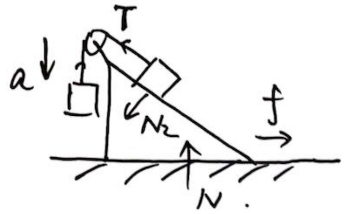
VP 160 HW3 Solutions

Solution 1.



a) $\begin{cases} m_1 a = m_1 g - T \\ m_2 a = T - m_2 g \sin\theta \end{cases}$.

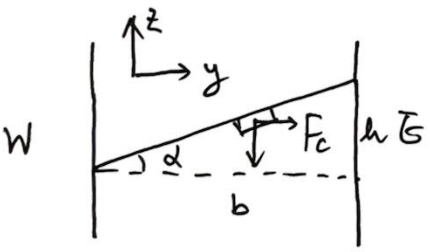
$\Rightarrow \begin{cases} a = \dfrac{m_1 - m_2 \sin\theta}{m_1 + m_2} g \\ T = \dfrac{m_1 m_2}{m_1 + m_2}(1+\sin\theta) g \end{cases}$.

b) Since $m_2 < m_1$, $N_2 \sin\theta < T\cos\theta$.

$\begin{cases} T\cos\theta - N_2 \sin\theta - f = 0 \\ T(1+\sin\theta) + N_2 \cos\theta + Mg - N = 0 \end{cases}$.

$$\Rightarrow \mu_{min} = \frac{m_2 \cos\theta\,(m_1 - m_2 \sin\theta)}{M(m_1+m_2) + 2m_1 m_2 (1+\sin\theta) + m_2^2 \cos^2\theta}$$

Solution 2.



Method 1.

$$\vec{F_c} = -2m\vec{\omega} \times \vec{v} = 2m\omega v \sin\lambda\,\hat{j}.$$

$$\frac{h}{b} = \tan\alpha = \frac{F_c}{mg}$$

$$\Rightarrow h = \frac{2b\omega v \sin\lambda}{g}.$$

Method 2.

$$\frac{dz}{dy} = \tan\alpha = \frac{F_c}{mg} = \frac{2\omega v \sin\lambda}{g}.$$

$$\Rightarrow h = \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{2\omega v \sin\lambda}{g}\,dy = \frac{2b\omega v \sin\lambda}{g}.$$

Solution 3.

First, calculate the total time T_K.

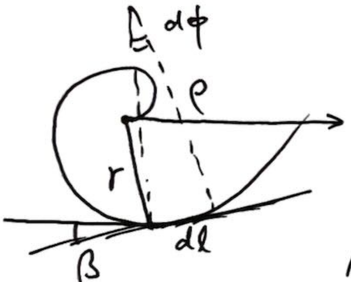
$$dl = \sqrt{(dr)^2 + (r d\theta)^2} = \frac{r_0}{\lambda}\sqrt{1+\theta^2}\, d\theta .$$

$$L_k = \int_0^{2k\lambda} dl = \frac{r_0}{\lambda}\left[k\lambda\sqrt{1+4k^2\lambda^2} + \frac{1}{2}\ln\left(2k\lambda + \sqrt{1+4k^2\lambda^2}\right)\right].$$

$$T_k = \frac{L_k}{v_0} = \frac{r_0}{\lambda v_0}\left[k\lambda\sqrt{1+4k^2\lambda^2} + \frac{1}{2}\ln\left(2k\lambda + \sqrt{1+4k^2\lambda^2}\right)\right].$$

Then, calculate the average $\overline{N}$.

$$N = \frac{mv^2}{\rho} \Rightarrow N dt = mv\frac{v dt}{\rho} = mv\frac{dl}{\rho} = mv\, d\phi .$$



$$\tan\beta = \frac{(r+dr)\,d\theta}{dr} = \theta . \qquad \Delta\phi = \theta + \beta .$$

$$\Delta\phi_k = 2k\lambda + \arctan(2k\lambda) .$$

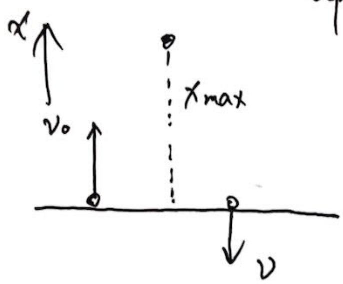
$$\overline{N} = \frac{\int_0^{T_K} N dt}{T_K} = \frac{m v_0 \Delta\phi_K}{T_K}$$

$$= \frac{\lambda m v_0^2}{r_0} \cdot \frac{2k\lambda + \arctan(2k\lambda)}{k\lambda\sqrt{1+4k^2\lambda^2} + \frac{1}{2}\ln\left(2k\lambda + \sqrt{1+4k^2\lambda^2}\right)}$$

when k is very large, $(k \gg 1)$.

$$\overline{N} \approx \frac{\lambda m v_0^2}{r_0} \frac{2k\lambda + \frac{\pi}{2}}{2k^2\lambda^2 + \frac{1}{2}\ln(4k\lambda)} \approx \frac{m v_0^2}{k r_0} .$$

Solution 4.



up: $\ddot{x} = -g - k\dot{x}^2$, $\ddot{x} = \frac{d\dot{x}}{dx}\dot{x}$

$$\Rightarrow \frac{\dot{x}\,d\dot{x}}{g+k\dot{x}^2} = -dx$$

$$\Rightarrow \frac{1}{k}\ln\frac{g}{g+kv_0^2} = -2x_{max}.$$

down: $\ddot{x} = -g + k\dot{x}^2$, $\ddot{x} = \frac{d\dot{x}}{dx}\dot{x}$

$$\Rightarrow \frac{\dot{x}\,d\dot{x}}{g-k\dot{x}^2} = -dx$$

$$\Rightarrow \frac{1}{k}\ln\frac{g-kv^2}{g} = -2x_{max}$$

Therefore, $\frac{g-kv^2}{g} = \frac{g}{g+kv_0^2}$

$$\Rightarrow v = \frac{v_0\sqrt{g}}{\sqrt{g+kv_0^2}}.$$

Since $-g+kv_t^2 = 0 \Rightarrow v_t = \sqrt{\frac{g}{k}}$.

$$\Rightarrow v = \frac{v_0 v_t}{\sqrt{v_0^2+v_t^2}}.$$

Solution 5:

Using Natural Coordinate System,

$$\begin{cases} m\frac{dv}{dt} = mg\sin\theta \\ m\frac{v^2}{\rho} = mg\cos\theta - N \end{cases} \Rightarrow \begin{cases} vdv = -g\,dz \\ N = mg\cos\theta - m\frac{v^2}{\rho}. \end{cases}$$

Since $\rho = \frac{(1+z'^2)^{3/2}}{|z''|} = (1+x^2)^{3/2}$

$\tan\theta = -\frac{dz}{dx} = x$.

we get $N = \frac{mg}{(1+x^2)^{3/2}}$

Solution 6.

$$m\frac{d\vec{v}}{dt} = \vec{F} - m\frac{d\vec{\omega}}{dt}\times\vec{r} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}$$

$$\Rightarrow \begin{cases} \ddot{x} = \omega^2 x + 2\omega\dot{y} \\ \ddot{y} = \omega^2 y - 2\omega\dot{x} \end{cases}$$

How to solve? - Using $(\omega t)^2$ can be ignored.

Assume
$$\begin{cases} x = A_x(t) + B_x(t)\,\omega + o(\omega^2) \\ y = A_y(t) + B_y(t)\,\omega + o(\omega^2) \end{cases}.$$

then
$$\begin{cases} \dot{x} = \dot{A}_x(t) + \dot{B}_x(t)\,\omega + o(\omega^2) \\ \dot{y} = \dot{A}_y(t) + \dot{B}_y(t)\,\omega + o(\omega^2) \end{cases}, \quad \begin{cases} \ddot{x} = \ddot{A}_x(t) + \ddot{B}_x(t)\,\omega + o(\omega^2) \\ \ddot{y} = \ddot{A}_y(t) + \ddot{B}_y(t)\,\omega + o(\omega^2) \end{cases}$$

$$\Rightarrow \begin{cases} \ddot{A}_x + \ddot{B}_x\,\omega + o(\omega^2) = \omega^2\left(A_x + B_x\,\omega + o(\omega^2)\right) + 2\omega\left(\dot{A}_y + \dot{B}_y\,\omega + o(\omega^2)\right) \\ \ddot{A}_y + \ddot{B}_y\,\omega + o(\omega^2) = \omega^2\left(A_y + B_y\,\omega + o(\omega^2)\right) - 2\omega\left(\dot{A}_x + \dot{B}_x\,\omega + o(\omega^2)\right) \end{cases}$$

$$\Rightarrow \begin{cases} \ddot{A}_x + \ddot{B}_x\,\omega + o(\omega^2) = 0 + 2\dot{A}_y\,\omega + o(\omega^2) \\ \ddot{A}_y + \ddot{B}_y\,\omega + o(\omega^2) = 0 - 2\dot{A}_x\,\omega + o(\omega^2) \end{cases}.$$

$$\Rightarrow \ddot{A}_x = \ddot{A}_y = 0. \quad \ddot{B}_x = 2\dot{A}_y. \quad \ddot{B}_y = -2\dot{A}_x$$

$$\Rightarrow \dot{A}_x = \dot{A}_x\big|_0 = -v. \quad \dot{A}_y = \dot{A}_y\big|_0 = 0.$$

$$\dot{B}_x = \dot{B}_x\big|_0 + 2\left(A_y - A_y\big|_0\right) = 2A_y.$$

$$\dot{B}_y = \dot{B}_y\big|_0 + 2\left(A_x\big|_0 - A_x\right) = 2(R - A_x).$$

$$\Rightarrow A_x = A_x\big|_0 - vt = R - vt. \quad A_y = A_y\big|_0 = 0.$$

$$B_x = B_x\big|_0 = 0. \quad B_y = B_y\big|_0 + vt^2 = vt^2.$$

$$\Rightarrow \begin{cases} x = R - vt + o(\omega^2) \\ y = vt^2\,\omega + o(\omega^2) \end{cases} \Rightarrow y \doteq \frac{\omega}{v}(x-R)^2.$$

Solution 7.

$$v_m = r\frac{d}{dt}(\alpha+\theta), \quad V_M = r\frac{d}{dt}(\alpha-\theta).$$

$$\begin{cases} m\frac{d}{dt}v_m = mg\sin(\alpha+\theta) - T\cos\alpha. \\ M\frac{d}{dt}v_M = Mg\sin(\alpha-\theta) - T\cos\alpha. \end{cases}$$

$$\Rightarrow T = \frac{2Mmg}{M+m}\tan\alpha\cos\theta.$$

Solution 8.

$$\vec{\Omega}: \quad \vec{v}_D = \cancelto{0}{\vec{v}_O} + \vec{\omega}_D \times \vec{r}_{D/O}$$

$$v_D\hat{j} = \omega_D\hat{k} \times d_1\hat{i} \Rightarrow \omega_D = \frac{v_D}{d_1} = 1.2 \text{ rad/s}.$$

$$\vec{\Omega} = \vec{\omega}_D = 1.2 \text{ rad/s } \hat{k}.$$

$$\dot{\vec{\Omega}}: \quad \vec{a}_D = \cancelto{0}{\vec{a}_O} + \dot{\vec{\omega}}_D \times \vec{r}_{D/O} - \omega_D^2\vec{r}_{D/O}$$

$$a_{Dx}\hat{i} + \cancelto{0}{a_{Dy}}\hat{j} = \dot{\omega}_D\hat{k} \times d_1\hat{i} - \omega_D^2 d_1\hat{i} \Rightarrow \dot{\omega}_D = 0.$$

$$\dot{\vec{\Omega}} = \dot{\vec{\omega}}_D = 0$$

$$(\vec{v}_{C/D})_{rel}: \quad \vec{v}_C = \vec{v}_D + \vec{\Omega} \times \vec{r}_{C/D} + (\vec{v}_{C/D})_{rel}$$

$$v_C\hat{j} = v_D\hat{j} + \omega_D\hat{k} \times d_2(-\hat{i}) + (\vec{v}_{C/D})_{rel}$$

$$(\vec{v}_{C/D})_{rel} = 6.6 \text{ m/s } \hat{j}.$$

$$(\vec{a}_{C/D})_{rel}: \quad \vec{a}_C = \vec{a}_D + \cancelto{0}{\dot{\vec{\Omega}}} \times \vec{r}_{C/D} - \Omega^2\vec{r}_{C/D} + 2\vec{\Omega} \times (\vec{v}_{C/D})_{rel} + (\vec{a}_{C/D})_{rel}.$$

$$(\vec{a}_{C/D})_{rel} = (22.32\,\hat{i} + 2\hat{j}) \text{ m/s}^2.$$