HW7

Solution 1

Consider $x = x'$ to $x = x' + dx'$.

$$dm = \frac{m}{l}dx', \quad v = \frac{x'}{l}\dot{x}$$

$$dE_k = \frac{1}{2}\frac{m}{l}dx'\,\frac{x'}{l}\dot{x} = \frac{1}{2}\frac{m x'^2}{l^3}\dot{x}^2 dx'$$

$$E_k = \int dE_k = \frac{1}{2}\left(\frac{1}{3}m\right)\dot{x}^2.$$

Energy $\frac{1}{2}M\dot{x}^2 + \frac{1}{6}m\dot{x}^2 + \frac{1}{2}kx^2 = Const.$

$$\Rightarrow T = 2\pi\sqrt{\frac{M + m/3}{k}}.$$

Solution 2.

$$P_0 + \rho g(h+H) = P_0 + \frac{1}{2}\rho v_2^2$$

$$P_2 + \frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho v_1^2 + \rho g H.$$

$$S_1 v_1 = S_2 v_2.$$

$$\Rightarrow h = \frac{S_1^2}{S_2^2 - S_1^2}\left(\frac{P_0}{\rho g} - \frac{S_2^2}{S_1^2}H\right).$$

Solution 3.

(a) $a = 1$ A.U. $\quad \varepsilon = 0.2$.

(b) $\dfrac{T^2}{\left(\frac{R_1+R_2}{2}\right)^3} = \dfrac{T_1^2}{R_1^3} \Rightarrow T = 1.40$ years.

(c) Along the tangent of orbit and in the same direction as the earth's rotation.

Solution 4.

$$\frac{mv^2}{R} = \frac{Gmm_s}{R^2}$$

$$-\frac{Gm_em_s}{R_a} + \frac{m_eV_a^2}{2} = -\frac{Gm_em_s}{R_p} + \frac{m_eV_p^2}{2}.$$

$$m_eR_aV_a = m_eR_pV_p.$$

$$\Rightarrow R_a = \frac{2Gm_s}{V_a(V_a+V_p)} = \frac{2RV^2}{V_a(V_a+V_p)} = 3\times10^8\ \mathrm{km}$$

Solution 5.

Method I: Choosing $(x_1, y_1, \theta_1, x_2, \theta_2, s)$.

Constraints:

① $y_1 = l\sin\theta_1$. ② $x_2 - R\theta_2 = 0$.

③ $\dot{x}_1 - s\dot{\theta}_1\sin\theta_1 = \dot{x}_2 + R\dot{\theta}_2\cos\theta_1$.

④ $\dot{y}_1 + s\dot{\theta}_1\cos\theta_1 = R\dot{\theta}_2\sin\theta_1$

⑤ $x_2 - x_1 + l\cos\theta_1 = l + s$

holonomic ①②⑤ $\Rightarrow$ 3 generalized coordinates

Method II: Choosing $(x_1, y_1, \theta_1, x_2, \theta_2)$.

Constraints:

① $y_1 = l\sin\theta_1$ ② $x_2 - R\theta_2 = 0$.

③ $\dot{x}_1\cos\theta_1 + \dot{y}_1\sin\theta_1 = \dot{x}_2\cos\theta_1 + R\dot{\theta}_2$

holonomic ①② $\Rightarrow$ 3 generalized coordinates

Solution 6.

Check the file on Canvas.

Solution 7.

(a) $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + (r\dot{\varphi}\sin\theta + \omega r\sin\theta)\hat{e}_\varphi$.

$T = \frac{1}{2}m\vec{v}\cdot\vec{v}$

$= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\varphi}^2\sin^2\theta + \omega^2 r^2\sin^2\theta + 2\omega r^2\sin^2\theta\dot{\varphi})$

$L = T - V = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2) - V(r)$

$+ \frac{1}{2}m(\omega^2 r^2\sin^2\theta + 2\omega r^2\sin^2\theta\dot{\varphi})$.

(b) Taking rotation ROF.

$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2)$.

Using generalized potential energy:

$U = -\frac{1}{2}m(\omega^2 r^2\sin^2\theta + 2\omega r^2\sin^2\theta\dot{\varphi}^2)$.

This should be introduced by.

$\vec{F}_1 = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$. $\vec{F}_2 = -2m\vec{\omega}\times\vec{v}$.

Prove it by yourself.

Then $L = T - V - U$

which is the same as (a).

Solution 8.

(a) $x_1 = \frac{l}{2}\cos\theta$. $x_2 = -\frac{l}{2}\cos\theta$

$y_1 = \frac{1}{2} l \sin\theta$ $y_2 = \frac{1}{2} l \sin\theta$.

$L = T - V = \frac{1}{3} m l^2 \dot{\theta}^2 - mgl\sin\theta$.

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \ddot{\theta} + \frac{3g}{2l}\cos\theta = 0$.

As $\dot{\theta} = 0$ when $\theta = 30°$.

$$\dot{\theta}^2 = \int 2\ddot{\theta}\, d\theta = \frac{3g}{2l}(1 - 2\sin\theta).$$

Hence when $\theta = 0$. $\dot{\theta} = -\sqrt{\frac{3g}{2l}}$.

$$|\vec{v}| = |l\dot{\theta}| = \sqrt{\frac{3}{2} g l}.$$

(b) $t = \int_{30°}^{0} \frac{d\theta}{\dot{\theta}} = \sqrt{\frac{2l}{3g}} \int_0^{30°} \frac{d\theta}{1 - 2\sin\theta}$.