \Delta S_2 = C_p \left(\frac{T_i}{T_f} - 1 + \ln \frac{T_f}{T_i} \right) > 0.$$

Solution 5.

$$\Delta S = n C_v \ln \frac{T_f}{T_i} + n R \ln \frac{V_f}{V_i}$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^g$$

$$\text{where } g = \frac{(n_1 + n_2)R}{n_1 C_{v1} + n_2 C_{v2}}$$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma$$

$$\text{where } \gamma = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$$

$$\Rightarrow T_f = 302 \text{ K}. \quad P_f = 2.0 \times 10^5 \text{ N/m}^2.$$

Solution 6.

$$(a) \quad C_p = \frac{1}{m} \left(\frac{\partial Q}{\partial T} \right)_p = \frac{1}{m} T \left(\frac{\partial S}{\partial T} \right)_p$$

$$(b) \quad \Delta S = \int_{T_1}^{T_2} \frac{C_p}{T} dT = C_p \frac{T_2 - T_1}{T_2} = 0.044 \text{ Cal/g} \cdot \text{K}$$

$$(c) \quad \Delta S' = \left[\int_{T_1}^{T_2} \frac{C_p}{T} dT \right] = \frac{1 \times (50 - 1)}{273 + 50} = 0.023 \text{ Cal/g} \cdot \text{K}$$

(d) Divide the range $[0^\circ\text{C}, 100^\circ\text{C}]$ into N parts.

($N \gg 1$). At every points, there exists large heat ~~the~~ reservoir, making the process thermal constant quasi-static $\Rightarrow \Delta S = 0$.

Solution 7.

$$(a) \quad Q = (1000 - 500) \frac{400 + 300}{2} = 1.75 \times 10^5 \text{ J}$$

$$W = (1000 - 500) \frac{400 - 300}{2} = 2.5 \times 10^4 \text{ J}$$

$$\eta = W/Q = 14.3\%$$

$$(b) \quad \text{Equilibrium } T_3 = \frac{T_1 + T_2}{2}$$

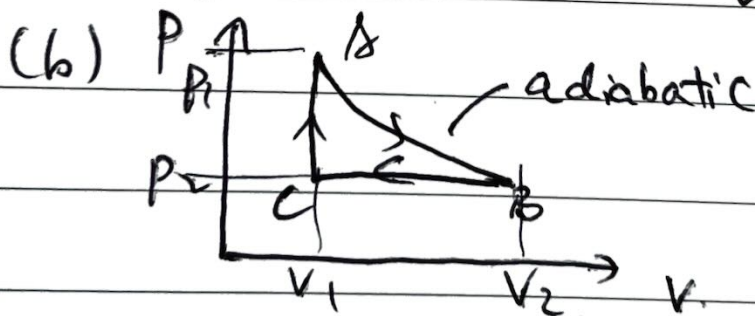
$$\Delta S_1 = \int_{T_1}^{T_3} \frac{Q dT}{T}, \quad \Delta S_2 = \int_{T_2}^{T_3} \frac{Q dT}{T}$$

$$\Delta S = C_p \ln \frac{(T_1 + T_2)^2}{4 T_1 T_2} \geq 0$$

Solution 8.

$$(a) \quad \text{From } dS = \frac{1}{T}(du + p dv) = \frac{1}{T}(C_v dT + p dv)$$

$$\Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$



$$W = \oint p dv = \int_{AB} p dv + P_2(V_1 - V_2)$$

$$= -C_v(T_2 - T_1) + P_2(V_1 - V_2)$$

$$= \frac{1}{1-\gamma} (P_2 V_2 - P_1 V_1) + P_2(V_1 - V_2)$$

$$Q = \int_{CA} T dS = \frac{1}{1-\gamma} V_1 (P_2 - P_1)$$

$$\eta = \frac{W}{Q} = 1 - \gamma \frac{V_2/V_1 - 1}{P_1/P_2 - 1}$$