Due: 11:59 pm on June 25, 2024

Problem 1 (10pts)

Water is poured from a height of h into a bucket placed on the ground at a rate of λ per second. The mass of the bucket is M. Find the force exerted by the ground on the bucket when the water is poured for time t.

Problem 2 (10pts)

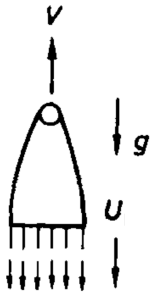
Two particles A and B with masses m_A and m_B are connected by an inextensible light rope and rest on a horizontal plane. When the rope is pulled tight, an impulse I is given to B, and the direction of I forms an angle α with AB, $\alpha < \pi/2$. Given the magnitude of B's velocity v_B after the impact, find the angle β which the direction of v_B forms with AB and the magnitude of the impulse I.

Problem 3 (15pts)

A light inextensible rope of length $2a$ is connected to a particle of mass M at its midpoint and to particles of mass m at each end, forming a straight line on a smooth horizontal plane. Give M an initial horizontal velocity v perpendicular to the rope. The particles at both ends collide after time τ. Find the distance b that M moves from the starting position, and the tension in the rope T just before the collision.

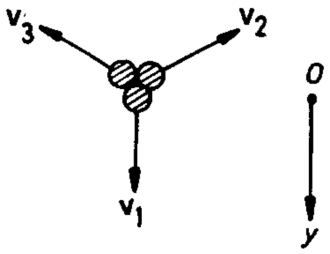
Problem 4 (10pts)

A rocket is fired straight up with no initial velocity. It is propelled by ejection of mass with a constant velocity of ejection u relative to the rocket and at a constant rate so determined that the initial acceleration is zero. Assuming constant acceleration due to gravity g. Find the acceleration of the rocket $a(t)$ and the height of the rocket $h(t)$ as functions of time.
Hint: $\int \ln(ax+b)\mathrm{d}x = \frac{ax+b}{a}\ln(ax+b) - x + C$.

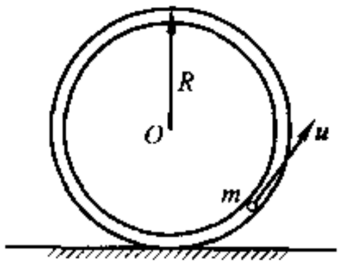


Problem 5 (10pts)

A rocket is projected straight up and explodes into three fragments of the mass m_1, m_2, m_3 just as it reaches the top of its flight. The fragment m_1 is observed to come straight down in a time t_1, while the other two m_2 and m_3 first move diagonally upwards then land at time t_2 and t_3 respectively, after the burst. Find the height $h(m_1, m_2, m_3, t_1, t_2, t_3, g)$ at which the fragmentation occurred.

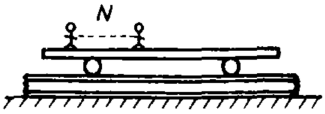


Problem 6 (15pts)



The mass of a ring is m, and its radius is R. The mass is evenly distributed along the radius, and the thickness is ignored. The ring stands upright on a smooth horizontal surface, and a beetle with a mass of m is crawling on the ring. At the beginning, the beetle is stationary at the lowest point of the ring and suddenly crawls along the ring at a relatively uniform speed u.
(a) Find the angular velocity of the ring ω_0 when the beetle starts moving.
(b) At what relative speed can the beetle climb to the same height as the center of the ring O?
(c) When the beetle climbs to the same height as the center of the ring, what is the force exerted by the ground on the ring?

Problem 7 (15pts)



A railroad flatcar of mass M can roll without friction along a straight horizontal track. N men, each of mass m, are initially standing on the car which is at rest.
(a) The N men run to one end of the car in unison; their speed relative to the car is V_r just before they jump off (all at the same time). Calculate the velocity of the car after the men have jumped off.
(b) The N men run off the car, one after the other (only one man running at a time), each reaching a speed V_r relative to the car just before jumping off. Find an expression for the final velocity of the car.
(c) In which case, (a) or (b), does the car attain the greater velocity?

Problem 8 (15pts)

A spherical dust particle falls through a water mist cloud of uniform density such that the rate of accretion onto the droplet is proportional to the volume of the mist cloud swept out by the droplet per unit time. If the droplet starts from rest in the cloud, find the value of the acceleration of the drop for large times.