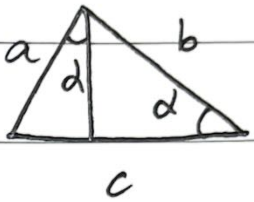
HW1

Solution 1.

(a) Consider the area A.



$[c] = L$, $[A] = L^2$. $[\alpha] = 1$.

Therefore, $A = k c^2 \phi(\alpha)$.

Then, $A = k a^2 \phi(\alpha) + k b^2 \phi(\alpha) = k c^2 \phi(\alpha)$

$\Rightarrow c^2 = a^2 + b^2$. proved.

(b) The variables related to T are:

l, m, g, $[l] = L$. $[m] = M$. $[g] = LT^{-2}$.

Assume $T = C l^{\alpha} m^{\beta} g^{\gamma}$.

Then, get $T = c\sqrt{\frac{l}{g}}$. proved.

Solution 2.

(a). First get $\begin{cases} x = r\sin\left(\frac{2\pi}{T}t\right) \\ y = r\left[1 - \cos\left(\frac{2\pi}{T}t\right)\right] \end{cases}$

Then, $\vec{v}(t) = \frac{2\pi r}{T}\cos\left(\frac{2\pi}{T}t\right)\hat{i} + \frac{2\pi r}{T}\sin\left(\frac{2\pi}{T}t\right)\hat{j}$.

$\vec{a}(t) = -\frac{4\pi^2 r}{T^2}\sin\left(\frac{2\pi}{T}t\right)\hat{i} + \frac{4\pi^2 r}{T^2}\cos\left(\frac{2\pi}{T}t\right)\hat{j}$.

when $t = T/3$. $\vec{r}\left(\frac{T}{3}\right) = \frac{\sqrt{3}}{2}r\hat{i} + \frac{3}{2}r\hat{j}$.

$\vec{v}\left(\frac{T}{3}\right) = -\frac{\pi r}{T}\hat{i} + \frac{\sqrt{3}\pi r}{T}\hat{j}$.

$\vec{a}\left(\frac{T}{3}\right) = -\frac{2\pi^2 r}{T^2}(\sqrt{3}\hat{i} + \hat{j})$.

(b) $\overline{\vec{v}} = \frac{\Delta\vec{r}}{\frac{1}{3}T} = \frac{3r}{T}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{3}{2}\hat{j}\right)$. $\overline{\vec{a}} = \frac{\Delta\vec{v}}{\frac{1}{3}T} = -\frac{3\pi r}{T^2}(3\hat{i} - \sqrt{3}\hat{j})$

Solution 3.

(a) $|\vec{r_1}| = 9.434$, $|\vec{r_2}| = 11.3578$.

(b) $\vec{r_{12}} = (-2, 7, -3)$. $|\vec{r_{12}}| = 7.874$. $\cancel{\hat{r}_{12} = (-0.254}$.

$\hat{r}_{12} = (-0.254, 0.889, -0.381)$.

(c) $\theta_{r_1, r_2} = 0.7555 \text{ rad} = 43.287^\circ$.

$\theta_{r_1, r_{12}} = 1.7194 \text{ rad} = 98.5158^\circ$.

$\theta_{r_2, r_{12}} = 0.9640 \text{ rad} = 55.231^\circ$.

(d) $\dfrac{\vec{r_2} \cdot \vec{r_1}}{|\vec{r_1}|^2} \cdot \vec{r_1} = (3.5056, 2.6292, 7.0112)$.

(e) $\vec{r_2} \times \vec{r_{12}} = (-65, -4, 34)$.

(f) $r = 5$, $\phi = 36.9^\circ$, $z = 8$.

Solution 4.

$$V_{cap} = \int_0^{2\pi} \int_0^{\arccos\frac{r-h}{r}} \int_{\frac{r-h}{\cos\varphi}}^{r} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \pi h^2 \left(r - \frac{h}{3}\right).$$

Since $r^2 = (r-h)^2 + a^2$. $r = \dfrac{a^2 + h^2}{2h}$.

other format: $V_{cap} = \frac{1}{6} \pi h (3a^2 + h^2)$

Solution 5.

Any methods are fine.

Solution 6.

Let $\overline{AH} = x$. $\overline{AF} = \xi$.

$$\xi(x) = \frac{ax+b}{cx+d}.$$

Since $\xi(0) = 0$, $b = 0$.

Let $\overline{AD} = 3e$, $\overline{AG} = 2e$.

$$\xi(3e) = 2\xi(2e) \quad [\overline{AC} = 2\overline{AB}].$$

$$\Rightarrow d = -6ec.$$

Therefore, $\xi(x) = \frac{a}{c} - \frac{6ae}{c(6e-x)}$.

$$v(x) = -\frac{6ae}{c(6e-x)^2}\dot{x}, \quad a(x) = -\frac{12ae}{c(6e-x)^3}\dot{x}^2$$

which means: $v(0) : v(2e) : v(3e) = 2^2 : 3^2 : 4^2$.

$$a(0) : a(2e) : a(3e) = 2^3 : 3^3 : 4^3.$$

Solution 7:

$$\tan\alpha = \frac{dx}{dy} = \frac{vt-x}{h-y}, \quad \dot{x}^2 + \dot{y}^2 = 4v^2.$$

let $q = \frac{dx}{dy}$, $\begin{cases} vt - x = q(h-y) \\ (1+q^2)\dot{y}^2 = 4v^2 \end{cases}$.

$$\Rightarrow (h-y)\frac{dq}{dy} = \frac{1}{2}(1+q^2)^{1/2}.$$

$$\Rightarrow \sqrt{1+q^2} = \sqrt{\frac{h}{h-y}} - q$$

$$x = \frac{1}{3\sqrt{h}}\left[(h-y)^{3/2} - h^{3/2}\right] + \sqrt{h}\left(\sqrt{h} - \sqrt{h-y}\right).$$

for $x = vt$, $y = h$, $t = \frac{2h}{3v}$.

Solution 8.

$v_{11} = v_{12} = \sqrt{c^2 - v^2}$, $t_1 = \frac{L}{v_{11}} + \frac{L}{v_{12}} = \frac{2L}{\sqrt{c^2 - v^2}}$

$v_{21} = c + v$. $v_{22} = c - v$. $t_2 = \frac{L}{v_{21}} + \frac{L}{v_{22}} = \frac{2Lc}{c^2 - v^2}$

$\frac{t_1}{t_2} = \sqrt{1 - \frac{v^2}{c^2}} < 1$. $\Rightarrow t_1 < t_2 \Rightarrow$ Boat 1 wins.